

Forecasting of monthly electricity generating for solar PV power plant by using Python based AI; case study of Thailand

Pornchai Chaweewat¹, Prof. Weerakorn Ongsakul¹, Dr. Kasem Pinthong²

Abstract— This paper develops forecasting tool by using Python language based artificial intelligent. The methodology starts with physical and statistic data collecting. The physical data includes solar PV power plant's capacity and location. The minimum installed capacity of solar PV power plant is 1 MW. The statistical data includes monthly solar PV power plant's electricity generation, daily and monthly maximum, minimum and average values of local temperature, solar irradiance and rain fall. The historical weather data are collected from SAM, Thai meteorology station and local meteorology on solar PV power plant. The collecting data are fed to train in ANN model. The results of ANN model are compared to the actual values. The contributions of this study is comparison of historical weather data sources.

Keywords— Artificial neural network, solar PV forecasting, Python, Neurolab library, Neural Network toolbox, Matlab

1. INTRODUCTION

Trend of solar PV power plant installation has been increasingly due to evidences on reducing GHG emission on electricity generation by using RES instead of fossil fuels. Integration of RES comes with great challenging in uncertainty in resources such as solar irradiation and wind flow. Thus, a number of research on RES focuses on forecasting.

The forecast horizon where most research has been done in the day-ahead. The reason for this behavior is that most of the energy is traded in day-ahead markets, when planning and unit commitment take place. As energy markets evolve, such as the case of EIM, intra-hour trading will become more important and thus, more research will focus on that time horizon and with a higher applicability in electricity markets. Traditionally, most a solar power forecasts were deterministic that is, for each forecast horizon they provided a single value. Nevertheless, state-of-the art papers are introducing probabilistic forecasts, which enable a better risk assessment and decision making. Compared to load or wind power forecasting, the state of probabilistic solar power forecasting is still immature and several challenges are yet to be solved [1]

In short term horizontal time, the impact of solar power forecasting improvements on solar power curtailment as well as on electricity generation, ramping, and starts and shutdowns of fossil fueled electricity generators resulted in impacts on operational electricity generation cost (summation of fuel costs, VO&M costs, and start and shutdown costs). Fig. above shows the economic value of uniform solar power forecasting improvement for different annual operational electricity generation cost. Moreover, the addition value of solar power forecasting improvement decreased for additional uniform improvements. For example, with a 13.5% solar power penetration, uniformly improving DA state-of-the-art solar power forecasting by

50% reduced annual operational electricity generation costs by \$13.22 M, while the additional value of improving solar power forecasting to 100% was only \$6.34 M [2].

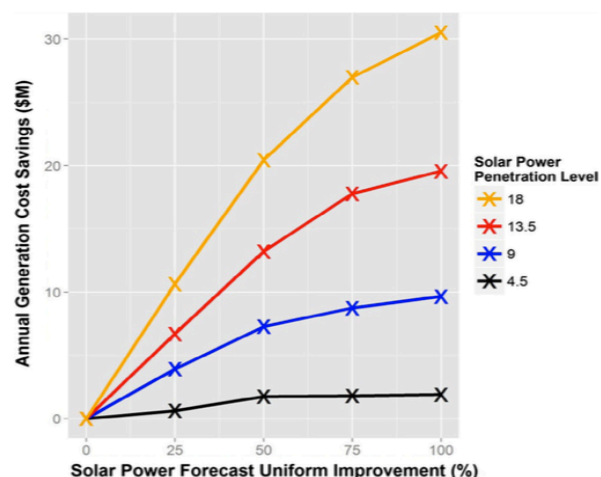


Fig. 1. Improvement in solar power forecasting and annual generation saving cost [2].

It is identified that, solar irradiance, temperature, wind speed and direction, humidity, cloud cover and aerosol index are major parameters to change of PV output power. It is also concluded that, solar irradiance is highly correlated with PV output power and follows the similar pattern. Therefore, the forecast accuracy of prediction models can be enhanced by optimizing and better selection of these correlated variables. Moreover, it is concluded that, endogenous stochastic methods such as AR, MA, ARMA and/or ARIMA can be used where less number of meteorological parameters are available as model input. In addition, different classification and clustering methods can be applied for improved training the forecasting model to enhance the forecast model performance. Intelligent Learning techniques such as artificial neural network

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(ANN) and fuzzy logic can have applied in dynamic environment to forecast the PV output, of adequate historical patterns are available to train the network [3]. In long term solar PV generating forecasting, the grid operator can perform a better operation and long term planning. The grid operator will also be able to perform reliable and safe maintenance planning and avoid any risks due to imbalance in power supply [4]. NREL provides access to TMY2 and TMY3 data sets and also uses these data sets in its online solar energy calculator PVWatts. There is only one TMY2 station in Thailand where locates in Bangkok as shown in Fig. 2.

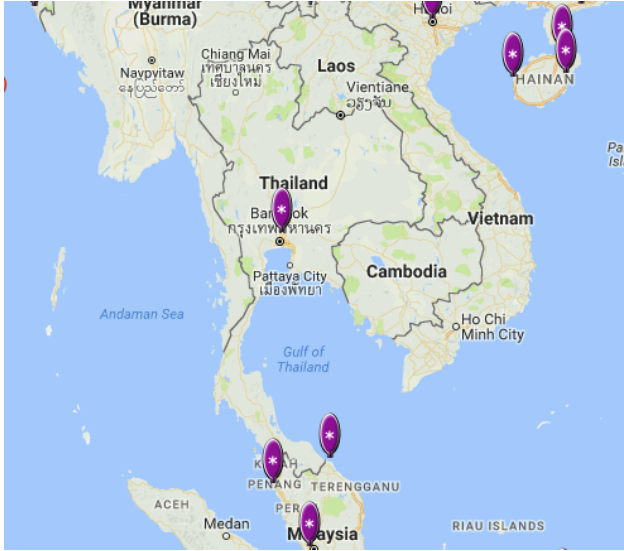


Fig.2. TMY2 stations locate in Thailand and nearby countries (source: PVwatt)

In summary, monthly solar PV generating forecasting provides several benefits as below;

- Accuracy solar PV generating forecasting can help the operator to plan safe maintenance and minimize risk on power balance. Moreover, solar PV owner can ensure their revenue and financial analysis.
- Underestimate solar PV generating can cause system's stability and reliability.
- Overestimate solar PV generating can cause financial damage to solar PV owners.

Research question:

- Since PVwatt accesses to only one meteorological station in Thailand, is it provide better to forecast data on monthly solar PV production than using local meteorological weather data from Thai Department of Meteorological?
- Is Neurolab library based Python language can perform better than neural network toolbox in Matlab in term of time of calculation and error of forecasting?

The contributions of this paper are listed below

- To forecast monthly output of solar PV power plant in Thailand using historical local weather data collected from Thai Department of Meteorological.
- To modify algorithm to forecast solar PV generating by using Neurolab library based on Python language.

The rest of this paper is organized as follows. The methods of data collection, construction of feed-forward neural network, learning algorithm and forecasting evaluation is described in chapter 2. Chapter 3 discusses numerical results and discussion of this paper. The conclusion and future work will be presented in Chapter 4. The collected

data of historical weather parameters and solar PV output will be shown in Appendices.

2. PROBLEM FORMULATION

2.1 Data description

This section describes the collected historical data and predictor data. The data is collected in 2016. The solar PV generation historical data are collected from Provincial Electricity Authority of Thailand. The historical solar PV generation data consists of location of plant, installation capacity (kW) and monthly solar PV generation (kWhr). The historical weather data is collected from Meteorological Department of Thailand. The historical weather data consists of location of measured station and monthly measured data which are cloud index, humidity, haze, fog, rain fall, minimum temperature, and maximum temperature. The average historical monthly solar PV generation and weather data are considered as input data to train forecasting model as shown in table 1.

Table 1. Data description and number of collected data

No. of variable	Variable name	No. of collected data
1	Location of plant	232
2	Size of plant	232
3	Monthly solar PV generation	2,784
4	Location of meteorological station	117
5	Monthly cloud index	1,317
6	Monthly humidity	1,371
7	Monthly haze	1,344
8	Monthly fog	1,344
9	Monthly rainfall	1,153
10	Monthly minimum temperature	1,393
11	Monthly maximum temperature	1,404



Fig.3. 4.2 kW solar PV grid-connected in AIT

The test data is collected from solar PV power plant in Energy building, Asian Institute of Technology and meteorological measured station in Pathumthani province where the site is located. The test data is collected during January to October, 2017. The tilt angle is 15 degrees.

2.2 Capacity factor of solar PV generation

The capacity factor (CF) of each solar PV generating represents the performance of power plant. In this paper, the CF is the total AC kWh of electricity generated by the system in a month divided by the system's rated capacity in DC kW divided by number of day in the month. The values of CF for each month are calculate by (1)

$$CF_{i,m} = \frac{AC_{i,m}}{DC_{i,m} \times \text{number of day}_m} \quad (1)$$

where i represent power plant i ; AC is total electricity produced in month m ; DC is overall installed capacity of the power plant; and number of day is number of day in the month.

2.3 Feed-forward neural network (FNN)

This section descripts basic structure of FNN which consists of three layers. They are input layer, hidden layer and output layer. In multi-layer FNN, neurons are allocated in distinct layered topology as shown in Fig.3. A FNN only allows data flow in a forward direction, i.e., the data flows from the input layer neurons, through the hidden layers' neurons, and finally reaching the output layer neurons. The widely used learning method in FNN is the backpropagation algorithm. Backpropagation is a form of supervised learning in which the network is provided with examples of inputs and a target output. The training starts with random weights and biases value. The objective is to adjust them to make sure that the error is minimal. The values of the hidden layer neurons can be expressed in (2)

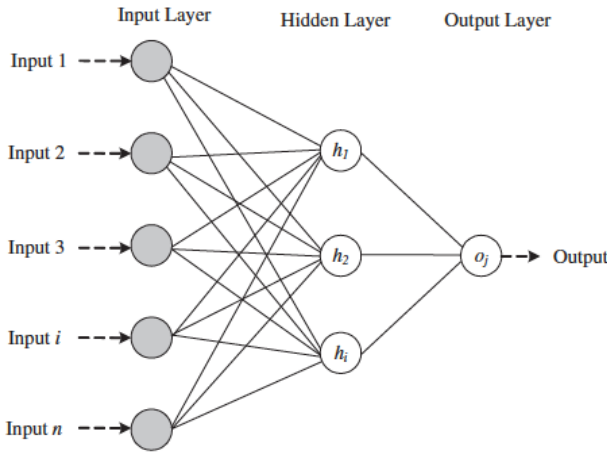


Fig.2. A simple three-layer feed-forward neural network [5]

$$h_j = f_1\left(\sum_{i=1}^n v_{ij}x_i + \theta_j\right) \quad (2)$$

where h_j is the values of the hidden layer neuron; $f_1(\cdot)$ is tangent sigmoid transfer function; x_i is the value of the input and hidden layers and θ_j is the bias of the hidden layer neuron.

The hidden layer will be considered as input to the output layer and the values of the output layer neurons is shown in (3).

$$o_j = f_2\left(\sum_{i=1}^n w_{ij}h_i + \gamma_j\right) \quad (3)$$

where o_j is the values of the hidden layer neuron; $f_2(\cdot)$ is a

linear transfer function; w_{ij} is the adjustable weight between the hidden and output layers and γ_j is the bias of the output layer neuron.

2.4 Levenberg-Marguardt algorithm (LMA)

This section illustrates how LMA can be used in FNN. Letting $e_k = R_k - Z_k$, $k = 1, \dots, N$, cost function would be defined to quantify the difference between R_k and Z_k in j -th epoch as

$$E_j = E_j(e_k, k = 1, \dots, N) = \frac{1}{2} \sum_{k=1}^N e_k^2 \quad (4)$$

where $R = [R_1 \dots R_N]^T$ is a $N \times 1$ vector as the target output, $Z = [Z_1 \dots Z_N]^T$ is a $N \times 1$ vector as the ANN output, $e = [e_1 \dots e_N]^T$ is a $N \times 1$ error vector, and N is the number of samples. All parameters of antecedent can be defined in a matrix as

$$W = [c_1 \ s_1 \dots c_{2n} \ s_{2n}]^T = [W_1 \dots W_{4n}]^T \quad (5)$$

where c_1 and s_1 indicate the center and standard deviation of first membership function, and so on. As mentioned before, all member ship functions have been selected as Gaussian functions.

LMA uses Jacobian matrix J_j which is a gradient matrix representing the partial derivatives of e_j with respect of W_j [6].

$$J_j = \frac{\partial e(W_j)}{\partial (W_j)} = \frac{\partial \begin{bmatrix} e_1 \\ \vdots \\ e_N \end{bmatrix}}{\partial \begin{bmatrix} W_1 \\ \vdots \\ W_N \end{bmatrix}_j} = \begin{bmatrix} \frac{\partial e_1}{\partial W_1} & \dots & \frac{\partial e_1}{\partial W_{4n}} \\ \vdots & \ddots & \vdots \\ \frac{\partial e_N}{\partial W_1} & \dots & \frac{\partial e_N}{\partial W_{4n}} \end{bmatrix}_j \quad (6)$$

The LMA update for the weights is expressed as

$$W_{j+1} = W_j - (J_j^T J_j + \mu I)^{-1} J_j^T e(W_j) \quad (7)$$

where the factor μ adjusts its value according to the rule depicted in the LMA flowchart in Fig. 3. The flowchart shows how the LMA formula adjusts μ to cleverly switch between the coarser delta rule update finer Gauss-Newton algorithm update. In this way, the parameters of antecedent which are $c_1, s_1, c_2, s_2, \dots$ are identified by LMA [7].

2.5 Mean absolute error (MAE) and mean absolute percentage error (MAPE)

This section descripts evaluation of developed algorithm using MAE and MAPE. The MAE define as the difference between the actual and forecasted monthly solar PV generation which is computed by (8). The MAPE expresses the scaled difference between the actual and forecasted monthly solar PV generation as a percentage of the actual solar PV generation. MAPE is scale independent and it can be used to compare forecast performance across different data sets. MAPE can be calculated by (9)

$$MAP = \frac{1}{N} \sum_{i=1}^N |P_a - P_f| \quad (8)$$

$$MAPE[\%] = \frac{100}{N} \sum_{i=1}^N \left| \frac{P_a - P_f}{P_a} \right| \quad (9)$$

where P_a and P_f are the actual and forecasted solar PV generation. N is the data size.

3. NUMERICAL RESULTS AND DISCUSSION

This chapter presents simulation results and evaluation of trained model using collected local weather data from Thai Meteorological Department. The training period starts from January to December, 2016. The testing period is during January to October, 2017. The solar PV generating data collected from PVwatt at tested site is used as benchmark.

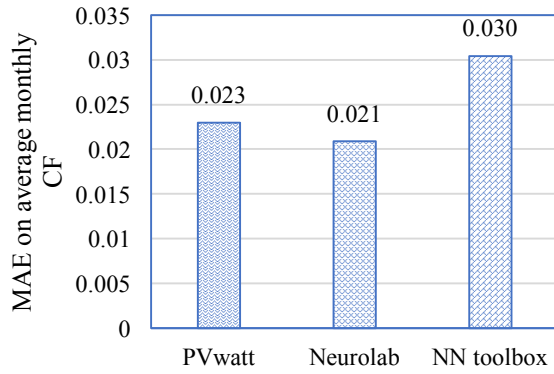


Fig. 3. MAE values on average monthly CF forecasting for PVwatt, Neurolab and NN toolbox

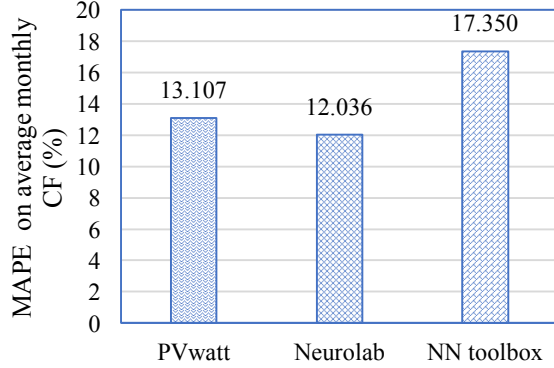


Fig. 4. MAPE values on average monthly CF forecasting for PVwatt, Neurolab and NN toolbox

According to average monthly capacity factor forecasting, MAE and MAPE results in Fig. 3 and 4, there are differences in results of PVwatts and proposed methods. Neurolab drops down these MAE on for 0.002 and 0.009 comparing to PVwatts and NN toolbox. Then, MAPE of Neurolab is found lower than PVwatt and NN toolbox. Thus, the proposed Neurolab based forecasting model performs the excellent results in AIT solar PV site.

The histograms, normalized distributed lines and cumulative normalized distributed lines plots of Neurolab and Neural Network toolbox (NN) are shown in Fig. 5. The comparison between Neurolab's and NN toolbox's results show that Neurolab consumes more calculating time but has less standard deviation of calculating time. However, Neurolab performs better in forecasting accuracy. The average monthly MAE is less than NN toolbox by 0.014 kWhr. Moreover, Neurolab provides lower MAPE by 7.78%.

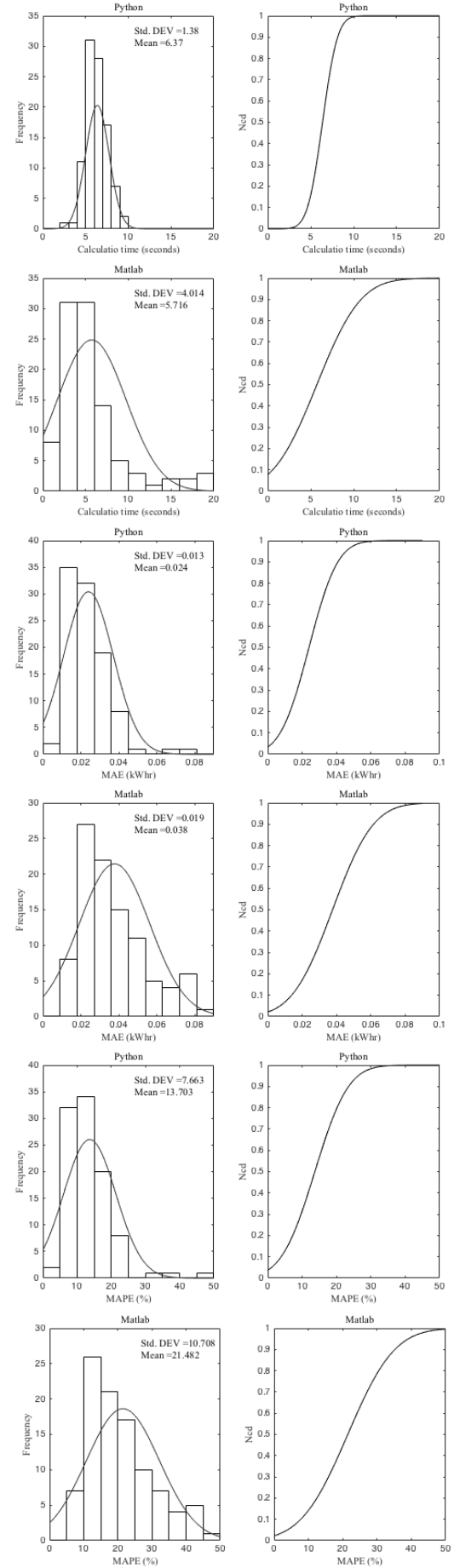


Fig. 5. Distribution histogram and cumulative distribution probability of calculating time, MAE and MRE for Neurolab and NN toolbox

4. CONCLUSION

This paper presents a modified neural network library and toolbox which are Neurolab based python and neural network toolbox base Matlab as monthly solar PV generation forecasting using local weather data from Thai Meteorological Department. The result of MAE shows the significant performance of using historical weather data better than PVwatt which using TMYs data. Moreover, Neurolab library based python shows excellent performance comparing to neural network toolbox based Matlab.

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Appendix

Table A-1. Historical monthly solar PV generating distributed by area and size for training the purposed model.

Area*	Size**	No. of site	Average monthly capacity factor											
			Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
1	small	18	0.144	0.173	0.167	0.178	0.160	0.147	0.146	0.151	0.150	0.139	0.160	0.145
	medium	7	0.187	0.205	0.199	0.215	0.195	0.172	0.177	0.179	0.171	0.177	0.197	0.183
	large	11	0.218	0.238	0.232	0.246	0.228	0.203	0.209	0.214	0.207	0.209	0.233	0.218
2	small	2	0.147	0.170	0.150	0.150	0.142	0.147	0.144	0.161	0.148	0.136	0.159	0.142
	medium	8	0.175	0.204	0.202	0.209	0.180	0.166	0.162	0.183	0.168	0.165	0.195	0.176
	large	1	0.157	0.176	0.166	0.184	0.155	0.149	0.150	0.125	0.093	0.091	0.169	0.165
3	small	1	0.249	0.247	0.248	0.275	0.221	0.221	0.211	0.207	0.210	0.203	0.224	0.223
	medium	5	0.216	0.218	0.220	0.241	0.228	0.198	0.202	0.207	0.199	0.200	0.205	0.199
	large	18	0.201	0.209	0.208	0.228	0.213	0.191	0.191	0.193	0.188	0.190	0.197	0.189
4	small	5	0.171	0.179	0.172	0.173	0.163	0.136	0.133	0.125	0.140	0.157	0.152	0.153
	medium	9	0.178	0.190	0.184	0.184	0.184	0.151	0.148	0.134	0.148	0.170	0.171	0.170
	large	1	0.208	0.215	0.205	0.209	0.209	0.201	0.183	0.188	0.195	0.200	0.197	0.210
5	small	0	None	None	None	None	None	None	None	None	None	None	None	None
	medium	1	0.183	0.143	0.198	0.218	0.199	0.180	0.176	0.177	0.174	0.203	0.209	0.200
	large	7	0.212	0.229	0.227	0.240	0.222	0.190	0.189	0.180	0.189	0.219	0.216	0.210
6	small	2	0.116	0.135	0.136	0.148	0.150	0.136	0.136	0.128	0.120	0.129	0.137	0.131
	medium	18	0.178	0.205	0.196	0.211	0.201	0.180	0.185	0.183	0.176	0.179	0.200	0.188
	large	7	0.183	0.208	0.197	0.210	0.199	0.175	0.181	0.183	0.172	0.187	0.201	0.195
7	small	1	0.114	0.148	0.127	0.133	0.138	0.132	0.123	0.118	0.112	0.126	0.130	0.126
	medium	21	0.138	0.176	0.155	0.159	0.161	0.152	0.147	0.145	0.142	0.157	0.168	0.163
	large	19	0.196	0.248	0.222	0.219	0.215	0.199	0.195	0.193	0.185	0.209	0.224	0.221
8	small	1	0.198	0.264	0.230	0.243	0.235	0.214	0.171	0.221	0.203	0.234	0.242	0.233
	medium	9	0.180	0.222	0.188	0.191	0.193	0.180	0.176	0.174	0.162	0.194	0.202	0.193
	large	6	0.192	0.250	0.222	0.223	0.203	0.207	0.216	0.201	0.183	0.206	0.210	0.202
9	small	1	0.082	0.096	0.075	0.067	0.072	0.098	0.100	0.083	0.073	0.069	0.077	0.067
	medium	16	0.185	0.224	0.212	0.222	0.205	0.195	0.197	0.201	0.179	0.195	0.206	0.198
	large	20	0.213	0.255	0.240	0.243	0.223	0.213	0.219	0.220	0.199	0.220	0.228	0.223
10	small	6	0.176	0.191	0.190	0.196	0.171	0.142	0.137	0.132	0.131	0.121	0.139	0.142
	medium	4	0.200	0.201	0.202	0.214	0.190	0.168	0.166	0.164	0.156	0.148	0.171	0.165

	large	0	None	None	None	None	None	None	None	None	None	None	None	None	None
11	small	2	0.066	0.051	0.038	0.055	0.064	0.070	0.049	0.046	0.049	0.043	0.046	0.034	
	medium	0	None	None	None	None	None	None	None	None	None	None	None	None	
	large	0	None	None	None	None	None	None	None	None	None	None	None	None	
12	small	3	0.086	0.103	0.109	0.092	0.071	0.069	0.079	0.083	0.088	0.084	0.080	0.071	
	medium	0	None	None	None	None	None	None	None	None	None	None	None	None	
	large	0	None	None	None	None	None	None	None	None	None	None	None	None	

*Area

** Size of plant: small (less than 1 MW), medium (1-5 MW), large (above 5 MW)

Table A-2. Actual monthly solar PV generation at AIT and forecasted data from PVwatt[8] during January to October, 2017

Month	Number of days in month	Monthly solar PV generation (kWhr)	
		AIT	PVwatt [8]
January	31	508.66	505
February	29	544.07	486
March	31	522.94	576
April	30	574.76	513
May	31	502.9	470
June	30	559.16	458
July	31	537.28	484
August	31	554.77	416
September	30	550.55	438
October	31	549.12	446

Table A-3. Actual monthly capacity factor at AIT and calculaed capacity from PVwatt[8] during January to October, 2017

Month	Number of days in month	Capacity factor	
		AIT	PVwatt [8]
January	31	0.1628	0.1616
February	29	0.1741	0.1555
March	31	0.1674	0.1843
April	30	0.1839	0.1642
May	31	0.1609	0.1504
June	30	0.1789	0.1466
July	31	0.1719	0.1549
August	31	0.1775	0.1331
September	30	0.1762	0.1402
October	31	0.1757	0.1427
MAE			0.0229
MAPE (%)			13.107

Table A-4. Results of calculation time, MAE and MAPE on average monthly capacity factor using Neurolab based Python and NN toolbox based Matlab

Sample	Neurolab			NN toolbox		
	Time	MAE	MAPE	Time	MAE	MAPE
1	10.209	0.038	22.052	12.947	0.052	29.950
2	7.935	0.016	9.001	3.378	0.030	16.993
3	5.998	0.015	8.270	2.895	0.044	24.535
4	5.585	0.012	6.311	4.056	0.018	10.021
5	4.740	0.018	10.068	3.516	0.017	9.389
6	5.647	0.015	8.994	11.273	0.070	39.041

7	7.355	0.012	7.374	5.422	0.066	37.331
8	5.000	0.037	20.818	4.804	0.074	42.202
9	4.381	0.018	10.616	3.262	0.032	18.069
10	6.968	0.075	45.446	7.378	0.019	10.855
11	5.138	0.015	8.401	7.236	0.056	31.752
12	4.868	0.032	17.896	4.840	0.040	22.424
13	5.239	0.022	12.428	4.209	0.080	45.443
14	7.664	0.018	10.471	4.009	0.022	12.979
15	5.677	0.013	7.269	1.592	0.029	16.402
16	6.005	0.019	10.802	2.982	0.040	23.599
17	6.012	0.015	8.986	3.907	0.020	11.251
18	5.999	0.018	10.221	1.721	0.056	31.462
19	5.319	0.030	15.997	6.536	0.028	15.901
20	5.644	0.025	14.951	3.527	0.045	24.947
21	5.351	0.023	13.043	9.070	0.039	22.437
22	6.568	0.028	15.649	4.372	0.038	21.316
23	5.997	0.024	13.957	3.994	0.039	22.240
24	7.368	0.030	17.421	2.155	0.073	43.448
25	6.688	0.021	11.956	4.445	0.029	16.113
26	5.918	0.040	22.311	4.503	0.030	17.063
27	7.001	0.009	4.864	5.525	0.015	8.610
28	4.499	0.033	19.066	4.037	0.054	30.980
29	6.940	0.015	8.960	5.501	0.026	15.425
30	10.993	0.038	20.913	7.439	0.023	13.020
31	6.407	0.013	7.388	2.846	0.029	16.053
32	6.473	0.024	13.214	10.795	0.031	17.553
33	6.117	0.022	12.662	2.831	0.026	14.709
34	8.724	0.033	18.597	1.295	0.038	21.798
35	7.239	0.019	10.397	17.785	0.029	17.003
36	6.782	0.031	17.758	1.981	0.028	15.582
37	6.477	0.028	15.594	8.132	0.036	20.621
38	5.904	0.014	8.103	19.329	0.024	13.001
39	7.744	0.011	6.601	15.378	0.052	30.008
40	4.994	0.017	9.952	5.359	0.035	20.395
41	9.562	0.020	11.295	4.723	0.021	11.606
42	5.027	0.011	6.091	5.637	0.088	51.924
43	5.011	0.029	16.287	2.749	0.029	16.224
44	6.381	0.028	15.850	5.476	0.060	34.383
45	6.185	0.015	8.798	5.806	0.037	20.652
46	6.173	0.053	31.918	3.232	0.024	13.408
47	7.495	0.065	36.682	4.752	0.048	27.320
48	5.415	0.011	6.045	3.247	0.021	12.031
49	5.552	0.016	8.870	2.212	0.029	16.469
50	6.930	0.023	12.683	3.563	0.020	11.574
51	3.990	0.018	10.389	1.008	0.024	13.565
52	5.930	0.014	7.822	9.158	0.030	17.560
53	6.200	0.019	10.661	3.616	0.048	27.120
54	6.408	0.017	9.755	4.454	0.068	39.003
55	7.084	0.022	12.645	6.716	0.075	44.140
56	4.697	0.013	7.978	4.169	0.045	26.018
57	6.083	0.022	12.465	15.742	0.025	14.000
58	5.068	0.017	9.743	5.664	0.017	9.960
59	5.436	0.016	9.198	1.547	0.038	21.969
60	4.847	0.007	3.979	3.877	0.018	9.914
61	5.999	0.035	19.936	3.275	0.025	14.131
62	8.072	0.021	11.539	7.015	0.024	13.286
63	6.323	0.024	13.777	2.807	0.017	9.368
64	7.450	0.039	21.864	6.935	0.047	25.668
65	5.944	0.034	19.344	6.525	0.115	64.653

66	9.644	0.034	19.095	7.767	0.052	30.177
67	4.689	0.020	11.711	2.600	0.044	24.587
68	6.570	0.025	14.435	9.247	0.048	27.421
69	7.027	0.018	10.062	18.055	0.072	42.115
70	8.380	0.019	10.700	17.397	0.029	16.765
71	7.468	0.011	6.114	4.733	0.023	13.179
72	5.765	0.013	7.324	3.464	0.026	14.448
73	6.611	0.018	10.566	2.428	0.046	25.628
74	6.520	0.027	15.084	19.089	0.024	13.549
75	6.488	0.011	6.582	3.479	0.027	15.242
76	8.632	0.033	18.428	3.153	0.030	17.147
77	6.900	0.016	9.247	1.951	0.028	15.476
78	5.710	0.041	23.448	2.095	0.026	14.443
79	6.430	0.035	19.991	1.904	0.015	8.925
80	6.440	0.026	14.512	4.687	0.046	26.120
81	5.294	0.026	14.478	2.301	0.048	26.530
82	7.191	0.021	12.115	2.711	0.027	14.807
83	7.416	0.013	7.647	5.606	0.019	10.773
84	5.930	0.014	7.820	7.808	0.042	23.742
85	2.052	0.009	5.323	4.896	0.036	20.404
86	8.953	0.015	8.014	4.710	0.029	16.145
87	6.437	0.029	16.431	11.206	0.023	13.522
88	5.861	0.025	14.320	8.016	0.056	31.377
89	6.806	0.028	16.265	3.658	0.036	20.102
90	5.541	0.020	11.144	4.815	0.021	12.133
91	7.682	0.030	16.654	2.513	0.029	16.380
92	4.172	0.026	14.499	7.254	0.020	11.286
93	5.126	0.022	12.367	5.422	0.025	13.873
94	5.822	0.033	18.819	5.820	0.034	19.097
95	5.468	0.039	22.164	6.496	0.044	24.796
96	7.001	0.022	12.289	3.595	0.049	27.556
97	8.309	0.093	54.420	7.369	0.017	9.766
98	4.655	0.015	8.158	4.474	0.065	37.712
99	7.506	0.016	9.238	6.495	0.074	41.913
100	8.065	0.036	20.426	4.232	0.024	13.243
Mean	6.192	0.021	12.036	4.595	0.030	17.350
Std. DEV	1.369	0.013	7.625	3.994	0.019	10.655
Max	10.993	0.093	54.420	19.329	0.115	64.653
Min	2.052	0.007	3.979	1.008	0.015	8.610