Optimal Time-of-Use Pricing For Residential Load Control

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Abstract— **Demand response (DR) can be defined as change in electric usage by end-use customers from their normal consumption patterns in response to change in the price of electricity over time. Demand Response also refers to incentive payments designed to induce lower electricity use at times of high wholesale market prices. Time-of-use (TOU) power pricing has been shown to have a significant influence on ensuring a stable and optimal operation of a power system. This paper presents a novel algorithm for finding an optimum time-of-use electricity pricing in monopoly utility markets; definitions and the relations between supply and demand as well as different cost components are also presented. Further, the optimal pricing strategy is developed to maximize the benefit of society while implementing a demand response strategy. Finally, the effect of demand response in electricity prices is demonstrated using a simulated case study.**

I. INTRODUCTION

Demand-side management and electricity rate structures have been the subject of extensive study and a major focus area for utility companies. The wholesale power costs vary substantially across time while most customers pay flat rates. This force the utility company to absorb all the risks associated with price fluctuations keeping customer prices fixed. Thus customers have little monetary incentive to reduce usage when wholesale power costs are high, leading to higher bills than would occur if their loads were less coincident with the system peak. A well-designed dynamic pricing structure reflects differences in power costs across time periods, giving customers more appropriate price signals than static rates. Customers benefit under dynamic pricing if they can shift sufficient consumption from more costly peak period hours to lower-priced off-peak hours. Various rate structures have been proposed or implemented and have led to improvements in a wide range of social objectives including the cost of electricity, the reliability of supply, utility profits and customer benefits [1-5]. The nature of electricity generation and consumption, which are characterized by cyclic and random variations in both demand and supply, has affected the design of rate structures. Demand response changes the electric usage by end-use customers from their normal consumption patterns in response to changes in the price of electricity over time via incentive payments designed to induce lower electricity use at times of high wholesale market prices [1].

Price-based demand response (DR) programs include time-of-use (TOU) rates [1, 4-6] and real-time pricing (RTP) [7-9]. TOU is a type of static pricing scheme and usually only reflects long-term electricity power systems costs. RTP is the ideal pricing scheme; but the full implementation of RTP is difficult, due to the technical limitation of the demand side [7]. Choi et. al. [8] suggests a theory and simulation results of real-time pricing of real and reactive powers that maximizes social benefit. Conejo et. al. [9] present an optimization model to adjust the hourly load level of a given consumer in response to hourly electricity prices and maximizes the utility of the consumer, subject to several constraints such as minimum daily energy-consumption levels and limits on hourly load levels. A multi-objective optimization problem is proposed in [10] where the objective is to minimize the peak load and difference between the peak and valley loads. The multi-objective optimization problem is transformed into a single objective optimization problem and is solved by a fuzzy membership method. Case studies presented in [11] revealed that, by implementing real-time pricing, a demand reduction of between 8 and 11 GW at times of peak demand and low-wind could be achieved in the UK, due to the price elasticity and load-shifting.

This paper discusses an iterative linear programming based optimization problem formulation resulting in a solution that maximizes the consumer surplus by adjusting the electricity price and guaranteeing a fixed profit to the utility company. This solution presented adjusts the electricity price and keeps the load peaks within the power system constraints.

The rest of this paper is organized as follows. Section II provides the economic principles in demand-supply and discusses the concept of demand elasticity. Section III discusses the different costs associated to the power generation and formulation of the optimization problem. In Section IV, a case study is presented and results evaluated. Finally, Section V provides some concluding remarks and several interesting topics for further study.

II. THE ECONOMIC PRINCIPLE: DEMAND AND SUPPLY

Figure 1 shows a generalized relationship between the price of a good and the quantity which consumers are willing to purchase, at a given price. This is known as a simple demand curve [12]. Since many variables other than the price may influence the quantity demanded, it may be difficult to derive the relation between the price and the quantity. Economist often linearized this curve around a given point

Price elasticity of demand (PED) is a measure used in economics to show the responsiveness, or elasticity, of the quantity demanded of a good or service to a change in its price.

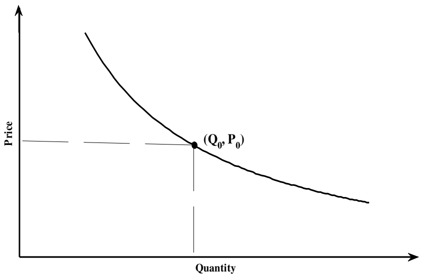

Figure 1. Typical demand curve.

The PED measures responsiveness of demand for a product following a change in price. The basic formula used to determine price elasticity is given as:

$$\text{PED}(\overline{\varepsilon}) = \frac{\%\text{ change in quantity demand}}{\%\text{ change in price}} = \frac{\Delta Q / Q_0}{\Delta P / P_0} \quad (1)$$

where ΔQ and ΔP are respective changes in demand and price; and Q_0 and P_0 are the base demand and price. If the price and quantity is normalized at a given equilibrium point (Q_0, P_0), the price elasticity of demand or the self elasticity can be expressed as [13]:

$$\varepsilon = \frac{\Delta Q}{\Delta P} \quad (2)$$

In some cases, a change in the price of one commodity will affect the demand for another commodity. For example, an increase in the price of coffee will reduce the demand for coffee but may increase the demand for tea. Elasticity of substitution shows to what degree two goods or services can be substitutes for one another. If the price and quantity is normalized at a given equilibrium point, the substitution elasticity or cross elasticity between two products 'a' and 'b' can be expressed as:

$$\varepsilon_{ab} = \frac{\Delta Q_a}{\Delta P_b} \text{ and } \varepsilon_{ba} = \frac{\Delta Q_b}{\Delta P_a} \quad (2a)$$

When the two goods are substitutes for each other, the cross elasticity of demand will be positive. The effect between the demands of product 'a' and the price of these two commodities is given by:

$$\Delta Q_a^s = \varepsilon_{aa} \Delta P_a;\ \varepsilon_{aa} \le 0 \quad (3)$$

$$\Delta Q_a^c = \varepsilon_{ab} \Delta P_b;\ \varepsilon_{ab} \ge 0 \quad (4)$$

where ΔQ_a^s and ΔQ_a^c represents the change in price of commodity due to self elasticity and cross elasticity, respectively.

With respect to the demand for electricity, a self-elasticity coefficient relates the demand during an hour period to the price during the same period. A rescheduling of appliances implies that the consumer reduces its electricity demand during some peak period and increases the demand during normal or off-peak periods. Cross-elasticity coefficients relate the demand in one hour to the price during other hours. The change in demand at an hour caused by a deviation of the published prices from the prices expected by the consumers is therefore given by the sum of individual effects. If the reciprocal effects between the two commodities are considered, the effect between price and demand can be defined as:

$$\begin{pmatrix} \Delta Q_a \\ \Delta Q_b \end{pmatrix} = \begin{pmatrix} \varepsilon_{aa} & \varepsilon_{ab} \\ \varepsilon_{ba} & \varepsilon_{bb} \end{pmatrix} \begin{pmatrix} \Delta P_a \\ \Delta P_b \end{pmatrix} \quad (5)$$

The diagonal elements of this matrix represent the self-elasticities and the off-diagonal elements correspond to the cross-elasticities. A column of this matrix indicates how a change in price during the single period affects the demand during all the periods. If the nonzero elements in this column are above the diagonal, the consumers react to a high price by shifting their consumption forward in time. If they are below the diagonal, they postpone their consumption until after the high price period. If consumers have the ability to reschedule their production over a long period, the nonzero elements will be spread widely over the column. On the other hand, if flexibility is limited, the nonzero elements will be clustered around the diagonal. Some customers may also decide that, if they have to reschedule their electricity consumption, they might as well take advantage of the lowest prices, which typically are in the early hours of the morning. For *m* commodities, the effect between price and demand can be defined as:

$$\Delta Q_i = \sum_{j=1}^{m} \varepsilon_{ij} \Delta P_j \quad (6)$$

Ramsey pricing or the Ramsey-Boiteux pricing principle [14] is a linear pricing scheme designed for the multiproduct natural monopolist. It is a policy rule focusing on what price a monopolist should set, in order to maximize social welfare, subject to a constraint on profit. As per this pricing rule, the consumer surplus should be maximized to guarantee a fixed profit to the utility company.

III. Mathematical Formulation of Costs

A typical electricity generation cost vs. generated power plot is illustrated in Figure 2. The curves can usually be adequately approximated using piece-wise linear, quadratic, or cubic functions. The pricing for the electricity from the generation side can be a function of amount of power generation or the amount of load.

For simplicity of design, a quadratic function as given in (7) is assumed between power generated and the cost of power:

$$C_g(P_g) = a + bP_g + cP_g^2 \quad (7)$$

where *a*, *b* and *c* are constants and P_g is the amount of power generated. Assume that the cost of electricity distribution is also included in the above relation. Using (7), the power generation cost during different hours of a day can be expressed as:

$$C_{g,i}(P_{g,i}) = a + bP_{g,i} + cP_{g,i}^2;\ i = 1,...24 \tag{8}$$

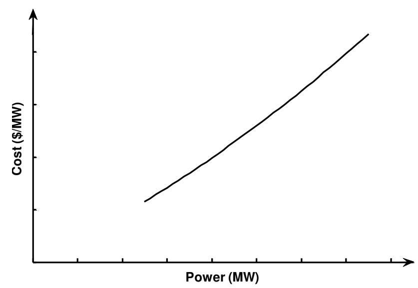

Figure 2. Power generation cost.

Hence the total power generation cost is:

$$C_g(P_{g,i}) = \sum_{i=1}^{24}(a + bP_{g,i} + cP_{g,i}^2) \tag{9}$$

If β_i is the electricity selling pricing to the customer during period i and $P_{L,i}$ is the total power delivered to the consumers, the total electricity cost paid by the consumers in a day is given by:

$$C_L(P_{L,i}) = \sum_{i=1}^{24}\beta_i P_{L,i} \tag{10}$$

A. Relation between Cost and Consumer Behavior

Consider that the whole day is divided into peak, normal and off-peak periods and the consumers are charged at different rates during these periods of time. The cost paid by the consumers can be represented as $\overline{\beta}_j\ \forall j = 1,2,3$ where $j = 1,...,3$ corresponds to the peak, normal and off-peak period, respectively. Consumers can make a choice between consuming power now or shifting the appliance operation time to a different period of the day when electricity will be presumably cheaper. If the electricity price changes from $\overline{\beta}_j$ to $\overline{\beta}_j(1+\Delta\overline{\beta}_j)$, then the demand for electricity will change from P_i to $P_i(1+\Delta P_i)$, where ΔP_i is the percentage change in power consumption. Hence the total generation cost is given by:

$$C_g(P_{gi}) = \sum_{i=1}^{24} a + bP_i(1+\Delta P_i) + c(P_i(1+\Delta P_i))^2 \tag{11}$$

Let ε_{ii} and ε_{ij} represents self-elasticity and substitution elasticity, respectively. The percentage change in power consumption is given by:

$$\Delta P_i = \sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j \tag{12}$$

If an increase in price does not modify the appliance operating schedule without a reduction in energy demand over a 24-hour scheduling period, the following relation holds between the elements of each column of the elasticity matrix:

$$\sum_{i=1}^{24}\varepsilon_{ij} = 0 \quad \forall j \tag{13}$$

On the other hand, if the consumer reduces its demand, this relation becomes:

$$\sum_{i=1}^{24}\varepsilon_{ij} < 0 \quad \forall j \tag{14}$$

By using the above relations, the total cost to the consumer can be written as:

$$C_g = \sum_{i=1}^{24}\sum_{j=1}^{3}\left\{a + bP_i(1+\varepsilon_{ij}\Delta\overline{\beta}_j) + c(P(1+\varepsilon_{ij}\Delta\overline{\beta}_j))^2\right\} \tag{15}$$

Similarly, the power generation cost can be written as

$$C_c = \sum_{i=1}^{24}\beta_i(1+\Delta\beta_i)P_i\left(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j\right) \tag{16}$$

B. Electricity Pricing for Welfare Maximization

Now consider the utility's pricing problem for the society's welfare maximization. The basic idea of this pricing method is that the electricity price of the generation side depends on the basis of a certain electricity consumption or power load, and the electricity demand of the demand side is closely relevant to the price. The pricing is relevant to the government, the utility company and the consumer. Hence the electric power system must be considered as public utility and the electricity should be priced using the Ramsey pricing rule. Based on the Ramsey pricing rule [15], the problem faced by the utility company is to maximize the consumer surplus and guarantee a fixed amount of profit to the utility company. This pricing rule is used for regulating the price for a multi-product monopolist. If the profit to the utility company is fixed to zero, then the cost of power generation (C_g) equals the cost paid by the consumer (C_c). Hence one obtains the following equality:

$$\sum_{i=1}^{24} a + bP_i(1+\Delta P_i) + c(P_i(1+\Delta P_i))^2 = \sum_{i=1}^{24}\beta_i(1+\Delta\beta_i)P_i(1+\Delta P_i) \tag{17}$$

$$\begin{aligned}\sum_{i=1}^{24} a + bP_i(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j) + c(P(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j))^2 \\ = \sum_{i=1}^{24}\beta_i(1+\Delta\beta_i)P_i(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j)\end{aligned} \tag{18}$$

There are few set of constraints which the utility faces on maximum power generation. During the different hours of the day, if the maximum power availability is $P_i^{\max};\ i = 1,...,24$ then the following constraint should be satisfied:

$$P_i(1+\Delta P_i) < P_i^{\max};\quad i = 1,...,24 \tag{19}$$

Using conditions (16) and (17), the pricing problem can be formulated for maximizing the consumer surplus. The problem can be easily solved if the equality condition in (16) is converted to some inequality. The term on the left hand side of the inequality is the power generation cost and the right hand side of the inequality is the cost paid by the consumer. If the consumer surplus needs to be maximized, then the cost of electricity should be kept as low as possible. At the same time, the total revenue from the consumer should be greater than or equal to power generation cost. Hence the equality condition is changed to inequality as shown in (20).

$$\sum_{i=1}^{24} a + bP_i(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j) + c(P(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j))^2 < \sum_{i=1}^{24}\beta_i(1+\Delta\beta_i)P_i(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j) \quad (20)$$

The objective of the problem is to maximize the consumer surplus. This can be written as $\max\sum_{i=1}^{24} P_i(\Delta P_i)$. Using the above inequalities (19) and (20), the pricing problem can be formulated as the following optimization problem:

$$\max_{\Delta\beta}\sum_{i=1}^{24}\sum_{j=1}^{3} P_i\varepsilon_{ij}\Delta\beta_j$$

subject to

$$\sum_{i=1}^{24} a + bP_i(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j) + c(P(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j))^2 < \sum_{i=1}^{24}\beta_i(1+\Delta\beta_i)P_i(1+\sum_{j=1}^{3}\varepsilon_{ij}\Delta\overline{\beta}_j) \quad (21)$$

$$P_i(1+\Delta P_i) < P_i^{\max}; i = 1,...,24 \quad (22)$$

The inequality in (21) is bilinear and hence it cannot be solved directly. If the value of $\Delta\overline{\beta}_j$ in the quadratic term is fixed, then it becomes a linear inequality and it can be solved directly. Then (21) and (22) can be solved by initially assuming a value of zero for the quadratic term and iteratively updating the value of the quadratic term.

IV. Illustrative Example

In order to show the effect of demand response on the load curve of the power distribution system, the power consumption profile, shown in Figure 3, is considered. In this figure, the power is normalized with respect to the maximum power capacity of the utility company. The times between 7AM to 2PM, 2PM to 9PM and 9PM to 7AM are taken as semi-peak, peak and off-peak periods, respectively. The initial prices (i.e., the prices if elasticity was not considered with a limited power generation capacity and without considering the demand response) assumed in this simulation during the peak, semi-peak and off peak are listed in Table I.

The constants associated with the power generation cost and the transmission cost are assumed as: a = 0, b = 0.13 and c = 0.004; the relation between these the two costs is given by:

$$C_g(P_{g,i}) = \sum_{i=1}^{24}(0.13P_{g,i} + 0.004P_{g,i}^2)$$

TABLE 1: POWER AVAILABILITY AND THE PRICE DURING DIFFERENT PERIODS OF THE DAY (CASE (I)).

	Semi-peak period	Peak period	Off-peak period
Available power	1.0	1.0	1.0
Initial price ($/kWh)	0.2	0.23	0.15
Calculated price ($/kWh)	0.1479	0.2866	0.1229

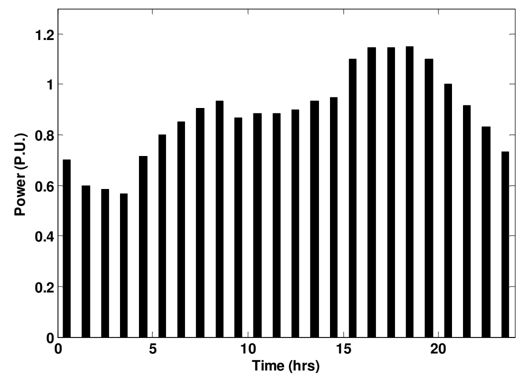

Figure 3. Power profile with the reference price.

A household's power consumption and associated response to price variation vary depending upon the set of appliances; therefore it is natural to expect the factors influencing electricity demand to differ between different households. For example, a household that uses central air conditioning for most of the summer might be willing to alter its thermostat setting in response to a small change in the price of electricity, which can yield a large change in its electricity consumption. In contrast, a small household that uses electricity to operate only a refrigerator and a few lights might exhibit little or no demand response even to large price changes. This suggests that both a household's electricity consumption and its price sensitivity may depend, in part, on the specific types of appliances it holds. Studies on price elasticity [16] show that the air condition ownership had a very significant influence on demand response, and the load reductions are more than twice for households with air conditioning than for those without. Certain cost and energy conscious consumer may react to the high price and reduce the energy consumption by switching off certain appliances when it is not necessary or reducing the lighting loads. Farugui and George [16] estimated substitution elasticities for different periods of the day and observed that the load reduction in peak period is between 10%-15% and the increase in load is less than 4% for the off peak period. In this paper, the price elasticities during different periods of the day are shown in Figure 4.

The optimization problem is solved using the Linear Matrix Inequality Toolbox available in MATLAB® and the results are presented in Table I and Figure 5. The profit to the utility company is assumed as zero. The calculated price is increased from the reference price during the peak period, since the required power during the peak period is greater than the grid capacity, i.e., the capacity that can be supplied by the utility company. By increasing the price during this period, the demand is adjusted to match the grid capacity.

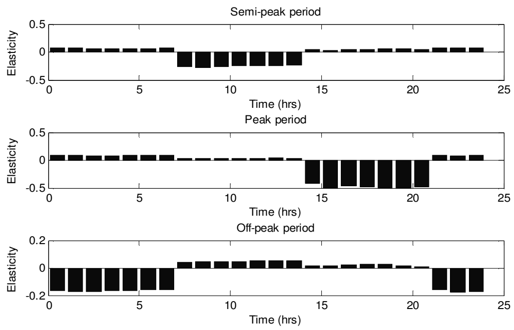

Figure 4. Price elasticities during different periods of the day.

Likewise, the demand during the off peak and the semi peak periods are less than the available capacity and the price during this period is reduced slightly so that the consumer will consume more power during this period and get more benefit. The profit gained by selling the electricity at high price during the peak period is used to give incentives to the consumers for the power consumption in off peak and semi peak period.

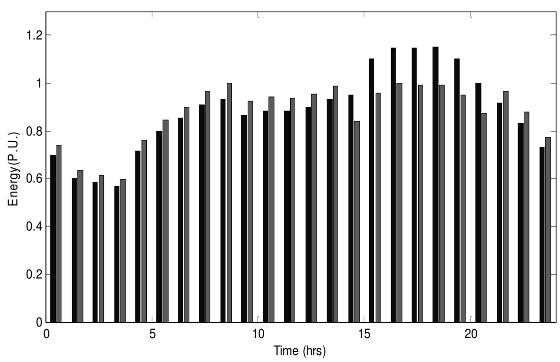

Figure 5. Power profile before and after demand response programs with the power generation limit given in Table 1 (dark shaded – before demand response, light shaded – after demand response).

Consider another case where the normalized power availability is limited to 0.9 during the semi peak period. The parameters considered in this simulation are given in Table II. The power requirement in this case is greater than the supply capacity during the semi peak and the peak period. The results obtained in this case are shown in Table II and Figure 6. The costs during the semi peak and the peak periods are increased to reduce the demand. The profits gained during this semi peak period and the peak periods are used to give incentive during the off peak period and the cost of electricity in this case is lower than that of case (i).

Next, consider the case with some amount of profit to the utility company. A profit amount equal to 2% of the total generation cost is assumed and the other parameters are assumed to be the same as in case (i). The problem is solved and the result obtained in this case is shown in Table III and Figure. 7. Since some amount of profit is included in the problem, the electricity cost is slightly higher than that of case (i). The electricity demands before and after the demand response is shown in Figure 7.

TABLE II. POWER AVAILABILITY AND THE PRICE DURING DIFFERENT PERIODS OF THE DAY (CASE (II)).

	Semi-peak period	Peak period	Off-peak period
Available power	0.9	1.0	1.0
Initial price ($/kWh)	0.2	0.23	0.15
Calculated price ($/kWh)	0.2214	0.2917	0.0914

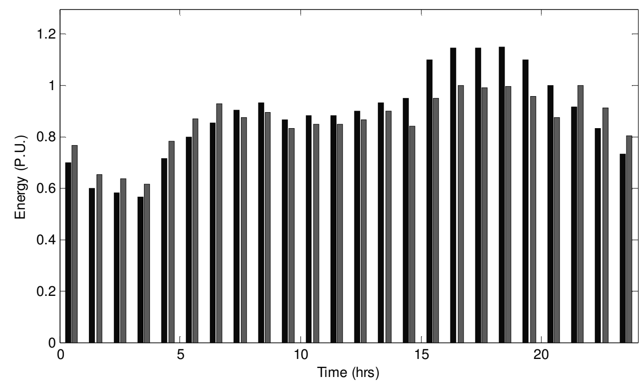

Figure 6. Power profile before and after demand response programs with the power generation limit given in Table 2 (dark shaded – before demand response, light shaded – after demand response).

TABLE III. POWER AVAILABILITY AND THE PRICE DURING DIFFERENT PERIODS OF THE DAY (CASE (III)).

	Semi-peak period	Peak period	Off-peak period
Available power	1.0	1.0	1.0
Initial price ($/kWh)	0.2	0.23	0.15
Calculated price ($/kWh)	0.1730	0.2873	0.1122

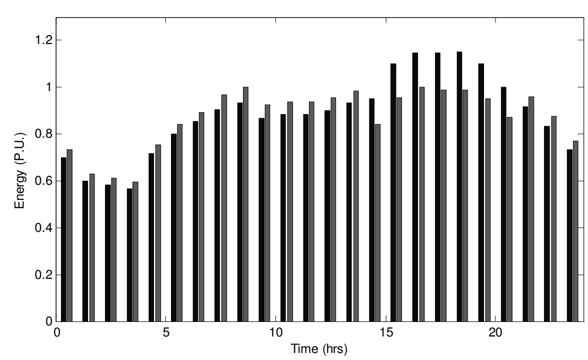

Figure 7. Power profile before and after demand response programs with the power generation limit given in Table 3 (dark shaded – before demand response, light shaded – after demand response).

Finally consider the electricity consumption pattern as shown in Figure 8 when it is desired to estimate the effect of change in price. In this figure, the energy is normalized with respect to the maximum capacity of the grid. Assume that the first two consumers are not responding to the change in price, the third consumer response is half when considering the overall elasticities and the last consumer response is the same as the elasticities given in Figure 4.

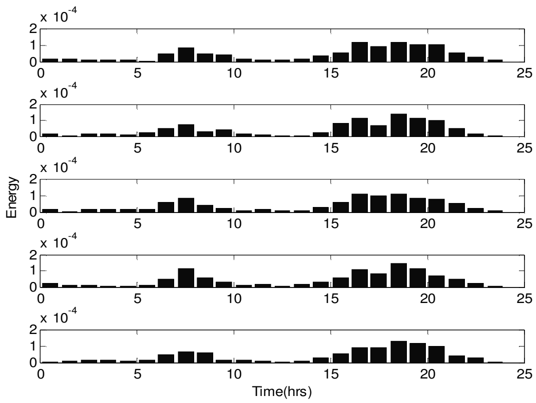

Figure 8. Typical electricity consumption patterns.

In this case, the energy consumption and the cost paid by the consumers before and after the demand response are shown in Table IV, where power consumption is in kW-hr and cost is in US dollars. The total power consumed by all the consumers is increased when the demand response is enabled and at the same time the total cost paid by them is reduced. This shows that the demand response is effective in maximizing the overall benefit for the consumers.

Table IV: Energy Consumption and Cost Profiles Before and After DR

Time segment	Energy Consumption before DR	Energy Consumption after DR	Cost before DR	Cost after DR
1	15.4777	15.4777	3.1136	2.4805
2	15.2003	15.2003	3.0218	2.3404
3	14.6859	15.0087	2.9168	2.3550
4	15.4891	16.1675	3.1250	2.6242
5	14.3645	15.0296	2.8746	2.3284

V. CONCLUSIONS

A demand response program benefits the electricity consumers and should rely on end users to deliberately alter their use of appliances and systems, which generally means lifestyle or comfort changes, or changes in operating procedures. Such changes would be acceptable to end users only if the consumer has a stake in the process either through financial compensation or through improved reliability of power supply. This paper proposes a method of finding the optimal pricing for electricity which can improve the grid reliability and reduce costs. An iterative linear programming based optimization problem is formulated for calculating the electricity price during different periods of the day. Demand response can reduce electricity prices, improve system reliability, and reduce price volatility. The illustrative example demonstrates the effect of the demand response strategy for reducing the peak power. The performance of demand response programs is demonstrated by peak load reduction and demand elasticity.

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