An optimal time-of-use pricing for urban gas: A study with a multi-agent evolutionary game-theoretic perspective

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HIGHLIGHTS

- A multi-agent system for urban gas market is developed.
- Evolutionary game-theoretic is adopted to determine the optimal TOU prices.
- Two types of TOU are assessed to reflect the variation of load and price.
- Significant potential for peak-shaving and load-shifting under TOU pricing.
- Industrial users is better in demand response than residential and commercial users.

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ABSTRACT

In energy markets, regulators are often tempted to use price schedules to improve economic efficiency and promote a reasonable resource allocation. Time-of-use pricing is very popular with economists, and many researchers have been written estimating and exploring the optimal time-of-use pricing for electricity markets. Yet, such pricing has rarely been used in the natural gas sector. In this paper, we propose an optimal time-of-use pricing in urban gas market based on an evolutionary game-theoretic perspective. As the urban gas market is a nonlinear complex economic system with several interacting agents, we use a multi-agent system comprising a government agent, a local gas distribution operator agent, and different types of end-user agents. A power structure demand response program is employed to simulate the user demand response. A mixed-integer linear programming is formulated to determine the time-of-use price-signal delivering maximum gas operator profit and the optimal load pattern for end-users. In an illustrative example, we simulate and compare the time-of-use block prices and time-of-use hourly prices with traditional fixed pricing using real-world data of Wuhan in China. The numerical results indicate that time-of-use pricing schedules have significant potential for peak-shaving and load-shifting for urban gas pipeline network systems and would thus lower operating costs. Furthermore, different gas users exhibit different demand responsiveness intensity. Finally, we evaluate the impact on total social welfare of regulation scenarios and find that welfare decreases with deregulation, implying that the urban gas market is immature and reasonable regulation is still necessary.

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1. Introduction

Price schedules can help to enhance the financial sustainability of urban energy systems [1–3]. In terms of the natural gas sector, a large part of the total cost is accounted for by the installation and maintenance of the distributed pipelines, metering infrastructure, and customer service running costs. Thus, the traditional marginal cost pricing rules, which mainly consider the operation costs, may not provide enough revenue to cover fixed costs. In developed countries, the two-part tariffs allowing for fixed access fees and market-oriented volumetric charges are commonly used [4,5]. In developing countries, on the other hand, the urban gas sector is still developing. The infrastructure of pipelines network and storage facilities is still expanding. As a result, gas supply systems cannot keep up with the rapidly growing demand in gas consumption. Consequently, the current government-regulated gas pricing schemes may fail to deliver allocation and financial objectives,

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Nomenclature

Acronyms

UGPN	urban gas pipeline network
DSM	demand-side management
TOU	time-of-use pricing
MAS	multi-agent system

Indices and parameters

t	index of model time unit
T	time periods in the optimization horizon
μ	cost coefficient of gas demand fluctuation

Decision variables

P_t	TOU price scheduling by operator
D_t	demands response of gas user

Random variables

P_f	original fixed price in time horizon T
P_c	gas purchase cost of operator
$P_c^{\uparrow}$	gas purchase cost if contract supply shortage
$P_c^{\downarrow}$	gas storage cost if contract supply overmuch
P^{up}	up-regulation price at the gas market
P^{down}	down-regulation price at the gas market
D_t^0	user nominal demand of gas at the time t
$D^{\uparrow}$	gas purchase amount if contract supply shortage
$D^{\downarrow}$	gas storage amount if contract supply overmuch
$\overline{D}$	mean of gas demand in time horizon T
D_t^{max}	maximum of gas demand at the time t
D_t^{min}	minimum of gas demand at the time t
ξ_t	price elasticity of gas demand at the time t

leading to financial losses and harming the prospects for healthy the development of urban gas sector in developing countries.

The natural gas market in China exemplifies this issue. Currently, the natural gas consumption has increased fast in China, growing at an annual rate of 9.6%, with its share in primary energy increasing from 2.6% to 5.1% between 2005 and 2013. To deal with the imbalance of demand and supply, the Chinese government launched a set of policies to encourage domestic and imported gas supply expansion, and promote efficient gas use, including the nationwide natural gas pricing reform program. However, the gas pricing reform progress is going on slowly, and gas shortages occur frequently due to the distortion gas pricing structure and insufficient gas sources [6–9].

To promote the construction of urban gas pipeline network (UGPN) and cover costs, the government has adopted two-part tariffs for urban gas markets, in which all consumers pay a fixed fee for access to the network once and a monthly fixed price volumetric charge based on average long run marginal cost [4,10]. However, the prices hardly reflect the changing operation costs or the imbalances between demand and supply throughout the day [11]. During peak hours, the gas demand is considerable higher than the average consumption. The gas pipeline network lacks pressure during peak demand and requires more storage gas to be injected, leading to running costs that are higher than the average marginal cost. While during off-peak hours, demand drops sharply and the pressure in the network is too high and raises safety concerns. We propose a time-varying gas pricing system to reflect the localized marginal cost in different periods. In the proposed scheme, the volumetric component of the two-part tariff would be replaced with a time-of-use pricing (TOU) while the access fee component of the tariff would remain unchanged.

TOU pricing is an efficient method of demand-side management (DSM) which can be employed by operators to influence user behavior [12–17]. By setting different prices in peak and off-peak periods of a day, the gas operators can encourage end-users to shift flexible demand components to off-peak hours. TOU pricing is commonly used in electricity markets. The TOU program is a well-known time-based demand response (DR) program which has the following advantages [18]: reduced operation costs; improved profits for gas operators; and reduction of peak load demand and the strain on the gas network.

There has been significant and beneficial work in the area of TOU pricing in recent years, highlighting the potential methods of TOU pricing in the DR program. The work in [19] adopts Monte Carlo simulation to quantify residential DR effectiveness of various

TOU scenarios based on survey results, it is important to be aware that the results are likely to be quite system specific. The work in [20] establishes a two-stage stochastic mixed-integer linear programming (SMILP) model to determine the optimum TOU rates based on grid reliability index, and solved it using CPLEX. This is, in effect, exploiting the optimal TOU prices to enhance system reliability. The work in [21] proposes a novel method to design feasible TOU tariff rates based on Gaussian mixture model, both energy cost reduction and peak shaving have been investigated to explore the effects of the TOU tariffs on demand response. To describe the interactive relationship of stakeholder, the work in [22] adopts a game theoretical model accounting for the relationship between retailers (leaders) and consumers (followers) in a dynamic price environment. They reformulate a bi-level program as a single-level mixed-integer linear program (MILP). Similarly, the work in [23] presented a game-theoretic approach to optimize TOU pricing strategies by building utility functions of both utility companies and users, which provides optimal prices and user response. Furthermore, the work in [24] introduces a comprehensive demand response model in an agent-based retail environment. Their purpose is to represent customer response to time-based and incentive-based demand response programs with Q-learning method. Papers [25,26] investigate the impacts of TOU tariffs on electricity demand, price savings, peak load shifting and peak electricity demand at sub-station level. All findings indicate that TOU tariffs bring about higher average consumptions and lower payments by end-users. But the demand responding policies are slow to emerge and limited by the knowledge on the scope of potential gains. All these studies are based on models using multidirectional information exchange, where the system operators choose a price sequence based on communicated demand schedules and corresponding load response to that price sequence.

However, since the urban natural gas market appeared relatively late in China, the studies on natural gas TOU pricing are still limited in number. Recently, with the rapid expansion of natural gas market and continuously increasing gas consumption, the price reform and peak shaving of natural gas become an urgent problem to be solved. The work in [27] establishes a static simulation model for natural gas TOU pricing by combining the grey relationship with a Monte-Carlo simulation. The work in [28] introduces the TOU pricing policy for natural gas industrial users, and designed a TOU price multi-agent system based on linear demand response model. This model has been used to simulate the running state of the UGPN in Zhengzhou, China. The results indicated that the peak–valley load difference in the UGPN can be reduced effectively with an optimal peak–valley price relationship.

This paper proposes a game theoretic model for optimal TOU pricing and demand response for an urban gas market with different types of end-users. The novelty of this approach is threefold. First of all, under the agent-based framework, we propose an approach using an evolutionary game model to simulate the interaction between the operator and end-users and an inverse elasticity demand response model to simulate the TOU pricing response. Using this approach, both optimal TOU pricing and demand response can be achieved simultaneously, and the conflicting economic interests of the operators and end-users can be captured directly. Secondly, we integrate the optimization problem of end-users into the optimization problem of operators. Therefore, the agent-based simulation problem has been transferred into a pure optimization problem. Finally, it has been recently argued that the TOU hourly pricing is more efficient in engaging customers in smart grids than the TOU block pricing [29]. Here we evaluate both these pricing schemes within a regulated and deregulated environments. This allows us to quantify and compare the advantages of these two TOU pricing schemes in the current urban gas market.

The remainder of this paper is organized as follows: Section 2 introduces the stakeholders in the UGPN and presents the formulation of the urban gas multi-agent system. Section 3 presents the mathematical formulation of the operator and the end-user problems; then an evolutionary game theory model is formulated as an optimization means to explore the optimal TOU price and demand response. Section 4 presents results from a numerical example based on real-world data. Finally, conclusions are drawn in Section 5.

2. Urban gas multi-agent system formulation

2.1. Urban gas market formulation

The structures of urban natural gas markets are relatively similar around the world. Depending on the varying local economical characteristics and infrastructure conditions, there are several categories of stakeholders such as governments, gas distribution operators and end-users. In the UGPN market of China, the government acts as a key ruler maker or policy maker to regulate the market and improve economic efficiency. The operator transports and distributes the gas to different users. The aim of the operator is to ensure reliability, stability, and security of the gas system. End-users differ by gas usage types and functions and can be divided into industrial, commercial and residential users. For residential users, their gas consumption is mainly for heating, cooling and cooking; and consumption patterns are difficult to modify. The relatively fixed patterns result in a small price elasticity of demand, especially during peak periods in the day. For commercial users, the demand for gas in business districts is high, and they are reluctant to reduce gas usage, which may harm the running of their businesses. As a result, they tend to have lower price elasticity compared to residential users. On the other hand, industrial users, especially those with high energy consumption facilities and huge demands, are more sensitive to gas prices. They can reschedule their production times to minimize gas costs and thus account for the most price elastic portion of the urban gas market.

2.2. Urban gas multi-agent system

UGPN is a complex economic system with non-linear interaction mechanisms [30,31]. The different modules of UGPN, including government, gas distribution operators, and multiple types of users, form a supply and demand network. This system can be transformed into a multi-agent system (MAS), wherein each module is regarded as an independent agent. The various interactions among customers, organizations, and systems can be described as independent activities between agents. An illustration of the TOU pricing multi-agent model of the UGPN is shown in Fig. 1.

In Fig. 1, there are three categories of agents in the TOU pricing model of UGPN. Firstly, the government agent, acting as a market regulator, specifies the TOU price bounds by considering the cost of natural gas production and transportation, relevant energy prices, and external environmental factors. The regulated price bounds will then be communicated to the operator agent. Then, the operator agent, determines the exact TOU prices that apply to end-users. The operator wishes to attract the end-users to participate in demand response and adjust their gas consumption behaviors by shifting load in peak periods. The object of the operator is to reduce the demand fluctuations throughout the day and maximize its benefits. The end-user agents (commercial, residential, and industrial) respond to the TOU prices with the objective of reducing their expenditures. These responses can involve behavioral adjustments where more gas is consumed during off-peak periods. The urban gas control center monitors changes in UGPN loading and provides feedback to the operator and the government.

In UGPN market, the financial risk to the operator stems from multiple stochastic variables: gas amounts from the upstream provider, the regulated market prices, shifts in the inflexible (must-serve) part of the load, and inaccuracies in predicting (modeling) end-user behavior. The specific market design allows the agents (operators and users) to make decisions a day ahead.

3. Model of time-of-use pricing from a game-theoretic perspective

In this section, we present the game-theoretic model of TOU pricing problem. In this multi-agent framework, the operator agent decides on the amount of gas purchased from the upstream gas provider taking into account the total end-user nominal demand predicted for the optimization horizon. The operator then specifies the different TOU prices it will apply to consumption by the different types of end-users based on gas demands expected from those end-users. As indicated above, these gas price decisions are done within the gas price bounds regulated by government agent. The end-user agents optimize their gas demands based on the price received from the operator. Finally, the operator and end-users will achieve a Nash equilibrium by finding an optimal TOU prices and optimal load response under continuous negotiations and interactions.

3.1. Urban gas operator problem

In an optimization horizon T, the objective function of the operator is the maximization of the expected market profits $\phi(P_t, D_t)$, with respect to both the TOU price P_t and end-user gas demand D_t, as

$$\phi(P_t, D_t) = \sum_{t=1}^{T} (P_t D_t - P_c D_t^0) - P_c^\uparrow D^\uparrow - P_c^\downarrow D^\downarrow - F(D_t) \qquad (1)$$

The objective function above is the sum of four terms. The first one includes the revenues obtained through charges from end-users $P_t D_t$ minus the cost of gas purchase $P_c D_t^0$. As the contract supply and user demands are not always balanced, the second and third terms respectively represent the extra purchasing cost $P_c^\uparrow D^\uparrow$ and extra storage cost $P_c^\downarrow D^\downarrow$ if the contract supply is not enough or is in excess, and $D^\uparrow \cap D^\downarrow = \varnothing$. The last term is the cost caused by the variation of user demands during the horizon, which can be measured by the sum of squared generation deviations from the mean demand [32], multiplied by a coefficient μ, i.e.,
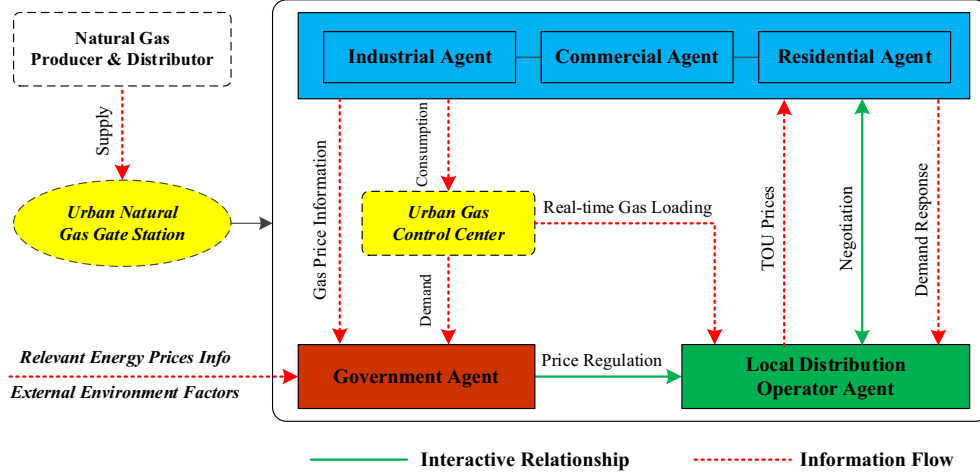
Fig. 1. Urban gas multi-agent framework of TOU pricing in UGPN market.

$$F(D_t) = \mu \sum_{t=1}^{T} \sqrt{(D_t - \overline{D})^2} \qquad (2)$$

With a fixed price P_f and a TOU price P_t, the changes of operator's benefit from P_f to P_t could be stated as following,

$$\hat{\phi}(P_t, D_t) = \phi(P_t, D_t) - \phi(P_f, D_t^0) \qquad (3)$$

In order to ensure the market competitiveness of TOU prices, there are some constraints to the contracts between operator and price-responsive end-users under the regulation by government. That is, the TOU prices should satisfy the following constraints,

$$P^{down} \leqslant P_t \leqslant P^{up}, \quad \frac{1}{T} \sum_{t=1}^{T} P_t = P_f \qquad (4)$$

Constraint (4) enforces the TOU price signal must have a fixed daily average. Notice that this constraint is necessary to ensure a sufficient number of low-price periods [24]. Without this constraint, the operator would charge the maximum price to the end-users within the regulation prices. Constraint (4) consents a straightforward comparison between operators in a competitive market, basing on some parameters such as the maximum and minimum values of average hourly price, the price level, and the price volatility.

Substituting Eqs. (1) and (2) into Eq. (3), the operator problem in the optimization horizon T can be denoted as:

$$\hat{\phi}(P_t, D_t) = \arg\max_{P} \sum_{t=1}^{T} (P_t D_t - P_f D_t^0) - P_c^{\uparrow} D^{\uparrow} - P_c^{\downarrow} D^{\downarrow} - F(D_t) + F(D_t^0) \qquad (5)$$

In Eq. (5), P_t is the operator decision variable constrained by (4). It should be noticed that the objective function of operator is a linear programming problem, and the optimal solutions vary with end-users' gas demands.

3.2. Urban gas end-user problem

Hereon, we consider a flexible demand response environment, where the end-users can optimize its future consumptions based on a dynamic TOU price schedule delivered by the operator. For the sake of simplicity, we initially formulate the model with a single type of user to describe the user problem. We assume that the end-user nominal demand remains at a fixed level. After charging with a flexible TOU pricing, end-users will maximize their benefits

of the gas consumption by shifting as much flexible demand as possible to low price periods, without giving up too much on their comfort or utility. Like the operator, the end-users face an optimization problem which can be described as follows.

Since the income effect is weak in natural gas demand [33], we assume the end-users gas demand is a power structure function of price [14], sated as $D_t = \lambda \times (P_t)^{\xi_t}$, where the ξ_t is the price elasticity of demand. Let $\beta(P_t, D_t)$ be the benefit the end-user obtains during t-th hour obtained with a demand of D_t. Then, the user's utility $\varphi(P_t, D_t)$ for the same period can be stated as:

$$\varphi(P_t, D_t) = \beta(P_t, D_t) - P_t D_t \qquad (6)$$

The benefit function $\beta(P_t, D_t)$ can be obtained by Taylor expansion as following [34]:

$$\beta(P_t, D_t) \cong \beta(P_f, D_t^0) + \frac{P_t^0 D_t}{1 + (\xi_t)^{-1}} \left(\frac{D_t}{D_t^0}\right)^{(\xi_t)^{-1}} \qquad (7)$$

Substituting (7) in (6), the end-user utility is

$$\varphi(P_t, D_t) = \beta(P_f, D_t^0) + \frac{P_t^0 D_t}{1 + (\xi_t)^{-1}} \left(\frac{D_t}{D_t^0}\right)^{(\xi_t)^{-1}} - P_t D_t \qquad (8)$$

After the implementing of TOU pricing, the changes of end-user's utility from D_t^0 to D_t is

$$\hat{\varphi}(P_t, D_t) = \varphi(P_t, D_t) - \varphi(P_f, D_t^0) \qquad (9)$$

Substituting Eq. (8) into Eq. (9), the optimization problem for an end-user is

$$\hat{\varphi}(P_t, D_t) = \arg\max_{D} \sum_{t=1}^{T} \left(\frac{P_t^0 D_t}{1 + (\xi_t)^{-1}} \left(\frac{D_t}{D_t^0}\right)^{\xi_t^{-1}} - \frac{P_t^0 D_t^0}{1 + (\xi_t)^{-1}} - P_t D_t + P_t^0 D_t^0 \right) \qquad (10)$$

In any optimization horizon $t \in T$, the end-user's gas consumption should satisfy $D_t \in [D_t^{\min}, D_t^{\max}]$. As the TOU price P_t is scheduled by the operator, the end-user objective function is a nonlinear optimization problem of the P_t. Thus, the end-user will participate in the demand response when the price P_t is determined.

3.3. Urban gas multi-agent interaction relationship

In this segment, we propose an evolutionary game model to present the interaction between the operator and the end-user.

Since this is a multi-stage game, the operator decides the TOU price P first, and then the end-user decides the actual consumption of gas D according to the price. All their interactions depend on both P and D. To maximize those two agents' utility functions $\hat{\phi}(P, D)$ and $\hat{\varphi}(P, D)$ in one horizon T, the optimization problem is formulated as:

$$\hat{\phi}(P, D) = \arg\max_P \sum_{t=1}^{T} (P_t D_t - P_f D_t^0) - P_c^\uparrow D^\uparrow - P_c^\downarrow D^\downarrow - F(D_t) + F(D_t^0) \quad (11)$$

$$\hat{\varphi}(P, D) = \arg\max_D \sum_{t=1}^{T} \left(\frac{P_t^0 D_t}{1 + (\xi_t)^{-1}} \left(\frac{D_t}{D_t^0}\right)^{\xi_t^{-1}} - \frac{P_t^0 D_t^0}{1 + (\xi_t)^{-1}} - P_t D_t + P_t^0 D_t^0 \right) \quad (12)$$

Let $\mathcal{P}$ denotes the strategy set of the operator, representing all the possible TOU prices. Let $\mathcal{D}$ denotes the strategy set of the end-user, representing all the possible loads. The strategy sets can be defined as follows:

$$\mathcal{P} = \{P | P \in \mathbb{R}^N, P^{down} \leqslant P \leqslant P^{up}\} \quad (13)$$

$$\mathcal{D} = \{D | D \in \mathbb{R}^N, D^{min} \leqslant D \leqslant D^{max}\} \quad (14)$$

In order to find the optimal price $P^* \in \mathcal{P}$ and optimal load response $D^* \in \mathcal{D}$, a Nash equilibrium $(P^*, D^*) \in \mathcal{P} \times \mathcal{D}$ is achieved between the operator and the end-user. Specifically, a Nash equilibrium is achieved when the following conditions are satisfied [35]:

$$\forall P \in \mathcal{P}, \quad P \neq P^* : \hat{\phi}(P^*, D^*) \geqslant \hat{\phi}(P, D^*) \quad (15)$$

$$\forall D \in \mathcal{D}, \quad D \neq D^* : \hat{\varphi}(P^*, D^*) \geqslant \hat{\varphi}(P^*, D) \quad (16)$$

Here, the interaction relationship between the operator and the end-user is a leader–follower structure typical of Stackelberg games.

3.4. Optimal time-of-use pricing and demand responding

Since this is a multi-stage game, we use backward induction to solve the equilibrium of the two agents. The upper-level operator takes action first by setting the TOU price, and then the end-user adjusts its gas demand. Therefore, according to the backward induction principle, we first maximize $\hat{\varphi}(P, D)$ with respect to D^*, and then plug the optimal load response D^* into $\hat{\phi}(P, D)$ and optimize it with respect to P^*.

In order to find a user's optimal demand response to the TOU prices set by the operator, we consider the TOU prices of different time periods P_t as given. According to the classical optimization rules, we take the first-order derivatives of $\hat{\varphi}(P, D)$ with respect to D as

$$\frac{\partial \hat{\varphi}}{\partial D_t} = -P_t + P_t^0 \left(\frac{D_t}{D_t^0}\right)^{\frac{1}{\xi_t}} \quad (17)$$

The second-order derivative of $\hat{\varphi}(P, D)$ is

$$\frac{\partial^2 \hat{\varphi}}{\partial D_t \partial D_k} = \begin{cases} \frac{P_t^0 (D_t)^{(\xi_t^{-1}-1)}}{\xi_t^{-1}(D_t^0)^{\xi_t^{-1}}} < 0 & when \ t = k \\ 0 & when \ t \neq k \end{cases} \quad (18)$$

Since $\xi_t < 0$, the diagonal elements of the Hessian matrix are all negative, and the off-diagonal elements are all zero. The Hessian matrix is negative definite [36]. Setting Eq. (17) equal to zero, we determine that

$$D_t = D_t^0 \left(\frac{P_t}{P_t^0}\right)^{\xi_t} \quad (19)$$

Therefore, Eq. (19) is the optimal user demand response function to the TOU prices scheduled by the operator. After

estimating the user's demand response, the operator can maximize its objective by finding the optimal TOU price. From Eq. (19), we obtain:

$$P_t = P_t^0 \left(\frac{D_t}{D_t^0}\right)^{1/\xi_t} \quad (20)$$

Since Eq. (20) is a decreasing function of D_t, the constraints to TOU prices can be rewritten as

$$\max\left\{ P^{down}, P_t^0 \left(\frac{D_t^{min}}{D_t^0}\right)^{1/\xi_t} \right\} \leqslant P_t \leqslant \min\left\{ P^{up}, P_t^0 \left(\frac{D_t^{max}}{D_t^0}\right)^{1/\xi_t} \right\} \quad (21)$$

The constraints of operator optimization problem are linear if D_t is determined, and the operator could schedule an optimal TOU price by employing a single-level MILP [37].

The end-user optimization problem described in Section 3.2 is limited to one single type. It is clear that not all users respond to a price change in the same way because that their gas price elasticity and demand during the optimization horizon are different [38]. Therefore, we extend the single type user model to a scenario with multiple types of agents: residential users, commercial users and industrial users.

Let P_{rt}, P_{ct} and P_{it} represent the TOU prices for residential, commercial, and industrial user, respectively, and D_{rt}^0, D_{ct}^0, and D_{it}^0 are the residential, commercial, and industrial user nominal demands. Though all the users' nominal demands and TOU prices are different, the demand response depends on their gas price elasticity according to their objective function types in the form of Eq. (12). Thus, according to the classical optimization rules, their optimal demands responding to prices are similar with the single user type in Eq. (19), and can be donated as follows:

$$D_{rt} = D_{rt}^0 \left(\frac{P_{rt}}{P_{rt}^0}\right)^{\xi_{rt}} \quad (22)$$

$$D_{ct} = D_{ct}^0 \left(\frac{P_{ct}}{P_{ct}^0}\right)^{\xi_{ct}} \quad (23)$$

$$D_{it} = D_{it}^0 \left(\frac{P_{it}}{P_{it}^0}\right)^{\xi_{it}} \quad (24)$$

It should be noted that the optimal demand response is not always in the above form when a different utility function is chosen basing on user characteristics.

After obtaining the optimal response of different users to gas TOU prices, we maximize the objective function of operator by finding the optimal TOU prices for the three types of users. The operator's objective function is

$$\hat{\phi}(P_t, D_t) = \arg\max_P \sum_{t=1}^{T} \Big[(P_{rt} D_{rt} - P_{rf} D_{rt}^0) + (P_{ct} D_{ct} - P_{cf} D_{ct}^0) + (P_{it} D_{it} - P_{if} D_{it}^0) \Big] - P_c^\uparrow D^\uparrow - P_c^\downarrow D^\downarrow - F(D_t) + F(D_t^0)$$

The total user gas demand D_t is the sum of the loads of all three types of users. The cost $F(D_t)$ caused by the fluctuation of total user demand could be measured as:

$$F(D_t) = \mu \sum_{t=1}^{T} \sqrt{\left[(D_{rt} + D_{ct} + D_{it}) - (\overline{D}_{rt} + \overline{D}_{ct} + \overline{D}_{it})\right]^2} \quad (26)$$

As all users have participated in demand response in an optimization horizon, all the user gas demands D_{rt}, D_{ct}, and D_{it} are regarded as known variables in Eq. (25). The operator optimization problem is linear, and the decision variables P_{rt}, P_{ct} and P_{it} could be solved by linear programming.

4. Numerical results and discussion

In this section, we analyze the numerical results obtained by running the model on a test-case using data from a middle-scale UGPN market in the City of Wuhan, China. Wuhan, the capital of Hubei province, is located in the national geographic center of China, and is an important financial, cultural, and transportation center. The current total population of Wuhan is about 10 million. The main energy products consumed here include coal, oil, electricity, natural gas, and heat. The annual natural gas consumption has exceeded one billion cubic meters, accounting for 6% of the total energy consumption [39]. At present, 4 million cubic meter gas is consumed daily. The residential, commercial, and industrial users consume about 25%, 30%, and 45% of the total gas, respectively.

4.1. Data and assumptions

In this numerical example, we adopt a time-block pricing by dividing a day into peak period, semi-peak period and off-peak period with three price levels and an hourly-based pricing by dividing a day into equal time periods. Fig. 2 presents the different types of users' load fluctuation and their nominal demands during a typical day according to the statistical data of the local gas distribution company. The time-block period is divided by the total user gas nominal demand. The peak periods refer to the periods between 08:00 and 13:00 and between 17:00 and 20:00. The semi-peak periods refer to the period between 06:00 and 08:00, between 13:00 and 17:00, and between 20:00 and 22:00. The off-peak period refers to the period between 22:00 and 06:00.

Before running the model, a number of input variables need to set fact-oriented. Hereon, all the input value of price and cost is from the local gas distribution company: P_{rf} = 2.53, P_{cf} = 4.085, P_{if} = 3.41, P_c = 2.62, $P_c^{\uparrow}$ = 3.10, $P_c^{\downarrow}$ = 2.62, and the unit is in Yuan/m³. As for the cost coefficient of gas demand fluctuation, the value is selected to be μ = 1.0. In addition to these data, it seems that the price elasticity ξ is the key to estimating the users' demand response in the model proposed above. However, we could not estimate the short-term price elasticity of all periods in a day precisely because of the limited data. So analyzing the gas consumption characteristic of different types of users is particularly needed. In general, among these three types user, the commercial user demand is most

inelastic, the industrial is most elastic, and the residential is in the middle. Thus, the overall price elasticity should be satisfied with $\xi_i < \xi_r < \xi_c < 0$ [33]. As for short-term gas price elasticity, it mainly depends on the gas consumption and the hours of consumption. Given that gas user consumption characteristics are similar to those of electric power consumers, we first compare the consumption features between gas and electricity, and then select the short-term gas price elasticity not only referring the existed electricity price elasticity [23], but also consulting some studies on natural gas price elasticity [40–42]. The recent work of [42] concludes that the gas price elasticity of urban resident was −0.28, the commercial user was −0.17, and the industrial user was −0.54. Hereon, we assume that the short-term gas price elasticity has a limited fluctuation actually within the means observed from [42]. For residential users, it is set $-0.53 \leqslant \xi_r \leqslant -0.15$ with mean −0.28. For commercial users, it is set $-0.32 \leqslant \xi_c \leqslant -0.11$ with mean −0.17. For industrial users, it is set $-0.65 \leqslant \xi_i \leqslant -0.33$ with mean −0.54. The three types of user gas price elasticity are final set as shown in Fig. 3.

4.2. Numerical results under regulation scenarios

Using data described above, we employ the game-theoretic model to find the optimal TOU pricing strategy in a regulated urban gas market. Under regulating, the TOU price should be no more than the limitation set by government agent. Thus, there is a single regulation price $P^{down} \leqslant P_t \leqslant P^{up}$ associated with each TOU price optimization. Because the user optimal demand response has been obtained by backward induction, the optimization of TOU price could be solved by linear programming with the fmincon function in Matlab® optimization toolbox. Through this approach, we not only obtain the optimal TOU block prices and TOU hourly prices of different types of end-user, but also gain the user optimal demand response. Considering the optimal results, we discuss how TOU prices imposed by the operator influence urban terminal gas consumption users and user behavior.

According to the gas wellhead price, urban gate station price, and other relevant energy prices in the market, the up and down regulations of price bounds are set as P^{down} = 1.7, P^{up} = 5.0. In order to compare the TOU block price and the TOU hourly price, the model is run twice on the same dataset. In the first run the price charged by the operator is set to be three levels corresponding to the time-block periods. In the second run the price is changed
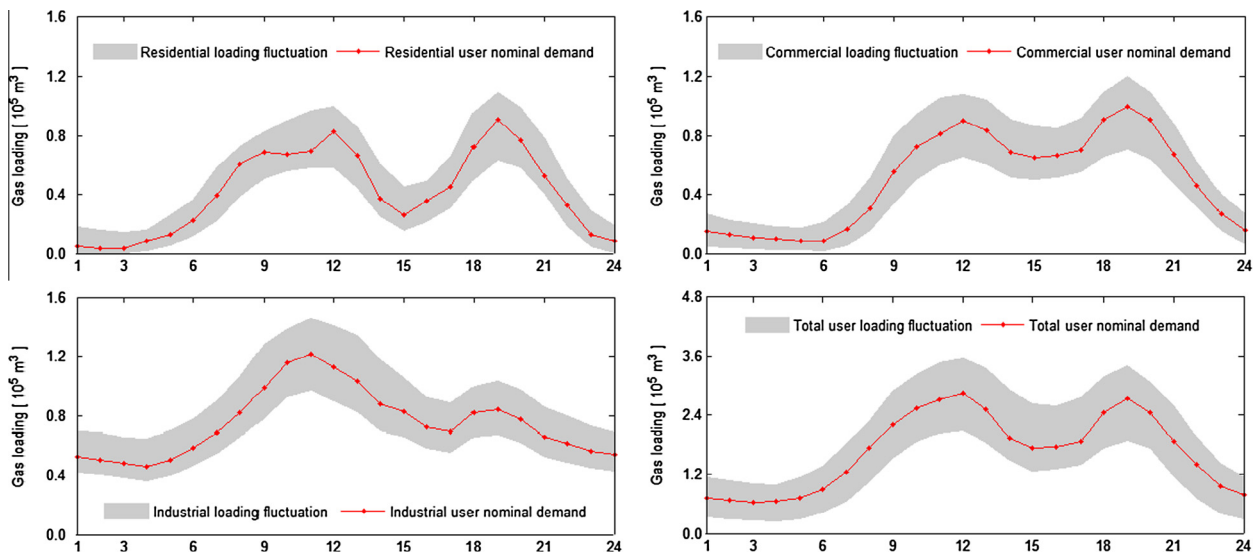

Fig. 2. Different type of users' fluctuation and their nominal gas demand in a typical day.
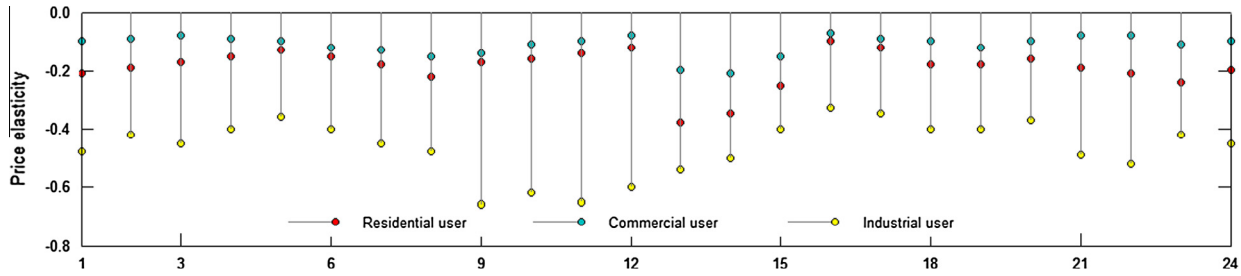
Fig. 3. Price elasticity for different types of users used in the numerical examples.

hourly, but the average TOU price should be equal to the fixed-price $P_{rf} = 2.53$, $P_{cf} = 4.085$, $P_{if} = 3.41$. All the three types of user optimal TOU prices and demand responses are shown in Figs. 4 and 5, respectively.

In the fixed-price case, there is no economic incentive for the users to modify their consumption schedules according to the price signal sent by the operator. In the optimal TOU block price case, the gas prices are considerably different in different time-block periods. It indicates the end-users prefer to allocate their flexible demands in off-peak hours because of the low prices, especially for industrial user. However, as the flexible demands of residential and commercial users are relatively low and not sensitive to price, their gas consumption change during the off-peak period is not particularly apparent. In the TOU hourly price case, the users will adapt to the price signal with dynamic load balancing. Since the commercial user's optimal TOU hourly price is located at the top and bottom of the regulation levels and the demand of commercial user is high, only a small part of the less time-urgent work can be

scheduled to other periods of day. So the commercial demand response is lower than that of other two types of users.

Compared with the fixed price throughout the day, all end-users will participate in demand response and reschedule their gas consumption to a certain degree under the TOU block price and the TOU hourly price scheme. Table 1 shows the load response results of all three types of end-users. It indicates that the total gas consumptions of all users have decreased, and the load response intensity is in a descending order as: industrial user, residential user and commercial user. It seems that TOU hourly pricing is better at reducing the maximum load and increasing the minimum load than the TOU block pricing, while the TOU block pricing will be more conducive to reduce the peak period load and increase the off-peak load than the TOU hourly pricing.

Fig. 6 presents the main results for operator in the simulations with fixed, TOU block and TOU hourly prices. All the average prices are the arithmetic mean of the prices of three end-users. Results show that the original fixed pricing has failed to reflect the
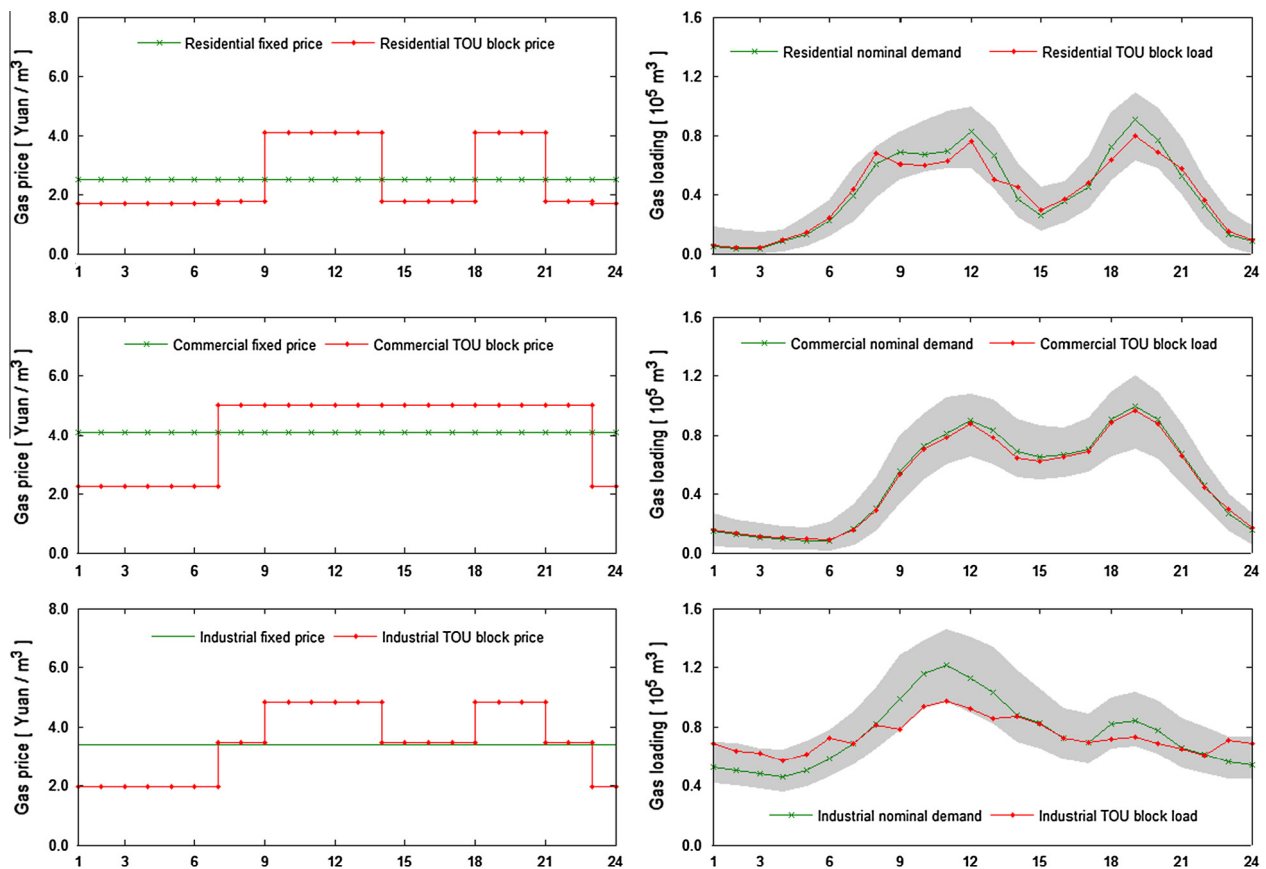

Fig. 4. Optimal TOU block price and response load for the end-users types with price regulation.
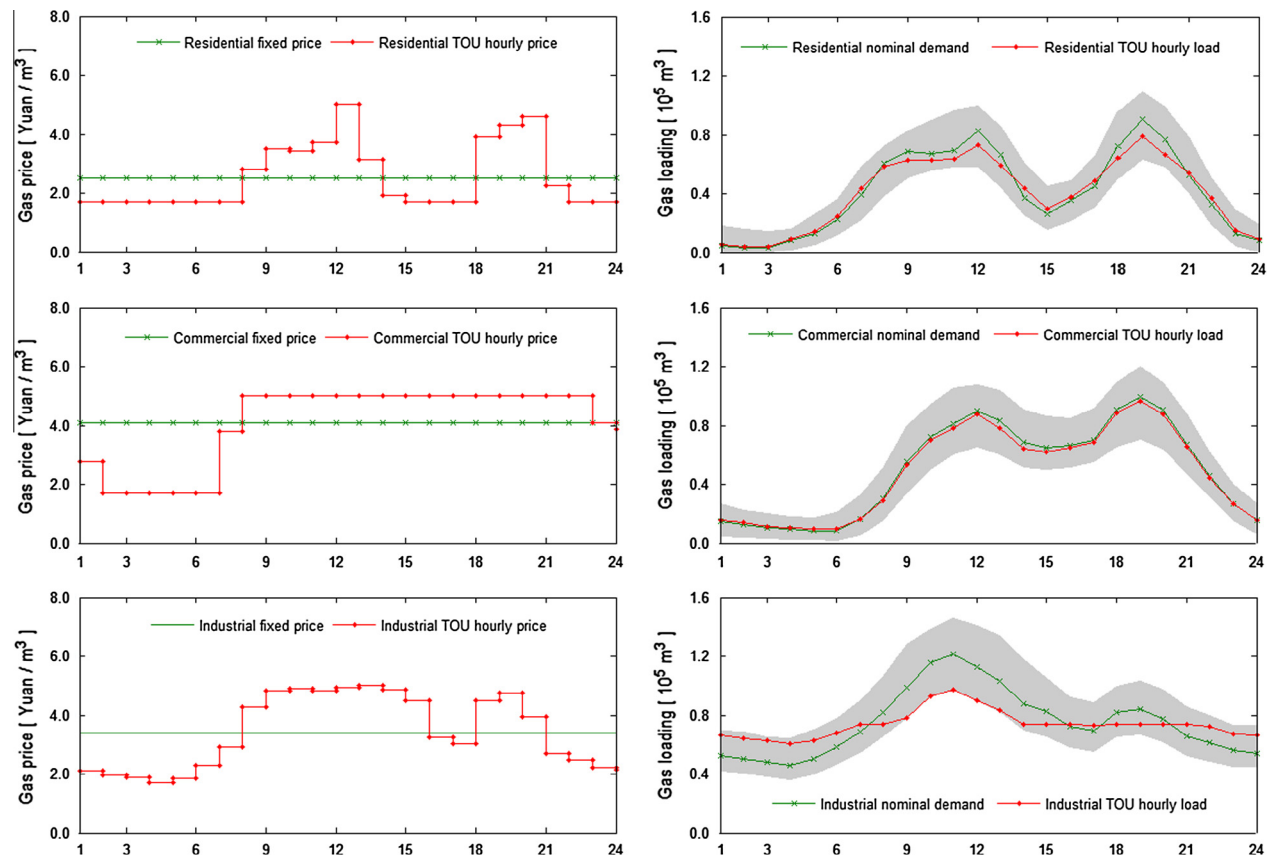
Fig. 5. Optimal TOU hourly price and response load for the end-users types with price regulation.

Table 1
User gas load response for regulation TOU block and TOU hourly price relative to original fixed price.

| User types	Gas load response for TOU block price (%)						Gas load response for TOU hourly price (%)					
	Total	Max	Min	Peak	Semi-peak	Off-peak	Total	Max	Min	Peak	Semi-peak	Off-peak
Residential user	−2.81	−12.28	12.24	−12.40	11.28	11.10	−3.06	−13.53	12.24	−10.69	7.41	11.10
Commercial user	−2.36	−4.57	11.28	−3.50	−3.58	9.48	−2.50	−4.57	17.09	−3.50	−2.38	7.22
Industrial user	−1.67	−20.00	24.66	−17.03	−0.61	26.25	−1.86	−20.00	32.00	−16.64	−0.61	24.63
Total end-users	−2.16	−10.54	20.21	−11.32	1.34	21.30	−2.35	−12.01	26.02	−10.67	0.46	19.77
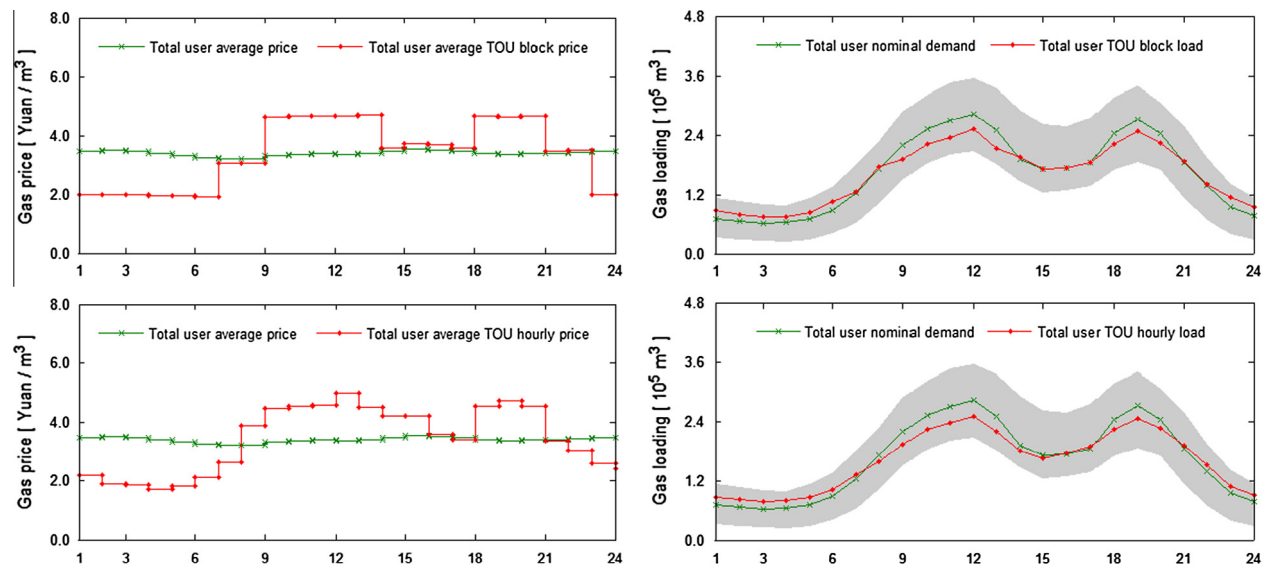
Fig. 6. Average TOU price and response load for the total end-users with price regulation.

variation of demand and the TOU block pricing just cloud reflect the peak and off-peak period of demand. Only the TOU hourly pricing has been consistent with the demand fluctuation tightly, which means the TOU hourly pricing is better to reflect the variation of gas marginal cost in a day. In terms of peak load shifting, both TOU block price and TOU hourly price cannot bring load shifting. So the operator is suggested to use the TOU block or hourly pricing to replace the fixed pricing.

As both the gas price and demand are changed, the end-user utility and operator benefit will change concurrently. The end-user utility and operator profit change results have been shown in Table 2. Table 2 shows that the end-user utility has been reduced, and the operator profit has been grown considerably. We calculate the total social welfare by adding the operator profit with the user utility. It is observed that TOU hourly pricing achieves the higher social welfare than TOU block pricing.

Since the transfer of money from end-users to operator cancels out in the social welfare calculation, we can interpret an increase of social welfare as the result of the reduction of the load fluctuation cost for the operator in the TOU pricing. So the increasing in operator profit can be interpreted as a proxy for the reduction of rescheduling cost. On the other hand, the increase in users' payments to the operator implies that the redistribution of this additional welfare between the agents might not be fair under this leader–follower games configuration.

4.3. Numerical results with deregulation scenarios

In Section 4.2, we have analyzed the optimal results under price regulation scenarios. Now we consider how different levels of TOU price influence users' demand response without the up and down price regulation in an open market environment. This is done by carrying out two additional game-theoretic models without the price regulation, and the optimal TOU block and hourly pricings and demand response are shown in Figs. 7 and 8.

Compared with the regulation scenarios, the optimal TOU pricing without regulation leads to more dramatic deviation than fixed pricing, especially in the peak and off-peak periods for commercial and residential users. It is indicated that the price is significantly dependent on demand in this open market environment, although commercial and residential users have less elastic demand in the peak and off-peak periods. As long as gas demand does not exceed the limited supply, the operator would rather cut-price in the off-peak period and increase it in the peak period to attract them to allocate more demand in a day. Table 3 illustrates the user gas demand response of deregulation TOU block and hourly pricing relative to the original fixed price. Table 4 compares the users' gas demand response changes relative to the regulation scenarios.

We can observe that the minimum load and off-peak period load have increased significantly, especially for commercial and residential users. This may be due to the lower prices in these time periods. As for the strong price elasticity of industrial users, the gas demand has already reached its maximum and minimum in the off-peak and peak periods, respectively. Thus, there is no need to reduce the price further. Both the TOU price and demand responses are not obviously changed compared to the regulation scenarios. It can be interpreted as the optimal gas price in an open market

environment mainly decided by demand intensity and demand elasticity. To lessen the gas demand fluctuations and realize the peak load shifting, the principle 'large amount of high price' is a very practical application for scarce resources.

Fig. 9 shows the average price and load change in hourly level. It indicates the TOU pricing without regulation also has a significant effect on peak load shifting, and the TOU hourly price is more effective. We calculate the user utility and operator profit basing on the optimal TOU price and demand response (Table 5).

Under deregulation, all the gas users' utility reduced because the amount of TOU price increases is bigger in the peak period than the amount of price reduction in off-peak period. Thus, although the user has participated in demand response, the cost of gas consumption is significant increased as the absolute demand is restricting. For the operator, the profit has improved since the users' payment is increased. On the other hand, a proxy for peak load shifting has reduced the cost caused by load fluctuation. For the total social welfare, it is also improved without regulation. However, the total social welfare has decreased compared with regulation scenarios. It is mainly resulted from the extravagant price without regulation, and the users' payments for the absolute part of the demand are highest.

4.4. Advantages of TOU gas pricing

Compared with the original fixed gas price, the TOU gas pricing has profound implications for the UGPN. Basing on the previous numerical results, the advantages of TOU gas pricing are summarized as following:

In terms of gas pricing, the original fixed price could not reflect the variation of marginal cost of scarce resources. In previous numerical, all users' fixed price remain unchanged during an optimum time horizon, while the TOU prices change greatly with the variation of load to reflect the changing gas operation cost. With the improvement of the urban gas infrastructure, the gas sources and prices become diversification. Fixed pricing commonly causes inefficiencies in an open market environment. However, the TOU pricing plays a leverage to balance the contradiction between gas supply and demand, which could be a more efficient pricing strategy for public resource sector like natural gas.

In terms of gas demand, when the prices are set to be the same throughout the day, users will consume the gas when they need. It causes waste and form the peak and off-peak periods. Just like the users' minimal demand in the previous numerical analysis, the rate of maximum load to minimum load has reached to 4.6. After implementing the TOU gas pricing, this rate dropped to 2.74, and the maximum peak–valley load difference decreased 32%. This indicates that TOU gas pricing is an effectively pricing strategy to realize the peak load shifting. Meanwhile, the TOU gas pricing is also an effective means for curbing natural gas waste. Compared with the fixed price, the user total gas demand is falling by 2% with TOU pricing. The reduction could be regarded as unnecessary demand, which is of positive significance under the natural gas shortage environment in China.

In terms of social welfare, the total social welfare with TOU pricing is higher than the fixed price. Higher social welfare is preferable as it indicates higher market efficiency. So the TOU

Table 2
User utility and operator benefit change for the regulation TOU block and hourly price relative to original fixed price.

Regulation Scenarios	TOU block price under regulation (10^5 Yuan)					TOU hourly pricing under regulation (10^5 Yuan)				
	Residential	Commercial	Industrial	Operator	SW	Residential	Commercial	Industrial	Operator	SW
Outcomes	−5.48	−7.77	−3.67	19.20	2.28	−5.78	−8.14	−4.07	20.57	2.58

Note: "SW" means the social welfare, which is the sum of the end-users utility and the operator profit.
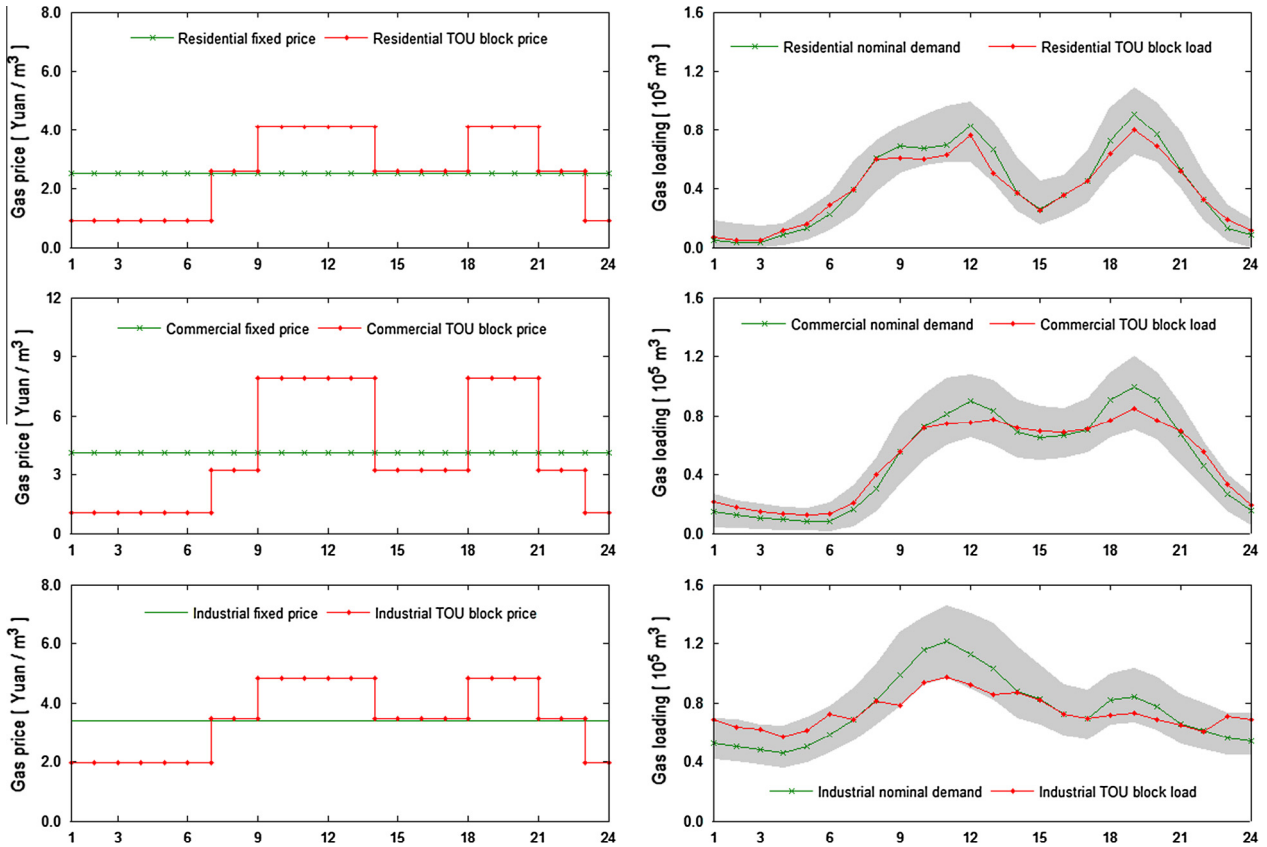
Fig. 7. Optimal TOU block price and response load for the end-users types with deregulation.
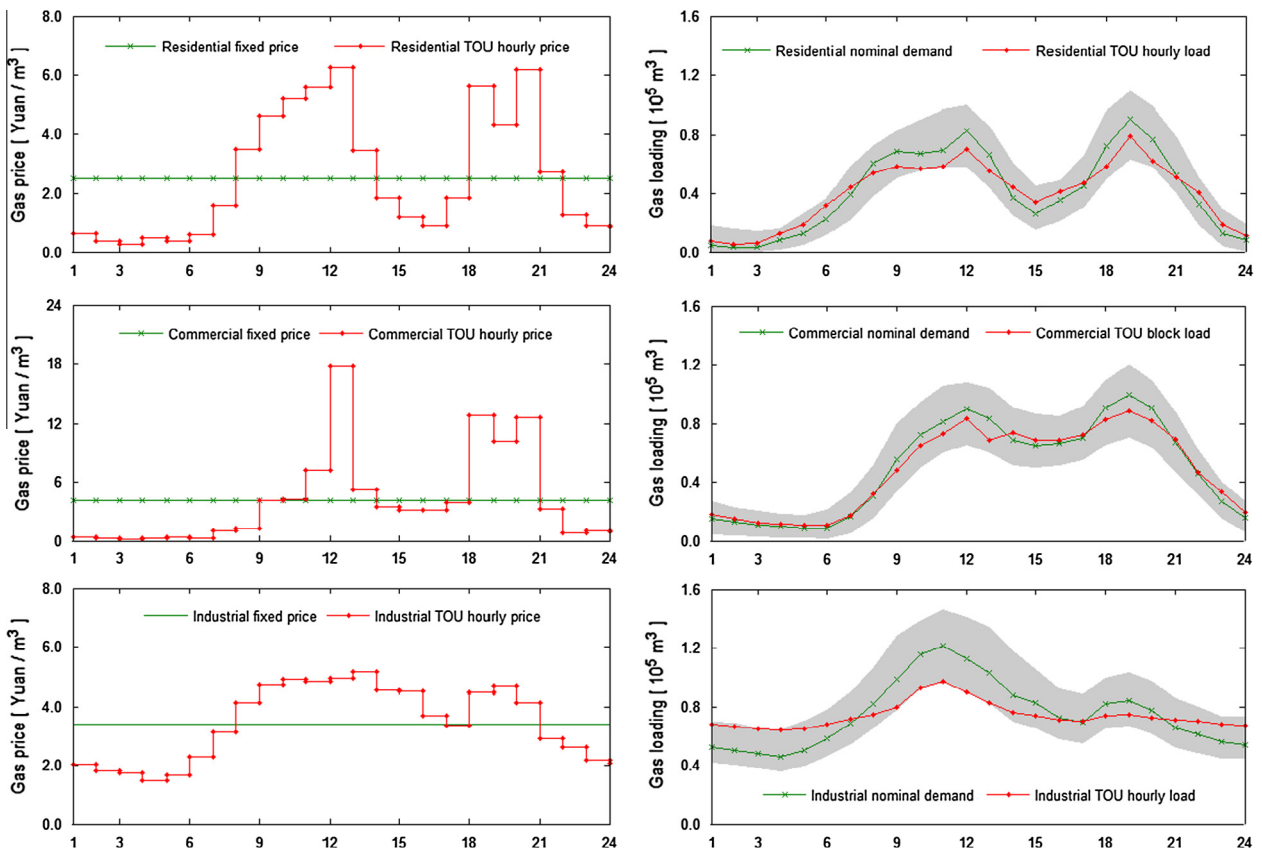

Fig. 8. Optimal TOU hourly price and response load for the end-users types with deregulation.
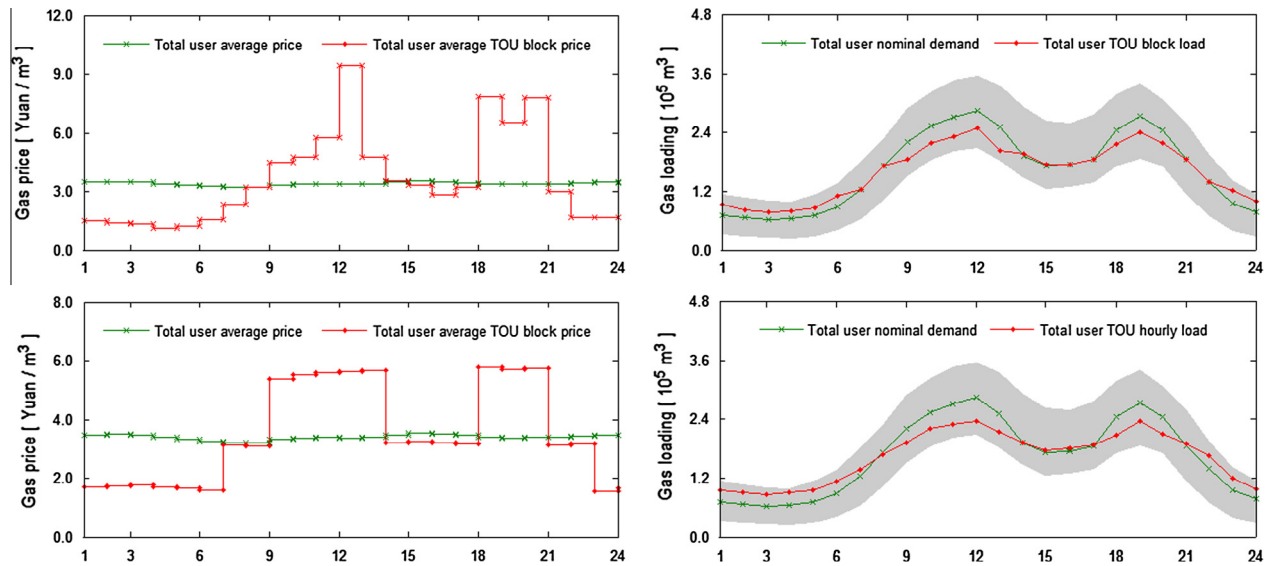
Table 3
User gas demand response for deregulation TOU block and hourly price relative to the original fixed price.

User types	Gas load response for TOU block price (%)						Gas load response for TOU hourly price (%)					
	Total	Max	Min	Peak	Semi-peak	Off-peak	Total	Max	Min	Peak	Semi-peak	Off-peak
Residential user	−5.12	−12.28	34.88	−12.40	−0.66	31.83	−3.02	−13.53	70.59	−16.08	9.01	46.20
Commercial user	−2.54	11.22	25.48	−10.97	4.30	22.17	0.50	−15.17	49.44	−10.73	8.40	38.24
Industrial user	−1.66	−20.00	24.66	−17.03	−0.61	26.26	−1.60	−20.00	32.42	−16.74	−2.01	27.94
Total end-users	−2.79	−12.18	26.45	−13.73	0.94	26.24	−1.33	−16.81	39.05	−14.61	3.40	32.14

Table 4
User gas demand response for deregulation TOU block and hourly price relative to the regulation scenarios.

User types	Gas load response for TOU block price (%)						Gas load response for TOU hourly price (%)					
	Total	Max	Min	Peak	Semi-peak	Off-peak	Total	Max	Min	Peak	Semi-peak	Off-peak
Residential user	−2.31	0.00	22.64	0.00	−11.94	20.73	0.04	0.00	58.35	−5.39	1.60	35.10
Commercial user	−0.18	15.79	14.20	−7.47	7.88	12.69	3.00	−10.60	32.35	−7.23	10.78	31.02
Industrial user	0.00	0.00	0.00	0.00	0.00	0.00	0.26	0.00	0.42	−0.10	−1.40	3.31
Total end-users	−0.63	−1.64	6.24	−2.41	−0.40	4.94	1.02	−4.80	13.03	−3.94	2.94	12.37

Fig. 9. Average TOU price and response load for the total end-users with deregulation.

Table 5
User utility and operator profit change for the regulation TOU block and hourly price relative to original fixed price.

TOU Pricing	TOU block price under deregulation (10⁵ Yuan)					TOU hourly price under deregulation (10⁵ Yuan)				
	Residential	Commercial	Industrial	Operator	SW	Residential	Commercial	Industrial	Operator	SW
Deregulation scenarios	−7.50	−16.65	−3.67	29.95	2.13	−10.61	−22.05	−4.09	38.09	1.34
Compared to regulation	−2.02	−8.88	0.00	10.75	−0.15	−4.83	−13.91	−0.02	17.52	−1.24

gas pricing could increase the market efficiency at some extent. Although the gas user utility has a certain decreased after implementing the TOU gas pricing, the operator's profit has increased, and the transfer of money from users to operator could be considered as an incentive for users to switch to TOU price. On the other hand, the increase in users' payments to the operator implies that the redistribution of this additional welfare between these agents might not be fair under this leader–follower configuration.

To conclude, TOU gas pricing is an effects strategy in current UGPN market environment. However, since the UGPN market of China is still under construction, the TOU prices will be beyond the users' affordability if introduced without price regulation. Thus, it is necessary to illustrate the TOU gas pricing under price regulation and without regulation.

5. Conclusions

This paper presents a game theoretical perspective of TOU pricing in an UGPN market. To explore the optimal TOU price and demand response, a single-level MILP is reformulated to solve the optimizing problem for the leader–follower structure of urban gas multi-agent system.

To verify the feasibility of the proposed model, we have illustrated it with an example by running the model on a small test-case based on real-world data. All the optimal results have indicated that TOU pricing schemes are validated to have significant potential for peak-shaving and load-shifting. In the case of load-shifting effectiveness, it seems that the TOU block prices is more conducive to reduce the peak period demand and increase the

off-peak demand, whereas the TOU hourly price is better at reducing the maximum load and increasing the minimum load throughout the day. To verify the regulation effect of urban gas market, we have run the proposed model both in regulation and deregulation market environment on the same dataset. The results indicate that the two kinds of optimal TOU pricing could reflect the running cost of demand fluctuation in some degree. Although the deregulating TOU pricing is better for peak-shaving and load-shifting, the total social welfare is lower than under TOU pricing in a regulated environment. It is mainly because of the deregulation that prices are tremendous changes in different periods. In peak periods, the prices will be over the end-user price tolerance, and in off-peak periods, the prices are far less than the average marginal cost, which is not conducive to the steady development of gas industry. Therefore, a reasonable price regulation level is essential for policy maker in the rapid development stage of urban gas market.

In the numerical example, because of urban gas market imperfection and data limitations, the user gas short-term price elasticity are referencing and replenishing from the electricity retail market. Although the gas users consuming behavior are similar with electrical users, there still exists some differences between the two groups, thus, these uncertain parameters should be carefully estimated based on historical data and possibly a survey of users.

Future extensions of this research could occur along several directions. Different utility functions to model the trade-off for the user between gas price and load response could be defined and simulated. For example, adopting the logarithmic and exponential structure of demand response functions, or establishing a satisfaction function of demand response. Furthermore, a different perspective obtaining the optimal solution to the leader–follower problem could be proposed so as to improve the controllability of the user demand from the operator perspective, such as hierarchical structure bilevel model. In addition, the gas generation could be introduced in the model, thus paving the way for an assessment of the value of demand response programs in the integration of energy conversion and complementation.

Acknowledgements

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