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Optimal placement of phasor measurement unit in distribution networks considering the changes in topology

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HIGHLIGHTS

- A generalized integer linear programming model of the PMU placement is proposed.
- Topology changes are considered to guarantee the observability with multiple topologies.
- The effects of existing measurements and their relevance are covered.

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ABSTRACT

Distribution networks usually have a meshed structure but are operated radially to improve the operating efficiency and ensure the power supply under an emergency situation. It is of great significance to guarantee the observability of the entire system under the topology changes in operation. This paper proposes an optimal placement method of phasor measurement unit (PMU) for distribution networks. A generalized binary integer linear programming (ILP) model for PMU placement is proposed. The changes in topology are considered to guarantee the observability under any possible operation mode. The existence of zero injection nodes (ZINs), the existing measurements, and their relevance are also covered in the proposed method to reduce the installed PMU number. The objective of the ILP problem is weighted by the degree of the corresponding node to obtain a scheme with higher measurement redundancy. The correctness and effectiveness of the proposed method are verified via case studies on the IEEE 33-node test feeder, PG&E 69-node test feeder, and a medium voltage distribution network in Southern China.

1. Introduction

With the development of phasor measurement units (PMUs) that are capable of acquiring synchronized high-precision phasor measurements of bus voltage and branch current, the conventional supervisory control and data acquisition (SCADA) system of power transmission system is evolving into a wide-area measurement system (WAMS) [1,2]. Meanwhile, the real-time monitoring performance of power distribution system is also improved with the configuration of micro-phasor measurement unit (μPMUs), which are progressively becoming less expensive and smaller [3,4]. The applications of μPMUs in distribution networks, such as state estimation [5], parameter and topology identification [6,7], and fault location detection [8] are showing promising prospects and provide data support for operational control [9,10] and energy management [11,12].

Compared with conventional measurements, the higher synchronism and accuracy of PMUs are achieved at a higher cost. Therefore, configuring PMUs in an economical and reasonable manner to meet the requirements on system observability is one of the focuses of recent researches [13].

With the integration of distributed energy resources, the complete observability of a network, that is, the ability to uniquely determine all the state variables of the system, is the basis of operation control and energy management for both transmission system and distribution system [14]. The objective of optimal PMU placement is to make the power system completely observable with the minimum number of PMUs [15]. Deterministic methods and meta-heuristics are two main methods to address such a problem. For deterministic methods, the configuration scheme of the PMUs is obtained directly from solving a programming problem in which the observability constraints are

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Nomenclature

Abbreviations

PMU	phasor measurement unit
SCADA	supervisory control and data acquisition
WAMS	wide-area measurement system
μPMU	micro-phasor measurement unit
ILP	integer linear programming
ZIN	zero injection node
CM	conventional measurement
IM	injection measurement
PFM	power flow measurement
IRN	indirectly related nodes
DRN	directly related nodes

Sets

Ω	set of all nodes
Ω_i	set of node i and its incident nodes
Ω_0	set of zero injection nodes
Ω_S	set of nodes of the sub-network
Ω_P	set of nodes configured with a PMU
Λ_U	set of branches with unknown parameters
Λ_L	set of branches with power flow measurements
Λ_W	set of branches with switches

Indices

i, j, m, n, k	indices of nodes

Variables

$\boldsymbol{f}$	vector of the observability degree
$\boldsymbol{g}$	vector of the revised observability degree
$\hat{\boldsymbol{f}}$	binary vector of the observability
$\hat{\boldsymbol{g}}$	binary auxiliary vector of $\boldsymbol{g}$
$\boldsymbol{x}$	vector of decision variables

Parameters

d_i	degree of node i in the distribution network
$d_{S,i}$	degree of node i in the sub-network
$\mathbf{A}$	adjacent matrix of the distribution network
$\mathbf{A}_T$	connectivity matrix weighted by the degree of the sub-network
$\boldsymbol{y}$	vector of the observability constraints
$\mathbf{1}$	vector with arrays equal to 1
$l_{i,j}$	branch that connects nodes i and j
$M_{0,j}$	number of branches related to zero injection node j

modeled as the optimization problem [16]. Meta-heuristics are mainly applied to conditions that are difficult to model or make the problem difficult to be solved, and they search for the scheme under certain heuristic rules, which are suitable for a larger system but are difficult to ensure the optimality [17]. Therefore, a simple and accurate mathematical model for PMU placement problem is of great importance to improve the efficiency of solving the problem [18].

An Integer linear programming (ILP) model of PMU placement was built in [19] using the adjacent matrix of the network. Multi-stage optimization and multi-objective model were used to select better feasible solutions respectively in [20] and [21], whereas criteria from the concept of graph theory were used in [22] to obtain a more redundant scheme by iteratively installing the PMUs at a high-priority location until the observability requirement was met.

The required number of PMUs can be reduced with the consideration of zero injection node (ZINs) and conventional measurements (CMs). A generalized ILP formulation was established in [23,24] considering ZINs, injection measurements (IMs) and power flow measurements (PFMs). However, their observable constraints were not sufficient under certain conditions [25]. Auxiliary variables were introduced to consider ZINs in the optimization model [26], but conventional measurements and the relevance of ZINs were neglected.

All of the studies above are for a transmission system. For distribution networks, an optimal PMU placement method for anomaly detection was proposed in [27]. The optimal PMU configuration model of the transmission system was applied to some typical distribution networks in [28], and a global search algorithm was used to solve the problem. The configuration method was further studied under a contingency condition in [29]. Differing from transmission networks, distribution networks usually have a meshed structure but operate radially to increase the operating efficiency and ensure the power supply in emergency situations [30,31]. In particular, with the high penetration of renewable energy resources [32,33], energy storage [34], and demand response resources [35], distribution networks are facing more frequent topology changes under both normal and fault operation conditions [36]. Therefore, PMU configuration schemes for distribution networks should guarantee the observability of the entire system under multiple operation topologies [37,38].

This paper proposes an optimal PMU placement method for distribution networks. The main contributions are summarized as follows:

1) A generalized binary ILP algorithm is proposed for the optimal PMU placement of distribution networks. By weighting the objective with node degrees, better solutions with a higher level of redundancy are obtained. Topology changes and unknown parameters of the branch are considered in the ILP based PMU placement optimization model to accommodate the characteristics in the network parameter of the distribution network. The observability of the distribution network is guaranteed with any possible operation topologies.
2) The effects of ZINs, existing measurements, and their relevance are covered in the ILP problem. Auxiliary binary variables are introduced to address the problem of constraint insufficiency faced by the existing methods. The number of PMUs required to achieve complete observability of the distribution network is reduced. The relevance of the existing measurements with topology changes and unknown line parameters is also considered to expand the applicability of the proposed method for distribution networks with complex conditions.

The remainder of this paper is organized as follows. Section 2 presents the ILP model of optimal PMU placement. The topology changes and unknown parameters of the branch are further considered in Section 3. ZINs and existing measurements are concerned in Section 4. The relevance of the existence of ZINs and measurements with model parameters are exploited in Section 5. Case studies are presented in Section 6, followed by the conclusion in Section 7.

2. Basic model of optimal PMU placement

With a sufficient number of available channels, a PMU is able to measure the phase angle and voltage magnitude of its installed node and the currents of its incident branch simultaneously. Therefore, with available line parameters, if a node is configured with a PMU, its neighboring nodes are also observable based on Ohm's law [17]. The objective of placing PMUs is to make the whole network observable with minimum economic cost.

2.1. Constraints

Assume a 6-node test distribution network, as shown in Fig. 1. The PMU at node 2 is able to make nodes 1, 2, 3, and 6 observable.

The entries of $\mathbf{A}$ and $\boldsymbol{x}$ can be obtained as shown in (1) and (2), respectively.

$$A_{i,j} = \begin{cases} 1 & i = j \\ 1 & i \neq j, j \in \Omega_i \\ 0 & i \neq j, j \notin \Omega_i \end{cases} \tag{1}$$

$$x_i = \begin{cases} 1 & i \in \Omega_P \\ 0 & i \notin \Omega_P \end{cases} \tag{2}$$

$\boldsymbol{f}$ is then calculated as follow:

$$\boldsymbol{f} = \mathbf{A}\boldsymbol{x} \tag{3}$$

When $\boldsymbol{f} \geq \mathbf{1}$, the distribution network is observable.

2.2. Objective

The objective of PMU placement is mainly to minimize the cost of configuration, which is identical to minimizing the number of PMUs when all of the PMUs are of the same price. Then, the objective function can be expressed as follows:

$$\min \sum_{i \in \Omega} x_i \tag{4}$$

2.3. Further consideration of the configuration scheme

Multiple solutions may be obtained by solving the above optimization problem. This paper aims to acquire more redundant configuration schemes with the same number of PMUs. For the 6-node distribution network in Fig. 1, two PMUs installed at nodes 2 and 4 or at nodes 2 and 5 are both able to make the network completely observable. Seven pairs of measurements are obtained for the former scheme, while only 6 pairs of measurements are obtained for the latter one. Node 4 has two incident branches, but node 5 only has one. d_i denotes the number of branches incident to node i, namely the degree of node i, which can be calculated based on the adjacent matrix as follows:

$$d_i = \sum_{j=1}^{N} A_{i,j} - 1 \tag{5}$$

The objective function is weighted by the degree of the corresponding node to choose a more redundant solution with the same number of PMUs. When setting M as a number much larger than the maximal degree of the network, the revised objective function is shown in (6).

$$\min \sum_{i \in \Omega} \left(1 - \frac{d_i}{M}\right) x_i \tag{6}$$

As shown in (6), nodes with higher degrees have a relatively small weight, which makes the PMUs more apt to be installed at these nodes to acquire more measurements with the same number of PMUs.

3. Topology changes and unknown parameters of branches

3.1. Topology changes

The topology of distribution network changes after reconfiguration but still operates in radial. The PMU placement scheme should guarantee complete observability under various operation topologies.

A tie switch is added to the distribution network in Fig. 1 to connect nodes 5 and 6, as shown by the dotted line in Fig. 2. Other branches are all with section switches. Node 4 is observable after installing a PMU at either node 3 or node 5 without the tie switch. However, node 4 is no longer observable when installing only one PMU at the two nodes due to the disconnection of branches $l_{3,4}$ and $l_{4,5}$ in operation. If PMUs are installed at nodes 3 and 5, node 4 is observable in any topology, as the switches on branches $l_{3,4}$ and $l_{4,5}$ will not open simultaneously to ensure the connectivity of the network. Therefore, node 4 is observable if a PMU is installed on node 4 or two PMUs are installed on nodes 3 and 5.

A sub-network, as shown in Fig. 3, is obtained by removing the nodes and branches that are not in the loop. Thus, the observability degrees for the nodes on the sub-network must be larger than their degrees of the sub-network to realize observability from the incident nodes.

To model the PMU placement problem of distribution networks, the observability constraint (3) is revised as shown in (7).

$$\boldsymbol{f} = \mathbf{A}_T\boldsymbol{x} \geq \boldsymbol{y} \tag{7}$$

$\boldsymbol{y}$ is obtained as follows:

$$y_i = \begin{cases} d_{S,i} & i \in \Omega_S \\ 1 & i \notin \Omega_S \end{cases} \tag{8}$$

The diagonal elements of $\mathbf{A}_T$ related to the sub-network are obtained as follows:

$$A_{T,i,i} = d_{S,i},\ i \in \Omega_S \tag{9}$$

As shown in Fig. 2, node 2 is always observable if a PMU is installed at node 1. Therefore, if node i is on the sub-network and its incident node j is not, $A_{T,i,j}$ is calculated as follows:

$$A_{T,i,j} = d_{S,i},\ i \in \Omega_S, j \in \Omega_i, j \notin \Omega_S \tag{10}$$

If nodes i and j are connected via the branch on the sub-network but branch $l_{i,j}$ does not have a switch, then the value of $A_{T,i,j}$ is as follows:

$$A_{T,i,j} = d_{S,i},\ i \in \Omega_S, j \in \Omega_S, l_{i,j} \notin \Lambda_W \tag{11}$$

$$A_{T,j,i} = d_{S,j},\ i \in \Omega_S, j \in \Omega_S, l_{i,j} \notin \Lambda_W \tag{12}$$

The values of other elements of matrix $\mathbf{A}_T$ are obtained as follows:

$$A_{T,i,j} = \begin{cases} 1 & i = j \\ 1 & i \neq j, j \in \Omega_i \\ 0 & i \neq j, j \notin \Omega_i \end{cases} \tag{13}$$

3.2. Branch with unknown parameters

The nodes that are incident to the PMU nodes are no longer observable when the line parameters of the corresponding branches are not available. Suppose that the parameters of $l_{1,2}$ in Fig. 2 are unknown. Then, a PMU at node 1 is unable to make node 2 observable and vice versa. Therefore,

$$A_{T,i,j} = A_{T,j,i} = 0,\ l_{i,j} \in \Lambda_U \tag{14}$$

4. Existence of ZINs and measurements

The number of PMUs required for complete observability can be reduced via considering the existing ZINs and CMs. The existing PMU measurements, ZINs, PFMs and IMs are all considered in this paper.

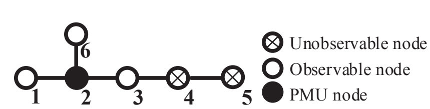

Fig. 1. Six-node distribution network.

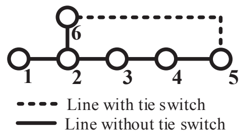

Fig. 2. Six-node distribution network with a tie switch.

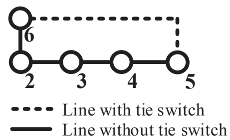

Fig. 3. Sub-network composed of the nodes and branches in loops.

4.1. Effect of existing PMU measurements

The decision variable of the related node should be set as 1 if a PMU is already installed at this node.

4.2. Effect of existing ZINs

As proposed in [21] and [22], the observability degree constraint of nodes i, j, k, and m is expressed as (15) if nodes j, k, and m are incident to ZIN i.

$$f_i + f_j + f_k + f_m \geq 3 \tag{15}$$

Eq. (15) is assumed to represent the situation that when any three nodes of i, j, k, and m are observable, the other node is also observable. However, feasible solutions that satisfy the observability constraints but cannot make the network observable may exist when the observability degrees of certain nodes in i, j, k, and m are larger than 1. As shown in Fig. 4, two PMUs are installed at nodes 3 and 4. If node 2 is a ZIN, then the observability constraint of nodes 1, 2, 3, and 6 is expressed as shown in (16).

$$f_1 + f_2 + f_3 + f_6 = 0 + 1 + 2 + 0 = 3 \geq 3 \tag{16}$$

Although the observability constraint (14) is satisfied, node 1 and node 6 are still unobservable as shown in Fig. 4 with dashed nodes.

When there exist nodes with observability degree larger than 1, the sufficiency of constraint (15) is no more guaranteed. To address this problem, an auxiliary vector $\hat{\boldsymbol{f}}$ composed of binary variables is introduced to represent the observability of each node as follows:

$$\hat{f}_i = \begin{cases} 1 & f_i \geq 1 \\ 0 & f_i = 0 \end{cases} \tag{17}$$

Then, the observability constraint (14) is revised as shown in (18).

$$\hat{f}_i + \hat{f}_j + \hat{f}_k + \hat{f}_m \geq 3 \tag{18}$$

The sufficiency of the corresponding constraints is ensured by introducing the auxiliary variable. Constraint (16) is changed to (19), and the PMU configuration scheme above no longer satisfies the constraint.

$$\hat{f}_1 + \hat{f}_2 + \hat{f}_3 + \hat{f}_6 = 0 + 1 + 1 + 0 = 2 < 3 \tag{19}$$

Therefore, if node i is a ZIN, the previous constraint in (20) is changed to (21).

$$f_j \geq 1, j \in \Omega_i, i \in \Omega_0 \tag{20}$$

$$\sum_{j \in \Omega_i} \hat{f}_j \geq d_i, i \in \Omega_0 \tag{21}$$

(1) Indirectly related ZINs

If there is a node that is not a ZIN but that is incident to the ZINs, then these ZINs are called indirectly related nodes (IRNs).

As shown in Fig. 5, if nodes 2 and 4 are both ZINs and nodes 1, 2, 5 and 6 are observable, the observability constraints are shown in (22) and (23) without considering the relevance of the ZINs.

$$\hat{f}_3 + \hat{f}_4 + \hat{f}_5 = 0 + 0 + 1 = 1 < 2 \tag{22}$$

$$\hat{f}_1 + \hat{f}_2 + \hat{f}_3 + \hat{f}_6 = 1 + 1 + 0 + 1 = 3 \geq 3 \tag{23}$$

As shown in (22) and (23), node 4 is not observable, as constraint (22) is not satisfied. However, according to (23), node 3 is already observable, and constraint (22) is then satisfied. Thus, node 4 is observable. The distribution network is observable when 4 of the total 6 nodes are observable. The network is unobservable when nodes 1, 2, 3, and 6 are observable. Therefore, at least one of nodes 4 and 5 should be observable. Similarly, at least two of nodes 1, 2, and 6 should be observable. Above all, if nodes m and n are IRNs, then the corresponding observability constraints are as follows:

$$\sum_{j \in \Omega_m \cup \Omega_n} \hat{f}_j \geq d_m + d_n - 1 \tag{24}$$

$$\sum_{j \in \Omega_m, j \notin \Omega_m \cap \Omega_n} \hat{f}_j \geq d_m - 1 \tag{25}$$

$$\sum_{j \in \Omega_n, j \notin \Omega_m \cap \Omega_n} \hat{f}_j \geq d_n - 1 \tag{26}$$

$$m \in \Omega_0,\ n \in \Omega_0,\ |\Omega_m \cap \Omega_n| \neq 0,\ m \notin \Omega_n \tag{27}$$

where $|\cdot|$ denotes the cardinality of set "·".

(2) Directly related ZINs

The two ZINs are called directly related nodes (DRNs) when they are connected with a branch. If nodes 2 and 3 are DRNs and nodes 1, 4, and 6 are observable in Fig. 5, the observability constraints of nodes 1, 2, 3, 4, and 6 is as follows:

$$\hat{f}_2 + \hat{f}_3 + \hat{f}_4 = 0 + 0 + 1 = 1 < 2 \tag{28}$$

$$\hat{f}_1 + \hat{f}_2 + \hat{f}_3 + \hat{f}_6 = 1 + 0 + 0 + 1 = 2 < 3 \tag{29}$$

Nodes 2 and 3 are unobservable since the two observability constraints are not satisfied. However, the state variables of nodes 2 and 3 can be calculated. All 5 nodes are observable when 3 of them are observable, but if the observable nodes are nodes 2, 3, and 4, then the other nodes are not observable. Therefore, at least one of nodes 1 and 6 that is incident to node 2 and does not belong to set Ω_3 should be observable. The number of observable nodes in the set of nodes that are incident to node 2 and does not belong to the set Ω_3 should not be less than its cardinality minus one. If nodes m and n are DRNs, then the corresponding constraints are as follows:

$$\sum_{j \in \Omega_m \cup \Omega_n} \hat{f}_j \geq d_m + d_n - 2 \tag{30}$$

$$\sum_{j \in \Omega_m, j \neq n, j \neq m} \hat{f}_j \geq |\Omega_m - (\Omega_m \cap \Omega_n)| - 1 \tag{31}$$

$$\sum_{j \in \Omega_n, j \neq n, j \neq m} \hat{f}_j \geq |\Omega_n - (\Omega_m \cap \Omega_n)| - 1 \tag{32}$$

$$m \in \Omega_0,\ n \in \Omega_0,\ m \in \Omega_n \tag{33}$$

where $\Omega_m - (\Omega_m \cap \Omega_n)$ denotes the set composed of nodes that belong

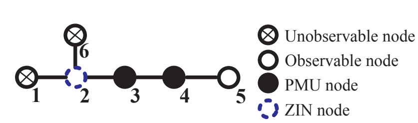

Fig. 4. Insufficiency of the constraints.

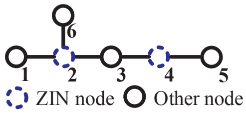

Fig. 5. Related zero injection nodes.

to set Ω_m and do not belong to set $(\Omega_m \cap \Omega_n)$.

By expanding the above condition, when nodes n and p are also DRNs, the corresponding nodes should satisfy observability constraint (34).

$$\sum_{j\in\Omega_m\cup\Omega_n\cup\Omega_p} \hat{f}_j \geq d_m + d_n + d_p - 3 \tag{34}$$

The incident nodes of m should satisfy observability constraint (35). The observability constraints of incident nodes of nodes n and p can be listed in the same manner.

$$\sum_{j\in\Omega_m, j\neq n, j\neq m, j\neq p} \hat{f}_j \geq |\Omega_m \cap ((\Omega_m \cap \Omega_p) \cup (\Omega_m \cap \Omega_n))| - 1 \tag{35}$$

The observability constraints of the nodes that are incident to either node m or n is (36). The observability constraints of nodes that are incident to node n or p can be listed in the same manner.

$$\sum_{j\in\Omega_m\cup\Omega_n, j\neq p} \hat{f}_j \geq |\Omega_m \cup \Omega_n| - 3 \tag{36}$$

If nodes 2, 3, and 4 are ZINs in Fig. 5, the corresponding constraints are as follows:

$$\hat{f}_1 + \hat{f}_2 + \hat{f}_3 + \hat{f}_4 + \hat{f}_5 + \hat{f}_6 \geq 3 \tag{37}$$

$$\hat{f}_1 + \hat{f}_2 + \hat{f}_3 + \hat{f}_6 \geq 1 \tag{38}$$

$$\hat{f}_3 + \hat{f}_4 + \hat{f}_5 \geq 0 \tag{39}$$

$$\hat{f}_1 + \hat{f}_6 \geq 1 \tag{40}$$

where constraint (38) is always satisfied and can be neglected. When constraint (39) is satisfied, constraint (37) is definitely satisfied. Thus, constraint (37) can be neglected as well.

When there are more than 3 directly related ZINs, the observability constraints can be listed in the same manner.

4.3. Effect of existing PFMs

Node i is observable if node j is observable when a PFM is installed on branch $l_{i,j}$. Node i can also make node j observable. The original constraints (41) and (42) can be replaced by (43).

$$f_i \geq 1 \tag{41}$$

$$f_j \geq 1 \tag{42}$$

$$f_i + f_j \geq 1,\ l_{i,j} \in \Lambda_{\mathrm{L}} \tag{43}$$

If two branches with PFM are related to a common node, then all of the nodes of the two branches are observable when one of them is observable as follows:

$$f_i + f_j + f_k \geq 1,\ l_{i,j},\ l_{j,k} \in \Lambda_{\mathrm{L}} \tag{44}$$

4.4. Effect of existing IMs

The influence of IMs is the same as that of ZINs.

4.5. Effect of the relevance of ZINs and PFMs

If node 2 is a ZIN and branch $l_{2,3}$ has a PFM in Fig. 5, then the corresponding observability constraints without considering their relevance are listed as follows:

$$\hat{f}_1 + \hat{f}_2 + \hat{f}_3 + \hat{f}_6 \geq 3 \tag{45}$$

$$\hat{f}_2 + \hat{f}_3 \geq 1 \tag{46}$$

However, if node 2 is observable, then node 3 is observable. Furthermore, either node 1 or node 6 is observable. Nodes 1, 2, 3, and 6 are all observable, even when the constraints are not satisfied. Hence, constraints (45) and (46) are changed to (47) and (48).

$$\hat{f}_1 + \hat{f}_6 \geq 1 \tag{47}$$

$$\hat{f}_2 + \hat{f}_3 \geq 1 \tag{48}$$

If branch $l_{2,3}$ also has a PFM, then the constraint will change to (49).

$$\hat{f}_2 + \hat{f}_3 + \hat{f}_6 \geq 1 \tag{49}$$

Above all, when the PFMs have relevance with ZINs, their corresponding constraints are revised as follows:

$$\sum_{i\in\Omega_j, i\neq m, i\neq j, i\neq k} \hat{f}_n \geq |\Omega_j| - M_{0,j} - 1 \tag{50}$$

$$f_j + f_k + f_m \geq 1 \tag{51}$$

5. Relevance between the existing measurements and network model parameters

5.1. Relevance of ZINs and topology changes

The topology change of the distribution network has no effect on the ZINs that are outside of the sub-network. The ZINs in the sub-network are considered as follows:

If node 3 in Fig. 3 is a ZIN, then the corresponding observability constraint is (52) when considering the status change of the switches.

$$\hat{f}_2 + \hat{f}_3 + \hat{f}_4 \geq 2 \tag{52}$$

However, one of the switches on branches $l_{2,3}$ and $l_{3,4}$ may open in operation. Node 4 is not always observable when nodes 2 and 3 are observable. The observability of node 2 cannot be guaranteed when nodes 3 and 4 are observable. Node 3 will always be observable when nodes 2 and 4 are observable, as the switches on branches $l_{2,3}$ and $l_{3,4}$ will not open simultaneously.

To consider the topology changes, the original adjacent matrix is modified such that the nodes of the sub-network may be not observable when their observability degrees are larger than 1. To model the relevance of the topology changes with the existing measurements in the optimization problem, the constraint (7) is changed to (53):

$$\mathbf{g} = \mathbf{B}\boldsymbol{x} \geq 1 \tag{53}$$

where

$$B_{i,j} = \begin{cases} \mathrm{A}_{\mathrm{T},i,j}/d_{\mathrm{S},i}\, i \in \Omega_{\mathrm{S}} \\ \mathrm{A}_{\mathrm{T},i,j}\, i \notin \Omega_{\mathrm{S}} \end{cases} \tag{54}$$

$\hat{g}_i = 1$ when $g_i \geq 1$, and node i is observable. The observability constraint of node 3 is expressed as shown in (55).

$$\hat{g}_2 + \hat{g}_4 + 2\hat{g}_3 \geq 2 \tag{55}$$

Therefore, the observability constraint of ZIN i on the sub-network is written as follows:

$$\sum_{j\in\Omega_i} \hat{g}_j + (d_{\mathrm{S},i})\hat{g}_i \geq d_{\mathrm{S},i} \tag{56}$$

If there is no sectional switch on $l_{3,4}$, node 4 will always be observable when nodes 2 and 3 are observable, the observability constraint of node 4 is expressed as (57).

$$\hat{g}_2 + \hat{g}_3 + 2\hat{g}_4 \geq 2 \tag{57}$$

Similarly, the observability constraint of node 2 can be expressed as (58) without sectional switch on $l_{2,3}$.

$$\hat{g}_3 + \hat{g}_4 + 2\hat{g}_2 \geq 2 \tag{58}$$

If there is no sectional switch on either $l_{3,4}$ or $l_{2,3}$, the relevance of ZINs and topology changes will not influence the observability constraint as (52).

For node p that is outside of the sub-network and incident to node i, is observable when other incident nodes of node i are all observable. The related constraints can be revised as shown in (59).

$$\sum_{j\in\Omega_i, j\neq p} \hat{g}_j \Big/ d_i + \hat{g}_p \geq 1 \tag{59}$$

Nodes m and n are all observable if they are DRNs, and the nodes incident to node m or node n are all observable, as shown below.

$$\sum_{j\in\Omega_m\cup\Omega_n, j\neq m, j\neq n} \hat{g}_j \Big/ (d_m + d_n - 2) + \hat{g}_m \geq 1 \tag{60}$$

$$\sum_{j\in\Omega_m\cup\Omega_n, j\neq m, j\neq n} \hat{g}_j \Big/ (d_m + d_n - 2) + \hat{g}_n \geq 1 \tag{61}$$

5.2. Relevance of PFMs and topology changes

If branch $l_{i,j}$ with the switch and PFMs is on the sub-network, the observability of node i cannot make node j observable. Constraint (44) is no longer efficient. If branch $l_{i,j}$ with PFMs is on the sub-network but does not have a switch, then the observability constraint is the same as that for (44).

If branches $l_{2,3}$ and $l_{3,4}$ in Fig. 3 have PFMs and are related, nodes 2 and 4 are both observable. Then, node 3 is observable. The observability constraint of node 3 should be revised as follows:

$$(\hat{g}_2 + \hat{g}_4)/2 + \hat{g}_3 \geq 1 \tag{62}$$

Therefore, the observability constraint of the related branches $l_{i,j}$ and $l_{j,k}$ with PFMs can be expressed as shown in (63).

$$(\hat{g}_i + \hat{g}_k)/d_{S,j} + \hat{g}_j \geq 1 \tag{63}$$

5.3. Network simplification with PMUs

The decision variables of the nodes with existing configured PMUs should be set as 1 in the optimization model. Furthermore, the distribution network may be simplified with the help of existing PMUs to reduce the scale and complexity of the optimization problem. If node i and its incident nodes are all observable with the existing PMUs, it can be deleted from the initial network. If node 2 in Fig. 5 already has a PMU installed, nodes 1, 2, and 6 can be deleted from the network. Node 3 is the border node that is already observable with $\hat{g}_3 = 1$. The observability constraint of the entire distribution network can be reduced to (64) when node 4 is a ZIN.

$$\hat{g}_4 + \hat{g}_5 \geq 1 \tag{64}$$

5.4. Branch with unknown parameters

The matrix element of the corresponding branches whose parameters are unknown should be set as 0.

$$\mathrm{B}_{i,j} = \mathrm{B}_{j,i} = 0,\ l_{i,j} \in \Lambda_{\mathrm{U}} \tag{65}$$

5.5. Generalized ILP model for the PMU placement of the distribution network

The generalized ILP model for the PMU placement of is shown as (66)–(70).

$$\min \sum_{i\in\Omega} \left(1 - \frac{d_i}{M}\right) x_i \tag{66}$$

$$\boldsymbol{g} = \mathbf{B}\boldsymbol{x} \tag{67}$$

$$M\hat{\boldsymbol{g}} \geq \boldsymbol{g} \tag{68}$$

$$\hat{\boldsymbol{g}} \leq \boldsymbol{g} \tag{69}$$

$$\boldsymbol{h}(\hat{\boldsymbol{g}}) \leq 0 \tag{70}$$

Eq. (65) is the objective function of the PMU placement model weighted by the node degrees of the original network. Equality constraint (66) is used to obtain the decision variables that denote the observability degree. (67) and (68) are used to transform the observability degrees into the binary decision variables that denote node observability. Eq. (69) denotes the observability constraint. For each node, the observability constraint is determined according to its related influence factors that are shown in Table 1.

The overall flowchart of the optimal PMU placement for the distribution network is shown in Fig. 6.

6. Case studies and analysis

Case studies and analysis are performed on IEEE 33-node test feeder [39], PG&E 69-node test feeder [40], a practical medium voltage distribution network feeder and its corresponding feeder group in Southern China. M is set as 100.0. Simulations are executed on a desktop with Intel Core i5-6500 3.20 GHz CPU, 8.00 GB RAM and Windows 10 operating system. The integer linear programming problems are solved by the "intlinprog" function of MATLAB (version R2016b).

6.1. Further selection of the configuration scheme

One of the configuration schemes of the IEEE 33-node test feeder is shown in Fig. 7. Solid nodes are installed with PMUs, and hollow nodes are not.

The configuration scheme extracted from the optimization model weighted by the node degree is shown in Fig. 8. The number of PMUs is 11 for both schemes, but the redundancy of the scheme in Fig. 8 is 1.03 compared with 1.0 in Fig. 7. Therefore, better schemes with higher redundancy will be selected by weighting the objective function with the node degree.

6.2. Effect of unknown parameters of the branches

When the line parameters of $l_{3,4}$ are unknown, the configuration scheme of IEEE 33-node test feeder is shown in Fig. 9. Eleven PMUs are

Table 1
Node observability constraint with difference influence factors.

Related influence factors				Constraints
Topology change	Unknown parameter	ZIN	PFM	
Yes	No	No	No	(7)–(12)
No	Yes	No	No	(13)
No	No	Yes	No	(23)–(26) and (29)–(39)
No	No	No	Yes	(43)
No	No	Yes	Yes	(49)–(50)
Yes	No	Yes	No	(52)–(60)
Yes	No	No	Yes	(62)

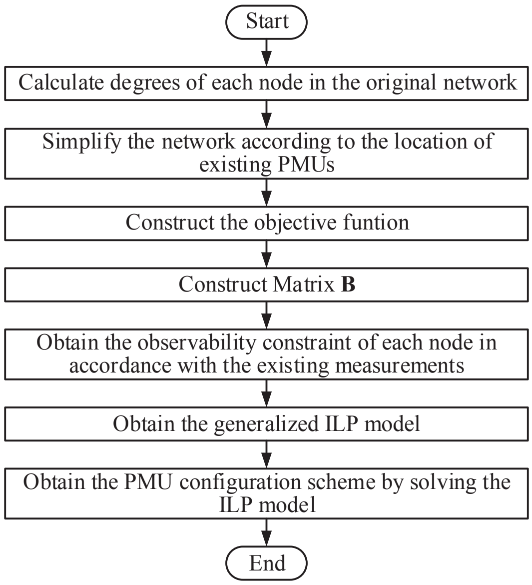

Fig. 6. Flowchart of the proposed optimal PMU placement.

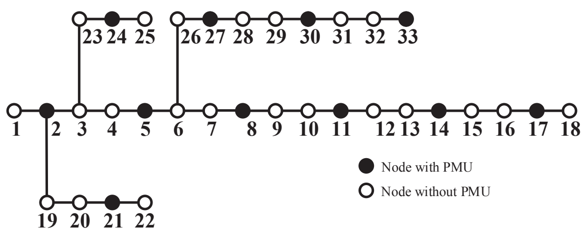

Fig. 7. Initial configuration scheme without considering the degree of nodes.

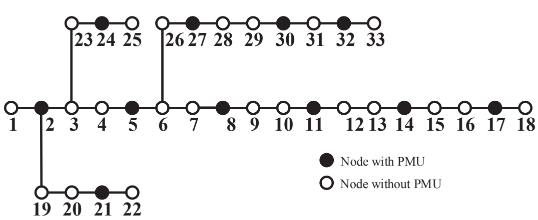

Fig. 8. Modified configuration scheme according to the degree of nodes.

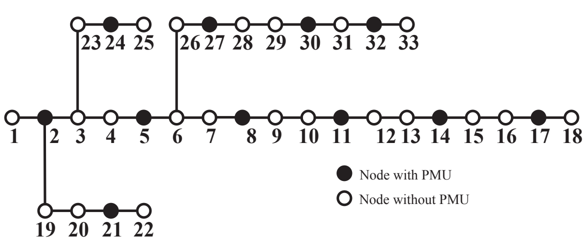

Fig. 9. Configuration scheme under unknown parameter of branch $\boldsymbol{l}_{3,4}$.

still sufficient to make the entire network observable. When parameters $l_{3,4}$ and $l_{4,5}$ are both unknown, the configuration is shown in Fig. 10. One more PMU is needed to realize complete observability. As the number of branches with unknown parameters increases, the required number of PMUs also increases.

6.3. Effect of topology changes

The existence of tie switches is shown as dashed lines in Fig. 11, and all of the other branches are configured with section switches. The configuration scheme of the IEEE 33-node test feeder with only tie switch TS1 is shown in Fig. 12. Thirteen PMUs are needed to achieve complete observability.

If the configuration scheme of Fig. 11 is still applied to this network with TS1, node 7 is no longer observable when the switch on branch $l_{7,8}$ is open and TS1 is closed, as shown in Fig. 13. Without considering the topology changes in the PMU configuration, some nodes will be unobservable after network reconfiguration.

The number of PMUs needed for the IEEE 33-node test feeder with more tie switches is shown in Table 2. The topology changes are more diverse with more tie switches, and the number of PMU increases to ensure complete observability of the distribution network under various operation topologies. The increase in computation time is relative small with the increase in switch number.

6.4. Effect of ZINs

ZINs exist in the distribution network which will reduce the requirement of PMUs to make the whole distribution network observable. To analyze the effect of ZINs, further case studies are performed on PG&E 69-node test feeder.

As shown in Figs. 14, 15, and 16, nodes 2, 3, 4, 5, 15, 19, 23, 25, 30, 32, 36, 45, 46, 47, 49, 52, 61, 65, and 67 in PG&E 69-node test feeder are ZINs and shown as dashed nodes.

The PMU configuration scheme without considering ZINs is shown in Fig. 14. Twenty-four PMUs are needed. The calculation time is 0.018 s. The configuration in which ZINs are considered but their relevance is not considered is shown in Fig. 15. Nineteen PMUs are needed. The calculation time is 0.018 s. The configuration in which ZINs and their relevance are both considered is shown in Fig. 16. Only 18 PMUs are needed. The calculation time is 0.024 s. Therefore, the ZINs in the distribution network can reduce the number of PMUs required, and their relevance can further reduce the demand of PMUs with little increase in computation time.

6.5. Case study on practical medium voltage distribution networks

To analyze the relevant factors that influence the results of PMU placement, case studies are performed on a practical distribution network in Southern China with 50 nodes. The locations of the switches, ZINs, and other existing measurements are shown in Table 3 and Fig. 17. The solid nodes in Fig. 17 are the nodes already installed with PMUs for fault location detection and distributed generator monitoring. The dashed lines are the branches with switches. Nodes 7 and 15 are connected to other feeders via tie switches. The topology changes with the two switches are considered in the analysis. The influence on the observability of other feeders is not considered in this case study. The decision variable of the node that is already configured with PMU is set as 1 in the programming model.

Six scenarios are used to analyze the influence of the different factors:

Scenario 1: system configured without switches, ZINs, and existing measurements;

Scenario 2: system configured with switches but without ZINs and existing measurements;

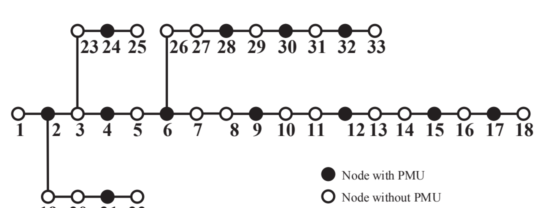

Fig. 10. Scheme with unknown parameters of branches $\boldsymbol{l}_{3,4}$ and $\boldsymbol{l}_{4,5}$.

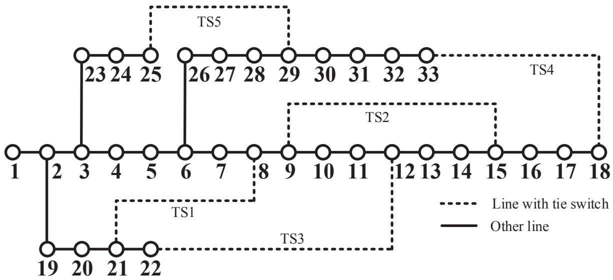

Fig. 11. IEEE 33-node test feeder with tie switches.

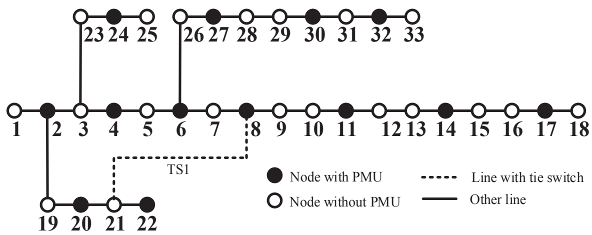

Fig. 12. Configuration scheme with tie switch TS1.

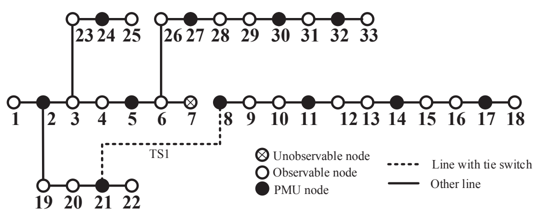

Fig. 13. Unobservable condition of IEEE 33-node test feeder.

Table 2
Number of PMUs configured with various tie switches.

Tie switch condition	Number of PMUs	Calculation Time (s)
Without tie switch	11	0.018
With only TS1	13	0.022
With TS1, TS2, and TS3	15	0.025
With all of the tie switches	17	0.029

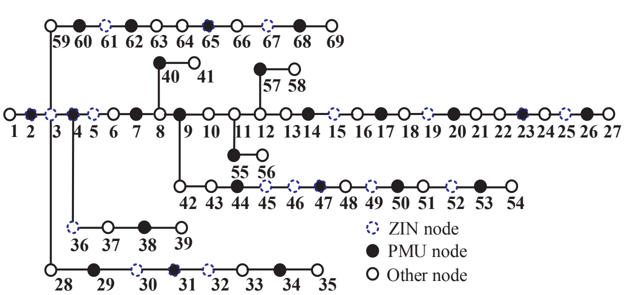

Fig. 14. Configuration scheme without considering ZINs.

Scenario 3: system configured with only existing PMU measurements;

Scenario 4: system configured with switches and existing PMU measurements;

Scenario 5: system configured with only ZINs and the existing measurements;

Scenario 6: system configured with switches, ZINs, and existing measurements.

The configuration schemes of the different scenarios are shown in Table 4.

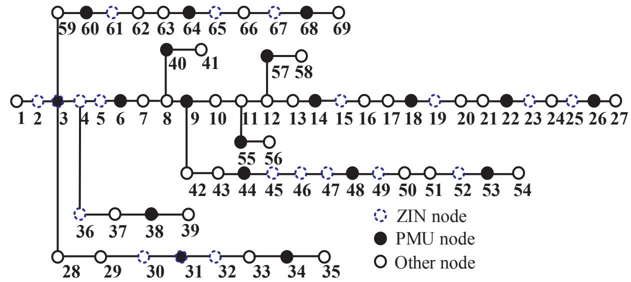

Fig. 15. Configuration scheme without considering the relevance of ZINs.

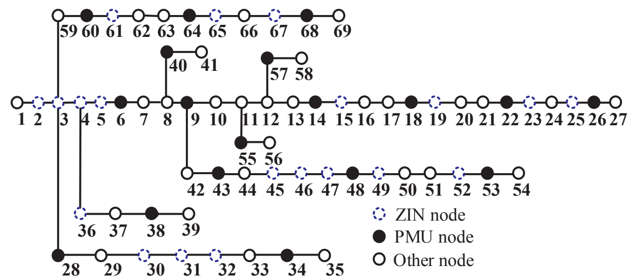

Fig. 16. Configuration scheme with the proposed method.

Table 3
System measurement configuration in the 50-node practical feeder.

Type	Locations	Number
PMU	1, 16, 17, 18, 19, 23, 25, 26, 29, 30, 42	11
PFM	2–17, 3–18, 5–6, 8–9	4
ZIN	2, 3, 4, 6, 7, 8, 10, 12, 13, 15, 20, 22, 37, 39, 43, 46	16
Switch	1–2, 5–6, 8–9, 7-, 15-	5

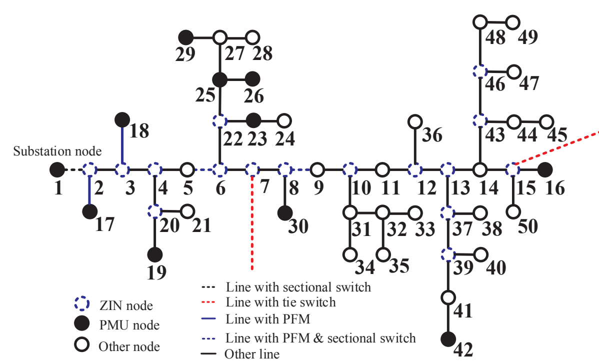

Fig. 17. Test case of a 50-node practical feeder in China.

The comparison of the schemes of scenarios 1, 3, and 5 with the scenarios 2, 4, and 6 illustrates that more PMUs are needed to guarantee complete observability in various operation topologies with tie switches. The comparison of the schemes of scenarios 1 and 2 with scenarios 3 and 4 indicates that the existence of PMUs will reduce the number of PMUs installed. The comparison of the schemes of scenarios 3 and 4 with scenarios 5 and 6 illustrates that CMs and ZINs can help considerably reduce the number of PMUs needed, with the number of PMUs reduced from 13 and 14 to 6 and 7, respectively.

To further test the scalability of the proposed method in practical project, the other two connected feeders are considered. The whole feeder group has 211 nodes in total. The locations of the switches, ZINs, and other existing measurements are shown in Fig. 18. The aforementioned six scenarios are used to analyze the influence of different factors. The configuration schemes of different scenarios are shown in Table 5. From Table 5, the conclusions above still hold. Although the

Table 4
Configuration schemes of the different scenarios in the 50-node practical feeder.

Scenario	PMU Locations	Number of PMUs	Calculation Time (s)
Scenario 1	2, 3, 6, 8, 12, 15, 20, 23, 25, 27, 31, 32, 37, 39, 41, 44, 46, 48	18	0.015
Scenario 2	1, 2, 3, 4, 6, 8, 10, 12, 15, 20, 23, 25, 27, 31, 32, 37, 39, 41, 44, 46, 48	21	0.016
Scenario 3	6, 8, 12, 15, 20, 27, 31, 32, 37, 39, 44, 46, 48	13	0.016
Scenario 4	4, 6, 10, 12, 15, 20, 27, 31, 32, 37, 39, 44, 46, 48	14	0.015
Scenario 5	27, 31, 32, 37, 44, 48	6	0.022
Scenario 6	12, 27, 31, 32, 39, 44, 48	7	0.023

scale of the network is considerable large, all the configuration schemes can be obtained within 1 s.

7. Conclusions

An optimal PMU placement method for distribution networks is proposed considering the topology changes in operation. ZINs, IMs, PFMs, existing PMUs, and their relevance are also considered. Better configuration schemes with higher redundancy are selected by weighting the objective function of the PMU placement optimization problem with the node degrees. Case studies performed on IEEE 33-node test feeder, PG&E 69-node test feeder, a practical medium voltage distribution network feeder and its corresponding feeder group in Southern China. To demonstrate that more PMUs should be configured when topology changes of the distribution network are considered. ZINs and CMs will reduce the required number of PMUs. By considering the relevance of ZINs and CMs, the number of PMUs needed to guarantee the complete observability of the distribution network is further reduced.

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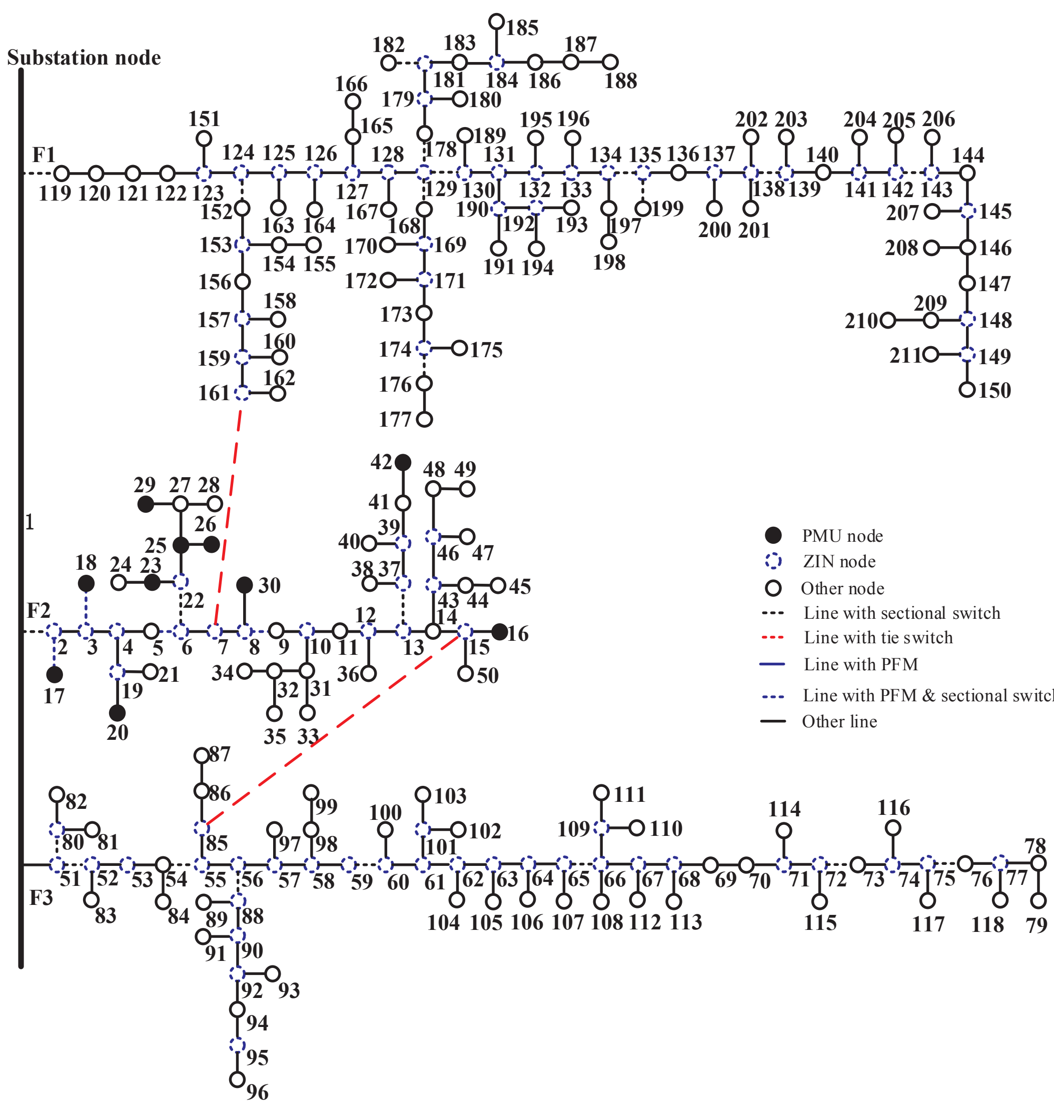

Fig. 18. Test case of a 211-node practical feeder group in China.

Table 5
Configuration schemes of the different scenarios in the 211-node practical feeder group.

Scenario	PMU Locations	Number of PMUs	Calculation Time (s)
Scenario 1	2, 3, 6, 8, 12, 15, 19, 23, 25, 27, 31, 32, 37, 39, 41, 44, 46, 48, 52, 54, 57, 60, 62, 63, 64, 65, 66, 67, 68, 71, 72, 74, 75, 77, 78, 80, 86, 88, 90, 92, 95, 98, 101, 109, 120, 123, 125, 126, 128, 130, 132, 133, 135, 137, 138, 139, 141, 142, 143, 145, 146, 149, 153, 154, 157, 159, 161, 165, 169, 171, 174, 176, 179, 181, 184, 187, 190, 192, 197, 209	80	0.063
Scenario 2	1, 2, 3, 4, 6, 8, 10, 12, 15, 19, 23, 25, 27, 31, 32, 37, 39, 41, 44, 46, 48, 52, 54, 57, 60, 62, 63, 64, 65, 66, 67, 68, 71, 72, 74, 75, 77, 78, 80, 85, 86, 88, 90, 92, 95, 98, 101, 109, 120, 123, 125, 126, 128, 130, 132, 133, 135, 137, 138, 139, 141, 142, 143, 145, 146, 149, 152, 154, 157, 159, 161, 165, 169, 171, 174, 176, 179, 181, 184, 187, 190, 192, 197, 209	84	0.125
Scenario 3	6, 8, 12, 15, 19, 27, 31, 32, 37, 39, 44, 46, 48, 52, 54, 57, 60, 62, 63, 64, 65, 66, 67, 68, 71, 72, 74, 75, 77, 78, 80, 86, 88, 90, 92, 95, 98, 101, 109, 120, 123, 124, 125, 126, 128, 130, 132, 133, 135, 137, 138, 139, 141, 142, 143, 145, 146, 149, 154, 157, 159, 161, 165, 169, 171, 174, 176, 179, 181, 184, 187, 190, 192, 197, 209	75	0.047
Scenario 4	5, 7, 9, 12, 15, 19, 27, 31, 32, 37, 39, 44, 46, 48, 52, 54, 55, 57, 60, 62, 63, 64, 65, 66, 67, 68, 71, 72, 74, 75, 77, 78, 80, 86, 88, 90, 92, 95, 98, 101, 109, 120, 123, 125, 126, 128, 130, 132, 133, 135, 137, 138, 139, 141, 142, 143, 145, 146, 149, 152, 154, 157, 159, 161, 165, 169, 171, 174, 176, 179, 181, 184, 187, 190, 192, 197, 209	77	0.093
Scenario 5	27, 31, 32, 37, 44, 48, 54, 61, 63, 66, 69, 72, 75, 78, 80, 86, 88, 94, 98, 101, 109, 121, 124, 127, 132, 135, 138, 141, 143, 146, 149, 154, 159, 165, 171, 176, 178, 183, 187, 192, 197, 209	42	0.200
Scenario 6	12, 27, 31, 32, 39, 44, 48, 54, 58, 63, 66, 69, 72, 75, 78, 80, 86, 90, 94, 98, 101, 109, 120, 123, 126, 132, 135, 138, 141, 143, 146, 149, 154, 157, 161, 165, 171, 176, 178, 183, 187, 192, 197, 209	44	0.4970

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