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Application of heuristic algorithms to optimal PMU placement in electric power systems: An updated review

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ABSTRACT

Phasor measurement unit (PMU) plays an important role in operation, protection, and control of modern power systems. PMU provides real time, synchronized measurements of bus voltage and branch current phasors. It is neither economical nor possible to place all the buses of the system with PMUs because of their high cost and communication facilities. Attaining the minimal number of PMUs to access an observable power system is the main objective of optimal PMU placement (OPP) problem, which is solved by utilizing different techniques. Graph theoretic and mathematical programming procedures have been first introduced to solve OPP problem, aiming to access power system observability. Heuristic method as an experience-based technique is defined as a quick method for obtaining solutions for optimization problems, in which optimal solutions are not achievable using mathematical methods in finite time. This paper provided the literature review on different heuristic optimization methods to solve the OPP problem. Then, the available methods were classified and compared with different points of views. Results from the tests of researches on heuristic algorithms with and without the consideration of zero-injection buses were compared and superiorities of the introduced heuristic concepts were demonstrated with relative to each other.

Contents

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1. Introduction

State estimation (SE) is an imperative process for monitoring power system and ensuring the system security considering contingency experiments and optimal power flow [1]. Previously, remote terminal units (RTUs) were responsible for collecting measurements like real/reactive power flows, power injections, and magnitude of bus voltages and branch currents for supervisory control and data acquisition (SCADA) system as the source provider of SE [2]. Synchronized phasor measurements (SPM) are introduced as an efficient tool to operate, protect, and control the power system [3]. Phasor measurement unit (PMU) was introduced in the 1990s utilizing in wide area monitory systems (WAMS) for producing synchronized measurements of bus voltage and branch current phasors in real-time [1,2]. Employing a synchronized signal obtained by global positioning system (GPS) with the accuracy of better than 1 μs provides synchronized measurements by PMU usage. A high value of sampling for PMUs and obtaining linear state estimators by PMU measurements ensures high speed of voltage control system compared to conventional SCADA/EMS systems [4].

At the beginning, local measurements were utilized to control power system until real-time phasor measurement technology was introduced. Phasor measurements enable the control of power system using remote measurements. Advantages of remote measurements include imputing measurements to the controlled device at high speed and being utilized as a feed-back signal in the controller [3]. Local signals, local measurements, and a mathematical model of the external world such as external equivalents were the base of control process. Processing phasor measurements in the 0.2–2.0 Hz range, in which the time tagged phasor data make the capability of providing actual state of the system control a short time in the past. To motivate the control process of the power system, 15–60 Hz frequency is required for the measurements [5]. Synchrophasors have the strength of monitoring angles to discover probable instabilities and discrete switching controls in order to militate against these events in the control of power system [6,7]. Early applications with continuous feedback aimed at the problems in which the control objective was global in nature: for example, an HVDC controller may be called upon to damp electromechanical oscillations between two widely separated areas of a power system [5]. A model based on PMU data was presented in [8] for small signal stability analysis of power systems. A prony analysis based method was used in [9] for online inter-area oscillation monitoring. Application of wavelet transform and Hilbert–Huang transform for identifying the inter-area modes utilizing PMU data was also studied in [10]. Some other applications of PMUs in power system include power system state estimation [4,11], wide area control and monitoring [12,13], fault location and detection [14,15], wide area protection [16], transient stability analysis and prediction [17], thermal monitoring of transmission lines [18], and online steady state angle stability monitoring [19].

A PMU installed in a bus can provide synchronized measurement value of voltage phase of that bus and also current phasors of some or all the adjacent and connected lines to that bus. Reaching an observable system needs enough measurements of state estimations, which makes the placement problem [20]. The system is completely and directly observable if all the buses are PMU installed; but, it is neither economical nor possible due to the high cost of PMUs [2]. Therefore, obtaining the optimum number of PMUs and their configuration in the system is propounded as a considerable challenge called optimal PMU placement (OPP) problem.

To solve the OPP problem, different optimization techniques have been presented in the literature which can be generally divided into two main groups of conventional techniques and heuristic algorithms. Linear programming (LP), non-linear programming (NLP), dynamic programming, and combinational optimization are the main employed methods of the first group. Advanced heuristic algorithms, not only analyze the system observability, but also dominate some difficulties of conventional methods such as PMU failure or branch outage. Sensitivity constraint [21], lack of communication in substation constraint [22], critical measurements [23,24], fault observability [25], and mean square error (MSE) [26,27] are other objects that have been considered by heuristic optimization algorithms.

This paper reviewed the most popular heuristic optimization tools for solving the OPP problem. Section 2 describes the formulation of the OPP problem. Section 3 provides an explanation for the heuristic methods utilized to solve OPP problem and a multi-dimension comparison between the presented algorithms. A comprehensive comparison from different aspects is provided in Section 4. And, Section 5 exhibits the conclusion of this paper.

2. Formulation of optimal PMU placement problem

Numerical and topological observability are two major techniques for analyzing system observability. The former suffers from high matrix calculation difficulty; therefore, it is not greatly employed for the observability analysis of systems. A system is called topologically observable when a full rank of spanning tree is obtained. There are some efficient rules which can simplify and improve topological analysis. These rules are illustrated in the following.

1. Voltage phasor of a PMU-equipped bus and current phasors of all joint lines are available as illustrated in Fig. 1. These measurements are called direct measurements. In Fig. 1, bus-1 is a PMU installed bus; so, the voltage phasor of bus-1 and current phasors of joint branches are known following rule 1.
2. Knowing both voltage and current phasors at one end of a line ensures the observability of the other end by providing voltage phasor, as presented in Fig. 2. These measurements are called pseudo-measurements. In Fig. 2, voltage phasor of bus-1 and

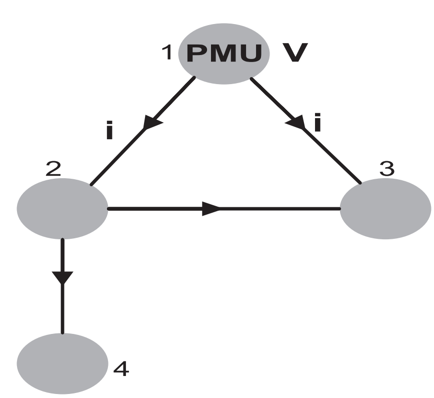

Fig. 1. PMU placement rule 1: observability with direct measurements.

current phasor of the line between bus-1 and bus-3 are calculated; so, by utilizing rule 2, voltage phasor of bus-3 can be obtained.

3. Considering a line with the known voltage phasors for both ends, the current phasor of this line can be calculated as shown in Fig. 3. These types of measurements are also called pseudo-measurements. In Fig. 2, voltage phasors of bus-2 and bus-3 are known; so, following rule 3 current phasor of the branch between bus-2 and bus-3 will be available.
4. Considering a zero-injection bus, if current phasors of all joint lines are known, except one, the current phasor of the unknown line can be calculated utilizing KCL equations. This situation is illustrated in Fig. 4. In this figure, bus-3 is zero-injection bus and current phasors of the line between bus-3, and bus-1, and also current phasor of the line between bus 3, and bus-2 is available. So by using rule 4 current phasor of the line between bus-3 and bus-4 will be known as mentioned in the fourth rule of observability rules.
5. Considering a zero-injection bus with unknown voltage phasor, if voltage phasors of all adjacent buses are known, by utilizing node equations, the voltage phasor of zero-injection bus can be obtained as presented in Fig. 5. In this figure, bus-3 is a zero-injection bus and voltage phasors of three adjacent buses including bus-1, bus-2 and, bus-4 are available. By utilizing node equations, voltage phasor of bus-3 is known.
6. Considering a group of adjacent zero-injection buses with unknown voltage phasors in which the voltage phasor of all the adjacent buses to the group are known, the zero-injection buses are observable utilizing both KCL and KVL equations. This condition is presented in Fig. 6, in which a group of zero-injection buses including bus-3 and bus-4 has unknown voltage phasor; but, the voltage phasors of adjacent buses to the mentioned group which includes bus-1, bus-2, bus-5 and, bus-6 are known. Following rule 6, the voltage phasors of bus-3 and bus-4 will be available.

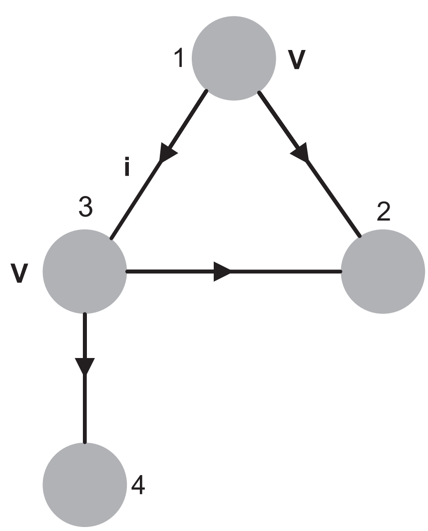

Fig. 2. PMU placement rule 2, observability of bus voltage using pseudo measurements.

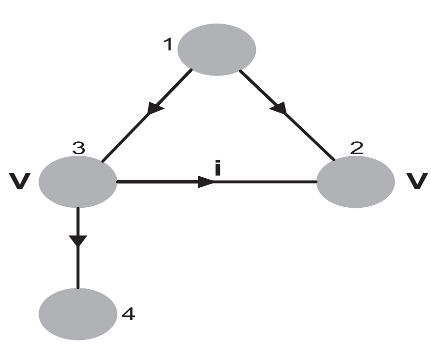

Fig. 3. PMU placement rule 3, observability of line current using pseudo measurements.

Considering different objective functions for investigating the system observability, optimal placement of PMUs in power system has been presented in many works in this area. The objective functions which have been handled by utilizing heuristic algorithms include minimizing number of PMUs, maximizing measurement redundancy, handling contingency constraint such as one PMU/line outage or failure of one PMU and, one line outages in the system. Definition of each objective function that could be considered in solving OPP problem is given in the following section.

Minimum number of PMUs: The main objective of the OPP problem is to determine the minimum number of PMUs and their appropriate placements to ensure full observability of power system. The constraint of the problem is in accessing a completely observable power network. So, the main objective function can be mathematically presented as follows:

$$\begin{aligned} &\text{Min} \quad \left(\sum_{j}^{Nbus} S_{ij} \right) \\ &\quad \text{s.t.} \quad \text{A.S} \geq \text{I} \\ &\quad I = [1\ 1\ 1\ \cdots\ 1]^T_{N \times 1} \end{aligned} \tag{1}$$

$$S(i) = \left\{ \begin{array}{ll} 1, & \text{if bus } i \text{ is a PMU equiped bus} \\ 0, & \text{otherwise} \end{array} \right\} \tag{2}$$

$$A(i,j) = \left\{ \begin{array}{ll} 1, & \text{if } i = j \\ 1, & \text{if buses } i \text{ and } j \text{ are connected} \\ 0, & \text{other wise} \end{array} \right\} \tag{3}$$

Measurement redundancy: Another aspect of solving the OPP problem is measurement redundancy which has been considered as objective function in some works. Typically, the number of redundant measurement of each bus or the number of times each bus is observed, either directly or indirectly, is defined as measurement redundancy. So, to ensure full observability of

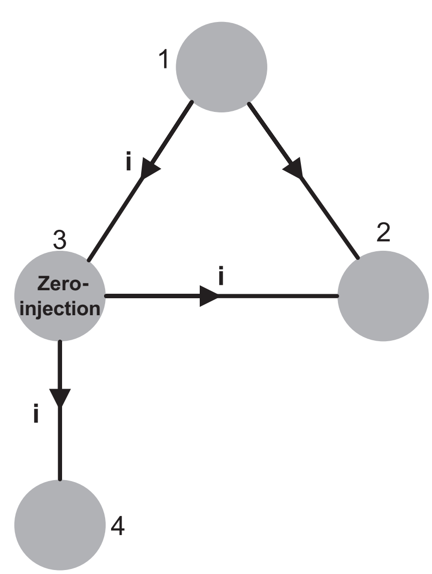

Fig. 4. PMU placement rule 4, observability of ZI buses using KCL equations.

power system, measurement redundancy value for each bus of the system should be at least one. Thus, another objective function could be defined as maximizing measurement redundancy in the electric network.

Contingency constraints: Different kinds of contingencies could occur in the power system. Power system instability usually occurs after contingencies. So, it is essential to analyze the system observability in these conditions which include line outage and PMU failure. To find the minimum number of PMUs and their configuration, while tolerating one line outage or one PMU failure in power system, several heuristic concepts have been utilized [28].

3. Heuristic algorithms applied to the OPP problem

3.1. Genetic algorithm (GA)

Modeling natural selection is the base of genetic algorithm (GA) which does not need any secondary functions such as derivatives computation. Some positive characteristics of GA which make it more usable in optimization problems are as follows: (a) probability of local minimum trapping is decreased, (b) computations of going from one state to another is declined, and (c) evaluation of the fitness of each string guides the search [29].

Pareto-optimal solutions obtained by a non-dominated sorting genetic-based algorithm (NSGA) which is a combination of graph-theoretical concept and GA was adapted in [30] to reach the minimum number of PMUs installed in a power system. The proposed method considered two competing objectives including minimizing the number of PMUs and maximizing the measurement redundancy in the OPP solution. Unlike most of the applied procedures, the entire Pareto-optimal solutions exist for the OPP problem, instead of a single point solution. Important steps including crossover, mutation, and population are mentioned to be problem-dependent, where crossover values are in high probable value; regardless of mutation probability, crossover would be a good choice for NSGA parameters. Different crossover and mutation probabilities are applied to reach several and common Pareto-optimal fronts. Repairing infeasible solutions which confront computation analysis with difficulty narrows the application of this method in plenty of optimization problems.

In [31], formulation of the OPP problem was taken by a topology-based algorithm and GA was used as a solution for this problem. This method considered zero-injection buses in a power network for solving the OPP problem. A comparison of the results between utilized method and earlier applied solutions was also made. Achieving completely observable power system utilizing GA algorithm was presented in [32], which could successfully provide a solution for the OPP problem considering two important objectives including (i) one PMU/branch outage and (ii) maximum redundancy in the system observability. Crossover and mutation were applied as two operators of GA method to cause the accurate number of PMUs for solving the OPP problem. Observing maximized redundancy in the number of buses was the result of optimum location of PMU determination. There was an increase in the number of PMUs which was needed to make the system observable when it had branch/PMU failure.

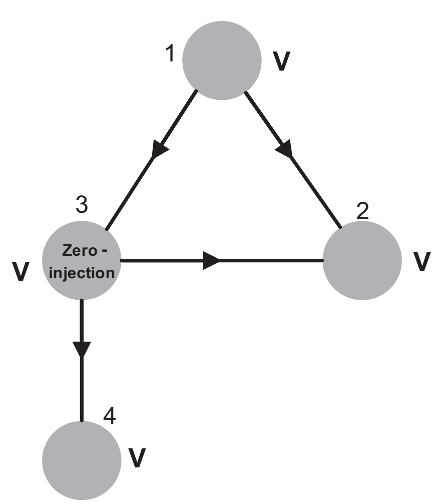

Fig. 5. PMU placement rule 5, observability of a zero-injection bus with unknown voltage phasor.

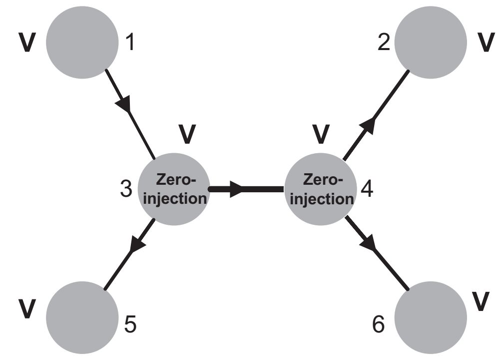

Fig. 6. PMU placement rule 6, observability of adjacent zero-injection buses with unknown voltage phasors.

A solution for the OPP problem using genetic algorithm-based procedure was presented in [33]to make the system observable for utilizing in linear state estimation. A new generation with fitness evaluation for a new population, started by opting crossover and mutation of individuals from the old population.

3.2. Particle swarm optimization (PSO)

A similar method to GA in which a population of random solutions is initially given to the system is particle swarm optimization (PSO). Particles remark the individuals that are flown through the multi-dimensional space. The best position for each particle is obtained by the best solution (fitness) faced by itself and its neighbors. As mentioned, the process of this algorithm starts with an initial position and velocity for each particle, in which the velocities are bounded due to not flying in unusable fields and also overflowing forbiddance [34].

A new concept for solving the OPP problem and reaching a completely observable network which satisfies the constraints of PMU loss or a transmission line outage was presented in [35], which was marked utilizing a modified binary particle swarm optimization (BPSO) method. BPSO algorithm is a discrete binary version of PSO in which variables can only take 0 and 1 values. The rules presented in topological observability in the majority of papers have been completed in the presented paper by developing the new rule based on analyzing the observability of a group of zero-injection buses to reach maximum usage of the existing data. As mentioned in the formulation of the OPP problem in the present paper, this rule ensures the observability of zero-injection buses whose adjacent buses had known values. Results of the presented method and different algorithms were compared in different situations including normal condition and a PMU/branch outage.

Difference in the cost of different PMU installation in a power system and the dependent factors including line adjacency numbers at the bus and communication conditions was introduced in [36]. In this paper, the latter factor was introduced in the presented optimization tool to find the best solution and prove that this tool was better than the conventional methods. So, this paper not only found the maximum number of PMUs, but also computed the cost of different installations of the minimum number of PMUs and opted the best one with minimum installation cost.

An observability analysis considering PMU loss based on control reconfigurability criterion was introduced in [37], which employed the PSO method to reach the minimum number of additional PMUs installed in a power network. Control reconfigurability method that formulates fault-tolerant PMU installation problem in a power system at the given numbers of PMUs, utilized this method as a constraint to install PMUs in the network. A rectification version of BPSO was used to solve the OPP problem as a higher optimization tool.

Solving the OPP problem for making a power system totally observable and maximizing measurement redundancy at the buses of the system was accomplished by a BPSO in [38]. This method was presented considering measurements with and without injections such as zero-injection or measured injections; also, flow measurements were considered in another phase.

3.3. Simulated annealing (SA)

Simulated annealing (SA) is a procedure for solving complicated combinatorial optimization in which the current solution is randomly altered. The new solution is the worse alteration with the probability that is reduced as the computation proceeds. An optimal solution for a large combinatorial optimization problem needs a fit perturbation mechanism, cost function, solution space, and cooling schedule to be solved by SA. Sufficiency of SA can be found by searching a large-scale system and obtaining good speed in terms of finding an optimal or near-optimal solution [29].

A solution for pragmatic communication-constrained PMU placement problem is SA method which is utilized to solve the OPP problem based on incomplete observability in [39]. Optimal locations of PMUs based on a desired depth of unobservability and impact of depth of unobservability on the number of PMUs were presented in this paper. There was a relationship between the certitude of state estimation of unobserved buses and a given unobservability depth. Lower depths of unobservability caused a particulate state estimation. This method provided optimal PMU installation for estimation with available communication facilities and certified that the unobserved buses were far from the observed buses.

To detect bad data in a power network which turns the measurement to critical measurement (CM), utilizing PMU installation is considered. Any bad data detection incidentally needs a critical measurement free. To identify critical measurement, several methods have been proposed; this paper used residual analysis to identify critical measurement. The absence of critical measurement means a power system that loses single measurement. A solution for the OPP problem to make a power system topologically observable, considering bad data revelation using SA, was presented in [23].

A similar method as a stochastic concept of simulated annealing (SSA) was introduced in [24]. A hybrid genetic algorithm and simulated annealing (HGS) was used as a solution for the OPP problem and a comparison organ with the results of SSA method. Difference between a system in its normal situation and with critical measurement free is observable, when for the second system, losing any single branch does not impact the observability of the power system.

Better usage of specific PMU measurement values and accessing highly sensitive system data were considered in [21] to optimally install PMUs for making power network completely observable. Reaching initial PMU configuration to have a system with full observability was analyzed by an observability topology algorithm based on incidence matrix. Sensitivity constrained OPP detection and completely observable power system were solved by applying simulated annealing (SA) method. Dynamic character of a network could be better defined by the data with high sensitivity.

3.4. Differential evolution (DE)

Differential evolution (DE) concept employs N-dimensional element vectors to minimize ongoing space functions. Mutation, crossover, and selection are the principle operators utilized to carry out the global optimization. This heuristic method could be widely used in different cost function problems such as non-differentiable, non-linear and, multi-modal functions. Parallel computations, easy usage, and good convergence properties are other benefits of this approach [40].

In [41], the authors presented multi-objective OPP using a non-dominated sorting differential evolution (NSDE) algorithm which is an organic integration of Pareto non-dominated sorting operation and differential evolution algorithm (NSDE). In addition to solving the OPP problem this concept considered maximum measurement redundancy and voluntary PMU failure to reach a completely observable network. Usage of DE algorithm obtained from GA led to proposing NSDE algorithm procedure. Achieving particular and complete Pareto front and finding many Pareto-optimal solutions were mentioned as the betterment of this procedure.

A minimal PMU placement method by DE was presented in [25], which analyzed network fault observability. Reaching the minimum number of PMUs required for system observability was discussed by utilizing integer linear programming (ILP), which provided an optimal solution by DE method. Three operators containing mutation, recombination, and selection process were functioned in this concept until the stopping criteria was accessed. Finally, solutions for the OPP problem considering fault observability were given considering the system with and without zero-injection.

A DE concept for optimally placing PMU to access state estimation with minimum mean square error (MSE) was discussed in [26], which considered the power system with and without conventional measurements. Utilizing conventional measurements moreover than PMU usage in the system, is to reach lower cost and also, get more accurate state estimation. This presented algorithm optimally provided a global solution in test systems which were benchmarked by state estimation. Also, the best solution was opted using the formulated procedure.

The presented DE procedure in [27] provided a method for minimal PMUs and their configuration in the power system to analyze the observability of the system. Reaching minimum MSE for the system was also considered by DE method, which employed mutation, recombination, and selection as main operators. Results of the proposed model were compared with other methods to show the minimal reached number of PMUs compared to others.

3.5. Tabu search (TS)

Tabu search (TS) is an adaptive algorithm that utilizes many other methods such as linear programming algorithms and heuristic concepts. This procedure is presented to solve the combinational optimization problems in scheduling and covering. Tabu list which is one of the main elements of TS consists of the number of recently visited states plus a number of unwanted states. Other main elements of TS are aspiration, diversification, and definition of a state and the surrounding area. There is a reset in TS when it is not converging [42].

A Solution for the OPP problem solution in terms of reaching a completely observable power system and enough redundancy using TS based on the linear state estimator model of a system was presented in [43]. This fast method of topologically observability analysis needed loss computation function based on incidence matrix for solving the OPP problem and was highly robust. Comfortableness and high speed of accessing an observable power system by the manipulation of integer numbers is also concerned by this method.

Most of the observability analysis techniques utilize topological method, while a combination of a numerical method with tabu search (TS), called recursive tabu search (RTS), was presented to reach a completely observable network with maximum redundancy in [44]. This optimization method found the best solution of the executions where the initial solution always obtained from greedy algorithm was utilized as executed recursively. This procedure considered the methods including MTS approach to reach the minimum number of PMUs as a solution for the OPP problem. The simulation results on IEEE networks held out that RTS was more empirical than MTS. A comparison of this solution with others was also made.

A new parallel tabu search (TS) for solving the OPP problem providing a shorter process time was presented in [22], which introduced four parallel spaces created by state division. Each of the newly obtained spaces was analyzed by Tabu list. In this method, a graph theory concept was utilized to reach an initial configuration for power system. Considering a constraint called lack of communication in substations was another phase of this paper. Two other methods called “Step Elite Solution” and “PMU Site Selector” were operated to calculate the functioned energy. Finally, an optimal solution for the OPP problem was obtained for the system considering the system with and without constraint.

The objective in [45] was to solve the OPP problem using different methods to reach the minimal PMUs while analyzing complete observability of a power system by considering of observability constraints. Moreover than TS, PSAT software was utilized to compare different results of each method. Different algorithms have been applied in PSAT to analyze the system observability and obtain the minimum number of PMUs, which have illustrated the efficiency of the proposed method.

3.6. Ant colony optimization (ACO)

Another concept utilized for presenting a solution to optimization problems is ant colony optimization (ACO) which initially uses a population of ants. Role of the colony of ants is to move through adjacent states of the problem by applying a stochastic local decision optimal controller (policy), which results in the solution for the optimization problem. Pheromone trail evaporation and daemon actions are other processes in ACO. Computational problems can be reduced using ACO to find good paths through graphs [46].

In [47], optimal PMU placement problem for obtaining an observable power system with the minimum number of PMUs and considering maximum measurement redundancy was solved by utilizing an improved ACO. Depth first search as a graph theoretic method was applied to build a measurement tree so that the network observability could be analyzed. Efficient calculation and equivalency between the exploration of new solution and that of aggregated problem learnt were mentioned as characteristics of ant colony system (ACS). Development of ACS by an adaptive stochastic perturbing ACS (ASPACS) was proposed in this paper to adaptively conduct the pheromone trail persistence coefficient (PTPC) and stochastic perturbing progress (SPP).

Providing the OPP solutions containing approximate solutions and global solutions considering maximum measurement redundancy in a power system was presented in [48] using a recursive method. An adaptive clonal concept (CLONALG) utilized recombination which could increase process velocity is presented. Feasible solutions scope was propagated by proposing a function which simplified an extended scheme access for engineers. Finally, a comparison was made between results of this method and adaptive GA and SGA.

3.7. Mutual information (MI)

An information-theoretic approach for solving OPP problem considering, not only accessing a completely observable network, but also modeling the uncertainties in the system states, which used mutual information (MI) between the PMU measurement values and network states was presented in [49]. DC model was assumed in this paper, since the analytical DC model of the power system was the base of MI criterion. Analyzing the power system with and without conventional measurements and PMU loss was also considered in this paper.

3.8. Iterated local search (ILS)

Searching a smaller subspace defined by the solutions which are local optima, instead of the whole space of solution, is the main viewpoint of iterated local search (ILS). By utilizing an embedded heuristic, a sequence of solutions is provided in which the best solution is obtained if one were to utilize repeated random trials of that heuristic [50].

Optimal PMU placement concept presented in [51] has two steps including an initial PMU dispensation to access an observable system by utilizing an iterated local search (ILS) to find the minimum number of PMUs needed to make a network completely observable. In this method, page Rank placement algorithm (PPA) is used to evaluate the importance of each node. To denote an initial configuration for network and then the OPP problem, a repeated process introduced in which removing one of the PMUs maintains full observability to access the minimized number of PMUs. In another phase, a greedy algorithm which has high performance, low complexity, and easy usage in large-scale networks is presented.

3.9. Immune genetic algorithm (IGA)

Immune genetic algorithm (IGA) was used in [52] to solve the optimal PMU installation using three impactful vaccines to make a power network topologically observable. Vaccination and immune options are the two steps that appear in an IA method to protect against bacteria and viruses. The incorporation of local knowledge and prior information of OPP problem is the base of vaccination. IGA which is used to make the results more optimal considers two operators including crossover and mutation. A remarkable growth in converging speed via this algorithm and its efficiency was displayed when familial reproduction was prevented by studying a new effect in the algorithm.

3.10. Imperialistic competition algorithm (ICA)

Imperialistic competition algorithm (ICA) is a newly developed method for solving different optimization problems. Similar to other heuristic algorithms, ICA starts with an initial population which is called country and is in two kinds of colonies and imperialistic. Competition between these countries results in the minima of the problem. Ability of ICA in usage in a wide scale of optimization problems has been confirmed by testing on different benchmark functions [53].

Two competitive objectives including different placement solutions to find the minimum number of PMUs to access an observable power system and providing maximum redundancy of the system that provides more accurate measurement values was presented by the new method based on binary imperialistic competition algorithm (BICA) as the OPP problem solving concept. This method considered different cautionary situations such as single branch outage or PMU disturbance and also considerations such as zero-injection buses. In this paper, an additional rule was proposed to topological observability rules such as [23], which ensures the observability of a zero-injection buses group in which the phasor of adjacent buses are observed and this rule results in the lesser number of PMUs needed for accessing the observable network. Fast isotropy, small running time, and zero standard deviation were mentioned as the advantages of the proposed method [28].

3.11. Biogeography based optimization (BBO)

Biogeography based optimization (BBO) mathematically models the migration quality of species from one island that is technically called habitat to another, arising and extinction circumstance of species. This procedure is utilized to solve the optimization problems by dynamic fitness landscapes and also introduce emigration and immigration quality of species within the habitat. This method works under two operators of migration and mutation [54].

A multi-objective method which tries to minimize the number of PMUs to reach a completely observable system and maximize the measurement redundancy due to state estimation was presented as a multi-objective biogeography based optimization (MO-BBO) in [55]. Since there was no single optimal solution in this optimization method, a non-dominated sorting and crowding function was applied to provide Pareto-optimal solutions and a fuzzy-based operator was used to achieve the best compromise solutions.

A similar method was introduced in [20] considering normal and contingency situations which included line outage and PMU failure. Recognition of strategic locations and usage of virtual reduction technique were mentioned to decrease the number of system nodes.

3.12. Matrix reduction

A coverage matrix exists for all the placement problems in which the coverage range is demonstrated by the matrix when a facility is installed in different positions. Scale of the problem is obtained by coverage size. Matrix reduction method tries to provide the optimal solution by reducing the incidence matrix [56].

Obtaining the minimum number of PMUs to access a fully observable network and simplify computation function by utilizing an algorithm based on eliminating a virtual data and a matrix reduction algorithm was presented in [56]. As mentioned, the first step is the elimination of virtual buses because there is big optimization on problem magnitude based on the coverage matrix size and increase in system criterion in the essence of enormous size of virtual buses. The OPP problem was ideally solved when a full empty coverage matrix is obtained. Finally, PMUs were optimally installed by a combination of matrix reduction algorithm with greedy algorithm. Lagrangian relaxation was also utilized to demonstrate the close relationship of the obtained minimum set with the optimal one.

3.13. Chemical reaction optimization (CRO)

A recently established heuristic method called chemical reaction optimization (CRO) based on population was introduced to solve optimization problems. Obtaining a lower energy stable state by simulating the action and reaction of molecules in a chemical reaction was the main process of CRO, which aimed to reach the minimum state of free energy. Applying the CRO to benchmarks and practical problems showed its high efficiency [57].

Table 1
Test systems data.

Test system	Number of branches	Number of zero-injection buses	Location of zero-injection buses
IEEE_14	20	1	7
IEEE_30	41	5	6–9–11–25–28
IEEE_39	46	12	1–2–5–6–9–10–11–13–14–17–19–22
IEEE_57	78	15	4–7–11–21–22–24–26–34–36–37–39–40–45–46–48
IEEE_118	179	10	5–9–30–37–38–63–64–68–71–81

Table 2
Comparison between results of optimal PMU placement algorithms considerin test systems with zero-injection buses.

Method	Test systems				
	14-Bus	30-Bus	39-Bus	57-Bus	118-Bus
Genetic algorithm (GA) [31]	3	7	9	12	29
Genetic algorithm (GA) [33]	3	7	–	12	29
Binary particle swarm optimization (BPSO) [35]	3	7	8	11	28
Binary particle swarm optimization (BPSO) [38]	3	7	–	13	29
Simulated annealing (SA) [39]	3	7	–	11	–
Recursive tabu search (RTS) [44]	3	7	8	11	28
Immune genetic algorithm (IGA) [52]	3	7	–	11	29
Binary imperialistic competition algorithm (BICA) [28]	3	7		11	28
Matrix reduction [56]	3	8	–	12	29
Chemical reaction optimization (CRO) [58]	3	7		14	29

Table 3
Comparison between results of optimal PMU placement algorithms considering test systems without zero-injection buses.

Method	Test systems				
	14-Bus	30-Bus	39-Bus	57-Bus	118-Bus
Differential evolution (DE) [27]	4	10	13	17	–
Genetic algorithm (GA) [31]	4	10	13	16	32
Binary particle swarm optimization (BPSO) [38]	4	10	–	17	32
Iterated local search (ILS)	4	–	–	17	32

Table 4
Obtained minimum number of PMUs for different test systems considering the system with and without zero-injection buses.

Test systems	Minimum number of PMUs considering zero-injection buses	Percentage of buses equipped with PMU considering zero-injection buses	Minimum number of PMUS without considering zero-injection buses	Percentage of buses equipped with PMU without considering zero-injection buses
14-Bus	3	21.43	4	28.57
30-Bus	7	23.33	10	33.33
39-Bus	8	20.51	13	33.33
57-Bus	11	19.30	16	28.07
118-Bus	28	23.73	32	27.12

Table 5
Optimal PMU placements for obtained minimum number of PMUs considering the power system with zero-injection buses.

Test system	PMU placements (Bus no.)	Reference
14-Bus	2, 6, 9	[2], [20], [22], [28], [31], [35], [38], [41], [43], [44], [52], [58], [59], [60]
30-Bus	1, 5, 10, 12, 15, 18, 29	[20]
	1, 2, 10, 12, 15, 20, 27	[31]
	2, 3, 10, 12, 18, 24, 27	[35], [60]
	1, 7, 10, 12, 19, 24, 27	[38]
	1, 5, 10, 12, 18, 23, 27	[44]
	1, 5, 10, 12, 18, 24, 27	[44]
	3, 5, 10, 12, 18, 23, 27	[44]
	3, 5, 10, 12, 18, 24, 27	[44]
	1, 5, 10, 12, 19, 24, 27	[44]
	2, 4, 10, 12, 18, 24, 27	[44]
	1, 2, 10, 12, 18, 24, 27	[44]
	1, 5, 10, 12, 18, 24, 30	[52]
	2, 4, 10, 12, 15, 19, 27	[28]
	1, 7, 10, 12, 19, 23, 27	[58]
39-Bus	3, 8, 13, 16, 20, 23, 25, 29	[31]
	3, 8, 12, 16, 20, 23, 25, 29	[35]
57-Bus	1, 5, 13, 19, 25, 26, 32, 38, 41, 51, 54	[31], [35]
	1, 4, 13, 19, 25, 29, 32, 38, 41, 51, 54	[44]
	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56	[44], [59]
	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56	[44], [52]
	1, 6, 13, 19, 25, 29, 32, 38, 41, 51, 54	[44]
	1, 4, 13, 20, 25, 29, 32, 38, 41, 51, 54	[44]
	1, 5, 13, 19, 25, 29, 32, 38, 41, 51, 54	[28]
118-Bus	2, 8, 11, 12, 17, 21, 25, 28, 33, 34, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	[35], [60]
	3, 8, 11, 12, 17, 20, 23, 29, 34, 37, 40, 45, 49, 53, 56, 62, 73, 75, 77, 80, 85, 86, 91, 94, 101, 105, 110, 115	[44]
	3, 8, 11, 12, 19, 22, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	[44]
	3, 8, 11, 12, 17, 20, 23, 29, 34, 37, 40, 45, 49, 52, 56, 62, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 115	[44]
	3, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 42, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110	[44]
	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	[44], [59]
	3, 8, 11, 12, 19, 22, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	[44]
	3, 8, 11, 12, 17, 21, 25, 28, 34, 35, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114	[52]
	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	[28]

OPP problem solving for reaching a fully observable power system considering the network with and without zero-injection was discussed by utilizing a new heuristic concept called CRO and the simplified model of CRO (SCRO). On-wall ineffective collision was the only reaction introduced in SCRO, while four reactions were utilized in canonical CRO. More efficiency, well adaption, simple structure and shorter time requirement remarked as SCRO priority over CRO. Only approximate solution generation was obtained by an operator in SCRO, while infeasible solutions might be provided by decomposition and synthesis operations in canonical CRO method [58].

3.14. Bacterial foraging algorithm (BFO)

Tendency of natural selection to animal omission with poor foraging strategies which function to locate, handle, and ingest food and propagation of the genes of successes animal in foraging strategies since they are more likely to reproductive success enjoyment. Poor foraging strategies are either eliminated or redesigned after many generations. Chemotaxis, swarming, reproduction, and elimination and dispersal are four operators used in the proposed bacterial foraging optimization (BFO) [61].

Ref. [2] formulated OPP problem by utilizing a mathematical procedure called integer linear programming (ILP) and a new optimization concept based on BFO which analyzed complete observability of a power system. This method considers the system with and without conventional measurements such as zero-injections or flows and reaches a high measurement redundant value. Only one installation solution is exhibited per studied case obtained between several solutions to compare the results with those of other methods.

3.15. Artificial bee colony (ABC)

The basis of artificial bee colony (ABC) algorithm is to examine the behaviors of real bees in terms of finding nectar and sharing the information of food resources to the bees through waggle dance in the hive. Three essential components of this method include food sources, employed foragers and, unemployed foragers. The basic motivation factors of CLA algorithm are direction of food resources, distance of food resources, and information about the quality of food resources. The proposed algorithm in [62] could be utilized to solve uni-modal and multi-modal numerical optimization problems.

A multi-objective OPP (MOPP) algorithm, named as binary coded ABC, was proposed in [59] to solve OPP problem and achieve the minimum number of PMUs and maximum redundancy of the system. Contingency constraint situation such as single branch outage was taken into account and configurations of the minimum number of required PMUs were determined.

3.16. Cellular learning automata (CLA)

Cellular learning automata (CLA) algorithm is based on the usage of learning automata (LA) to the state transition probability adjustment of cellular automata (CA). This method starts by specifying the internal state of every cell. Then, the reinforcement signal for each LA is determined on the basis of the rule of CLA. Finally, as for supplied reinforcement signal and action chosen by the cell, each LA updates its action probability vector. The desired state will be reached by continuing this process [63].

Table 6
Optimal PMU placements for obtained minimum number of PMUs considering the power system without zero-injection buses.

Test system	PMU placements (Bus no.)	Reference
14-Bus	2, 6, 7, 9	[20], [27], [31], [32], [38], [48], [55], [58], [59]
	2, 7, 11, 13	[27], [48], [55]
	2, 7, 10, 13	[27], [48], [55]
	2, 6, 8, 9	[27], [48]
	2, 8, 10, 13	[27]
30-Bus	1, 5, 6, 9, 10, 12, 15, 18, 25, 29	[20]
	1, 5, 6, 9, 10, 12, 15, 19, 25, 29	[4], [31]
	1, 5, 8, 10, 11, 12, 19, 23, 26, 29	[27]
	1, 5, 10, 11, 12, 19, 23, 25, 27, 28	[27]
	1, 6, 7, 10, 11, 12, 18, 23, 26, 30	[27]
	1, 7, 10, 11, 12, 19, 24, 26, 28, 30	[27]
	1, 5, 9, 10, 12, 19, 23, 26, 27, 28	[27]
	2, 3, 6, 9, 10, 12, 18, 23, 25, 29	[27]
	1, 5, 8, 10, 11, 12, 19, 23, 26, 29	[27]
	1, 7, 10, 11, 12, 18, 24, 25, 28, 30	[27]
	3, 5, 9, 10, 12, 19, 24, 25, 27, 28	[27]
	3, 5, 8, 10, 11, 12, 18, 23, 26, 29	[27]
	3, 5, 10, 11, 12, 15, 18, 25, 27, 28	[27]
	1, 5, 9, 10, 12, 19, 24, 26, 27, 28	[27]
	1, 5, 8, 9, 10, 12, 18, 24, 26, 30	[27]
	1, 7, 10, 11, 12, 15, 20, 25, 27, 28	[27]
	1, 5, 8, 9, 10, 12, 15, 20, 25, 29	[27]
	1, 5, 6, 10, 11, 12, 18, 24, 26, 27	[27]
	1, 5, 8, 9, 10, 12, 19, 24, 25, 27	[27]
	1, 5, 6, 9, 10, 12, 18, 24, 25, 27	[27]
	1, 2, 6, 9, 10, 12, 15, 19, 25, 27	[31]
	2, 4, 6, 9, 10, 12, 15, 19, 25, 27	[32]
	2, 4, 6, 9, 10, 12, 15, 18, 25, 27	[38]
	3, 5, 8, 9, 10, 12, 15, 19, 25, 27	[55]
	2, 4, 6, 9, 10, 12, 19, 23, 25, 26	[58]
39-Bus	2, 6, 9, 10, 13, 14, 17, 19, 22, 23, 25, 29, 34	[4], [27], [31]
	2, 6, 9, 12, 14, 17, 22, 23, 25, 29, 32, 33, 34	[27]
	2, 6, 9, 12, 14, 17, 22, 23, 29, 32, 33, 34, 37	[27]
	2, 6, 9, 10, 11, 14, 17, 22, 23, 29, 33, 34, 37	[27]
	2, 6, 9, 11, 14, 17, 19, 22, 23, 29, 32, 34, 37	[27]
	2, 6, 9, 10, 12, 14, 17, 22, 23, 25, 29, 33, 34	[27]
	2, 6, 9, 12, 14, 17, 20, 22, 23, 25, 29, 32, 34	[27]
	2, 6, 9, 12, 14, 17, 20, 22, 23, 25, 29, 32, 33	[27]
	2, 6, 9, 10, 12, 14, 17, 20, 22, 23, 25, 29, 33	[27]
	2, 6, 9, 10, 12, 14, 17, 19, 22, 23, 25, 29, 34	[27]
	2, 6, 9, 13, 14, 17, 19, 20, 22, 23, 29, 32, 37	[27]
	2, 6, 9, 11, 14, 17, 19, 22, 23, 25, 29, 32, 34	[27]
	2, 6, 9, 10, 11, 14, 17, 19, 20, 22, 23, 25, 29	[31], [65]
57-Bus	1, 6, 9, 15, 19, 22, 25, 28, 32, 36, 38, 41, 47, 51, 53, 57	[31], [58]
118-Bus	3, 5, 9, 12, 25, 17, 21, 23, 28, 30, 36, 40, 44, 46, 51, 54, 57, 62, 64, 68, 71, 75, 80, 85, 86, 91, 94, 101, 105, 110, 114	[4], [31]
	3, 5, 9, 12, 15, 17, 21, 23, 28, 30, 36, 40, 44, 46, 51, 54, 57, 62, 64, 68, 71, 75, 80, 85, 86, 91, 94, 101, 105, 110, 114	[31]
	2, 5, 9, 12, 15, 17, 21, 25, 29, 34, 37, 42, 45, 49, 53, 56, 62, 63, 68, 70, 71, 75, 77, 80, 85, 86, 91, 94, 102, 105, 110, 114	[58]

Table 7
Optimal number and placement of PMUs using different methods aiming to reach maximum redundancy considering the system with zero-injection buses.

Test systems	PMU placements (Bus no.)	Average redundancy	Reference
14-Bus	2, 6, 9	1.143	[28], [59], [64]
30-Bus	2, 4, 10, 12, 15, 18, 27	1.467	[64]
	2, 4, 10, 12, 15, 19, 27	1.367	[28]
39-Bus	3, 8, 10, 16, 20, 23, 25, 29	1.103	[64]
57-Bus	1, 4, 13, 20, 25, 29, 32, 38, 51, 54, 56	1.105	[64]
	1, 5, 13, 19, 25, 29, 32, 38, 41, 51, 54	1.035	[28]
118-Bus	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110	1.322	[28]

Incorporation of two conflicting objective functions including minimum number of PMUs and maximum measurement redundancy to attain a fully observable power system was investigated in [64] using a CLA method. This work also considered contingency constraints including PMU/line outage plus conventional measurements and zero-injection buses.

3.17. Hybrid methods

A solution for optimally installing PMUs and RTUs for a large system or connected grids identified by a multi-area system state estimation was presented using a developed hybrid GA and SA in [66]. A PMU installation was added to a power system via RTU and

Table 8
Optimal number and placement of PMUs using different methods in one PMU failure mode considering the system with zero-injection buses.

Test systems	Method	Minimum number of PMUs	PMU placements (Bus no.)
14-bus	Biogeography based optimization (BBO) [20]	7	1, 3, 6, 7, 9, 10, 13
	Iterative method [68]	7	1, 2, 4, 6, 9, 10, 13
	Binary imperialistic competition algorithm (BICA) [28]	7	2, 4, 5, 6, 9, 10, 13
30-bus	Biogeography based optimization (BBO) [20]	15	1, 3, 5, 6, 9, 10, 12, 15, 16, 19, 21, 24, 25, 27, 29
	Iterative method [68]	15	1, 2, 3, 5, 6, 10, 12, 13, 15, 1, 18, 19, 24, 27, 30
	Binary imperialistic competition algorithm (BICA) [28]	13	2, 3, 4, 7, 10, 12, 15, 17, 18, 20, 24, 27, 29
57-bus	Iterative method [68]	26	1, 2, 4, 6, 9, 12, 15, 18, 19, 22, 24, 25, 27, 29, 30, 32, 33, 36, 38, 41, 47, 50, 51, 53, 54, 56
	Binary imperialistic competition algorithm (BICA) [28]	22	1, 3, 6, 9, 12, 15, 19, 20, 25, 28, 29, 30, 32, 33, 38, 41, 46, 50, 51, 53, 54, 56
	Cellular learning automata (CLA) [64]	25	1, 3, 4, 6, 9, 10, 12, 13, 15, 19, 20, 25, 27, 29, 30, 32, 33, 37, 38, 41, 49, 51, 53, 54, 56
118-bus	Iterative method [68]	64	1, 2, 5, 6, 8, 9, 11, 12, 15, 17, 19, 20, 21, 23, 25, 27, 28, 29, 32, 34, 35, 37, 40, 41, 43, 45, 46, 49, 50, 51, 52, 53, 56, 59, 62, 66, 68, 70, 71, 72, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 106, 108, 110, 111, 112, 114, 117
	Binary imperialistic competition algorithm (BICA) [28]	61	1, 3, 7, 8, 9, 11, 12, 15, 17, 19, 21, 22, 23, 24, 27, 29, 31, 32, 34, 35, 40, 42, 44, 45, 46, 49, 51, 52, 54, 56, 57, 59, 62, 66, 68, 70, 71, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 91, 92, 94, 96, 100, 101, 105, 106, 108, 110, 11, 112, 115, 117

Table 9
Optimal number and placement of PMUs using different methods in one line outage mode considering the system with zero-injection buses.

Test systems	Method	Minimum number of PMUs	PMU placements (Bus no.)
14-bus	Biogeography based optimization (BBO) [20]	7	1, 3, 6, 7, 9, 10, 13
	Exhaustive search [69]	7	2, 4, 5, 6, 9, 10, 13
	Binary imperialistic competition algorithm (BICA) [28]	7	2, 4, 5, 6, 9, 10, 13
30-bus	Biogeography based optimization (BBO) [20]	11	2, 3, 7, 8, 10, 12, 15, 18, 20, 24, 29
	Exhaustive search [69]	10	2, 3, 5, 10, 12, 15, 17, 19, 24, 27
	Binary imperialistic competition algorithm (BICA) [28]	11	2, 3, 6, 7, 10, 12, 15, 16, 19, 24, 29 1, 4, 5, 6, 10, 12, 15, 17, 19, 24, 30
57-bus	Binary imperialistic competition algorithm (BICA) [28]	19	1, 3, 6, 12, 14, 15, 19, 27, 29, 30, 32, 33, 38, 41, 49, 51, 53, 55, 56
	Cellular learning automata (CLA) [64]	19	1, 2, 6, 12, 14, 19, 21, 27, 29, 30, 32, 33, 41, 44, 49, 51, 53, 55, 56
118-bus	Binary imperialistic competition algorithm (BICA) [28]	53	1, 6, 10, 11, 12, 15, 17, 19, 21, 23, 24, 25, 27, 29, 32, 34, 35, 40, 42, 44, 46, 49, 51, 53, 56, 57, 59, 62, 6, 70, 73, 75, 76, 78, 80, 83, 85, 87, 89, 91, 92, 94, 96, 100, 102, 105, 106, 109, 109, 111, 112, 115, 166, 117

Table 10
Optimal number and placement of PMUs using different methods in one line/PMU outage mode considering the system with zero-injection buses.

Test systems	Method	Minimum number of PMUs	PMU placements (Bus no.)
14-bus	Binary particle swarm optimization (BPSO) [35]	7	1, 2, 4, 6, 9, 10, 13
	Biogeography based optimization (BBO) [20]	8	2, 4, 5, 6, 7, 9, 11, 13
30-bus	Biogeography based optimization (BBO) [20]	13	2, 3, 7, 10, 12, 15, 17, 19, 22, 23, 25, 27, 29
	Binary particle swarm optimization (BPSO) [35]	15	2, 3, 4, 8, 10, 12, 13, 15, 16, 18, 20, 22, 24, 27,30
39-bus	Binary particle swarm optimization (BPSO) [20]	17	3, 7, 8, 12, 13, 16, 20, 21, 23, 25, 26, 30, 34, 36, 37, 38
57-bus	Binary particle swarm optimization (BPSO) [35]	22	1, 2, 4, 9, 12, 15, 18, 19, 25, 28, 29, 30, 32, 33, 38, 41, 47, 50, 51, 53, 54, 56
	Cellular learning automata (CLA) [64]	25	1, 3, 4, 6, 9, 10, 12, 13, 15, 19, 20, 25, 29, 30, 32, 33, 37, 38, 41, 49, 51, 53, 54, 56
118-bus	Binary particle swarm optimization (BPSO) [35]	62	1, 3, 7, 8, 10, 11, 12, 15, 17, 19, 21, 22, 24, 25, 27, 28, 29, 32, 34, 35, 40, 41, 44, 45, 4, 49, 50, 51, 52, 54, 56, 59, 62, 66, 68, 72, 73, 74, 75, 76, 77, 78, 80, 83, 85, 86, 87, 89, 90, 92, 94, 96, 100, 101, 105, 107, 109, 110, 111, 112, 115, 117

Table 11
Investigation of optimal PMU placement problem for large scale power systems.

Method	Test system	Minimum number of PMUs	Percentage of buses equipped with PMU	Supplemental information
Simulated annealing (SA) [70]	Italy 129-bus	35	27.13	–
Simulated annealing (SA) [70]	WSCC 173-bus	34	19.65	–
Simulated annealing (SA) [71]	Taiwan 199-bus	39	19.60	–
Cellular learning automata (CLA) [64]	Iranian 242-bus	71	29.34	PMU placements: 4, 6, 9, 16, 18, 19, 23, 28, 36, 39, 43, 45, 56, 57, 60, 61, 62, 72, 78, 88, 93, 95, 97, 98, 99, 101, 102, 106, 108, 111, 115, 117, 126, 129, 133, 134, 138, 143, 147, 153, 154, 156, 160, 163, 164, 169, 177, 179, 183, 185, 187, 188, 192, 195, 197, 198, 201, 202, 203, 206, 207, 210, 211, 212, 217, 222, 225, 228, 232, 233, 240
Sequential elimination algorithm (SEA) [72]	Northern region power grid 246-bus Indian system	70	28.46	6, 7, 11, 24, 29, 34, 35, 40, 42, 45, 48, 54, 55, 57, 61, 62, 63, 65, 69, 73, 74, 7, 80, 83, 91, 93, 94, 95, 96, 98, 101, 106, 109, 119, 122, 125, 126, 128, 129, 132, 134, 141, 142, 14, 153, 157, 158, 160, 167, 168, 169, 174, 180, 181, 183, 185, 187, 190, 191, 194, 199, 201, 202, 203, 215, 216, 219, 234, 235, 242
Simulated annealing (SA) [71]	Taiwan 265-bus	61	23.02	–
Non-linear iterative technique [73]	270-Bus	90	33.33	Number of lines: 326
Chemical reaction optimization (CRO) [58]	300-Bus	87	29	PMU placements: 1, 2, 3, 11, 12, 15, 17, 20, 23, 24, 26, 33, 35, 39, 43, 44, 49, 55, 57, 61, 62, 63, 70, 71, 72, 74, 77, 78, 81, 86, 97, 98, 104, 105, 108, 109, 114, 119, 120, 122, 130, 132, 133, 134, 137, 139, 140, 143, 153, 154, 159, 160, 164, 166, 173, 178, 181, 184, 194, 198, 204, 208, 210, 211, 214, 217, 223, 225, 229, 231, 232, 234, 237, 238, 240, 243, 245, 249, 251, 252, 253, 254, 256, 257, 258, 259, 261
Non-linear iterative technique [73]	444-Bus	121	27.25	Number of lines: 574
Chemical reaction optimization (CRO) [58]	1180-Bus	144	12.20	With considering zero-injection buses
Chemical reaction optimization (CRO) [58]	1180-Bus	181	15.34	Without considering zero-injection buses
Matrix reduction [56]	Brazil 1495-bus	390	26.09	Number of lines: 1932 Number of zero-injection buses: 64
Tabu search (TS) [44]	2383-Bus	550	23.08	Number of lines: 2896 Number of zero-injection buses: 552
Immune genetic algorithm (IGA) [52]	2746-Bus	609	22.18	Number of lines: 3514 Number of zero-injection buses: 705

conventional measurement to make the estimated state more certitude and decrease the cost of conventional measurements and RTU cost. Control center of one area of a multi-area to access the state estimates needed just one PMU installation, because the phasor of the system bus voltage was obtained by PMU measurement. The bus with maximum connected branches was a consideration place for this PMU. Bad data detection was done by considering the critical measurement of each area.

A combination of minimum spanning tree (MST) algorithm with improved GA constitutes a hybrid approach called MST-GA proposed in [67] to reach the minimum number of PMUs needed for making a network completely observable and considering redundancy maximization. This method improved the operation of mutation considering topological knowledge of grid. Unfeasible solutions for OPP problem were repaired as a new consideration of this paper. Crossover and mutation were utilized as an operation to generate new individuals as the main and side steps. Decreasing the number of needed PMUs and diversity of solutions was the results of new consideration of this method [67].

4. Comparison of heuristic algorithms with different points of views

The IEEE 14-bus, 30-bus, 39-bus, 57-bus, and 118-bus test systems are mostly utilized for observability analysis studies using different optimization methods. Table 1 shows the data for the mentioned IEEE test systems including line numbers and number and location of zero-injection buses. Comparisons of the results of different heuristic algorithms used for solving OPP problem are shown in Tables 2 and 3, respectively. To access a completely observable power system considering zero-injection buses, the minimum number of required PMUs obtained by several methods is represented in Table 2. Table 3 tabulates the results of the minimum number of needed PMUs to obtain a fully observable power system without considering zero-injection buses. Table 4 provides the minimum number of PMUs required to access a completely observable system and percentage of the buses equipped with PMU in both conditions of zero-injection buses and non-zero-injection buses. Several configurations for reaching the minimum number of PMUs have been exhibited in different papers. Table 5 shows different installations of PMUs to reach a fully observable power system considering the system with zero-injection buses. Optimal PMU placement for the minimum number of PMUs considering the system without zero-injection buses can be seen in Table 6. The results of the works considering maximum redundancy in solving OPP problem in a power system are tabulated in Table 7. This table presents the optimal placement of PMUs for achieving maximum measurement redundancy and average obtained redundancy. Different contingency constraints including one PMU failure, one branch outage, and a PMU/line outage were handled by applying heuristic algorithms. Tables 8–10 present the minimum number of required PMUs and their configurations in the systems considering the mentioned constraints. To discuss the implementation of heuristic algorithms for placing the minimum number of PMUs in large-scale power systems by attaining

Table 12
Objective functions and contribution of analyzed papers.

Reference	Objective function(s) and main contributions	Method	Year	Main considerations	Advantage
[30]	Minimizing number of PMUs and maximizing the measurement redundancy. Pareto-optimal solutions are provided instead of a single optimal solution	Non-dominated sorting genetic algorithm (NSGA)	2003	Measurement redundancy, Zero-injection buses	Providing Pareto-optimal front for conflicting objectives, solution repair (correction of infeasible solutions)
[33]	Minimizing number of PMUs and finding their geographic distribution, attaining a complete observable power system. A GA-based method is utilized and equipping PMUs with current phasor measurements as the maximal number of concurrent lines in all buses of the system is highlighted	Genetic Algorithm (GA)	2003	Relationship between PMUs and the number of current phasors that must be measured	Considering required current channels in the optimization problem
[21]	Sensitivity constrained PMU placement to attain a full observable power system has been investigated by utilizing SA method. Obtained placements for the PMUs by this procedure not only ensure observability of the power system, but also provide more valuable dynamic data of power systems at the same time	Simulated annealing (SA)	2005	Zero-injection buses-Placement of PMUs on buses with higher parameter sensitivities	Considering parameter sensitivity
[39]	Minimizing number of the PMUs. Modeling depth of observability, using spanning trees of the power system graph	Simulated annealing (SA)	2005	Incomplete observability based on depth of unobservability	Communication-constrained PMU placement,
[43]	Two competing objectives including minimum number of PMUs and enough redundancy	Tabu Search (TS)	2006	Maximum measurement redundancy- Zero-injection buses	Solutions with high accuracy and less computational effort
[48]	Two opponent objectives which include minimum number of PMUs and maximum measurement redundancy of the system	An adaptive clonal algorithm (CLONALG)	2006	Maximum measurement redundancy	High velocity of process, Obtaining feasible schemes
[23]	Optimal placement of PMUs to enable bad data detection. SA algorithm with stochastic new solution generating is introduced	Simulated annealing (SA)	2007	Critical measurement recognition	Power system observable with critical measurement free
[4]	Minimizing number of the PMUs. Monitoring pilot buses required for secondary voltage control	Branch and bound (B and B)	2008	Zero-injection buses-improvement of secondary voltage control performance	Monitoring pilot buses for increasing velocity of voltage control scheme
[22]	Minimizing number of the PMUs. Proposing new parallel TS algorithm	A new parallel Tabu search (TS)	2008	Zero-injection buses, communication constraint, State estimation matrix condition	Less computational time.
[24]	Minimizing number of the PMUs using the proposed SSA algorithm	Stochastic simulated annealing (SSA)	2008	Critical measurement recognition is included as a penalty function.	Critical measurement recognition.
[56]	Minimizing number of PMUs. Using preprocessing method and solving using mathematical based methods	Matrix reduction	2008	Virtual data elimination preprocessing method and matrix reduction algorithm, using Lagrangian relaxation	Reducing the size of the placement model and the computational effort, applied to large scale system
[74]	Minimizing number of PMUs for full observability. Proposing hybrid algorithm based on BPSO method and immune mechanism	Binary particle swarm optimization (BPSO)	2008	Maximum measurement redundancy, single PMU and multi PMU fault	High speed of process and simplified function
[31]	Minimizing number of PMUs, topological based observability formulation	Branch and bound and genetic algorithm (GA)	2009	Zero-injection buses	Formulating as mixed integer linear and nonlinear programming
[47]	Minimizing number of PMUs. Proposing improved ant colony algorithm	Improved ant colony optimization	2009	Maximum measurement redundancy,	Escaping from stagnation behavior and high speed of process, applying a graph-theoretic procedure based on depth first search
[52]	Minimizing number of PMUs, proposing improved IGA which is based on utilization of the local and prior knowledge associated with the considered problem	Immune genetic algorithm (IGA)	2009	Zero-injection buses, considering three new impactful vaccines	A remarkable growth in process speed, applied to large scale system, prevention from familial reproduction
[66]	Minimizing the total number of PMUs and RTUs with critical measurement free	Hybrid Genetic algorithm and simulated annealing (HGS)	2009	Conventional measurement and remote terminal unit (RTU), bad data detection, current measurement loss	Applicable to current power systems monitored using RTUs.
[2]	Minimizing the number of PMUs and maximizing redundancy, considering conventional measurement	Bacterial Foraging algorithm (BFA)	2010	Zero-injection buses- -Maximum measurement redundancy	Proper for real word current power system due to modeling conventional measurements
[26]	Minimizing mean square error (MSE) by obtaining the minimum number of PMUs, with or without existence of conventional measurements	Differential evolution (DE)	2010	Conventional measurements-Minimum square error (MSE) of state estimation	Accurate, quick and simple process, capability of apply in multi-objective problems
[36]	Minimizing total PMU installation cost, modeling non-uniform cost of PMUs for different buses	Particle swarm optimization (PSO)	2010	Non-uniform cost of PMU placements	Considering realistic installation cost of PMUs, minimizing total cost instead of number of PMUs

Table 12 (*continued*)

Reference	Objective function(s) and main contributions	Method	Year	Main considerations	Advantage
[41]	Minimizing number of PMUs and maximizing measurement reliability of the power system, multi-objective optimization using NSDE algorithm	Non-dominated sorting differential evolution (NSDE)	2010	Zero-injection buses- Maximum measurement reliability	Accurate and complete Pareto front achievement- flexibility, diversity and practically of the method
[51]	Minimizing total number of PMUs using page rank placement algorithm (PPA) and ILS	Iterated local search (ILS)	2010	PMU failure	Considering contingencies, easy understanding and implementing
[20]	Minimizing number of meters and PMUs considering single branch/meter outage and single branch/PMU outage	Biogeography based optimization (BBO)	2011	Zero-injection buses- PMU failure- line outage- PMU/line outage- SCADA meter outage	Utilizing virtual bus reduction technique for reducing the scale of the system, robustness against the outages
[27]	Minimizing mean square error (MSE) of state estimation by minimum number of PMUs	Differential evolution (DE)	2011	Considering continuous changes in the power system's topology	Incorporating PMU placement problem into state estimation problem
[32]	Multi objective model, minimizing number of PMUs and maximizing measurement redundancy	Genetic algorithm (GA)	2011	One line/one PMU outage is considered	Very small population size and less number of iterations
[35]	Minimizing number of PMUs, proposing new rule of topological observability of the system	Modified binary particle swarm optimization (BPSO)	2011	Zero-injection buses- PMU/line outage	Introducing new rules of topological observability assessment for reducing number of the required PMUs
[38]	Minimizing number of PMUs in presence of conventional measurement such as injections and flows	Binary particle swarm optimization (BPSO)	2011	Zero-injection buses, conventional measurement, maximum measurement redundancy	Modeling different types of conventional measurements in OPP formulation
[55]	Two conflicting objectives as obtaining minimum number of PMUs and maximizing measurement redundancy	Multi objective biogeography based optimization (MO-BBO)	2012	Zero-injection buses, measurement redundancy	The proposed MO-BBO algorithm produces well distributed Pareto optimal solutions than NSGA-II and NSDE
[37]	Minimizing number of PMUs, fault tolerant PMU placement formulation	Binary particle swarm optimization (BPSO)	2013	Control reconfigurability criterion, modeling data loss at a given number of PMUs	Robustness of the solution against data loss at a given number of PMUs
[25]	Minimizing number of PMUs to attain both general and fault observability of the power system	Differential evolution (DE)	2013	Fault observability- Zero-injection buses	Using PMUs for power system observability and fault observability
[44]	Minimizing number of PMUs using a Recursive tabu search	Recursive tabu search (RTS)	2013	Zero-injection buses, maximum measurement redundancy	Applied to large scale power system
[49]	Minimizing the number of PMUs using an information theoretic concept, namely Mutual information, uncertainty modeling	Mutual information (MI)	2013	Conventional measurement, PMU failure	Modeling the uncertainties in the system states
[28]	Minimizing number of PMUs required for complete observability and maximize measurement redundancy. A new topological observability rule of zero-injection buses is also introduced	Binary imperialistic completion algorithm (BICA)	2013	Zero-injection buses- PMU failure- Line outage- PMU/line outage- Measurement redundancy	Fast convergence, small deviation, capability of finding global optimum and zero standard deviation
[58]	Minimum number of PMUs is obtained to reach full observability	Chemical reaction optimization (CRO) and simplified version of CRO (SCRO)	2013	Zero-injection buses, large scale power system	Efficiency, adaptability, simple structure and less computational time
[64]	A multi-objective optimal placement of Minimizing number of PMUs and maximizing measurement redundancy. PMU placement is also investigated in presence of conventional non-synchronous by introducing a generalized observability function	Cellular learning automata (CLA)	2013	PMU failure- Line outage- PMU/line outage, maximum measurement redundancy, conventional measurements	Good efficiency in large scale power systems
[67]	Minimizing number of PMUs using a hybrid method	Combination of Minimum spanning tree algorithm with improved genetic algorithm (MST-GA)	2013	Maximum measurement redundancy	This method has a capability of repairing infeasible solutions and well balances between efficiency of reparation and quality of solutions
[59]	ABC concept is applied to obtain minimum number of PMUs to attain a full observable power system, satisfying measurement redundancy	Artificial bee colony (ABC)	2014	Zero-injection buses- Single line outage	Feasibility and performance of the method demonstrated by comparing the simulation results with the earlier works
[75]	A mixed-integer programming model is applied to solve OPP problem in ac/dc systems	Mixed-integer programming	2014	Non-uniform cost of PMUs, Limited PMU measurement channels, Integration of dc transmission lines	Considering variable-cost for different installations of PMUs as a function of the number and type of measurement channels.

a completely observable system, Table 11 can be referred to. This table contains the minimum number of needed PMUs and percentage of the buses equipped with PMU along with the supplemental information of OPP available in the presented work. Finally, Table 12 determines different objective functions and their contributions to the OPP problem.

5. Conclusion

Optimal PMU placement (OPP) problem has been solved by utilizing numerous optimization methods. To obtain solutions for the OPP problem, two major techniques including conventional methods and heuristic algorithms exist; this paper provided a comprehensive literature review on heuristic algorithms. A general comparison of the results of introduced heuristic methods and some specific features of the works was shown. Detailed comparison of the obtained results of different methods applied to standard test systems like IEEE 14-bus, 30-bus, 39-bus, 57-bus, and 118-bus was also provided. The presented review of heuristic optimization techniques could largely help researchers in terms of employing new concepts to solve the OPP problem. Future works will contain new heuristic optimization approaches for multi-objective optimal PMU placement considering the constraints.

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