Demand Response Program in Smart Grid Using Supply Function Bidding Mechanism

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***Abstract*—In smart grid, customers have access to the electricity consumption and the price data via smart meters; thus, they are able to participate in the demand response (DR) programs. In this paper, we address the interaction among multiple utility companies and multiple customers in smart grid by modeling the DR problem as two noncooperative games: the supplier and customer side games. In the first game, supply function bidding mechanism is employed to model the utility companies' profit maximization problem. In the proposed mechanism, the utility companies submit their bids to the data center, where the electricity price is computed and is sent to the customers. In the second game, the price anticipating customers determine optimal shiftable load profile to maximize their daily payoff. The existence and uniqueness of the Nash equilibrium in the mentioned games are studied and a computationally tractable distributed algorithm is designed to determine the equilibrium. Simulation results demonstrate the superior performance of the proposed DR method in increasing the utility companies' profit and customers' payoff, as well as in reducing the peak-to-average ratio in the aggregate load demand. Finally, the algorithm performance is compared with a DR method in the literature to demonstrate the similarities and differences.**

***Index Terms*—Bidding mechanism, demand response (DR), Nash equilibrium, noncooperative game.**

I. Introduction

SMART GRID provides integration of bidirectional communication and advanced control technologies for supervision of power systems. It also enables automated energy management to improve load management and energy efficiency based on information gathered from energy utility companies and customers. In smart grid, the customers are active parts of the grid. They can participate in the demand response (DR) programs, consider the available information on the electricity price and make wise decisions regarding their daily electricity consumption [1]. The main objective of DR programs is to encourage customers to consume less power during peak times or to shift energy use to off-peak hours to flatten the demand curve. DR program can be used as an effective tool to accomplish different load shaping objectives, such as peak clipping, valley filling, and load shifting [2]. Combination of the mentioned techniques enables the load shape to follow generation as close as possible; nevertheless, it is an existing challenge to adopt effective DR programs which consider both utility companies' and customers' objectives. In this paper, we are interested in investigating how multiple utility companies would deal with DR in a smart grid system.

In the literature, there are several studies that address the DR. Bahrami and Parniani [3] proposed a DR program for residential customers. This paper models an optimization problem consisting of the electricity bill and customers' discomfort costs to determine the optimal start time of use for residential appliances. Guo *et al.* [4] proposed a decentralized aggregated control method to schedule residential appliances which can have multiple operation modes. The interaction among the appliances is modeled as a potential game. Arai *et al.* [5] proposed a game theoretic framework for analyzing the decentralized and centralized control in smart grids. The decentralized control scheme is formulated using a differential game, and the centralized control scheme is formulated as an optimal control problem. Baharlouei and Hashemi [6] studied the concept of customers' privacy in DR programs. They design a billing mechanism to minimize the aggregate cost of the system. A distributed algorithm is also designed that preserve the privacy of the customers. Deng *et al.* [7] modeled the DR problem as a coupled-constraint game. The coupled-constraint game is transformed into a decoupled one using dual decomposition method. Deng *et al.* [8] proposed a distributed real-time DR algorithm for smart grid with multiple utility companies to determine each user's demand and each utility company's supply simultaneously. By applying dual decomposition, each utility company and user locally solve sub-problems to perform energy allocation. Chen *et al.* [9] developed a billing mechanism to fairly charge customers for energy consumption. An aggregative game approach is used to model the strategic behaviors of the customers. Maharjan *et al.* [10] studied the DR program based on a Stackelberg game between utility companies and customers

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to maximize the revenue of each utility company and the payoff of each user. It is proved that a unique equilibrium exists. Chai *et al.* [11] studied the DR problem in the system with multiple utility companies and multiple residential customers. The problem is modeled as a two-level game. At the lower level, the interaction among residential users is modeled as an evolutionary game. At the higher level, the competition among the utility companies is modeled as a noncooperative game.

The DR problem has been studied widely in smart grid literature [3]–[11]. In most of the aforementioned studies, one supplier is considered to serve the multiple customers. However, in smart grid and regulated markets, it is possible to have multiple utility companies competing with each other to supply the customer side. The utility companies can communicate with the customers via two-way communication system and aware them with their prices in real time. Then, the customers can decide on their consumption during the day. The contributions of this paper are as follows.

1) We model the DR problem as a supplier side and the customer side games. In the utility companies interact with each other, and consider their opponent strategies to submit their bid. The customers decide on their load profile based on the electricity price. A distributed algorithm is developed to determine the Nash equilibrium. We show that the running time increases linearly with the number of utility companies and customers.
2) The DR algorithm is simulated on a system with ten customers that can be residential, commercial, or industrial. Simulations show that the proposed approach decreases can benefit both customers, by increasing their payoff, and the utility companies by reducing the daily cost of the utility companies and the peak-to-average ratio (PAR) in the aggregate load demand. We also compare the performance of our method with the one in the literature to show the differences and similarities.

Our method can be partly compared with the works in [11]. The problem addressed in this paper is different from [11] in two respects. First, in [11], the interaction among the customers is formulated as an evolutionary game. Each user select one utility company to purchase power and gradually adjust its strategy on the load demand and the selection of the utility companies. While, we use a classic noncooperative game to formulate the interaction among the customers. In our model, the customers will not select the utility companies and only determine their optimal load demand at each time. Second, in [11], the interaction among utility companies is modeled as a classic noncooperative game. While, in our model, the supply function bidding mechanism is employed to model the noncooperative game among the utility companies. Both approaches in [11] and this paper have their own advantages. We perform simulations to compare the results of both approaches and discuss their differences briefly.

The rest of this paper is organized as follows. In Section II, the DR problem for supplier and customer sides is formulated. In Section III, an iterative algorithm is given to determine the Nash equilibrium in both customer and supplier sides. In Section IV, simulation results are presented, and the performance of the algorithm is compared with the one in literature. The conclusion is drawn in Section V.

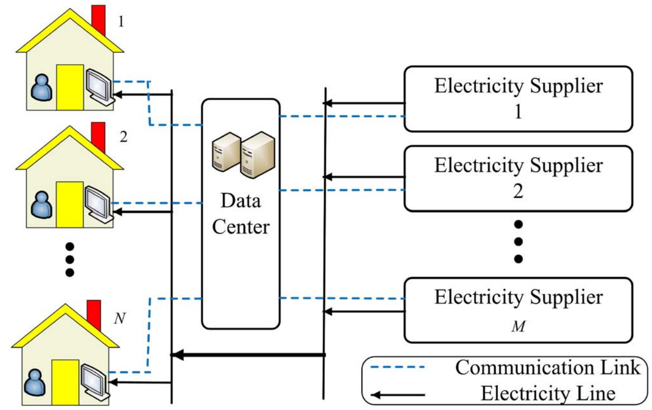

Fig. 1. Diagram of a smart grid system composed of M utility companies, N households with smart meters, data center, electricity lines, and communication links.

II. System Model

Consider a smart grid with M utility companies and N customers as shown in Fig. 1. A day is divided to T time slots. Let $\mathcal{T} = \{1, \ldots, T\}$ denote the set of time slots. In what follows, we will model the supplier and the customer sides in the DR program.

A. Supplier Side Model

We consider a market mechanism for the utility companies based on supply function equilibrium. We study a parameterized class of supply functions allowing utility companies to communicate information about their production cost by describing the amount of the electricity a utility company is willing to produce at any given price [12], [13]. Currently, many power markets actually operate in practice by having generators submit complete supply functions. The interested reader is referred to [14] for a more complete list of applications, and [15] and [16] for some discussion of power market design and bidding mechanism.

Let $\mathcal{M} = \{1, \ldots, M\}$ denote the set of utility companies. Let D_t denote the aggregate load demand of the customers at time slot $t \in \mathcal{T}$. Let $s_{j,t}$ denote the supply function of the electricity utility company j. $s_{j,t}$ is the electrical load supplied by the electricity utility company j at time slot t. It is assumed that supply function $s_{j,t}$ is chosen from the family of increasing and convex piece-wise linear functions of electricity price $p_e(t)$ [15]. Let $p_1, \ldots, p_H$ denote H break pints of the piece-wise linear supply function of all utility companies. Let $\beta^h_{j,t} \geq 0$ denote the slop of the function between the break point p_{h-1} and p_h. If $h = 1$, then $\beta^h_{j,t}$ denote the slop of the function between center and break point p_1. Fig. 2(a) shows an increasing and convex piece-wise linear supply function. Let vector $\boldsymbol{\beta}_{j,t} = (\beta^1_{j,t}, \ldots, \beta^H_{j,t})$ denote the bid profile of utility company $j \in \mathcal{M}$ at time slot t. Hence, we have

$$
\begin{aligned}
s_{j,t}\big(p_e(t), \boldsymbol{\beta}_{j,t}\big) &= \beta^0_{j,t} p_e(t), \quad 0 \leq p_e(t) \leq p_1 \\
s_{j,t}\big(p_e(t), \boldsymbol{\beta}_{j,t}\big) &= \beta^h_{j,t} p_e(t) + \beta^{h-1}_{j,t} p_{h-1}, \quad p_{h-1} \leq p_e(t) \leq p_h.
\end{aligned} \tag{1}
$$

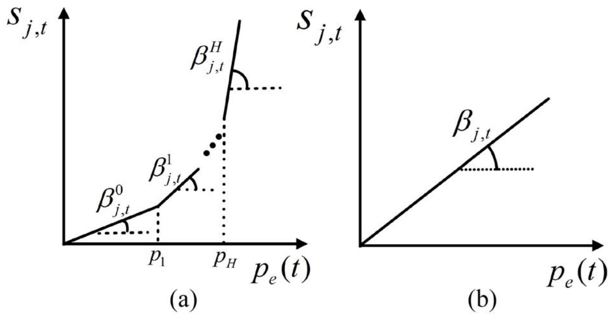

Fig. 2. (a) Piece-wise linear. (b) Affine supply functions.

In the proposed market mechanism, each electricity utility company submits $\boldsymbol{\beta}_{j,t}$ as a bid profile to the data center at time slot t. Let $\boldsymbol{\beta}_t = (\boldsymbol{\beta}_{1,t}, \ldots, \boldsymbol{\beta}_{M,t})$ denote the bids profile for all of the utility companies at time slot $t \in \mathcal{T}$. In response to the bids' submission by all of the utility companies, the data center sets the price $p_e(t)$ to clear the market. Hence

$$\sum_{j \in \mathcal{M}} s_{j,t}\big(p_e(t), \beta_{j,t}\big) = D_t, \quad t \in \mathcal{T}. \tag{2}$$

From (1) and (2), we have

$$p_e(t) = \frac{D_t}{\sum_{j \in \mathcal{M}} \Big(\beta^{h}_{j,t} + \beta^{h-1}_{j,t} p_{h-1}\Big)}, \quad p_{h-1} \le p_e(t) \le p_h. \tag{3}$$

In [15], the affine supply function $s_{j,t}(p_e(t), \beta_{j,t}) = \beta^{0}_{j,t} p_e(t)$ is used as a special case of aforementioned piece-wise linear functions. Almost all the results for affine supply functions can be generalized to the piece-wise affine supply functions [16]. For example, the electricity price function (3) can be expressed as follows for an affine supply function:

$$p_e(t) = \frac{D_t}{\sum_{j \in \mathcal{M}} \beta^{0}_{j,t}}, \quad t \in \mathcal{T}. \tag{4}$$

Comparing (3) with (4), we can conclude that considering the affine supply function is equivalent to considering the piece-wise linear supply function between two break pints. In fact, the terms $\beta^{h-1}_{j,t} p_{h-1}$ are fixed when we are between break points p_{h-1} and p_h. Hence, without loss of generality, we can study the affine supply functions and generalized the results to piece-wise linear functions. For sake of simplicity, we use the notation $\beta_{j,t}$ instead of $\beta^{0}_{j,t}$ for the affine supply function of utility company j. Moreover, we use $\boldsymbol{\beta}_t = (\beta_{1,t}, \ldots, \beta_{M,t})$ to denote the bids profile for all of the utility companies at time slot $t \in \mathcal{T}$. From (4), the electricity price is related to $\beta_{j,t}$, $j \in \mathcal{M}$, and D_t. Thus, we can denote the price function by $p_e(\boldsymbol{\beta}_t, D_t)$. From (1), supply function $s_{j,t}$ for utility company j is

$$s_{j,t}\big(p_e(\boldsymbol{\beta}_t, D_t), \beta_{j,t}\big) = \frac{\beta_{j,t} D_t}{\sum_{r \in \mathcal{M}} \beta_{r,t}}, \quad t \in \mathcal{T}. \tag{5}$$

Similar to the electricity price, the supply function can be denoted by $s_{j,t}(\boldsymbol{\beta}_t, D_t)$. We can model the interaction among the profit maximizer utility companies as a noncooperative game. Let $\boldsymbol{\beta}_{-j,t}$ denote the vector of the submitted bids of other utility companies and is defined as $\boldsymbol{\beta}_{-j,t} = (\beta_{1,t}, \ldots, \beta_{j-1,t}, \beta_{j+1,t}, \ldots, \beta_{M,t})$. Let $\pi_{j,t}(\beta_{j,t}, \boldsymbol{\beta}_{-j,t})$ denote the profit function of the electricity utility company j, and is determined as follows:

$$\pi_{j,t}\big(\beta_{j,t}, \boldsymbol{\beta}_{-j,t}\big) = s_{j,t}\big(\boldsymbol{\beta}_t, D_t\big) p_e\big(\boldsymbol{\beta}_t, D_t\big) - c_j\big(s_{j,t}\big(\boldsymbol{\beta}_t, D_t\big)\big) \tag{6}$$

where $c_j(.)$ is the generation cost function of electricity utility company j. We can use a quadratic cost functions to model the generation cost of the utility company. This class of cost functions has the form of $c(s_{j,t}) = a_2 s^2_{j,t} + a_1 s_{j,t} + a_0$, where a_2, a_1, and a_0 are positive coefficients. The cost function is assumed to be increasing and convex [11].

The bid for electricity utility company j at time slot t in the Nash equilibrium is denoted by $\beta^{\star}_{j,t}$. We substitute (4) and (5) into (6). In the equilibrium, the optimal bid $\beta^{\star}_{j,t}$, $j \in \mathcal{M}$ is the solution of the following optimization problem when other utility companies' bids are fixed:

$$\begin{aligned} \underset{\beta_{j,t}}{\text{maximize}} \quad & \frac{\beta_{j,t} D^2_t}{\big(\sum_{r \in \mathcal{M}} \beta_{r,t}\big)^2} - c_j\bigg(\frac{\beta_{j,t} D_t}{\sum_{r \in \mathcal{M}} \beta_{r,t}}\bigg) \\ \text{subject to} \quad & \beta_{j,t} \ge 0, \quad t \in \mathcal{T}. \end{aligned} \tag{7}$$

We show that supplier side game (7) has unique Nash equilibrium. First consider the following lemma.

Lemma 1: If $\boldsymbol{\beta}^{\star}_t$ denotes the bids profile in Nash equilibrium in time slot $t \in \mathcal{T}$, then we have $\beta^{\star}_{j,t} < \sum_{r \ne j} \beta^{\star}_{r,t}$ for all utility companies $j \in \mathcal{M}$.

Proof: Define function $\Phi_{j,t}(\beta_{j,t}, \boldsymbol{\beta}_{-j,t})$ as follows:

$$\Phi_{j,t}\big(\beta_{j,t}, \boldsymbol{\beta}_{-j,t}\big) = \frac{\beta_{j,t} D^2_t}{\big(\sum_{r \in \mathcal{M}} \beta_{r,t}\big)^2}. \tag{8}$$

As we can see, $\Phi_{j,t}(\beta_{j,t}, \boldsymbol{\beta}_{-i,t})$ is the first term in $\pi_{j,t}(\beta_{j,t}, \boldsymbol{\beta}_{-j,t})$. It is not difficult to show that $\Phi_{j,t}(\beta_{j,t}, \boldsymbol{\beta}_{-j,t})$ is increasing function when $0 \le \beta_{j,t} < \sum_{r \ne j} \beta_{r,t}$ and is decreasing function when $\beta_{j,t} \ge \sum_{r \ne j} \beta_{r,t}$. Hence, to maximize revenue, we should have $0 \le \beta_{j,t} < \sum_{r \ne j} \beta_{r,t}$. Consequently, in Nash equilibrium we also have $\beta^{\star}_{j,t} < \sum_{r \ne j} \beta^{\star}_{r,t}$ for $j \in \mathcal{M}$. ■

Now consider the following theorem. The proof is similar in part to the proof given in [14].

Theorem 1: The supplier side game has a unique Nash equilibrium. Furthermore, the equilibrium solves the following convex optimization problem:

$$\begin{aligned} \underset{0 \le s_{j,t} < \frac{D_t}{2}}{\text{maximize}} \quad & \sum_{j \in \mathcal{M}} -\Psi_j\big(s_{j,t}\big) \\ \text{subject to} \quad & \sum_{j \in \mathcal{M}} s_{j,t} = D_t \end{aligned} \tag{9}$$

where

$$\Psi_j\big(s_{j,t}\big) = \bigg(\frac{D_t - s_{j,t}}{D_t - 2 s_{j,t}}\bigg) c_j\big(s_{j,t}\big) - \int_0^{s_{j,t}} \frac{D_t c_j\big(\Phi_j\big)}{\big(D_t - 2\Phi_j\big)^2} d\Phi_j. \tag{10}$$

Proof: From Lemma 1, we can conclude that each utility company will supply a load of less than $D_t/2$ at time slot t in the Nash equilibrium. Let L denote the Lagrangian function of the optimization (9). We have

$$L = \sum_{j \in \mathcal{M}} -\Psi_j\big(s_{j,t}\big) + \lambda \Bigg(\sum_{j \in \mathcal{M}} s_{j,t} - D_t\Bigg) \tag{11}$$

where λ is the Lagrangian multiplier. From the first-order optimality condition, we have

$$\left(\frac{\partial L}{\partial s^{\star}_{j,t}}\right)\left(s_{j,t} - s^{\star}_{j,t}\right) \leq 0, \quad \forall j \in \mathcal{M} \tag{12}$$

where $s^{\star}_{j,t}$ is the supply function in equilibrium. $\lambda^{\star}$ is also the Lagrangian multiplier in equilibrium.

From (10), $(\partial L/\partial s^{\star}_{j,t})$ can be calculated as follows:

$$\frac{\partial L}{\partial s^{\star}_{j,t}} = \lambda^{\star} - \left(\frac{D_t - s^{\star}_{j,t}}{D_t - 2s^{\star}_{j,t}}\right)c'_j\left(s^{\star}_{j,t}\right). \tag{13}$$

On the other hand, from the first-order optimality condition for optimization problem (7), we have

$$\left(\frac{\partial \pi_{j,t}}{\partial \beta_{j,t}}\right)\left(\beta_{j,t} - \beta^{\star}_{j,t}\right) \leq 0, \quad \forall j \in \mathcal{M}. \tag{14}$$

We can calculate $(\partial \pi_{j,t}/\partial \beta_{j,t})$ and substitute it in (14) to achieve the following optimality condition for Nash equilibrium:

$$\left(p_e\left(\boldsymbol{\beta}_t, D_t\right) - \frac{D_t - s^{\star}_{j,t}}{D_t - 2s^{\star}_{j,t}}c'_j\left(s^{\star}_{j,t}\right)\right)\left(\beta_{j,t} - \beta^{\star}_{j,t}\right) \leq 0. \tag{15}$$

Comparing (12) and (15), we can conclude that the Lagrangian multiplier is in fact the electricity price $p_e(\boldsymbol{\beta}_t, D_t)$. Furthermore, both optimality conditions (12) and (15) are similar. Hence, the existence and uniqueness of the Nash equilibrium is equivalent to existence and uniqueness of the optimal point of problem (9). The proof is completed. ∎

In Theorem 1, it is proved that there exists a unique Nash equilibrium for the game among the utility companies. The strategy of the utility companies in the Nash equilibrium depends on the aggregate load D_t. For different level of the aggregate load, the utility companies will bid differently. The aggregate load is determined by the customers' load profiles. Hence, the actions of the customers will affect the utility companies' strategies. In the next section, we model the customer side in the DR program. We will show that the customers' load profiles depend on the utility companies' bids.

B. Customer Side Model

Let $\mathcal{N} = \{1, \ldots, N\}$ denote the set of customers. The customers' sectors are equipped with smart meter that can communicate with the data center via a communication network. Hence, the customers can receive the electricity price signals in real time, and they know it is calculated according to (4). Let $\mathbf{x}_i = (x_i(1), \ldots, x_i(T))$, $i \in \mathcal{N}$ denote the shiftable load profile for customer i. Each customer aims to determine its shiftable load profile to maximize its daily payoff. Again, the noncooperative game theory provides a framework to analyze the interaction among the price anticipating customers. The payoff function of customer $i \in \mathcal{N}$ is

$$u_i(\mathbf{x}_i, \mathbf{x}_{-i}) = \sum_{t\in\mathcal{T}}\Big(U_i(x_i(t) + a_i(t)) - (x_i(t) + a_i(t))p_e(\boldsymbol{\beta}_t, D_t)\Big) \tag{16}$$

where $\mathbf{x}_{-i}$ is the vector of the load profiles of other customers and it is defined as $\mathbf{x}_{-i} = (\mathbf{x}_1, \ldots, \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, \ldots, \mathbf{x}_N)$. Besides, $a_i(t)$ denote base load of customer i at time slot t, and it is known and fixed. In (16), $U_i(.)$ is the utility of customer i to show its satisfaction from consuming electricity. The utility is a function of $x_i(t) + a_i(t)$ at time t.

We use a quadratic utility function since it is nondecreasing and its marginal benefit is nonincreasing

$$U_i(x_i(t) + a_i(t)) = \nu_{i,t}(x_i(t) + a_i(t)) - \frac{\alpha_{i,t}}{2}(x_i(t) + a_i(t))^2 \tag{17}$$

where $\nu_{i,t}$ and $\alpha_{i,t}$, $i \in \mathcal{N}$ are time varying coefficients to measure the utility function in monetary units. The advantages of quadratic utility function in modeling customers' satisfaction is described in [11].

Let $\mathbf{x}^{\star}_i$ denote the load profile for customer $i \in \mathcal{N}$ in Nash equilibrium. As we defined earlier, $D_t = \sum_{j\in\mathcal{N}}(x_j(t) + a_j(t))$. We substitute (4) into (16). In equilibrium, the optimal load profile $\mathbf{x}^{\star}_i$, $i \in \mathcal{N}$ is the solution of the following optimization problem when other customers' profiles are fixed:

$$\begin{aligned}
\underset{\mathbf{x}_i}{\text{maximize}} \quad & \sum_{t\in\mathcal{T}}\Bigg(U_i(x_i(t) + a_i(t)) \\
& \qquad - (x_i(t) + a_i(t))\frac{\sum_{j\in\mathcal{N}}\big(x_j(t) + a_j(t)\big)}{\sum_{r\in\mathcal{M}}\beta_{r,t}}\Bigg) \\
\text{subject to} \quad & \sum_{t\in\mathcal{T}} x_{i,t} = L^{\text{total}}_i, \quad \forall i \in \mathcal{N} \\
& x_{i,t} \geq 0, \qquad\qquad \forall t \in \mathcal{T}
\end{aligned} \tag{18}$$

where L^{total}_i denotes the total daily shiftable load of customer i. Since, we consider shifting the load demand in the DR program; thus, L^{total}_i is fixed with and without DR. Considering interruptible loads that can be shut down in emergency conditions is beyond the scope this paper. Consider the following theorem in the customer side game.

Theorem 2: The customer side game (18) is in fact an n-person game, and has unique pure strategy Nash equilibrium.

Proof: Note that, utility function $U_i(.)$ is strictly concave, and the price function (4) is linear function of the consumed load. Hence, the payoff function of each customer is strictly concave of its load demand. Therefore, the customer side game is a strictly concave N-person game. In this case, existence of Nash equilibrium directly results from [17, Th. 1]. Moreover, the Nash equilibrium is unique due to [17, Th. 3]. The same approach is used in [11, Lemma 1] to proof the uniqueness of Nash equilibrium in the supplier side game. ∎

In the Nash equilibrium, for given companies' bids, neither customer has an incentive to deviate from its optimal load profile as long as the other customers' load profiles are unchanged. In the next section, a distributed algorithm is developed to determine the equilibrium point for both supplier and customer side's games.

III. Distributed Algorithm

In this section, a distributed algorithm is proposed to show the interaction among the customers and utility companies. Let k denote the iteration number. Let $x^k_i(t)$, $i \in \mathcal{N}$ denote the load profile of customer i in iteration k at time slot t.

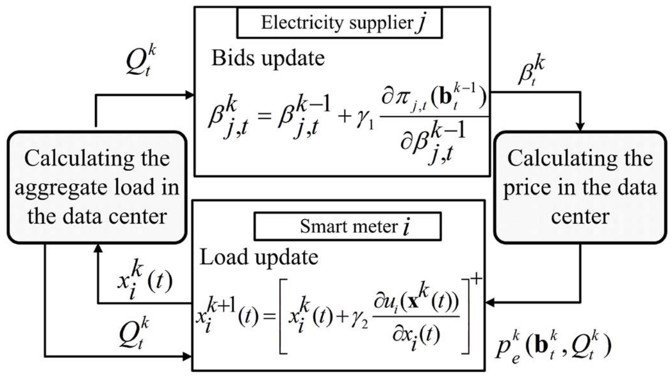

Fig. 3. Interactions between the utility companies and the smart meters.

Algorithm 1 Executed by Customer $i \in \mathcal{N}$

1: Initialization: $k = 0$, $\epsilon = 10^{-3}$.
2: **Repeat**
3: Send the load profile $x_i^k(t)$ to the data center and receive D_t^k from the data center.
4: Receive the updated $p_e^k(t)$ after updating the bids according to (19).
5: Update the load profile $x_i^k(t)$ according to (20).
6: $k := k + 1$.
7: **Until** $||x_i^k(t) - x_i^{k-1}(t)|| < \epsilon$.

Let $\mathbf{x}^k(t) = (x_1^k(t), \ldots, x_N^k(t))$ denote the load profile of all customer in iteration k. Let $\boldsymbol{\beta}_t^k = (\beta_{1,t}^k, \ldots, \beta_{M,t}^k)$ and D_t^k denote the utility companies' bid profile and the aggregate load in iteration k at time slot t, respectively. Also, let $p_e^k(\boldsymbol{\beta}_t^k, D_t^k)$ denote the electricity price in iteration k at time slot t. The utility companies and the customers interact with each other at the beginning of the day to determine optimal bids and load profiles for that day. In iteration k, the utility companies update their bids for all time slots $t \in \mathcal{T}$. Then, the electricity price $p_e^k(\boldsymbol{\beta}_t^k, D_t^k)$ for all time slots are updated in the data center according to (4). After that, the customers will update their load profile by updating $x_i^k(t)$ for all time slot. Note that, the customers will shift their loads from some time slots to the other time slots. As a result, their total daily load demand will be unchanged. The interaction between each household and each utility company is shown in Fig. 3. Besides, the distributed algorithm of smart meter i is summarized in Algorithm 1. Smart meter i randomly initializes the load profile at all time slots $t \in \mathcal{T}$. Within the loop in lines 2–6, smart meter i communicates the load consumption $x_i^k(t)$ to the data center. The data center sends the aggregate load for all time slots to the utility companies. Then, the electricity utility company $j \in \mathcal{M}$ updates its bid $\beta_{j,t}^k$ for all $t \in \mathcal{T}$ according to the following iterative equation:

$$\beta_{j,t}^{k} = \beta_{j,t}^{k-1} + \gamma_1 \frac{\partial \pi_{j,t}\left(\boldsymbol{\beta}_t^{k-1}\right)}{\partial \beta_{j,t}^{k-1}}, \quad \forall t \in \mathcal{T} \tag{19}$$

where parameters γ_1 denote the step size. Note that, utility company $j \in \mathcal{M}$ does not need to know the other utility companies bids. In fact, the utility companies know the electricity price calculated by the data center. Hence, they can calculate the sum of bids submitted by all utility companies for each time slot according to (4). Then, they can use this information for their update in (19). After updating the bids, the data center calculates the updated value of $p_e^k(\boldsymbol{\beta}_t^k, D_t^k)$ according to (4). In this step, the updated value of $p_e^k(\boldsymbol{\beta}_t^k, D_t^k)$ will be sent to the smart meter $i \in \mathcal{N}$. In line 5, the smart meter updates the load profile as follows:

$$x_i^{k+1}(t) = \left[x_i^k(t) + \gamma_2 \frac{\partial u_i\left(\mathbf{x}^k(t)\right)}{\partial x_i(t)} \right]^+, \quad \forall t \in \mathcal{T} \tag{20}$$

where parameters γ_2 denote the step size. $[.]^+$ is the projection onto the feasible set. The projection $[.]^+$ guarantees that the total load of the customer will be unchanged during a day. Considering (18), smart meter i needs the updated values of $p_e^k(\boldsymbol{\beta}_t^k, D_t^k)$ and D_t^k to determine $(\partial u_i(\mathbf{x}^k(t))/\partial x_i(t))$. However, it does not need the load profile of the other customers since $(\partial u_i(\mathbf{x}^k(t))/\partial x_i(t))$ only depends on its own load profile and the electricity price. This fact guarantees the privacy of the customers. The iteration number is updated until the stopping criterion for the algorithm is satisfied. The proposed algorithm is based on the projected gradient method; and hence, it will converge to the optimal point of the optimization problems (9) and (15) for all utility companies and customers for sufficiently small step sizes γ_1 and γ_2 [18]. In [18], it is shown that why the projected gradient method converges to the optimal point of an optimization problem. Since Nash equilibrium of both supplier and customer sides games are unique and solve optimization problems (9) and (15), the iterative processes (19) and (20) will converge to the Nash equilibrium of the games. In fact, the algorithm converges to a point in which utility companies are playing their equilibrium strategy based on the customers' load profile data, and the customers also select their equilibrium strategy based on the utility companies' submitted bids. In the equilibrium, none of the customers and utility companies gains benefit by deviating from their chosen strategy.

IV. Performance Evaluation

In this section, we evaluate the performance of the iterative Algorithm 1 to show its efficiency. We also compare the results with the proposed DR method in [11]. To make the comparison, the simulation setup is chosen similar to the one given in [11]. We consider a smart grid system with $M = 3$ utility companies, and $N = 10$ customers, which are accepted to participate in the DR program. The results can be easily generalized to higher customers' and utility companies' numbers. The day is divided to $T = 24$ 1-h time slots. The utility companies' are assumed to be thermal generation units. Generation cost function for utility company $j \in \mathcal{M}$ is modeled as a quadratic function, $c_j(x)|_{x=D_t} = \lambda_{j2}x^2 + \lambda_{j1}x + \lambda_{j0}$, where the generation is measured in kilowatts and we have λ_{j2}, $\forall j \in \mathcal{M}$ is selected $[0.2, 0.3]$. Besides, $\lambda_{j1} = 0.1$ and $\lambda_{j0} = 0$, $\forall j \in \mathcal{M}$ [11]. On the customer side, $\nu_{i,t}$ for each customer is randomly selected from [4, 10], and $\alpha_{i,t}$ for each customer is 0.5.

To model the customers' consumption pattern, we use real data from [19] for 25th of April, 2015. The aggregate and base load demand profiles without DR are shown in Fig. 4.

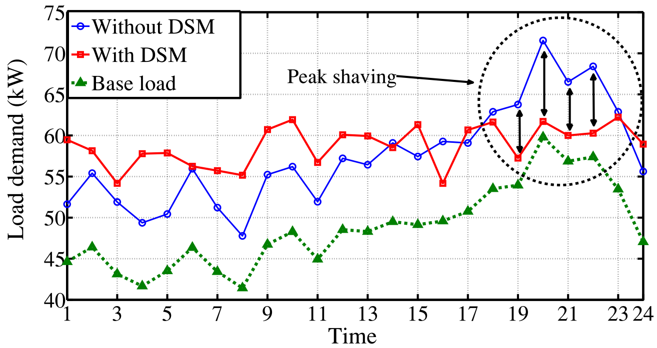

Fig. 4. Base and total load profiles with and without DR program. The peak shaving is achieved by using DR program (the dashed circle).

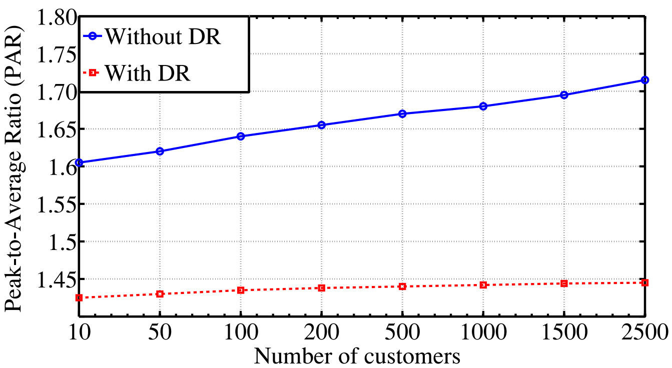

Fig. 5. PAR for different number of customers with and without DR program.

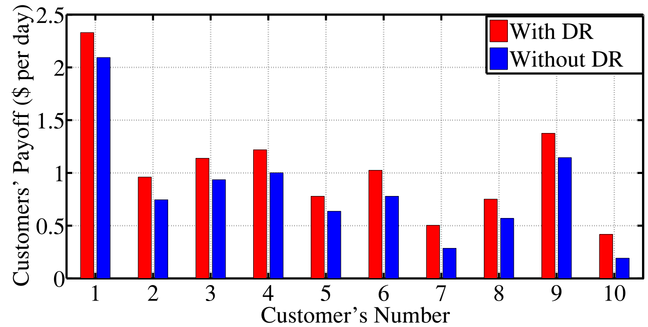

Fig. 6. Daily payoff for customers 1–10 with and without DR program. Increase in daily payoff is demonstrated.

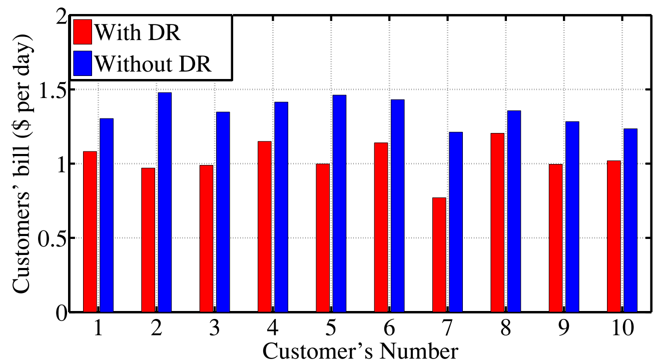

Fig. 7. Daily electricity bill for customers 1–10 with and without DR program. Reduction in electricity bill is shown.

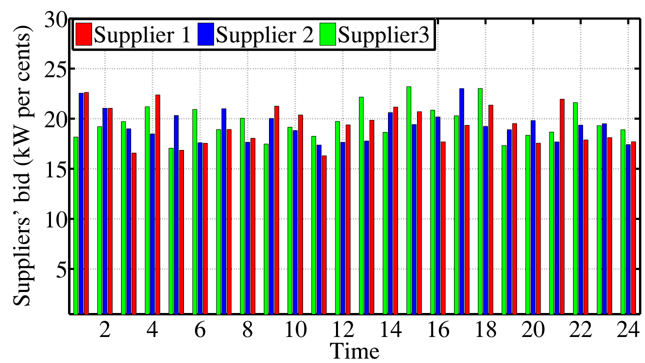

Fig. 8. Utility companies' bids at different time slots.

The shiftable load of each customer is assumed to be chosen randomly from 5% to 25% of its total consumption. For simulations, the initial state of the customers' electricity consumption is assumed to be the load profile without DR. Without DR, the peak load is 73 kW. However, with the DR Algorithm 1, the load demand is shifted to off-peak time slots, and the peak load decreases to 62 kW. Furthermore, the load demand for each customer will shifted to time slots with higher $\nu_{i,t}$. We also perform simulations for different number of customers. In Fig. 5, PAR index with and without DR are shown. Without DR, we have high peak load and low average load; hence, PAR index is growing gradually. Whereas, with DR, the peak shaving is achieved and the PAR index is low even for high number of customers. Consequently, Algorithm 1 has high performance in reducing the PAR in aggregate load demand even for high number of customers with random characteristics.

By participating in the modeled DR program, the daily payoff of the customers increases. Fig. 6 shows the daily payoff of the customers. The customers have different payoff because they have different utility functions. Furthermore, the customers' daily bill decreases with DR. Fig. 7 shows the daily bill for customers 1–10. The daily bill of each customers with DR is around 75% of its value without DR. Therefore, the customers can save around 25% by participating in the DR program. On the supplier side, the utility companies play a game to determine optimal bids in each time slot. Fig. 8 shows the utility companies' bids in different times of a day. The submitted bid for each utility company in each time slot depends on its generation cost in that time slot. Therefore, we have seen a random pattern in the optimal bids. Furthermore, the daily revenue of the utility companies depends on the aggregate load demand and their bids. Fig. 9 shows the utility companies' daily cost (minus revenue) with and without DR. As we can see, the daily cost of the utility companies decreases with DR because the aggregate load profile becomes flatten; and hence, the utility companies' generation cost decreases. Furthermore, the utility companies submit an optimal bid that maximize their revenue at each time slot.

In practical case studies, the DR Algorithm 1 will be deployed for a large number of customers. Therefore, the running time of the algorithm is an important factor to evaluate its efficiency. Figs. 10 and 11 show the convergence of the utility companies' bids and the customers load at time slot 10, respectively. The step sizes γ_1 and γ_2 are 0.08 and 0.08. The utility companies' bids and the load demands start from the initial values and converge to the equilibrium values. The algorithm converges

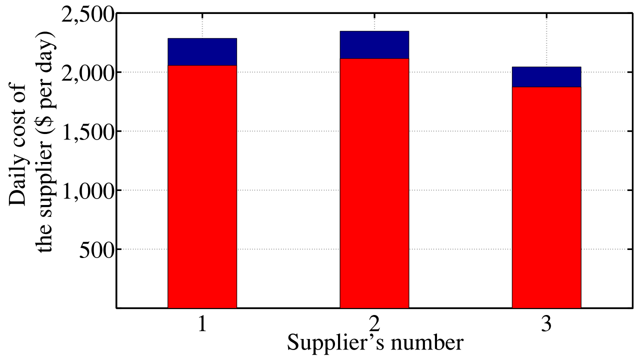

Fig. 9. Daily cost of the utility companies with DR (the red bars) and without DR (the blue bar).

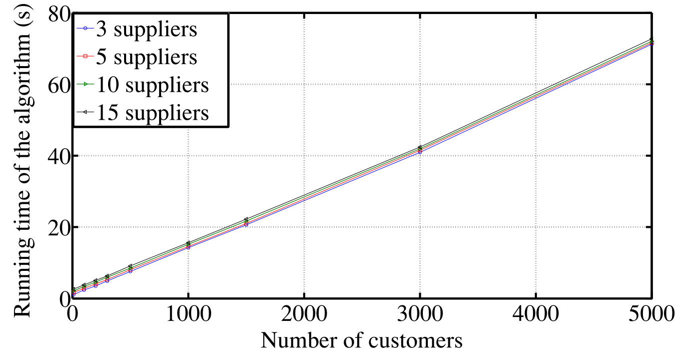

Fig. 12. Running time of Algorithm 1 for different number of customers and utility companies.

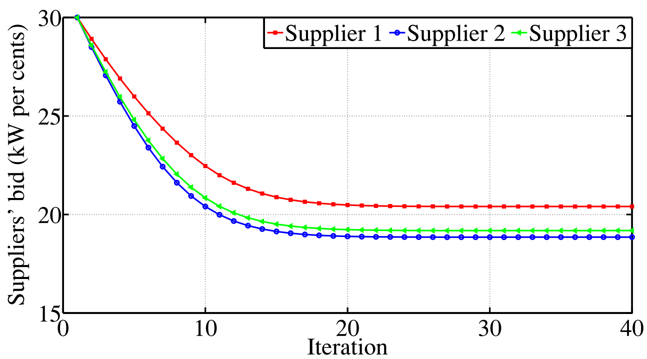

Fig. 10. Convergence of utility companies' bid at time slot 10.

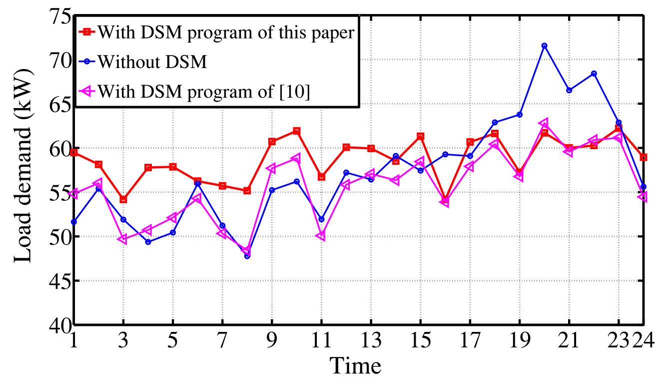

Fig. 13. Load profile with the DR program in this paper and the one in [11].

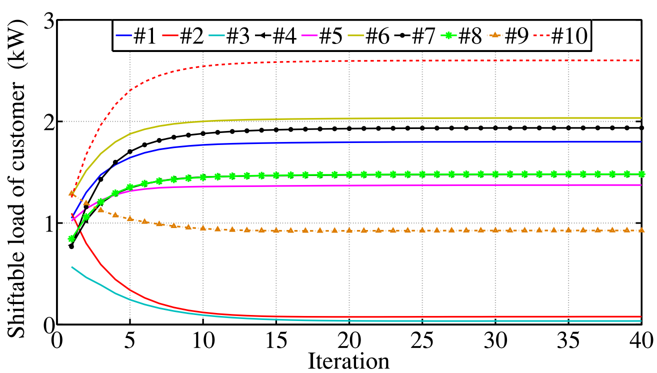

Fig. 11. Convergence of shiftable loads for all customers at time slot 10.

in about 20 iterations in 0.55 s. The number of iterations depends on the step sizes, and might not give a complete information about the running time of the algorithm. To show the computational complexity of the algorithm, we evaluate the running time of the algorithm for different number of customers and utility companies. Fig. 12 shows that the running time of the algorithm increases linearly with the number of customers N and it is almost independent of M. The reason is that by increasing the number of customers and utility companies, the number of updates for the customers and the utility companies will increase proportional to N and M, respectively. However, the update process for the utility companies takes much lower time comparing with the updates for the customers since the customers need to consider load shifting (the projected gradient) which increases the complexity of the update process. We can see that the algorithm is efficient even for large number of customers and can be implemented in scenarios with large number of customers and utility companies.

Finally, we compare our DR algorithm with the one given in [11]. The DR method in [11] is based on two level games. In the first level, the customers will chose the utility company they buy from, and the load demand based on an evolutionary game model. In the second level, the utility companies will determine their generation level and the electricity price. In Fig. 13, the results for both DR approaches are shown. We can see, both methods have the same performance in reducing the peak load demand. However, they are not the same in distributing the load demand in different time slots. The DR approach in [11] is based on increasing or decreasing the load demand in each time slots regardless of the other time slots. While, the DR algorithm of this paper is based on shifting the load demand from one time slot to other time slots. As a result, we can see in Fig. 13 that the DR method of [11] does not guarantees unchanged total daily consumption for the customers. While, our DR algorithm guarantees according to constraint (14) and the projected gradient method. However, the daily bill for the customers in DR method of [11] is lower than our DR method since they have lower consumptions in different time slots. We cannot say anything about the customers' payoff in general because the daily bill is higher in DR method of [11], but the utility of the customers is lower. We also cannot say anything about the utility companies' revenue in general. Both algorithms have high run time according to our simulations and both can be used for high number of customers and utility companies.

V. Conclusion

In this paper, we proposed a DR algorithm for customers in smart grid to reduce peak load of the system. It was assumed that there exist multiple utility companies in the system. The DR problem is formulated as two games to maximize the utility companies' profit and customers' payoff. Furthermore, the electricity price is obtained by employing supply function bidding mechanism model and by solving the profit maximization problem of the utility companies. It was shown that the pure strategy Nash equilibrium for both games exists and is unique. We proposed a linear-time distributed algorithm to determine the equilibrium. Simulation results show that the algorithm increases the customers payoff and reduces the peak load by shifting the load demand to off-peak periods. Furthermore, the utility companies' profits are also increased by participating in the DR game. Comparing our algorithm with the one in the literature, we demonstrate that the DR algorithm has acceptable performance with its own advantages, and can be employed in smart grid with multiple customers and multiple utility companies. For future work, we will consider industrial customers and a detailed load model instead of a general daily load profile.

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