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Emission reduction and profit-neutral permit allocations☆

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ABSTRACT

The present paper addresses two policy objectives: to implement a market for pollution permits and to make regulation acceptable for businesses. Profit-neutral permit allocations are defined as the number of permits that the regulator should give for free so that post-regulation profits (i.e. a firm's profits in the products market plus the value of the allowances granted for free) are equal to pre-regulation profits. The proposed model is developed by assuming that firms use polluting technologies and compete "à la Cournot". The paper demonstrates that a low number of free allowances is sufficient to meet these two goals. Moreover, the regulator can fully offset losses, even when the reduction in emissions is high, provided that the sectors concerned are not monopolies, both for isoelastic and linear demand functions.

1. Introduction

In 2005, the EU set up the European Emission Trading Scheme (EU-ETS), a large cap-and-trade system designed to enforce its international commitments to reducing CO2 emissions. The first two phases (2005–2007 and 2008–2012) were characterized by the distribution of free allocations, the use of grandfathered allocations, and a low percentage reduction in emissions. These mechanisms induced an increase in profits but also generated loss of competitiveness and leakage - i.e. the substitution of emissions from environmentally regulated countries to countries without effective environmental policies. In light of these results, the EU decided to revise the allocation rules for its third phase (2013–2020). Although it has abandoned grandfathering in favor of innovative distribution measures linked to production capacities, the EU has only achieved a low percentage reduction in emissions. This positioning is highly problematic since it has been proven that emissions must be drastically reduced to respond to climate change issues (Meyer et al., 2014; Raftery et al., 2017). The EU must therefore find a way to make more

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stringent environmental policies acceptable to firms.

It appears crucial to secure firms' acceptance of environmental regulations so that they do not lobby against their application or limit their enforcement. The regulator faces a participation constraint in that it needs to induce firms to support environmental regulation by protecting their profits, while also ensuring the desired outcome of regulation, i.e. effective emission reduction. To reconcile acceptability for the businesses concerned with a strengthened EU environmental policy, a different approach could involve retaining grandfathering while also introducing higher emission reductions.

Profit-neutral permit allocations are used to determine the conditions under which firms will accept regulation. Profit-neutral permit allocations are defined as the number of permits the regulator should give for free so that post-regulation profits (i.e. a firm's profits in the products market plus the value of the allowances granted for free) are equal to pre-regulation profits. This paper aims to determine the maximum amount of permits to grant for free while satisfying the participation constraint. To do so, it analyzes the relationship between profit-neutral permit allocations and the reduction in emissions in a partial equilibrium framework.

The EU-ETS covers oligopolistic sectors (such as cement, electricity, and steel) and encompasses more than three thousand firms.[1] Although some firms dominate the market for permits, the three largest emitters - RWE, E.ON, and Vattenfall - emit only 7.1%, 4.7%, and 4.2% of total emissions, respectively. Even power companies are not big enough to manipulate the permit price alone.[2] It is hence assumed that firms are price-takers in the permits market and compete "*à la Cournot*" in the products markets.

We first assume a general demand function, although we specify this function in a later step for some of our results. There is very little empirical evidence regarding the curvature and the steepness of the demand curve for the products covered by pollution permits markets. Most papers focusing on emission trading schemes use linear demand. Some papers have, however, demonstrated that the pass-through may be higher than 100% for the electricity sector, which is impossible with linear demand. Hintermann (2016) demonstrates that carbon cost pass-through is more than complete during hours of high demand, while the average level of cost pass-through based on all hours is less than 100%. Such results are not sufficient to prove that demand is curved, but justify widening the scope of study beyond linear demand functions. For this reason, we use both linear and isoelastic demand functions.

Our paper also considers that the regulator reduces emissions by applying an emission reduction rate, such that total post-regulation emissions are equal to initial emissions multiplied by one minus the emission reduction rate. This is the approach used by policy makers, who first determine the desired percentage reduction in emissions and then set pollution caps. We further assume that firms can only abate emissions by reducing production.

We firstly demonstrate that when the elasticity of the demand slope is negative and sufficiently high in absolute values, free allowances are not required for profit neutrality. The paper shows that when demand is either isoelastic or linear, the percentage of profit-neutral allowances given for free in fact increases with the percentage reduction in emissions and decreases with the number of firms. These results are consistent with Hepburn et al. (2013) who examine the impact of pollution permits on profits in a generalized Cournot model. They identify a formula for the number of emission permits that must be freely allocated to firms to neutralize the impact on profits of pollution permits and show that it is lower than the Herfindahl index. The authors also use an exogenous permit price to demonstrate that free allowances are not required for profit neutrality in some cases. The present paper is complementary to Hepburn et al. (2013) since it endogenizes the permit price and analyzes the profit-neutral policy in function of the level of emission reduction. This approach requires us to specify the demand function, which differs from that adopted by Hepburn et al. (2013). Hepburn et al. (2013) use the Taylor expansion of the profits function to determine the grandfathering rate that offsets the loss in profit, while the present paper calculates the equilibrium permit price and derives the percentage of profit-neutral allowances.

The second contribution of our paper in relation to Hepburn et al. (2013) is to highlight the constraint satisfied by the regulator: the number of free allowances should be lower than the number of permits put into circulation.[3] If not, the permit price would be equal to zero.[4] Hepburn et al. (2013) state that, to achieve profit-neutrality, an industry may receive more permits than it needs, although they do not prove this assertion. In the present paper, we show that the regulator cannot offset firms' losses in either a monopoly or a duopoly with high emission reductions. We further determine the maximal percentage reduction in emissions that a regulator could set if its goal is to offset losses in profits in order to obtain firms' assent. A crucial policy implication is deduced from this finding: the regulator may implement stringent regulations while satisfying firms' participation constraints.

In the case of international competition, however, unilaterally implementing pollution permits induces a loss of competitiveness and generates leakage. The present paper establishes that the conditions required to make environmental regulation acceptable are more stringent and suggests that if there are large numbers of domestic firms and few foreign firms, then offsetting losses in profits may be possible. We extend our analysis to a market for permits covering several sectors and assess the way different sectors are affected by the implementation of pollution permits.

[1] More specifically, the EU-ETS applies to more than eleven thousand plants.

[2] It has, however, been shown that when firms are not price-takers, they may have incentives to over-purchase permits. See Hintermann (2011).

[3] This condition is equivalent to the percentage of permits lower than one hundred percent.

[4] This type of constraint was highlighted by Martin et al. (2014) in a relocation risk context.

The remainder of the paper is structured as follows. Section 2 presents the modeling assumptions. Section 3 focuses on a representative sector not exposed to international competition. Section 4 analyzes the robustness of the model, extends the paper to several different sectors, and considers successively the possibility of firms exiting the market and the presence of international competition. Section 5 concludes.

2. The model

The present section introduces the model.

Firms. There are n symmetric firms competing in a market and producing a homogeneous good. The production technology is polluting. Let c be the marginal cost and assume that the emissions intensity is equal to f. In other words, one unit of production generates f units of pollution. Firms can only abate emissions by reducing production. The emission intensity indicates how polluting a sector is. Firms compete "*à la Cournot*", simultaneously choosing their production quantity in order to maximize profits.

Consumers. Firms face an inverse demand function $P(Q)$, where Q is the total quantity produced. The inverse demand function is twice differentiable, positive or null, and strictly decreasing when positive, and $P(0) > 0$. Moreover, let us assume that $P(Q) + P'(Q)q_i$ for any firm i is decreasing in q_i and that $P(Q) + P'(Q)Q/n$ is decreasing in Q. Let $E = P''Q/P'$ be the elasticity of the demand slope. Moreover, two specific demand functions will be analyzed: an isoelastic demand function and a linear one.

- The linear demand function that we use is given by:

$$P(Q) = a - bQ, \tag{1}$$

  where a is market size and b is the slope of the demand function. Obviously, b is assumed to be positive.
- The isoelastic demand function is assumed to be:

$$P(Q) = \alpha Q^{-\frac{1}{\beta}}, \tag{2}$$

  where α is market size and β is the elasticity of demand. In order to ensure the existence of the equilibrium, let us assume that $\beta > 1/n$. Moreover, in order to be sufficiently realistic, we assume that the elasticity of demand is lower than 10.

When demand is isoelastic, the elasticity of the demand slope is equal to $1 + \frac{1}{\beta}$. When demand is linear, the elasticity of the demand slope is equal to 0.

Regulation. In order to cut pollution, the regulator implements a market for permits. A firm must own a permit in order to pollute one unit. Assume that there are many identical oligopolistic markets, each producing a different product, although the market for permits is common to all of these industries. Firms are price-takers in the market for permits. The permit price is denoted by σ and clears when supply equals demand. When the permit price is equal to σ, total emissions are equal to $fQ(\sigma)$. The goal of the regulator is to reduce emissions such that:

$$fQ(\sigma) = (1 - z)fQ(0), \tag{3}$$

where $0 < z < 1$. The emissions before regulation are denoted by $Q(0)$. In other words, the percentage reduction in emissions is given by $100z$. The number of permits put into circulation is equal to $(1 - z)fQ(0)$.

3. Profit-neutral allocations

The regulator distributes free allowances ε_i to firm i and auctions the remaining permits. Profits may be written as the sum of the profits in the products market and the gain from free allowances.

$$\pi_i(\sigma) = (p(Q) - c - f\sigma)\, q_i + \varepsilon_i\sigma.$$

Since allowances are grandfathered, they are merely a lump-sum transfer from the regulator to the firms and do not affect firms' decisions. Free allowances do, however, increase firms' profits. We now define profit-neutral allowances.

Definition 1. *Profit-neutral allowances ϵ_i^N are defined as the number of free allowances required to level out firms' profits such that profits are identical with or without environmental regulation*: $\pi_i(0) = \pi_i(\sigma) + \epsilon_i^N\sigma$.

We first analyze the effect on profits of implementing a market for permits. We then determine the number of profit-neutral allowances, i.e. the number of free allowances that would level out firms' profits with or without environmental regulation.

The first order conditions for profits satisfy:

$$P(Q) + P'(Q)q_i = f\sigma + c. \tag{4}$$

By summing the first order conditions, we obtain:

$$nP(Q) + P'(Q)Q = n\,(f\sigma + c)\,. \tag{5}$$

There exists a single level of production satisfying equation (5) since we assume that the expression on the left (the marginal revenue) is decreasing in Q. Let $M_i : R_+^n \to R$ be the marginal revenue of firm i, so that $M_i(q_1, \ldots, q_i, \ldots, q_n) = P(Q) + P'(Q)q_i$ and let $M : R_+ \to R$ be the marginal revenue of any firm in the case where output is identical across firms, i.e., $M(q, \ldots, q) = P(nq) + P'(nq)q$. Let $L : R_+^2 \to R$ be the function defined by $L(c', n) = Q$, where Q solves (5) and $c' = f\sigma + c$. Note that L decreases with c', and hence with the permit price, emission intensity, and the marginal cost.

Once we have defined total production, we determine the equilibrium permit price on the market for permits. The aggregate demand for permits is equal to the total amount of permits that firms need and that they have not been granted for free, i.e., $fQ(\sigma) - \Sigma_{i=1}^n \epsilon_i$. The total supply is the number of permits that the regulator is ready to sell, i.e., $(1-z)Q(0) - \Sigma_{i=1}^n \epsilon_i$. The perfectly competitive permits market therefore clears when supply equals demand, or:

$$\begin{aligned} fQ(\sigma) - \Sigma_{i=1}^n \epsilon_i &= (1-z)fQ(0) - \Sigma_{i=1}^n \epsilon_i \\ &\Leftrightarrow fQ(\sigma) = (1-z)fQ(0). \end{aligned} \tag{6}$$

Note that the permit price is independent of the amount of permits granted for free. Free allowances reduce supply and demand in the same way. Thus, grandfathered free allowances modify neither firms' decisions, nor the equilibrium permit price. This result comes from the assumption that firms are price-takers in the market for permits. Inserting equation (6) into equation (5) and rewriting this latter we get:

$$\sigma = \frac{M((1-z)q(0)) - M(q(0))}{f}. \tag{7}$$

The permit price is equal to the marginal abatement cost.[5] Reducing production from $q(0)$ to $(1-z)q(0)$ induces a reduction in marginal revenue. The following lemma characterizes the equilibrium permit price:

Lemma 1. *The equilibrium permit price increases with the percentage reduction in emissions and decreases with emission intensity. The equilibrium permit price increases with the marginal cost if marginal revenue is sufficiently convex. When $L(c, n) = l(c)h(n)$, where $l(c)$ and $h(n)$ are two independent functions, the equilibrium permit price does not depend on the number of firms.*

Proof. Proof in Appendix A.1. □

A rise of z denotes that the environmental policy is more stringent and induces a decrease in supply. The permit price therefore increases with the percentage reduction in emissions. Emissions obviously increase with emission intensity. The higher the initial emissions, the lower the abatement cost will be. An increase in marginal cost generates a lower initial production. The evolution of the permit price in function of the marginal cost depends on whether marginal revenue is convex. Under isoelastic demand, the permit price increases with marginal cost while it decreases under linear demand.

If we break down the function $L(c, n)$ as the product of two independent functions, one dependent on the number of firms and the other dependent on the marginal cost, we find that the equilibrium permit price does not depend on the number of firms. This means that there exists a single value of the marginal cost that corresponds to the required level of production. The difference between the targeted marginal cost and the initial marginal cost does not depend on the number of firms. This decomposition of the function $L(c, n)$ holds for both isoelastic and linear demand functions. For instance, under a linear demand function $L(n, c') = n\frac{a-c'}{b(n+1)}$ and under an isoelastic demand function $L(n, c') = \left(\frac{\alpha(\beta-1/n)}{\beta c'}\right)^{\beta}$. It is therefore deduced that under linear demand, the equilibrium permit price is given by:

$$\sigma = z\frac{a-c}{f}. \tag{8}$$

Calculations of the linear case are provided in Appendix A.2. Under an isoelastic demand function, the equilibrium permit price is equal to:

$$\sigma = ((1-z)^{-\frac{1}{\beta}} - 1)\frac{c}{f}. \tag{9}$$

Calculations of the isoelastic case are provided in Appendix A.3. The result that the permit price does not depend on the number of firms is surprising. However, if the regulator fixes the cap instead of the percentage reduction in emissions, the permit price will depend on the number of firms. Moreover, relaxing symmetry also modifies this result.

Deriving the first order condition with respect to σ gives:

$$P'q_i' = f - [1 + \frac{E}{n}]P'Q'. \tag{10}$$

Taking the derivative of this equation with respect to σ gives:

$$P'Q' = nf/(n+1+E). \tag{11}$$

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[5] Since there is no abatement technology, the abatement cost is the cost required to reduce production.

We analyze the derivative of function π_i with respect to σ, and obtain:

$$\frac{\partial \pi_i}{\partial \sigma} = q_i[P'(Q' - q_i')] - f. \tag{12}$$

Inserting (10) and (11) into (12) gives:

$$\frac{\partial \pi_i}{\partial \sigma} = \frac{P' q_i Q'}{n}\left[-E - 2\right]. \tag{13}$$

Since the permit price increases with the percentage reduction in emissions, we deduce the following lemma.

Lemma 2. *When the elasticity of the slope of the demand function is lower than minus two, profits increase with the percentage reduction in emissions and free allowances are not required to achieve profit-neutrality.*

This phenomenon is well known from Seade (1985). The permit price helps firms to coordinate their activities so as to decrease production and consequently boost the product price. Note that the elasticity of the demand slope is constant with an isoelastic demand function and equal to one plus the inverse of the elasticity of demand. Thus, when demand is isoelastic, profits increase with the percentage reduction in emissions when the elasticity of demand is lower than one. When demand is linear, profits do not increase with the percentage reduction in emissions.[6]

From now on, we consider a scenario in which the elasticity of the demand slope is sufficiently high ($E > -2$). In other words, we focus on the case where profits decrease with the permit price.

Profits may be written as a function of the percentage reduction in emissions. Inserting equation (22) into equation (3), we get:

$$\pi_i(z) = -P'(Q) q_i^2, \tag{14}$$

$$= -P'((1-z)Q(0))(1-z)^2 q_i(0)^2. \tag{15}$$

The implementation of a market for permits has two effects: a reduction in output and a reduction in the mark-up. The total losses for a firm due to the implementation of pollution permits are:

$$\pi_i(z) - \pi(z=0) = -P'((1-z)Q(0))(1-z)^2 q_i(0)^2 + P'(Q(0)) q_i(0)^2, \tag{16}$$

$$= q_i(0)^2 \left(P'(Q(0)) - (1-z)^2 P'((1-z)Q(0))\right). \tag{17}$$

Profit-neutral allowances, which level out firms' profits such that profits are identical with or without environmental regulation (ϵ_i^N), are given by:

$$\epsilon^N = \frac{Q(0)^2(-P'(zQ(0))z^2 + P'(Q(0)))}{n^2 \sigma}.$$

We now define the ratio of profit-neutral free allowances.

Definition 2. *The ratio of profit-neutral free allowances is defined as profit-neutral allowances over permits, i.e.* $\gamma_p = \frac{n\epsilon_i^N}{fQ(\sigma)}$.[7]

We calculate the ratio of profit-neutral free allowances for linear and isoelastic demand functions. Under a linear demand function, the ratio of free allowances to the permits required to offset losses is equal to:

$$\gamma_p = \frac{1}{n+1}\frac{2-z}{1-z}.$$

Under an isoelastic demand function, the ratio is given by:

$$\gamma_p = \frac{1}{n\beta - 1}\left(\frac{(1-z)^{-1} - 1}{(1-z)^{-\frac{1}{\beta}} - 1} - 1\right).$$

The characteristics of the profit-neutral policy are given by the following proposition.

Proposition 1. *When demand is either isoelatic or linear, the ratio of free allowances to permits (γ_p) increases with the percentage reduction in emissions,* $\frac{\partial \gamma_p}{\partial z} > 0$, *and decreases with the number of firms,* $\frac{\partial \gamma_p}{\partial n} < 0$.

The ratio of profit-neutral allowances increases with the percentage reduction in emissions, $\frac{\partial \gamma_p}{\partial z} > 0$. A rise in the percentage reduction in emissions indicates that environmental policy is more stringent and consequently the permit price is higher. The ratio of free allowances to permits should be higher to offset losses. The percentage of profit-neutral allowances decreases with

[6] Hintermann (2017) also shows that profits may increase when pollution permits are implemented.
[7] The percentage of permits given freely that neutralizes profits is equal to 100 multiplied by.

the number of firms. Losses in profit decrease with the number of firms, while the gain from free allowances does not depend on the number of firms. The equilibrium permit price is in fact independent of the number of firms under both linear and isoelastic demand. When the number of firms increases, the number of allowances required decreases.

Figure 1 illustrates the percentage of profit-neutral allowances in function of the number of firms and the percentage reduction in emissions for both linear demand and isoelastic demand. Note that in both cases the percentage of free allowances diminishes rapidly with the number of firms. Moreover, in both cases, the percentage of profit-neutral allowances is higher than 100% for a monopoly. When the number of firms is equal to five, the orders of magnitude are low, even for large percentage reductions in emissions (for $z = 0.5$ the percentage of profit-neutral allowances is lower than fifty percent). In a duopoly scenario, the percentage of profit-neutral allowances is lower than 100%, even for large percentage reductions in emissions.

The ratio of free allowances to permits (γ_p) should be lower than one. Otherwise, firms will receive more free allowances than there are permits in circulation. In such a case, firms have no incentive to reduce pollution and the permit price will be equal to zero. We determine the conditions under which this constraint is satisfied.

Definition 3. *Let $\overline{z}$ be the maximal percentage reduction in emissions that the regulator can implement while giving all permits for free and neutralizing profits.*

From the previous results, the two following propositions are deduced.

Proposition 2. *When demand is linear, the maximal percentage reduction in emissions that the regulator can implement while giving all permits for free and neutralizing profits depends on the number of firms and is given by $\overline{z}(n) = 1 - \frac{1}{n}$.*

Under linear demand, when there is only one firm, it is impossible to offset the loss in profit. When there is a duopoly, the regulator can reduce emissions by fifty percent while keeping profits at their initial level. The maximal percentage reduction in emissions that the regulator can implement, while giving all permits for free and neutralizing profits, decreases with the number of firms.

Proposition 3. *When demand is isoelastic, the maximal percentage reduction in emissions that the regulator can implement, giving all permits for free and neutralizing profits, depends on both the elasticity of demand and the number of firms, and is characterized by:*

(i) $\frac{\partial \overline{z}}{\partial \beta} < 0$ *and* $\frac{\partial \overline{z}}{\partial n} < 0$,
(ii) $\overline{z}(\beta, 1) < 0$,
(iii) $\overline{z}(\beta, 2) > 0.75$.

Proof. The proof is provided in Appendix A.4. □

In the isoelastic case, the threshold $\overline{z}(\beta, n)$ also decreases with the number of firms and is impossible to offset in a monopoly situation. In the case of a duopoly, the maximal percentage reduction in emissions that the regulator can implement is higher than in the linear demand case. This stems from the fact that the percentage of profit-neutral allocations decreases more rapidly under isoelastic demand than under linear demand.

Proposition 3 allows us to study how the maximal percentage reduction in emissions achieved by the regulator evolves in function of the elasticity of demand. It decreases with the elasticity of demand. In fact, the greater the elasticity of demand, the lower the initial production, and the higher the permit price will be.

In a monopoly situation and in both cases, i.e. under linear and isoelastic demand functions, the regulator cannot offset the losses generated by implementing a market for permits. The losses suffered by a monopoly are too significant to be compensated. These results are consistent with Hepburn et al. (2013), who consider a monopoly scenario and show that the monopolist receives more free allowances than it needs. In the case of a duopoly, the regulator can fully compensate firms for high reductions in emissions. From a policy point of view, these two propositions are crucial. Even when the emission reduction is high, the regulator can only fully offset losses when the sectors are not monopolies. Thus, the regulator may be more ambitious when fixing the percentage reduction in emissions, even if it wants to obtain firms' assent.

4. Extensions

The main assumptions of the paper are: a fixed number of firms, identical sectors, and the absence of international competition. In this section, these three assumptions are relaxed and the robustness of the previous results is analyzed.

4.1. Endogenous market structure

Assume that firms bear a fixed cost denoted by F. We consider that the market structure is endogenous and, more precisely, that it depends on the stringency of environmental regulation. The goal is to study the profit-neutral distribution of free allowances when the market structure is endogenous. As previously, we consider that total emissions are reduced such that $fQ(\sigma) = (1 - z)fQ(0)$. We assume that n firms are active in the products market before the market for permits is implemented. A firm is active if $\pi(n, z) - F > 0$. We denote by $n(z)$ the number of active firms in the products market after the market for permits has been implemented, i.e. $\pi(n(z), z) - F = 0$. The number of active firms depends on the percentage reduction in total emissions.

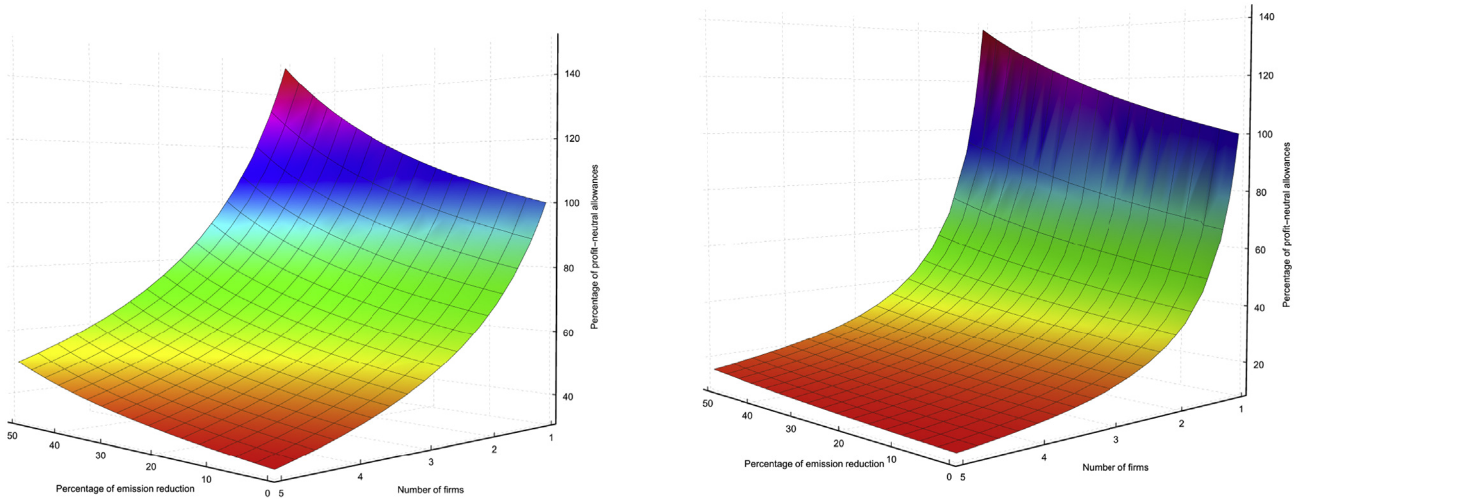

Fig. 1. Illustration of the percentage of profit-neutral allowances in function of the number of firms and the percentage reduction in emissions for linear demand (the figure on the left) and for isoelastic demand with elasticity of demand equal to 2 (the figure on the right).

Let us denote by z' the percentage reduction in individual emissions, i.e. $q_i(n(z), z) = (1 - z')q_i(n, z = 1)$. When the number of active firms decreases with the percentage reduction in emissions, each firm reduces its production by a lower percentage than the reduction in global emissions. We can easily show that $1 - z = \frac{n(z)}{n}(1 - z')$. In other words, $1 - z' > 1 - z$. Indeed, since there are fewer firms, each firm produces more than in the case where all firms remain active. We now analyze the following two cases: first, free allowances are given only to active firms and second, free allowances are given to all firms.

Allowances given only to active firms. In the EU-ETS, free allowances are only given to active firms. We consider the case where free allowances are given to $n(z)$ firms. We compare this case with the situation where $n(z)$ firms, whose emissions are reduced by z', are granted profit-neutral permit allocations. The permit price and the total number of permits are identical in the two cases. However, pre-regulation profits are higher in the second case than in the first one, $\pi(n, z = 0) < \pi(n(z), z = 0)$. When the number of firms increases, individual profits fall. We can thus show that:

$$\frac{\pi(n(z), z = 0) - \pi(n(z), z)}{Q(n(z), z)\sigma(n(z), z)} > \frac{\pi(n, z = 0) - \pi(n(z), z)}{Q(n(z), z)\sigma(n(z), z)} \tag{18}$$

In other words, the percentage of permits that the regulator should give for free to offset the loss in profits when allowances are given to active firms is lower than in the case where the number of firms is exogenous. Thus, if offsetting is possible in the second case it is also possible in the first case. Since $z' < z$, we know from the previous section that if offsetting losses in profit is possible when the number of firms is exogenous, it is also possible when the market structure is endogenous, under both linear and isoelastic demand functions, except in the case $n(z) = 1$.

Allowances given to all firms. Let us consider the case where even non-active firms receive free allowances. The number of active firms decreases with the implementation of the market for permits. Let us focus on the effect on the equilibrium permit price when firms drop out. Consider for a moment that the permit price is exogenous. When one firm drops out, the others automatically produce more. However, total output decreases. The equilibrium permit price required to clear the market is therefore lower. In other words, the permit price decreases in function of firms dropping out. The number of active firms has decreased with the implementation of the market for permits. Since individual profits increase with the number of firms, we deduce that $\pi(n(z), z) > \pi(n, z)$. Two effects are thus in play. Losses in profit are lower and the value of permits is also lower.

Let us denote by z'' the percentage reduction in emissions such that the permit price for n active firms is equal to the permit price after firms have dropped out, i.e. $\sigma(n, z'') = \sigma(n(z), z)$. Obviously, $z'' < z$ since the permit price is lower than in the case where firms remain active and the percentage reduction in emissions is equal to z. Moreover, we deduce that $\pi(n(z), z) > \pi(n, z)$. These two situations in fact lead to the same permit price. Furthermore, $\pi(n(z), \sigma) > \pi(n, \sigma)$ since profits decrease with the number of firms. Thus, the case in which the percentage reduction in emissions is given by z'' and the n firms remain active is an upper bound of the case in which the regulator gives free allowances to all the firms even if only $n(z)$ firms are still active. Since $z'' < z$, we deduce from Section 3 that it is always possible to offset losses in profits if it is possible when the number of firms is exogenous.

4.2. International competition and unilateral regulation

Assume two geographical areas H and F. Trade between the two zones is permitted and there is no trade barrier. However, transportation from one area to the other is costly. Firms bear a unit transportation cost denoted by τ. Assume that there are $n = n^H + n^F$ firms producing a homogeneous good, where n^H and n^F are the number of domestic and foreign firms, respectively. For the sake of simplicity, assume that emission intensity is equal to one. Consumers are only present in area H and firms face an inverse demand function $P(Q)$. We assume that the elasticity of the demand slope is sufficiently high ($E > -2$). In other words, we focus on the case where profits decrease with the permit price, even in the closed economy situation.

To reduce pollution, the regulator in area H implements a market for permits in the domestic area. The regulator in area H reduces global emissions by a percentage z. This approach is close to reality, where scientists detail the percentage reduction in emissions required to avoid irreversibly damaging the environment. We can then determine the conditions under which a country may reach acceptability and global objectives. Finally, we analyze the extent to which international competition prevents the regulator from offsetting losses in profits. Domestic firms must own pollution permits to produce, while foreign firms are not subject to this regulation. Domestic firms' profits may be written as the sum of the profits in the products market and the gain from free allowances.

$$\pi_{iH}(\sigma) = (p(Q) - c - \sigma)\, q_{iH} + \varepsilon_i \sigma.$$

Foreign firms' profits are given by:

$$\pi_{jF}(\sigma) = (p(Q) - c - \tau)\, q_{jF}.$$

The first order condition for a domestic firm is equal to:

$$P(Q) + P'(Q)q_{iH} = \sigma + c. \tag{19}$$

The first order condition for a foreign firm is equal to:

$$P(Q) + P'(Q)q_{jF} = c + \tau. \tag{20}$$

By summing the first order conditions, we obtain:

$$(n_H + n_F)P(Q) + P'(Q)Q = n_H\sigma + (n_H + n_F)c + n_F\tau. \quad (21)$$

Since the inverse demand function is twice differentiable, strictly decreasing when positive, and $P(0) > 0$, we deduce that there exists a single level of production verifying equation (21). Moreover, we show in Appendices A.5.1 and A.5.2 that total production decreases with the permit price. Following Février and Linnemer (2004), who focus on an oligopoly and idiosyncratic shocks, we also show that domestic profits always decrease with the permit price and, in order to comply with the profit-neutrality criterion, free allowances should be granted.

We now consider the equilibrium in the market for permits. The perfectly competitive permits market is such that $Q_H(\sigma) + Q_F(\sigma) = (1 - z)(Q_H(0) + Q_F(0))$.

The main difference here compared with Lemma 1 is that the equilibrium permit price, even for linear and isoelastic demand functions, depends on the number of domestic and foreign firms. In Appendices A.5.1 and A.5.2, we demonstrate that the equilibrium permit price increases with the number of foreign firms and decreases with the number of domestic firms, for both linear and isoelastic demand functions. Foreign firms do not need permits to produce. Thus, the higher the number of foreign firms, the lower the initial pollution, and the lower the permit price will be. The mechanism is different for domestic firms, however, since they must bear the permit price to produce. Because the regulation is unilateral, domestic firms' production decreases with the permit price. The lower the number of domestic firms, the higher the initial individual domestic production, and the higher the permit price will be.

Obviously, to reach a given percentage reduction in global emissions, the regulator should implement a higher percentage reduction in domestic emissions. We denote by z_H the percentage reduction in emissions that the regulator must apply in the domestic area in order to reduce global emissions by a percentage, z. In other words, $1 - z_H = \frac{Q_H(\sigma)}{Q_H(0)}$. In Appendices A.5.1 and A.5.2, we show that the percentage reduction in domestic emissions decreases with the number of domestic firms, under both linear and isoelastic demand functions. Indeed, the higher the number of domestic firms, the lower the initial individual domestic production, and the lower the percentage reduction in domestic emissions will be. Moreover, the percentage reduction in domestic emissions must be lower than 100%, which reduces the percentage reduction in global emissions that the regulator can implement. This constraint is clearly different from the one analyzed in Section 3, which is a participation constraint. However, the above-mentioned constraint represents the maximal percentage reduction in global emissions that the regulator can implement in order for domestic production to be strictly positive. For instance, under linear demand, the higher the number of domestic firms, the higher the percentage reduction in emissions the regulator will be able to fix.

As in Section 3, we determine the percentage of free allowances that needs to be given to offset losses in profits and the maximal percentage reduction in domestic emissions that the regulator can implement while giving all permits for free and neutralizing profits. The following two propositions are deduced.

Proposition 4. *When demand is linear, the maximal percentage reduction in domestic emissions that the regulator can implement while giving all permits for free and neutralizing profits is given by* $\overline{z_H}(n_H, n_F)) = 1 - \frac{n_F+1}{n_H}$. *The maximal percentage reduction in global emissions is given by* $\overline{z}(n_H, n_F) = \frac{n_H}{n_F+1} - \frac{a-c-n_F\tau}{(n_H+n_F)(a-c)-n_F\tau}$.

Proof. The proof is provided in Appendix A.5.1. □

Proposition 2 is a special case of Proposition 4. Proposition 2 corresponds to the situation where there are no foreign firms ($n_F = 0$ and $z_H = z$). The maximal percentage reduction in domestic emissions increases with the number of domestic firms and decreases with the number of foreign firms. As in Section 3, when the number of domestic firms is lower, initial profits will be higher, and more free allowances will be required to offset losses in profits. Regardless of the number of foreign firms, if there is only one domestic firm, offsetting losses in profits will be impossible. Exposure to international competition makes the conditions for offsetting more stringent. Thus, at least three domestic firms are required in order for $\overline{z_H}(n_H, n_F)$ to be positive. The maximal percentage reduction in global emissions increases with the number of domestic firms and the marginal cost, and decreases with market size. When initial production is higher, a lower percentage of permits will be required to offset losses in profits.

Proposition 5. *When demand is isoelastic, the maximal percentage reduction in domestic emissions that the regulator can implement while giving all permits for free satisfies:*

$$A^2 < (1-z)(n_H(A) - \frac{1}{n_H}((n_F+n_H)\beta - 1)(1-(1-z)^{\frac{1}{\beta}}))(\frac{1}{\beta} - n_F - n_H)c + (n_Hc + n_F\tau)(1-z)^{\frac{1}{\beta}})$$

where $A = n_F\tau + (\frac{1}{\beta} - n_F)c$.

Proof. The proof is provided in Appendix A.5.2. □

To illustrate Proposition 5, we use the parameters of Nicolai and Zamorano (2018) who consider isoelastic demand and sectors exposed to international competition. They focus on a different issue, namely the redistributive effects of output-based allocation. They calibrate their model for the first two phases of the EU-ETS. Using their parameters for the number of domestic firms, the number of foreign firms, the elasticity of demand, the unit cost, and the transportation cost, we show that offsetting losses in profits for the cement sector is possible only for a percentage reduction in global emissions lower than 9.98%,[8] which corresponds to a percentage reduction in domestic emissions equal to 19.8%. For the steel sector, we demonstrate that the maximal percentage reduction in global emissions is 5.8%,[9] which corresponds to a percentage reduction in domestic emissions of 7.2%. In these two cases, the elasticity of demand is particularly low.

To conclude, offsetting is possible under certain conditions. However, the conditions are more stringent than in Section 3.

4.3. Multi-sector market for permits

We now consider two sectors, called A and B, that are covered by the same market for permits. The two sectors are not identical. Inside each sector, all firms are symmetric. The goal of the regulator is now assumed to be to reduce global emissions, such that $f_A Q_A(\sigma) + f_B Q_B(\sigma) = (1-z)\left(f_A Q_A(0) + f_B Q_B(0)\right)$. Firms are price-takers in the market for permits. Firms take into account the permit price σ as exogenous. The effective marginal cost is equal to the sum of the marginal cost of production and the permit price weighted by emission intensity. Let $z_A = 1 - \frac{Q_A(\sigma)}{Q_A(0)}$ and $z_B = 1 - \frac{Q_B(\sigma)}{Q_B(0)}$ be the percentage reduction in emissions of sector A and sector B, respectively.

At the symmetric equilibrium, as in Section 3, the first order condition of the profits of sector j satisfies:

$$P(Q_j) + P'(Q_j)q_j = f_j\sigma + c_j. \tag{22}$$

As in Section 3, production decreases with the permit price, emission intensity, and marginal cost.

On the market for permits, the aggregate demand for permits is equal to the total number of permits that firms need and have not been granted for free. Thus, the perfectly competitive permits market clears when supply equals demand, or:

$$\begin{aligned} &f_A Q_A(\sigma) + f_B Q_B(\sigma) = (1-z)\left(f_A Q_A(0) + f_B Q_B(0)\right) \\ &\qquad \Rightarrow f_A Q_A(0) * (z - z_A) + f_B Q_B(0) * (z - z_B) = 0. \end{aligned} \tag{23}$$

We note that the percentage reduction in emissions in one of the two sectors is obviously lower than the percentage reduction in global emissions, whereas the other is higher. Moreover, as in Section 3, the permit price is independent of the way in which the permits are distributed. The approach here is different from that used above. The goal is not to determine the permit price but to determine the sector-based reduction in each sector. The sector-based reduction in sector j is given by:

$$\sigma = \frac{M_j((1-z_j)q_j(0)) - M_j(q_j(0))}{f_j}. \tag{24}$$

As noted previously, the permit price is equal to the marginal abatement cost. A difference compared with Section 3 is that the percentage reduction in emissions for a sector is endogenous. However, all firms make decisions based on the same permit price. The goal of a market for permits is that firms may trade permits until they equalize their marginal abatement costs. Therefore, at the equilibrium, the following equation is satisfied:

$$\frac{M_A((1-z_A)q_A(0)) - M_A(q_A(0))}{f_A} = \frac{M_B((1-z_B)q_B(0)) - M_B(q_B(0))}{f_B}. \tag{25}$$

We rewrite the percentage reduction in emissions in sector B according to the one in sector A:

$$z_B = 1 - \frac{1}{q_B(0)} L_B\left(\frac{f_B}{f_A}(M_A((1-z_A)q_A(0)) - M_A(q_A(0)) + M_B(q_B(0))\right). \tag{26}$$

Since there is no abatement technology, a multi-sector market for permits is equivalent to several independent markets for permits with different percentage reductions in emissions. Inserting equation (26) into equation (23), we get:

$$f_A Q_A(0) * (z_A) + f_B n_B L_B\left(\frac{f_B}{f_A}(M_A((1-z_A)q_A(0)) - M_A(q_A(0)) + M_B(q_B(0)))\right) = z(f_A Q_A(0 + (1-z)(f_B Q_B(0)). \tag{27}$$

At the equilibrium, z_A satisfies the previous equation. The analytical results under isoelastic demand are not tractable. For the remainder of this section, we therefore focus on the linear demand case. Under linear demand, equation (27) may be rewritten as:

$$z_A = \frac{f_A n_A \frac{a_A - c_A}{n_A+1} + f_B n_B \frac{a_B - c_B}{n_B+1}}{(a_A - c_A)\left(\frac{f_A n_A}{n_A+1} + \frac{f_B^2}{f_A}\frac{n_B}{n_B+1}\right)} z. \tag{28}$$

The following proposition is then deduced:

[8] The calibrations of Nicolai and Zamorano (2018) for the cement sector give $n_H = 7, n_F = 1, \beta = 0.5, c = 46.8$ and $\tau = 35$.

[9] The calibrations of Nicolai and Zamorano (2018) for the steel sector give $n_H = 7, n_F = 1, \beta = 0.6, c = 247$ and $\tau = 31$.

Proposition 6. *Under linear demand, the percentage reduction in emissions in sector A decreases with market size in sector A, the number of firms in sector A, the marginal cost in sector B, and emission intensity in sector B, and increases with emission intensity in sector A, marginal cost in sector A, and market size in sector B.*

Under linear demand, the percentage reduction in emissions in sector A is higher than the percentage reduction in emissions in the whole economy if $f_A(a_B - c_B) > f_B(a_A - c_A)$. When the initial production of one sector is higher, the emission percentage in the other sector will be higher. When the emission intensity of one sector is lower, the emission percentage in the other sector will be higher. A low emission intensity induces a high marginal abatement cost. Finally, the greater the number of firms in a sector, the higher the initial production, and the lower the percentage reduction in emissions will be in that sector.

Assume that the regulator allocates the adequate number of free allowances to each sector. Let us consider linear demand. The ratio of free allowances to permits for the whole economy is given by:

$$\gamma_{ps} = \frac{n_A f_A(2-z_A)\frac{1}{n_A+1}q_A(\sigma=0) + n_B f_B(2-z_B)\frac{1}{n_B+1}q_B(\sigma=0)}{n_A f_A q_A(\sigma) + n_B f_B q_B(\sigma)}. \tag{29}$$

Let us assume that $n_A = 1$. The following proposition determines whether the maximal percentage reduction in emissions that the regulator can implement while offsetting losses in profits is positive.

Proposition 7. *Under linear demand, the maximal percentage reduction in emissions that the regulator can implement while offsetting losses in profits is positive if there are at least two firms in the other sector. This percentage increases with the number of firms in the second sector.*

Proof.

- If $n_B = 1$, $\gamma_{ps} = \frac{2-z}{2(1-z)}$, since $f_A Q_A(0)^*(z - z_A) + f_B Q_B(0)^*(z - z_B) = 0$. $\gamma_{ps} \le 1$ induces $z = 0$.
- If $n_B > 1$, $\gamma_{ps} = \frac{2-z}{2(1-z)} - \frac{n_B f_B q_B(\sigma=0)(2-z_B)(\frac{1}{2}-\frac{1}{n_B+1})}{(1-z)(f_A q_A(\sigma=0)+n_B f_B q_B(\sigma=0))}$. Since $\frac{n_B f_B q_B(\sigma=0)(2-z_B)(\frac{1}{2}-\frac{1}{n_B+1})}{(1-z)(f_A q_A(\sigma=0)+n_B f_B q_B(\sigma=0))} > 0$, there exists a threshold $\bar{z}$, strictly positive, such that $z < \bar{z}$ implies $\gamma_{ps} < 1$. □

Offsetting the loss in profits for a monopoly is possible if there are at least two firms in the other sector.

5. Conclusion

The present paper addresses two policy objectives: to implement a market for pollution permits and to make regulation acceptable for businesses. It shows that a low number of free allowances is sufficient to meet these two goals. Moreover, the regulator can fully offset losses, even when the reduction in emissions is high, provided that the sectors concerned are not monopolies. The existence of international competition is one of the main factors limiting the regulator's ability to offset losses in profits, although when the number of domestic firms is sufficiently great and the number of foreign firms is sufficiently low, offsetting losses in profits remains possible. Nonetheless, including a polluting sector not exposed to international competition in the market for permits allows the regulator to offset losses in profits in sectors exposed to international competition.

In Europe, grandfathering has been replaced by capacity-based allocation, which lowers the effective marginal production cost but impedes attempts to offset losses in profits. In light of these findings, we argue that the use of grandfathering coupled with a significant reduction in carbon emissions should be promoted instead of promoting capacity-based allocation and a weak percentage reduction in emissions.

Under a profit-neutral allocation, the cost of environmental regulation is entirely borne by consumers and the state. Regulators should limit the number of free allowances to this upper bound.

A. Appendixes

A.1. Proof Lemma 1

From equation (7), we immediately deduce that the equilibrium permit price increases with the percentage of emission reduction and decreases with emission intensity. Taking the derivative of equation (7) according to the marginal cost, we get:

$$\frac{\partial \sigma}{\partial c} = \frac{1}{f}((1-z)\frac{\partial M((1-z)q(0))}{\partial q(o)} - \frac{\partial M(q(0))}{\partial q(o)})\frac{\partial q(o)}{\partial c}. \tag{30}$$

We deduce that $\frac{\partial \sigma}{\partial c} > 0$ if M is sufficiently convex.

The perfectly competitive permits market clears when supply equals demand, or:

$$fQ(\sigma) = (1-z)fQ(0). \tag{31}$$

We rewrite this equation as

$$fL(f\sigma + c, n) = (1-z)fL(c, n). \tag{32}$$

Assume that $L(c, n) = l(c)h(n)$, where l and h are two independent functions. Equation (32) may rewritten as

$$fl(f\sigma + c)h(n) = (1 - z)fl(c)h(n). \tag{33}$$

We deduce that there exists a single σ satisfying equation (33) and that σ does not depend on n.

A.2. Linear demand function

Firms face a demand given by:

$$P(Q) = a - bQ \quad with \quad Q = \Sigma_{i=1}^{n} q_i. \tag{34}$$

Profits may be written as the sum of the profits in the market for products and the gain due to free allowances.

$$\pi_i(\sigma) = (p(Q) - c - f\sigma)\, q_i + \varepsilon_i \sigma.$$

The quantities produced, the product price and the mark-up are given by:

$$q_i(\sigma) = \frac{a - c - f\sigma}{b(n+1)}, \quad p(\sigma) = \frac{a + n(c + f\sigma)}{n+1}, \quad p(\sigma) - c - f\sigma = \frac{a - c - f\sigma}{(n+1)}.$$

Since $q_i(\sigma) = (1 - z)q_i(0)$, we get:

$$\sigma = z\frac{a - c}{f}. \tag{35}$$

The profit-neutral allowances are given by $\epsilon\sigma = \pi_i(0) - \pi_i(\sigma)$ and may be formulated as:

$$\epsilon^N = \frac{1}{n\beta - 1}\frac{1}{\sigma}\left(q_i(0)c - q_i(\sigma)(c + f\sigma)\right).$$

Using $\pi_i(\sigma) = b(q_i(\sigma))^2$ and $q_i(\sigma) = (1 - z)q_i(0)$, we get:

$$\epsilon^N = f(2 - z)\frac{1}{n+1}q_i(\sigma = 0).$$

The ratio of free allowances over permits ($\gamma_p = \frac{\epsilon}{fq_i(\sigma)}$) which is required to offset losses is equal to:

$$\gamma_p = \frac{1}{n+1}\frac{2 - z}{1 - z}.$$

Free allowances cannot exceed the number of permits in circulation and then $\gamma_p < 1$. This condition is equivalent to $z < 1 - \frac{1}{n}$.

A.3. Isoelastic demand function

Assume an iso-elastic demand function. Let β be the elasticity of the demand. Firms face a demand given by:

$$P(Q) = \alpha Q^{-\frac{1}{\beta}} \quad with \quad Q = \Sigma_{i=1}^{n} q_i, \tag{36}$$

where α is the market size. Assume $\beta > 1/n$. This assumption states that elasticity is higher than $\frac{1}{n}$ and it is shown below that this ensures the existence of the equilibrium.

The quantities produced, the product price and the mark-up rate are given by:

$$q_i(\sigma) = \frac{1}{n}\left(\frac{\alpha(\beta - 1/n)}{\beta(c + f\sigma)}\right)^{\beta}, \quad p(\sigma) = \frac{c + f\sigma}{1 - 1/(n\beta)}, \quad \frac{p(\sigma) - c - f\sigma}{c + f\sigma} = \frac{1}{n\beta - 1}.$$

Moreover, $q_i(\sigma) = (1 - z)q_i(0)$ and we get:

$$\sigma = ((1 - z)^{-\frac{1}{\beta}} - 1)\frac{c}{f}. \tag{37}$$

The profit-neutral allowances are given by $\varepsilon\sigma = \pi_i(0) - \pi_i(\sigma)$.

Using $\pi_i(\sigma) = q_i(\sigma)\frac{c+f\sigma}{n\beta-1}$ and $q_i(\sigma) = (1 - z)q_i(0)$, we get:

$$\epsilon^N = f(\frac{1}{n})^{\beta+1}(\frac{\alpha}{\beta})^{\beta}(n\beta - 1)^{\beta-1}\left(\frac{1 - (1 - z)^{1-1/\beta}}{(1 - z)^{-1/\beta} - 1}\right)c^{-\beta}.$$

The ratio of free allowances over permits ($\gamma_p = \frac{\epsilon}{fq_i(\sigma)}$) which is required to offset losses is equal to:

$$\gamma_p = \frac{1}{n\beta - 1}\left(\frac{(1 - z)^{-1} - 1}{(1 - z)^{-\frac{1}{\beta}} - 1} - 1\right). \tag{38}$$

A.4. Proof of Proposition 3

Let compare now γ_p with 1 under isoelastic demand function. Free allowances cannot exceed the number of permits put into circulation. When $n = 1$, β is higher than 1 by assumption. γ_p may then be rewritten as follows:

$$\gamma_p = \frac{1}{n\beta - 1}\Sigma_{k=1}^{\beta-1}(1-z)^{(-\frac{1}{\beta})^k}.$$

γ_p is the quotient of a sum of β terms higher than 1 over $\beta - 1$ which is higher than 1. To conclude, if $n = 1$, then $\gamma_p > 1$. However,

$$\gamma_p = \frac{1}{n\beta - 1}\left(\frac{(1-z)^{-1}-1}{(1-z)^{-\frac{1}{\beta}}-1} - 1\right) < 1 \Leftrightarrow \left(\frac{(1-z)^{-1}-1}{(1-z)^{-\frac{1}{\beta}}-1}\right) < n\beta.$$

When $n = 2$, the constraint is given by $(\frac{(1-z)^{-1}-1}{(1-z)^{-\frac{1}{\beta}}-1}) < 2\beta$. For $\beta < 10$, this corresponds to $\bar{z} = 0.75$. When $n = 3$, the constraint is given by $(\frac{(1-z)^{-1}-1}{(1-z)^{-\frac{1}{\beta}}-1}) < 3\beta$. For $\beta < 10$, this corresponds to $\bar{z} = 0.873$.

Since $\bar{z}$ increases with elasticity of demand and decreases with the number of firms, it can be concluded that, when $n = 2$, $0.75 < \bar{z}(\beta, 2)$ and when $n > 2$, $\bar{z}(\beta, n) > 0.87$.

A.5. International competition and unilateral regulation

A.5.1. International competition and linear demand function

Firms face a demand given by:

$$P(Q) = a - bQ \quad with \quad Q = \Sigma_{i=1}^{n} q_i. \tag{39}$$

We show that, at equilibrium, the total production is equal to:

$$Q = \frac{(n_H + n_F)(a-c) - n_F\tau - n_H\sigma}{b(n_H + n_F + 1)}, \tag{40}$$

and the domestic individual quantity is given by:

$$q_H = \frac{a - c - (n_F + 1)\sigma + n_F\tau}{b(n_H + n_F + 1)}. \tag{41}$$

The perfectly competitive permits market is such that $Q_H(\sigma) + Q_F(\sigma) = (1 - z)(Q_H(0) + Q_F(0))$, which leads to

$$\sigma = z\frac{(n_H + n_F)(a-c) - n_F\tau}{n_H}. \tag{42}$$

The percentage of domestic emission reduction that the regulator should apply in the domestic area in order to reduce global emissions by a percentage z is denoted by z_H, where $1 - z_H = \frac{Q_H(\sigma)}{Q_H(0)}$.

$$z_H = 1 - \frac{q_h(\sigma)}{q_h(0)}, \tag{43}$$

$$= \frac{\frac{(n_F+1)\sigma}{b(n_H+n_F+1)}}{\frac{a-c+n_F\tau}{b(n_H+n_F+1)}}, \tag{44}$$

$$= \frac{(n_F+1)((n_H+n_F)(a-c) - n_F\tau)}{n_H(a-c+n_F\tau)}z. \tag{45}$$

Since z_H should be lower than one, z should be lower than $\frac{n_H(a-c+n_F\tau)}{(n_F+1)((n_H+n_F)(a-c)-n_F\tau)}$.

We determine the level of free-allowances which offset profits' losses:

$$\epsilon^N\sigma = \pi_H(\sigma = 0) - \pi_H(\sigma),$$

$$\epsilon^N\sigma = b(q_H(\sigma = 0))^2 - b(q_H(\sigma))^2,$$

$$\epsilon^N\sigma = b(q_H(\sigma = 0))(2 - z_H)(\frac{-(n_F+1)\sigma}{n_H + n_F + 1}.$$

The profit-neutral allowances are equal to:

$$\epsilon^N = (2 - z_H)\frac{n_F + 1}{n_H + n_F + 1}q_H(\sigma = 0).$$

The ratio of free allowances over permits ($\gamma_p = \frac{n_H \epsilon}{n_H q_H(\sigma)}$) is equal to:

$$\gamma_p = \frac{(2 - z_H)}{1 - z_H} \frac{n_F + 1}{n_H + n_F + 1}.$$

γ_p should be lower than one. This constraint may rewritten as

$$z_H < 1 - \frac{n_F + 1}{n_H}. \tag{46}$$

The constraint may also be expressed as a function of the percentage of global emission reduction,

$$z < \frac{n_H}{n_F + 1} - \frac{a - c - n_F \tau}{(n_H + n_F)(a - c) - n_F \tau}. \tag{47}$$

A.5.2. International competition and isoelastic demand function

Assume an iso-elastic demand function. Let β be the elasticity of the demand. Firms face a demand given by:

$$P(Q) = \alpha Q^{-\frac{1}{\beta}} \quad with \quad Q = \Sigma_{i=1}^{n} q_i, \tag{48}$$

where α is the market size. Moreover, assume that $\beta > \frac{1}{n_H + n_F}$. As previously, this assumption ensures the existence of an equilibrium. The individual quantities produced by domestic and foreign firms are given by:

$$q_{i,H} = (\frac{\alpha}{\beta})^{\beta} \frac{((n_F + n_H)\beta - 1)^{\beta}}{(n_H(c + \sigma) + n_F \tau)^{\beta+1}} \left(\beta n_F \tau + (1 - n_F \beta)(c + \sigma)\right),$$

$$q_{i,F} = (\frac{\alpha}{\beta})^{\beta} \frac{((n_F + n_H)\beta - 1)^{\beta}}{(n_H(c + \sigma) + n_F \tau)^{\beta+1}} \left(\beta n_H (c + \sigma) + (1 - n_H \beta)\tau\right).$$

Total production is equal to:

$$(\frac{\alpha}{\beta})^{\beta} \frac{((n_F + n_H)\beta - 1)^{\beta}}{(n_H(c + \sigma) + n_F \tau)^{\beta}}. \tag{49}$$

The perfectly competitive permits market is such that $Q_H(\sigma) + Q_F(\sigma) = (1 - z)(Q_H(0) + Q_F(0))$. The equilibrium permit price is then equal to:

$$\sigma = ((1 - z)^{-\frac{1}{\beta}} - 1)(c + \frac{n_F}{n_H}\tau).$$

The percentage of domestic emission reduction that the regulator should apply in the domestic area in order to reduce global emissions by a percentage z is denoted by z_H, where $1 - z_H = \frac{Q_H(\sigma)}{Q_H(0)}$.

We calculate $\frac{Q_H(\sigma)}{Q_H(0)}$ and show that it is equal to:

$$\frac{Q_H(\sigma)}{Q_H(0)} = (1 - z)\left(1 - \frac{1}{n_H} \frac{((n_F + n_H)\beta - 1)}{(\frac{1}{n_F} - \beta)\frac{c}{\tau} + \beta}(1 - (1 - z)^{\frac{1}{\beta}})\right). \tag{50}$$

The threshold $\widetilde{z}_H$, such that $z < \widetilde{z}_H$ implies $z_H < 1$ is given by:

$$\widetilde{z} = 1 - \left(1 - \frac{n_H(\frac{1}{n_F} - \beta)\frac{c}{\tau} + \beta)}{(n_F + n_H)\beta - 1)}\right)^{\beta}.$$

The profit-neutral allowances for a domestic firm (ϵ_{iH}) are given by:

$$\begin{aligned}
\epsilon_{iH} &= \frac{1}{(n_H + n_F)\beta - 1}\left((q_{iH}(\sigma = 0) - q_{iH}(\sigma))\frac{\beta n_F \tau + (1 - n_F \beta)c}{\sigma} - (1 - n_F \beta) q_{iH}(\sigma)\right), \\
&= \frac{q_{iH}(\sigma)}{(n_H + n_F)\beta - 1}\left(((1 - z_H)^{-1} - 1)\frac{\beta n_F \tau + (1 - n_F \beta)c}{\sigma} - (1 - n_F \beta)\right), \\
&= \frac{q_{iH}(\sigma)}{(n_H + n_F)\beta - 1}\left(\frac{((1 - z_H)^{-1} - 1)}{((1 - z)^{-\frac{1}{\beta}} - 1)} \frac{\beta n_F \tau + (1 - n_F \beta)c}{c + \frac{n_F}{n_H}\tau} - (1 - n_F \beta)\right).
\end{aligned}$$

It is immediately apparent that the share of permits that the regulator grants for free (denoted by γ_{pH}) is equal to:

$$\gamma_{pH} = \frac{1}{(n_H + n_F)\beta - 1}\left(\frac{((1-z_H)^{-1}-1)}{((1-z)^{-\frac{1}{\beta}}-1)}\frac{\beta n_F \tau + (1-n_F\beta)c}{c + \frac{n_F}{n_H}\tau} - (1-n_F\beta)\right). \quad (51)$$

Assume that $z > \tilde{z}_H$. The following equation indicates that profit-neutral allowances are lower than the total number of permits if and only if

$$\frac{((1-z_H)^{-1}-1)}{((1-z)^{-\frac{1}{\beta}}-1)} < \frac{n_H c + n_F \tau}{n_F\tau + (\frac{1}{\beta} - n_F)c}. \quad (52)$$

This condition can be rewritten as

$$(1-z_H)^{-1} < \frac{(\frac{1}{\beta} - n_F - n_H)c + (n_H c + n_F\tau)(1-z)^{\frac{1}{\beta}}}{n_F\tau + (\frac{1}{\beta} - n_F)c}. \quad (53)$$

Inserting equation (50) in equation (53), we get:

$$A^2 < (1-z)(n_H(A) - \frac{1}{n_H}((n_F+n_H)\beta - 1)(1-(1-z)^{\frac{1}{\beta}}))(\frac{1}{\beta} - n_F - n_H)c + (n_H c + n_F\tau)(1-z)^{\frac{1}{\beta}})$$

where $A = n_F\tau + (\frac{1}{\beta} - n_F)c$.

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