In [5]:

```
import numpy as np
import pandas as pd
import seaborn as sns
from matplotlib import pyplot as plt
import matplotlib.axes as ax
from IPython.display import clear_output
```

In [6]:

```
df = pd.read_csv("data_for_lr.csv")
df.head()

# Drop null values
df = df.dropna()
df.head()
```

Out[6]:

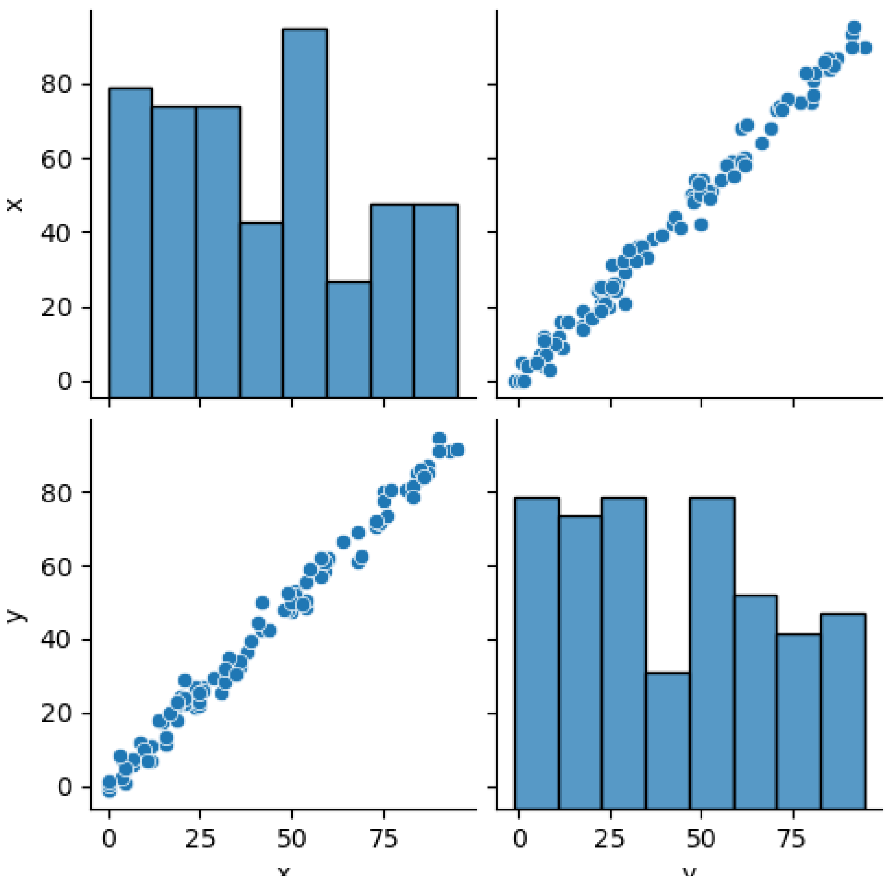
	x	y
0	24.0	21.549452
1	50.0	47.464463
2	15.0	17.218656
3	38.0	36.586398
4	87.0	87.288984

In [7]:

```
sns.pairplot(
    df[:100]
)
```

Out[7]:

<seaborn.axisgrid.PairGrid at 0x161e928ed90>



Train-Test Split

In [8]:

```
# skipping cross-validation part for now for convenience
N = len(df)
training_data_len = int (N*0.8)
x_train, y_train = np.array(df['x'][0:training_data_len]).reshape(training_data_len, 1),
np.array(df['y'][0:training_data_len]).reshape(
    training_data_len, 1
)
x_test, y_test = np.array(df['x'][training_data_len:N]).reshape(N - training_data_len, 1
), np.array(
    df['y'][training_data_len:N]).reshape(N - training_data_len, 1)
```

In [9]:

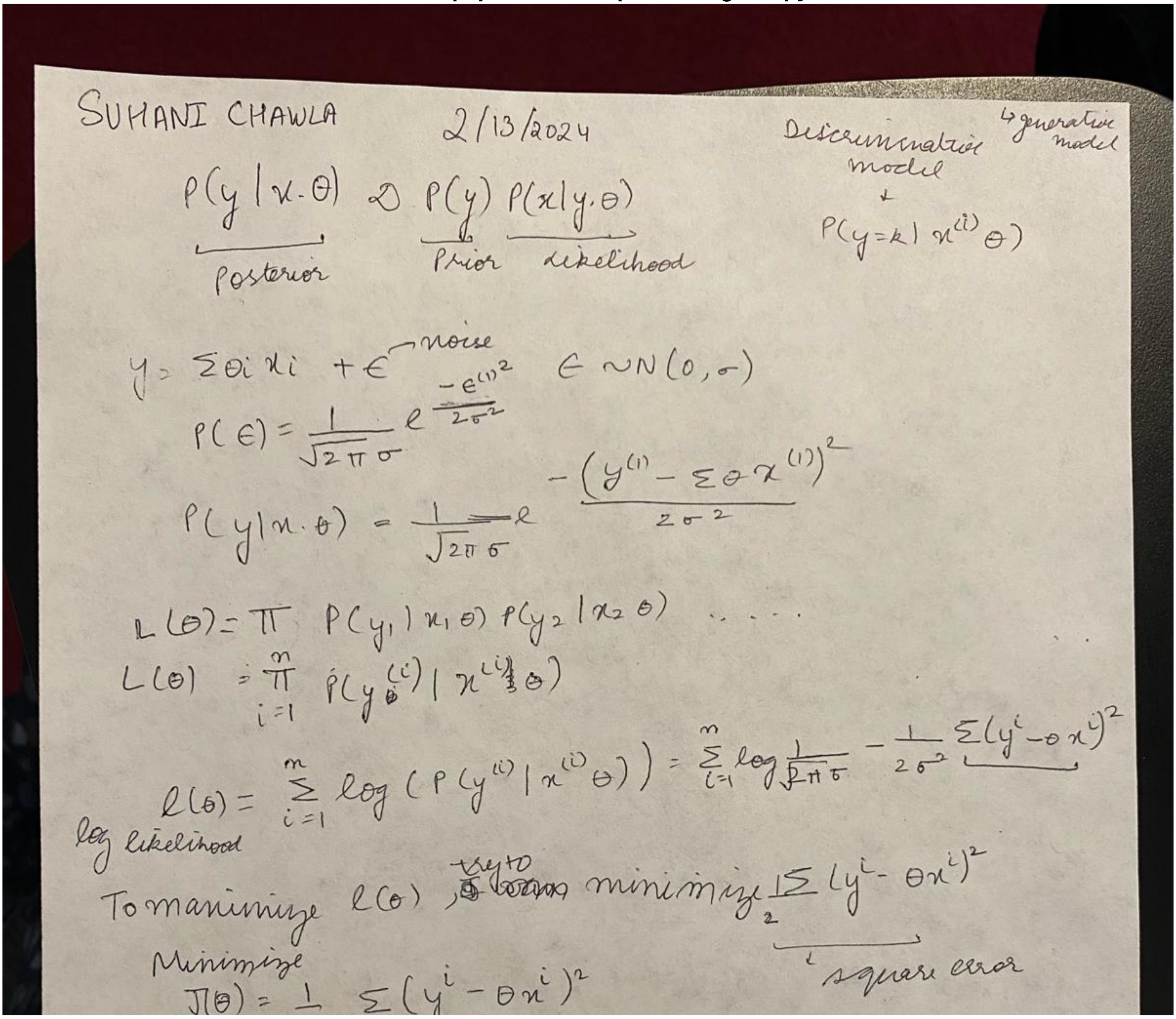
```
N
```

Out[9]:

```
699
```

linear regression

here's the mathematical derivation I did on paper before implementing it in python



SUHANI CHAWLA 2/13/2024

Discriminative model ↓ $P(y=k \mid x^{(i)}\, \theta)$

↳ generative model

$$\underbrace{P(y \mid x.\theta)}_{\text{Posterior}} \propto \underbrace{P(y)}_{\text{Prior}}\ \underbrace{P(x \mid y.\theta)}_{\text{likelihood}}$$

$$y = \sum \theta_i x_i + \overset{\text{noise}}{\epsilon} \qquad \epsilon \sim N(0, \sigma)$$

$$P(\epsilon) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-\epsilon^{(i)2}}{2\sigma^2}}$$

$$P(y \mid x.\theta) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(y^{(i)} - \sum \theta x^{(i)})^2}{2\sigma^2}}$$

$$L(\theta) = \prod P(y_1 \mid x_1\, \theta)\, P(y_2 \mid x_2\, \theta) \ldots$$

$$L(\theta) = \prod_{i=1}^{m} P(y^{(i)} \mid x^{(i)}\, \theta)$$

$$\ell(\theta) = \sum_{i=1}^{m} \log\left(P(y^{(i)} \mid x^{(i)}\, \theta)\right) = \sum_{i=1}^{m} \log \frac{1}{\sqrt{2\pi}\,\sigma} - \frac{1}{2\sigma^2} \underbrace{\sum (y^i - \theta x^i)^2}$$

log likelihood

To maximize $\ell(\theta)$, try to minimize $\frac{1}{2}\underbrace{\sum (y^i - \theta x^i)^2}_{\text{square error}}$

Minimize

$$J(\theta) = \frac{1}{2} \sum (y^i - \theta x^i)^2$$

$$J(\theta) = \frac{1}{2} \sum (x\theta - y)^T (x\theta - y)$$

$$d\theta J(\theta) = x^T x \theta - x^T y = 0 \rightarrow \text{for minimizing}$$

$$\boxed{\theta = (x^T x)^{-1} x^T y}$$

$$\theta_J^t = \theta_J^{t-1} - d \frac{\alpha}{\alpha \theta_J} J(\theta)$$

$$\theta_J^t = \theta_J^{t-1} - \underset{\downarrow}{d} \sum_{i=1}^{m} (x^i \theta - y^i) x_J$$

learning rate

Train → Validation data → Test data

stop on validation error

Shuffle

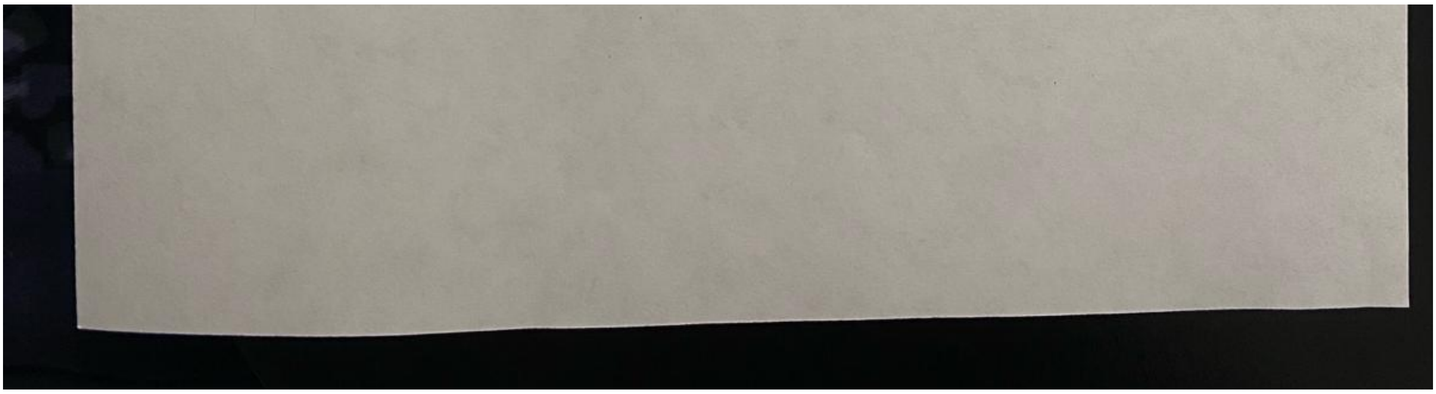
$$SST = \sum_{i=1}^{n} (y^i - \bar{y})^2$$

$$SSR = \sum (x^i \theta - y^i)^2$$

$$r\text{-}2 \text{ score} = 1 - \frac{SSR}{SST}$$

higher the r-2 score, better the model

list mean square error $= \frac{1}{m} \sum_{i=1}^{n} \frac{1}{2} (x\theta - y)^2$



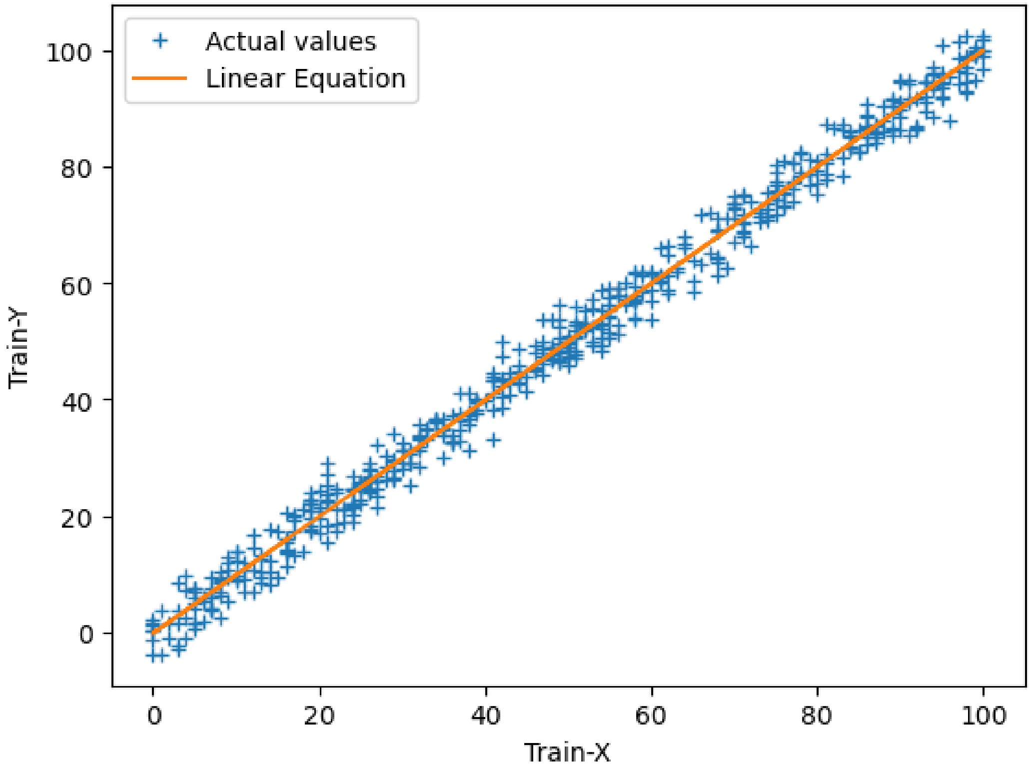
In [10]:

```
# the same calculation in python

class LinearRegression:
    def __init__(self):
        self.Q0 = np.random.uniform(0, 1) * -1  # Intercept
        self.Q1 = np.random.uniform(0, 1) * -1  # Coefficient of X
        self.losses = []  # Storing the loss of each iteration
        self.r2s = []

    def forward_propogation(self, training_input):
        predicted_values = np.multiply(self.Q1, training_input) + self.Q0  # y = mx + c
        return predicted_values

    def cost(self, predictions, training_output):
        lmse = np.mean((predictions - training_output) ** 2)  # Calculating the cost
        ssr = np.sum((predictions - training_output)**2)
        sst = np.sum((training_output - np.mean(training_output))**2)
        r2_score = 1 - (ssr/sst)
        return lmse, r2_score

    def finding_derivatives(self, cost, predictions, training_input, training_output):
        diff = predictions - training_output
        dQ0 = np.mean(diff)  # d(J(Q0, Q1))/d(Q0)
        dQ1 = np.mean(np.multiply(diff, training_input))  # d(J(Q0, Q1))/d(Q1)
        return dQ0, dQ1

    def train(self, x_train, y_train, lr, itrs):
        for i in range(itrs):
            # Finding the predicted values (Using the linear equation y=mx+c)
            predicted_values = self.forward_propogation(x_train)

            # Calculating the Loss
            loss, r2 = self.cost(predicted_values, y_train)
            self.losses.append(loss)
            self.r2s.append(r2)

            # Back Propagation (Finding Derivatives of Weights)
            dQ0, dQ1 = self.finding_derivatives(
                loss, predicted_values, x_train, y_train
            )

            # Updating the Weights
            self.Q0 = self.Q0 - lr * (dQ0)
            self.Q1 = self.Q1 - lr * (dQ1)

            # It will dynamically update the plot of the straight line
            line = self.Q0 + x_train * self.Q1
            clear_output(wait=True)
            plt.plot(x_train, y_train, "+", label="Actual values")
            plt.plot(x_train, line, label="Linear Equation")
            plt.xlabel("Train-X")
            plt.ylabel("Train-Y")
            plt.legend()
            plt.show()
        return (
            self.Q0,
```

```
            self.Q1,
            self.losses,
            self.r2s
        )   # Returning the final model weights and the losses
```

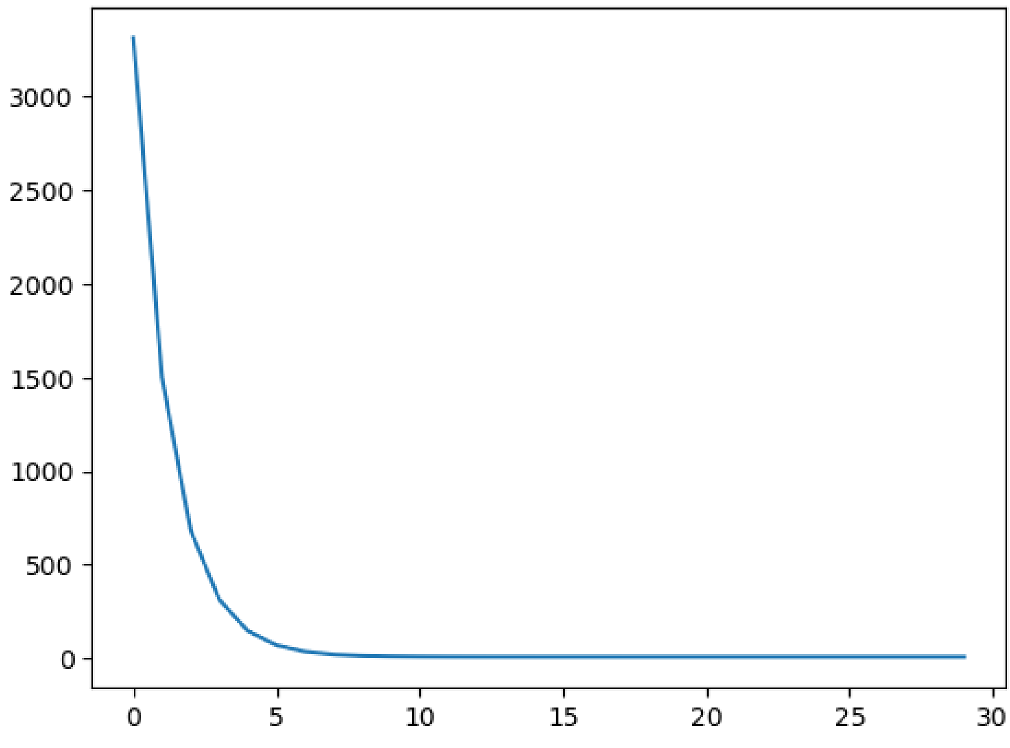
In [11]:

```
lr = 0.0001   # Learning Rate
itrs = 30   # No. of iterations
model = LinearRegression()
Q0, Q1, losses,r2s = model.train(x_train, y_train, lr, itrs)
```



In [12]:

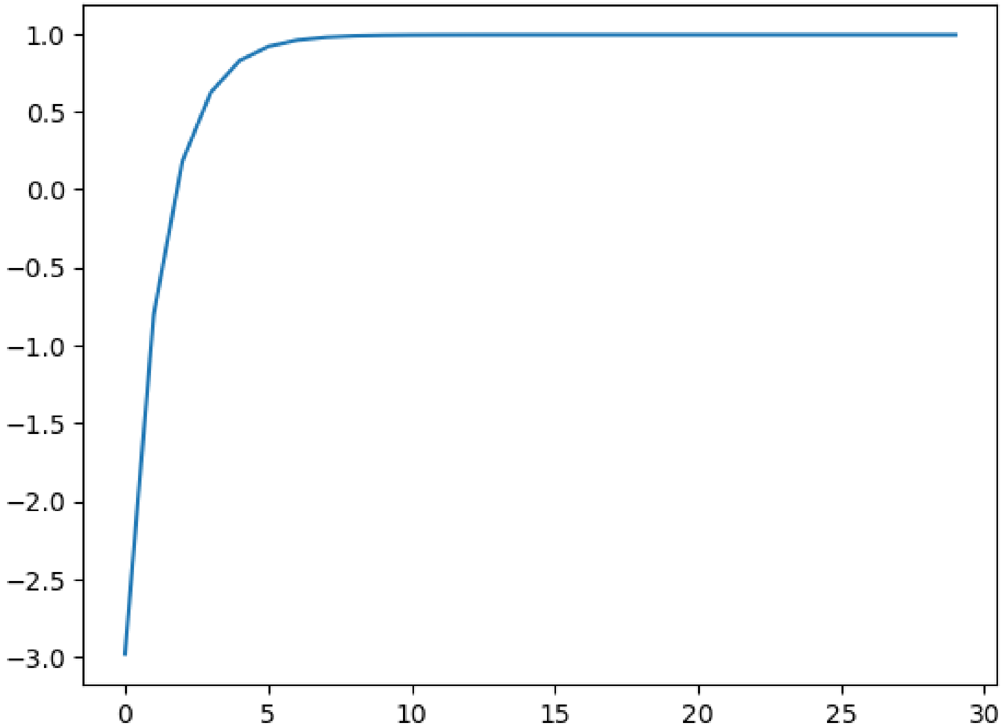
```
plt.plot(losses)
plt.show()
print("from this graph we can see that after 10 iterations the loss is stabilized, theref
ore we could stop roughly after 10 iterations")
```



from this graph we can see that after 10 iterations the loss is stabilized, therefore we could stop roughly after 10 iterations

In [13]:

```
plt.plot(r2s)
plt.show()
```



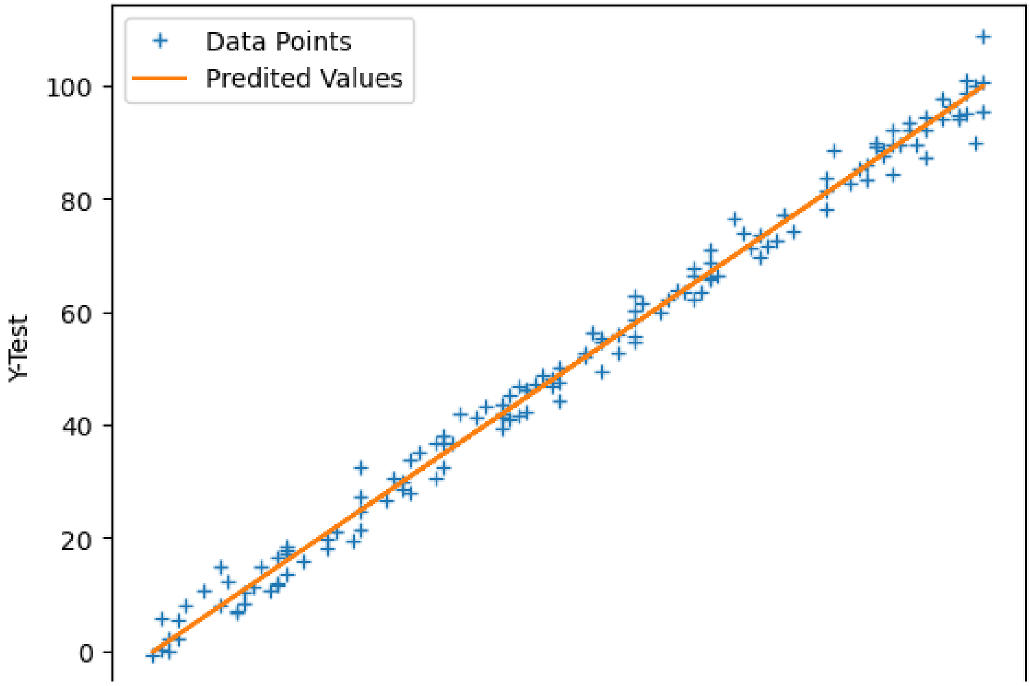
performance on test data

In [14]:

```
y_pred = model.forward_propogation(x_test)
```

In [15]:

```
plt.plot(x_test, y_test, "+", label="Data Points")
plt.plot(x_test, y_pred, label="Predited Values")
plt.xlabel("X-Test")
plt.ylabel("Y-Test")
plt.legend()
plt.show()
```



2nd way

the manual implementation was for my own understanding, now using sklearn LinearRegression and train_test_split

In [16]:

```
df2 = pd.read_csv("data_for_lr.csv")
df2.head()

# Drop null values
df2 = df2.dropna()

Y = df2['y']
X = df2.drop("y", axis=1)
len(X)==len(Y)
```

Out[16]:

```
True
```

In [17]:

```
X.head()
```

Out[17]:

	x
0	24.0
1	50.0
2	15.0
3	38.0
4	87.0

In [18]:

```
Y.head()
```

Out[18]:

```
0    21.549452
1    47.464463
2    17.218656
3    36.586398
4    87.288984
Name: y, dtype: float64
```

In [21]:

```
### do train test split using sklearn
from sklearn.model_selection import train_test_split
X_train, X_test, Y_train, Y_test = train_test_split(X, Y, test_size = 0.25)
```

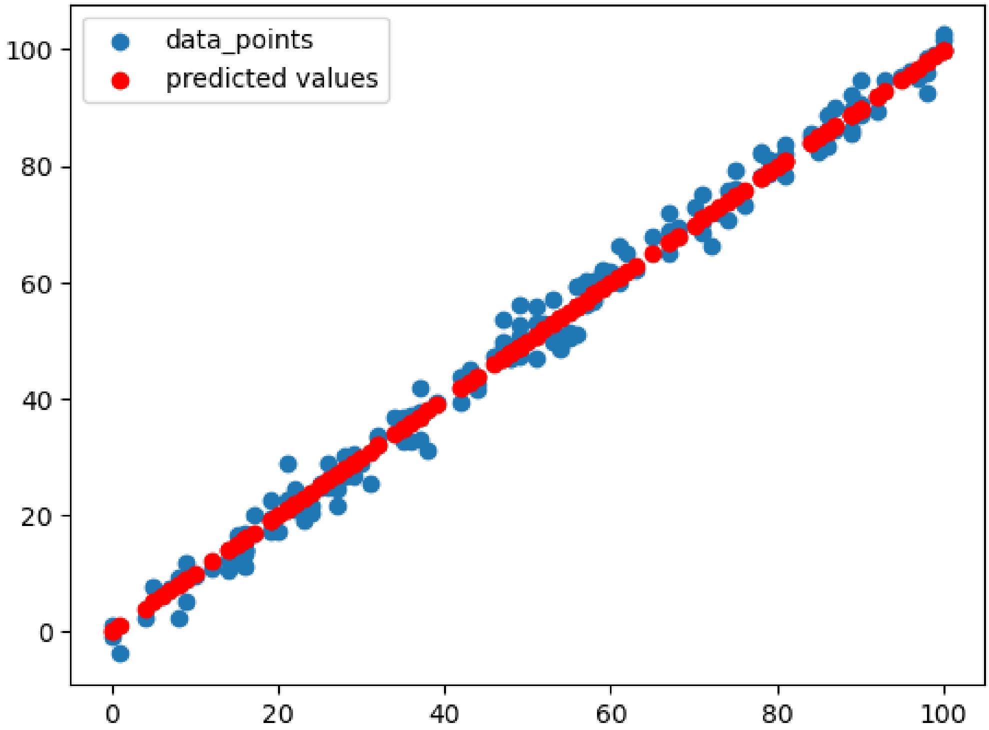
In [24]:

```
### fit a regression model on train data
### predict values of on test data
from sklearn.linear_model import LinearRegression
regr = LinearRegression()
regr.fit(X_train, Y_train)
print(regr.score(X_test, Y_test))
```

```
0.9900697525613219
```

In [42]:

```
# graphing results
import matplotlib.pyplot as plt
y_pred = regr.predict(X_test)
plt.scatter(X_test, Y_test, label = 'data_points')
plt.scatter(X_test, y_pred, color = 'r', label = 'predicted values')
plt.legend()
plt.show()
```



In [ ]:

```
```

In [ ]:

```
```