Homework # 6

Chapter 6 Kittel
Phys 175A
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SJSU

Prob. 1 – Kinetic Energy of electron gas

Adam Gray

Kinetic energy of electron gas.

Show that the kinetic energy of a three-dimensional gas of N free electrons at 0 K is

$$U_0 = N\left(\frac{3}{5}\right)E_f .$$

In Kittel, we're given the energy of a free electron in this model to be:

$$E_k = \left(\frac{\hbar^2}{2m}\right)K^2$$

To find the mean value of E over the volume of the sphere in K space we use the definition:

$$\langle E \rangle = \left(\frac{1}{Vol\ (C)}\right)\int_C E$$

For our problem we have

$$\langle E \rangle = \left(\frac{\hbar^2}{2m}\right)\left(\frac{1}{(4/3)\pi K_f^3}\right)\iiint K^2 \bullet K^2 \sin\theta dk d\theta d\phi$$

$$\langle E \rangle = \left(\frac{\hbar^2}{2m}\right)\left(\frac{1}{(4/3)\pi K_f^3}\right)\left((4/5)\pi K_f^5\right)$$

Which results in the following mean energy per electron of

$$\langle E \rangle = \left(\frac{3}{5}\right)\left(\frac{\hbar^2}{2m}\right)K_f^2$$

Comparing to our initial energy equation (12), we can rewrite this as

$$\langle E \rangle = \left(\frac{3}{5}\right)E_f$$

So for N number of electrons, the total energy would be

$$U_0 = N\left(\frac{3}{5}\right)E_f$$

Prob. 2 – Pressure & Bulk modulus

Victor Chikhani

(a) **Derive a relation connecting the pressure (p) and volume *(V)* of an electron gas at 0 K.**

At 0 Kelvin entropy is constant (3rd Law of Thermo.)

When S=0 we can solve for the pressure (*p*) by taking the partial of the internal energy (U_o) with respect to volume (*V*)

$$p = \left(-\frac{\partial U_o}{\partial V} \right) \qquad U_o = \frac{3N\hbar^2}{10m}\left(\frac{3\pi^2 N}{V}\right)^{2/3}$$

Solution:

$$p = \left(-\frac{\partial U_o}{\partial V} \right) = -\frac{3N\hbar^2}{10m}\frac{2}{3}\left(\frac{3\pi^2 N}{V}\right)^{-1/3}\left(-\frac{3\pi^2 N}{V^2}\right) = \frac{2U_o}{3V}$$

(b) Show that the bulk modulus *(B)* of an electron gas at 0 K is $B = \frac{5p}{3} = \frac{10U_o}{9V}$

$$B = -V\left(\frac{\partial p}{\partial V}\right) \qquad p = \frac{2U_o}{3V}$$

Solution:

$$B = -V\left(\frac{2}{3V}\frac{\partial U_o}{\partial V} + U_o \frac{\partial}{\partial V}\left(\frac{2}{3V}\right)\right) = -V\left(\frac{2}{3V} - \frac{2U_o}{3V} + U_o - \frac{2}{3V^2}\right)$$

$$B = \left(\frac{4U_o}{9V} + \frac{6U_o}{9V}\right) = \frac{10U_o}{9V}$$

(c) Estimate for potassium (K), using Table 1, the value of the electron gas contribution to *B*

Solution:

$$B = \frac{10U_o}{9V} = \frac{10}{9}\frac{3N\varepsilon}{5V} = \frac{10}{9}\frac{3}{5}\left(\frac{1.40\times10^{22}}{cm^3}\right)(2.12eV) = 1.97*10^{22}\frac{eV}{cm^3} = \frac{3.15*10^{10}dyn}{cm^2}$$

Answer agrees with value from Table 3, chapter 3.

Prob. 3 – Chemical potential in 2D

Jason Thorsen

Show that the chemical potential of a Fermi Gas in two dimensions is given by:

$$\mu(T) = k_B T ln[\exp\left(\frac{\pi n \hbar^2}{m k_B T}\right) - 1]$$

for n electrons per unit area. Note: the density of orbitals of a free electron gas in two dimensions is independent of energy:

$$D(\epsilon) = \frac{m}{\pi \hbar^2}$$

Solution:

We know the Fermi-Dirac distribution is given by

$$f(\epsilon) = \frac{1}{\exp\left[\frac{\epsilon-\mu}{k_B T}\right]+1}$$

And we know the identity

$$n = \int_0^\infty d\epsilon\, D(\epsilon) f(\epsilon)$$

We know that D(ε) is a constant so take it out of integral:

$$\frac{n}{D(\epsilon)} = \int_0^\infty d\epsilon\, f(\epsilon)$$

Plug in values and evaluate:

$$\frac{\pi n \hbar^2}{m} = \int_0^\infty \frac{1}{\exp\left[\frac{\epsilon - \mu}{k_B T}\right] + 1} d\epsilon$$

$$\frac{\pi n \hbar^2}{m} = -\left[\ln\left(e^{\frac{\epsilon}{k_B T}} + e^{\frac{\mu}{k_B T}}\right) k_B T - \epsilon\right]_0^\infty$$

$$\frac{\pi n \hbar^2}{m} = \ln\left[1 + e^{\frac{\mu}{k_B T}}\right] k_B T$$

QED

$$\mu = k_B \mathrm{T} \ln\left[e^{\frac{\pi n \hbar^2}{m k_B T}} - 1\right]$$

Prob. 4 – Fermi gas in astrophysics

Michael Tuffley

4. *Fermi gases in astrophysics.* (a) Given $M_\odot = 2 \times 10^{33}$ g for the mass of the Sun, estimate the number of electrons in the Sun. In a white dwarf star this number of electrons may be ionized and contained in a sphere of radius 2×10^9 cm; find the Fermi energy of the electrons in electron volts.

$$M_\odot = 2 \times 10^{33}\ g$$

$$H \cong 75\%$$

$$He \cong 25\%$$

$$M_H = (0.75)(2 \times 10^{33}\ g) = 1.5 \times 10^{33}\ g$$

$$N_{e^-} \cong \left(\frac{1\ e^-}{atom}\right)\left(\frac{6.022 \times 10^{23}\ atom}{mol}\right)\left(\frac{1\ mol}{g}\right)(1.5 \times 10^{33}\ g) = 9 \times 10^{56}\ e^-$$

$$M_{He} = (0.25)(2 \times 10^{33}\ g) = 0.5 \times 10^{33}\ g$$

$$N_{e^-} \cong \left(\frac{2\ e^-}{atom}\right)\left(\frac{6.022 \times 10^{23}\ atom}{mol}\right)\left(\frac{1\ mol}{4\ g}\right)(0.5 \times 10^{33}\ g) = 1.5 \times 10^{56}\ e^-$$

$$N_{e^-} \cong 9 \times 10^{56}\ e^- + 1.5 \times 10^{56}\ e^- \approx 10^{57}\ e^-$$

$$\epsilon_F = \left(\frac{\hbar^2}{2m_{e^-}}\right)\left(\frac{3\pi^2 N_{e^-}}{V}\right)^{\frac{2}{3}} \cong \left[\frac{(1.054 \times 10^{-34}\ J \cdot s)^2}{2(9.1 \times 10^{-31}\ kg)}\right]\left[\frac{3\pi^2(10^{57})}{\frac{4}{3}\pi(2 \times 10^9\ cm)^3}\right]^{\frac{2}{3}}$$

$$= (5.62 \times 10^{-15}\ J)\left(\frac{6.24 \times 10^{18}\ eV}{1\ J}\right) = 3.5 \times 10^4\ eV$$

(b) The energy of an electron in the relativistic limit $\epsilon \gg mc^2$ is related to the wavevector as $\epsilon \cong pc = \hbar kc$. Show that the Fermi energy in this limit is $\epsilon_F \approx \hbar c(N/V)^{1/3}$, roughly.

$$\epsilon \gg mc^2$$

$$\epsilon \cong pc = \hbar kc$$

$$k_F = \left(\frac{3\pi^2 N_{e^-}}{V}\right)^{\frac{1}{3}}$$

(Wavevector is invariant in relativistic limit)

$$\epsilon_F \cong \hbar k_F c = \hbar c\left(\frac{3\pi^2 N_{e^-}}{V}\right)^{\frac{1}{3}} \approx \hbar c\left(\frac{N_{e^-}}{V}\right)^{\frac{1}{3}}$$

(Off by a factor of ~3)

(c) If the above number of electrons were contained within a pulsar of radius 10 km, show that the Fermi energy would be $\approx 10^8$ eV.

$$\epsilon_F \approx \hbar c\left(\frac{N_{e^-}}{V}\right)^{\frac{1}{3}} = (1.054 \times 10^{-34}\,J \cdot s)(3 \times 10^8\,m/s)\left[\frac{10^{57}}{\frac{4}{3}\pi(10\,km)^3}\right]^{\frac{1}{3}}$$

$$= (1.96 \times 10^{-11}\,J)\left(\frac{6.24 \times 10^{18}\,eV}{1\,J}\right) \approx 10^8\,eV$$

Prob. 5 – Liquid H^3

Daniel Wolpert

6.5 Liquid He^3. The atom He^3 has spin ½ and is a fermion. The density of liquid He^3 is $0.081 g/cm^3$ near absolute zero. Calculate the Fermi energy ε_F and the Fermi temperature T_F.

$(.081 g/cm^3/3.016(g/mole))(6.02\times10^{23} atoms/mole) = 1.625\times10^{22}\ atoms/cm^3$

2 electrons per molecule = 3.25×10^{28} electrons/m^3

$\varepsilon_F = \hbar^2/2m\ (3\pi^2 N/V)^{2/3}$

$= (1.055\times10^{-34})^2/(2(9.11\times10^{-31})) * (29.6*3.25\times10^{28})^{2/3} = 5.97\times10^{-19}\ J$

$T_F = \varepsilon_F/k_b = 5.97\times10^{-19}\ J/1.38\times10^{-23}\ J/K$

$T_F = 4.3\times10^4\ K$

Prob. 6 – Frequency dependence of σ

John Anzaldo

Use the equation $$m\left(\frac{dv}{dt}+\frac{v}{\tau}\right)=-eE$$

for the electron drift velocity to show that the conductivity at frequency **w** is:

$$\sigma=\frac{\sigma_0(1+i\omega\tau)}{1+(w\tau)^2}$$

where $$\sigma_0=\frac{ne^2\tau}{m}$$

Chapter 6 #6: Oscillation of Conductivity

- Because E oscillates with **w** as $E = E_0 e^{i\omega t}$, we find that $v = v_0 e^{i\omega t}$, where $v_0 = \frac{E_0 e\tau}{m}$ (From Eq. 42 on page 147)
- Solving $$m\left(\frac{d}{dt}\left(v_0 e^{i\omega t}\right) + \frac{v_0 e^{i\omega t}}{\tau}\right) = -eEe^{i\omega t}$$

gives $m\left(-\omega i v_0 e^{-i\omega t} + \frac{v_0 e^{-i\omega t}}{\tau}\right) = -eE_0 e^{-i\omega t}$

- Plugging in $v = v_0 e^{i\omega t}$ and $E = E_0 e^{i\omega t}$

gives $m\left(-\omega i v + \frac{v}{\tau}\right) = -eE$.

Chapter 6 #6: Oscillation of Conductivity

- Solving for v gives $v = \dfrac{-eE}{m\left(\frac{1}{\tau} - \omega i\right)}$
- Multiplying by $\dfrac{\tau}{\tau}$ and $\dfrac{1+i\omega\tau}{1+i\omega\tau}$ gives

$$v = -\frac{\frac{eE\tau}{m}(1+i\omega\tau)}{1+(\omega\tau)^2}$$

- Recall that J=σE=nqv, where q=-e (p. 147-148)

Chapter 6 #6: Oscillation of Conductivity

- This gives the relation

$$\sigma E = -nev = \frac{ne^2 E\tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)}$$

- Solving for σ gives

$$\sigma = \frac{ne^2 \tau(1 + i\omega\tau)}{m(1 + (\omega\tau)^2)}$$

which is what we set out to prove.

Prob. 7 – Dynamic Magnetoconductivity tensor for free electron

Michael Tuffley

(a) Solve the drift velocity equation (51) to find the components of the magnetoconductivity tensor.

$$m\left(\frac{d}{dt}+\frac{1}{\tau}\right)v_x = -e\left(E_x + \frac{B}{c}v_y\right) \qquad \frac{eB}{mc} = \omega_c$$

$$\frac{dv_x}{dt} + \frac{v_x}{\tau} = -\frac{eE_x}{m} - \omega_c v_y$$

$$v_x = v_{0x}e^{-i\omega t}$$

$$\frac{dv_x}{dt} = -i\omega v_{0x}e^{-i\omega t} = -i\omega v_x$$

$$v_x\left(-i\omega + \frac{1}{\tau}\right) = -\frac{eE_x}{m} - \omega_c v_y$$

$$\omega \gg \frac{1}{\tau}$$

$$-i\omega v_x = -\frac{eE_x}{m} - \omega_c v_y$$

$$v_x = -\frac{ieE_x}{\omega m} - \frac{i\omega_c v_y}{\omega}$$

$$v_y = -\frac{ieE_y}{\omega m} + \frac{i\omega_c v_x}{\omega}$$

$$v_x = -\frac{ieE_x}{\omega m} - \frac{i\omega_c}{\omega}\left(-\frac{ieE_y}{\omega m} + \frac{i\omega_c v_x}{\omega}\right)$$

$$v_x(\omega_c^2 - \omega^2) = \frac{ie\omega E_x}{m} + \frac{e\omega_c E_y}{m}$$

$$\omega \gg \omega_c$$

$$v_x = -\frac{ieE_x}{\omega m} - \frac{e\omega_c E_y}{\omega^2 m}$$

$$v_y = \frac{ieE_y}{\omega m} - \frac{e\omega_c E_x}{\omega^2 m}$$

$$j = -nev$$

$$j_x = -nev_x = \frac{ine^2 E_x}{\omega m} + \frac{ne^2 \omega_c E_y}{\omega^2 m}$$

$$\sigma = \frac{ne^2 \tau}{m}$$

$$j_x = \frac{i\sigma E_x}{\omega\tau} + \frac{\sigma\omega_c E_y}{\omega^2 \tau}$$

$$\omega_p^2 = \frac{4\pi n e^2}{m} = \frac{4\pi\sigma}{\tau}$$

$$j_x = \frac{i\omega_p^2 E_x}{4\pi\omega} + \frac{\omega_c \omega_p^2 E_y}{4\pi\omega^2}$$

$$j_y = -\frac{i\omega_p^2 E_y}{4\pi\omega} + \frac{\omega_c \omega_p^2 E_x}{4\pi\omega^2}$$

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$

$$\begin{bmatrix} j_x \\ j_y \end{bmatrix} = \begin{bmatrix} \frac{i\omega_p^2}{4\pi\omega} & \frac{\omega_c\omega_p^2}{4\pi\omega^2} \\ -\frac{\omega_c\omega_p^2}{4\pi\omega^2} & \frac{i\omega_p^2}{4\pi\omega} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} j_x \\ j_y \end{bmatrix} = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\sigma_{xx} = \sigma_{yy} = \frac{i\omega_p^2}{4\pi\omega}$$

$$\sigma_{xy} = -\sigma_{yx} = \frac{\omega_c\omega_p^2}{4\pi\omega^2}$$

QED

(b) Show that the dispersion relation for this wave in the medium is:

$$c^2k^2 = \omega^2 - \omega_p^2 \pm \frac{\omega_c \omega_p^2}{\omega}$$

electromagnetic wave equation in a nonmagnetic isotropic medium

$E = E_{0}e^{i(kr-\omega t)} = E_{0}e^{i(kz-\omega t)}$

$$\frac{\partial^2 D}{\partial t^2} = c^2\nabla^2\mathbf{E} \qquad\qquad D = \varepsilon E$$

$$\varepsilon\frac{\partial^2 E}{\partial t^2} = c^2\nabla^2\mathbf{E}$$

$$E = E_{0}e^{i(k\cdot r-\omega t)} = E_{0}e^{i(kz-\omega t)}$$

$$\varepsilon\frac{\partial^2}{\partial t^2}\left(e^{i(kz-\omega t)}\right) = c^2\frac{\partial^2}{\partial z^2}\left(e^{i(kz-\omega t)}\right)$$

$$\varepsilon\omega^2 = c^2k^2 \qquad\qquad \varepsilon = 1 + i\left(\frac{4\pi}{\omega}\right)\sigma$$

This gives rise to a set of four linear, homogeneous, algebraic equations. For a nontrivial solution to exist, the determinant must vanish (Thornton 497.)[1]

$$\begin{vmatrix} \varepsilon_{xx}\omega^2 - c^2k^2 & \varepsilon_{xy}\omega^2 \\ \varepsilon_{yx}\omega^2 & \varepsilon_{yy}\omega^2 - c^2k^2 \end{vmatrix} = 0$$

$$(\varepsilon_{xx}\omega^2 - c^2k^2)(\varepsilon_{yy}\omega^2 - c^2k^2) = (\varepsilon_{xy}\omega^2)(\varepsilon_{yx}\omega^2)$$

$$\varepsilon_{xx} = \varepsilon_{yy} \qquad \varepsilon_{xy} = -\varepsilon_{yx}$$

$$\varepsilon_{xx}^2\omega^4 - 2\varepsilon_{xx}\omega^2c^2k^2 + c^4k^4 = -\varepsilon_{xy}^2\omega^4$$

$$\varepsilon_{xx} = 1 + i\left(\frac{4\pi}{\omega}\right)\sigma_{xx} = 1 + i\left(\frac{4\pi}{\omega}\right)\left(\frac{i\omega_p^2}{4\pi\omega}\right) = 1 - \frac{\omega_p^2}{\omega^2}$$

$$\varepsilon_{xx}^2 = \left(1 - \frac{\omega_p^2}{\omega^2}\right)^2 = 1 - \frac{2\omega_p^2}{\omega^2} + \frac{\omega_p^4}{\omega^4}$$

$$\varepsilon_{xy} = i\left(\frac{4\pi}{\omega}\right)\left(\frac{\omega_c\omega_p^2}{4\pi\omega^2}\right) = \frac{i\omega_c\omega_p^2}{\omega^3} \qquad \varepsilon_{xy}^2 = \left(\frac{i\omega_c\omega_p^2}{\omega^3}\right)^2 = -\frac{\omega_c^2\omega_p^4}{\omega^6}$$

$$\left(1-\frac{2\omega_p^2}{\omega^2}+\frac{\omega_p^4}{\omega^4}\right)\omega^4-2\left(1-\frac{\omega_p^2}{\omega^2}\right)\omega^2c^2k^2+c^4k^4=-\left(-\frac{\omega_c^2\omega_p^4}{\omega^6}\right)\omega^4$$

$$\omega^4-2\omega_p^2\omega^2+\omega_p^4-2\omega^2c^2k^2+2\omega_p^2c^2k^2+c^4k^4=\frac{\omega_c^2\omega_p^4}{\omega^2}$$

$$\left(\omega_p^2-\omega^2\right)^2+2c^2k^2\left(\omega_p^2-\omega^2\right)+c^4k^4=\frac{\omega_c^2\omega_p^4}{\omega^2}$$

$$\left(\omega_p^2-\omega^2+c^2k^2\right)^2=\frac{\omega_c^2\omega_p^4}{\omega^2}$$

$$\omega_p^2-\omega^2+c^2k^2=\pm\frac{\omega_c\omega_p^2}{\omega}$$

$$c^2k^2=\omega^2-\omega_p^2\pm\frac{\omega_c\omega_p^2}{\omega}$$

QED

[1]Thornton, Stephen T. Classical dynamics of particles and systems. Belmont, CA: Brooks/Cole, 2004.

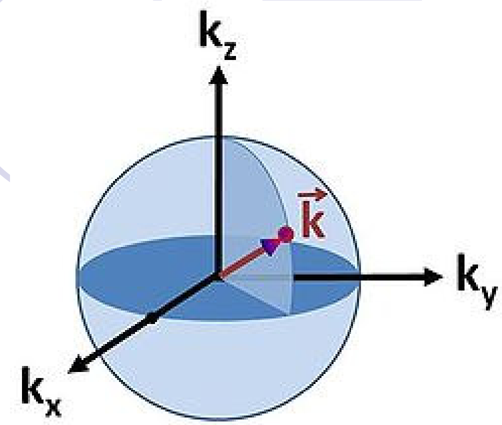
Prob. 8 – Cohesive energy of free electron Fermi gas

Gregory Kaminsky

Cohesive energy of free electron Fermi gas. We define the dimensionless length r_s as r_0/a_H, where r_0 is the radius of a sphere that contains one electron, and a_H is the Bohr radius h^2/e^2m.

a) Show that the average kinetic energy per electron in a free electron Fermi gas at 0 K is $2.21/r_s^2$, where the energy is expressed in rydbergs, with 1 Ry = $me^4/2h^2$.

Average kinetic energy: Integral of the energy inside the sphere in k space, divided by the volume of the sphere in k space.



$$\frac{\hbar^2}{2m}\frac{\int_0^{k_F} 4\pi * k^4 dk}{\int_0^{k_F} 4\pi * k^2 dk} = \frac{3}{5}\varepsilon_F$$

Where k_F is the wavevector at the Fermi surface and ε_F is the Fermi energy.

$$\varepsilon_F = \frac{\hbar^2}{2m}k_F^2$$

Need to show that (3/5) $\varepsilon_F = 2.21/r_s^2$

$$\frac{3}{5}\varepsilon_F = \frac{3\hbar^2}{5*2m}k_F^2 = \frac{3\hbar^2}{5*2m}\left(\frac{3\pi^2}{\frac{4}{3}\pi * r_0^3}\right)^{2/3} = \frac{3}{5}*\frac{\hbar^2}{2mr_0^2}\left(\frac{9\pi}{4}\right)^{2/3}$$

$$\frac{2.21}{r_s^2} = \frac{2.21a_H^2}{r_0} = \frac{2.21*\hbar^4}{e^4m^2r_0^2}$$

Converting from Rydbergs.

$$\frac{2.21*\hbar^4}{e^4m^2r_0^2}*\frac{me^4}{2\hbar^2} = 2.21*\frac{\hbar^2}{2mr_0^2}$$

Setting them equal to each other and canceling out similar factors.

$$2.21 = \frac{3}{5}\left(\frac{9\pi}{4}\right)^{2/3}$$

That's true, you can check on the calculator.

Q.E.D

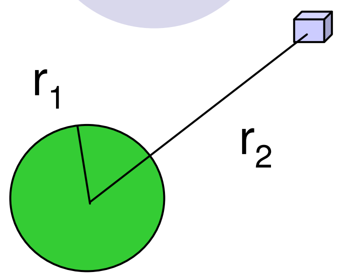
b)Show that the coulomb energy of a point positive charge e interacting with the uniform electron distribution of one electron in the volume of radius r_0 is $-3e^2/r_0$, or $-3/r_s$ in rydbergs.

Formula for the energy of electron due to point positive charge is just the integral of the potential multiplied by the charge density within the sphere, were interaction is occurring.

Charge density = $\delta = \dfrac{e}{\frac{4}{3}\pi * r^3}$ $\qquad$ *Potential* $\rightarrow V = \dfrac{e}{r}$

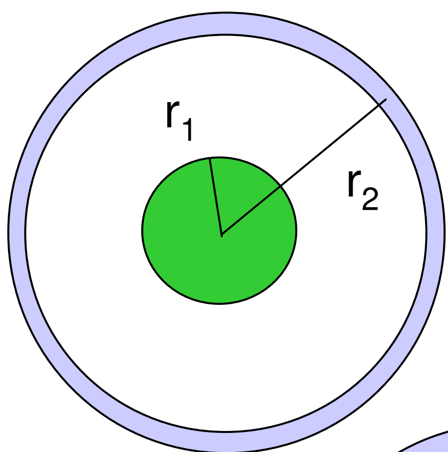
$$U = \iiint r^2 \sin\theta dr d\theta d\phi \cdot \rho(r)\left(\frac{-e}{r}\right) = \int_0^{r_o} 4\pi r^2 dr \left(\frac{e}{\frac{4}{3}\pi r_o^3}\right)\left(\frac{-e}{r}\right) = \frac{-3e^2}{r_o^3}\int_0^{r_o} r dr = -\frac{3e^2}{2r_o}$$

c) Show that the coulomb self-energy of the electron distribution in the sphere is $3e^2/5r_0$, or $6/5r_s$ in rydbergs.



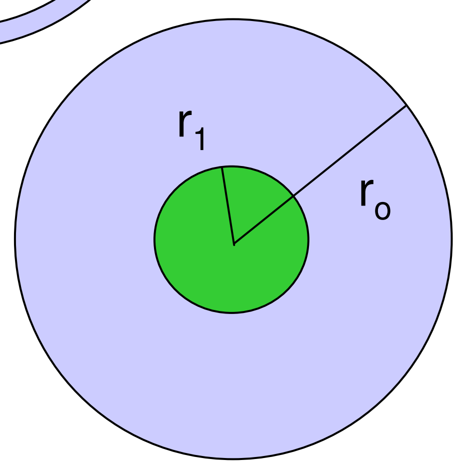
Energy required to bring a point charge close to a sphere is

$$\frac{\rho(d^3r_1)\rho(d^3r_2)}{r_2}$$



Bring a ring close to a charged sphere is

$$\frac{\rho(d^3r_1)\rho(4\pi r_2^2 dr_2)}{r_2}$$



Stack up all the shell up to $r = r_o$

$$\int_{r_2=r_1}^{r_o} \frac{\rho(d^3r_1)\rho(4\pi r_2^2 dr_2)}{r_2} = 2\pi\rho^2 d^3r_1(r_o^2 - r_1^2)$$

This is the potential between a charged sphere and the thick shelf enclosed it.

Total energy stored between the 2 charge-density is just the integral of it:

$$U = \int_0^{r_o} 2\pi\rho^2 (4\pi r_1^2 dr_1)(r_o^2 - r_1^2) = 8\pi^2\rho^2 [\frac{r_o^3 r_o^2}{3} - \frac{r_o^5}{5}] = \frac{16}{15}\pi^2\rho^2 r_o^5$$

With $\rho = \frac{-e}{\frac{4}{3}\pi r_o^3}$

$$U = \frac{3e^2}{5r_o}$$

d) The Sum of (b) and (c) gives $-1.80/r_s$ for the total coulomb energy per electron. Show that the equilibrium value of r_s is 2.45. Will such a material be stable with respect to separated H atoms?

Factoring in the Kinetic energy of the electron in a free Fermi gas at 0 K, basically adding in the answer from part (a) to (b) and (c)

$$Energy = \frac{2.21}{r_s^2} - \frac{1.80}{r_s}$$

To find the equilibrium value of r_s value I took the derivative of Energy with respect to r_s

$$\frac{d}{dr_s}(E) = -\frac{4.42}{r_s^3} + \frac{1.80}{r_s^2}$$

Setting this equation to zero: $$\frac{d}{dr_s}(E) = -\frac{4.42}{r_s^3} + \frac{1.80}{r_s^2} = 0$$

$r_s = 2.45$

To check whether this is a stable equilibrium, I took the second derivative.

$$\frac{d^2}{dr_s^2}(E) = \frac{13.3}{r_s^4} - \frac{2.6}{r_s^3}$$

At $E(r_s = 2.45)$, this second derivative is positive, so it is a stable equilibrium.

$$\frac{d}{dr_s}(E) = -\frac{13.3}{2.45^4} + \frac{2.6}{2.45^3} = 0.19$$

Thus since the second derivative is positive, It is a stable equilibrium, and such a metal will be stable with respect to the separated H atoms at zero Kelvin.

The end.

Prob. 9 – Static magnetoconductivity tensor

Wanshan Li

Q: Let a static magnetic field B lie along the z axis,

a) find the static current density j in a static electric field E.

b) under strong magnetic field ($\omega_c \tau >> 1$), find σ_{yx} , σ_{xy}

A:

a) From (52) p153, the drift velocity is

$$v_x = -\frac{e\tau}{m} E_x - \omega_c \tau v_y [1]$$

$$v_y = -\frac{e\tau}{m} E_y - \omega_c \tau v_x [2]$$

$$v_z = -\frac{e\tau}{m} E_z$$

Problem 9: Current Density (2/3)

Plug [2] into [1], and solve for v_x

$$v_x = \frac{-\frac{e\tau}{m}E_x + \frac{\omega_c e\tau^2}{m}E_y}{1+(\omega_c\tau)^2}$$

plug [1] into [2], and solve for v_y.

$$v_y = \frac{\frac{-\omega_c\tau^2 e}{m}E_x - \frac{e\tau}{m}E_y}{1+(\omega_c\tau)^2}$$

From (42) p147, electric current density

$$j_x = n(-e)v_x$$

Plug in v_x, and define electrical conductivity

$$\sigma_o = \frac{ne^2\tau}{m}$$

Then,

$$j_x = \frac{\sigma_o(E_x - \omega_c\tau E_y)}{1+(\omega_c\tau)^2}$$

Problem 9: Hall conductivity (3/3)

Similarly for j_y, j_z.

In matrix form,

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_o}{1+(\omega_c\tau)^2}\begin{pmatrix} 1 & -\omega_c\tau & 0 \\ \omega_c\tau & 1 & 0 \\ 0 & 0 & 1+(\omega_c\tau)^2 \end{pmatrix}\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

b) Hall conductivity

$$\sigma_{yx} = \frac{\sigma_o \omega_c \tau}{1+(\omega_c\tau)^2}$$

When in high magnetic field,

$$\omega_c\tau >> 1$$

We have

$$\sigma_{yx} = \frac{\sigma_o}{\omega_c\tau}$$

Plug in frequency

$$\omega_c = \frac{eB}{mc}$$

and σ_o , then

$$\sigma_{yx} = \frac{nec}{B} = -\sigma_{xy}$$

Prob. 10 – Maximum surface resistance

Nabel Alkhawlani

Consider a square sheet of side L, thickness d, and electrical resisitvity ρ. the resistance measured between opposite edges of the sheet is called the surface resistance Rsq = ρ L/Ld = ρ/d, which is independent of the area L^2 of the sheet. If we express ρ by 44, then

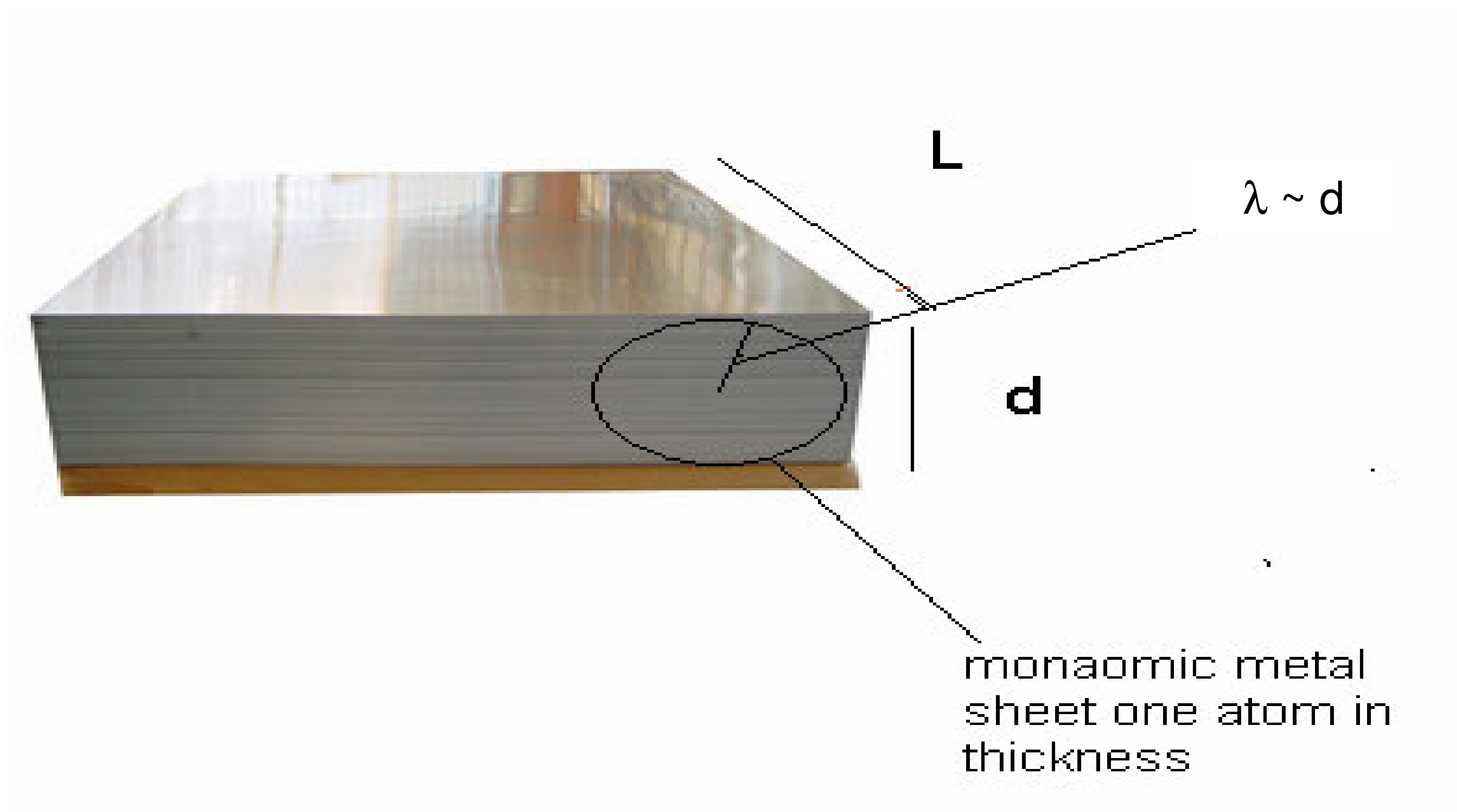
$$Rsq = \frac{m}{nde^2\tau}$$

Suppose now that the minimum value of the oscillation time is determined by the scattering from the surfaces of the sheet, so that, $\tau \approx \frac{d}{v_f}$ where v_f is the fermi velocity. Thus the maximum surface

resistvity is $Rsq \approx \frac{mv_f}{nd^2e^2}$ show for a monatomic metal sheet one atom

in thickness that $Rsq \approx \frac{\hbar}{e^2}$

Sketch of the problem



$$R_{sq} = \frac{\rho L}{A} = \frac{\rho L}{Ld} = \frac{\rho}{d}$$

$$\sigma = \frac{ne^2\tau}{m} = \frac{1}{\rho}$$

$$\tau = \frac{d}{v_f}$$

d^3 = vol/atom

$n = 1/d^3$ = # of atoms/vol

$$R_{sq} = \frac{m}{ne^2\tau d} = \frac{mv_f}{ne^2 d^2} = \frac{mv_f d}{e^2}$$

With $\lambda \sim d$, $p = mv = h / \lambda \sim h / d$

$$R_{sq} \approx \frac{h}{e^2} = (6.6 \times 10^{-34})/(1.6 \times 10^{-19})^2 = 26\ \text{k}\Omega$$