Homework Assignment #2

(due 9/24/14) Physics 601

Reading: (i) Simon (S) chapter 4, (ii) Alloul (A) 4.1, 4.2 (iii) "A Fermi Gas of Atoms," by D. Jin , Physics World (2002) (please find this reading on the course website)

Problems/Questions:

1. Simon Problem 4.2 Velocities in the Free Electron Theory

2. Simon Problem 4.4 Another Review of Free Electron Theory.

3. Consider a metal at uniform temperature in a static uniform electric field $\vec{E}$. An electron experiences a collision, and then, after a time t, a second collision. In the Drude model, energy is not conserved in collisions, for the mean speed of an electron emerging from a collision does not depend on the energy that the electron acquired from the field since the time of the preceeding collision.

   a) Show that the average energy lost to the ions in the second of two collisions separated by a time t is $(eEt)^2/2m$. (The average is over all directions in which the electron emerged from the first collision.)

   b) Show, using a result from HW#1, that the average energy loss to the ions per electron per collision is $(eE\tau)^2/m$, and hence that the average loss per cubic centimeter per second is $\left(\frac{ne^2\tau}{m}\right)E^2 = \sigma E^2$. Deduce that the power loss in a wire of length L and cross section A is I^2R, where I is the current flowing and R is the resistance of the wire.

4. Fermi gases in astrophysics.

Fermi gases in astrophysics. (a) Given $M_\odot = 2 \times 10^{33}$ g for the mass of the Sun, estimate the number of electrons in the Sun. In a white dwarf star this number of electrons may be ionized and contained in a sphere of radius 2×10^9 cm; find the Fermi energy of the electrons in electron volts. (b) The energy of an electron in the relativistic limit $\epsilon \gg mc^2$ is related to the wavevector as $\epsilon \cong pc = \hbar kc$. Show that the Fermi energy in this limit is $\epsilon_F \approx \hbar c(N/V)^{1/3}$, roughly. (c) If the above number of electrons were contained within a pulsar of radius 10 km, show that the Fermi energy would be $\approx 10^8$ eV. This value explains why pulsars are believed to be composed largely of neutrons rather than of protons and electrons, for the energy release in the reaction $n \rightarrow p + e^-$ is only 0.8×10^6 eV, which is not large enough to enable many electrons to form a Fermi sea. The neutron decay proceeds only until the electron concentration builds up enough to create a Fermi level of 0.8×10^6 eV, at which point the neutron, proton, and electron concentrations are in equilibrium.

5. Liquid He(3)

Liquid He^3. The atom He^3 has spin $\frac{1}{2}$ and is a fermion. The density of liquid He^3 is 0.081 g cm^{-3} near absolute zero. Calculate the Fermi energy ϵ_F and the Fermi temperature T_F.

6. Please describe the challenges associated with observing quantum degeneracy in Fermi gases of cold atoms and how this was eventually achieved; please base your summary on your reading of Jin's Physics World article and use a minimum of four sentences in your response.