

Physics 4243/5243  
 Problem Set #3  
 Due Friday February 6<sup>th</sup>

Problem 1:

- a) Calculate the 1-D and 2-D analogues to equations 4.3, 4.6, 4.7 and 4.10 in Simon
- b) Sketch the density of states in 1, 2 and 3D.

Problem 2: Simon 4.2

Problem 3: Simon 4.7

For Graduate Students only (5243):

Problem 4: Taken from Kittel

A metal with a concentration  $n$  of free electrons of charge  $-e$  is in a static magnetic field,  $B\hat{z}$ . The electric current density in the  $xy$  plane is related to the electric field by

$$\mathbf{j}_x = \sigma_{xx} E_x + \sigma_{xy} E_y ; \mathbf{j}_y = \sigma_{yx} E_x + \sigma_{yy} E_y$$

Assume that the frequency  $\omega \gg \omega_c$  and  $\omega \gg 1/\tau$ , where  $\omega_c \equiv eB/m$  and  $\tau$  is the collision time.

- (a) Solve the drift velocity equation to find the components of the magnetoconductivity tensor:

$$\sigma_{xx} = \sigma_{yy} = i\omega_p^2 / 4\pi\omega ;$$

$$\sigma_{xy} = -\sigma_{yx} = \omega_c \omega_p^2 / 4\pi\omega^2$$

$$\text{where } \omega_p^2 \equiv 4\pi n e^2 / m$$

- (b) Note from a Maxwell equation  $[c^2 \nabla^2 E = \epsilon \partial^2 E / \partial t^2]$  that the dielectric function tensor of the medium is related to the conductivity tensor as  $\epsilon = 1 + i(4\pi/\omega)\sigma$ . Consider an electromagnetic wave with wavevector  $k = k\hat{z}$ . Show that the dispersion relation for this wave in the medium is

$$c^2 k^2 = \omega^2 - \omega_p^2 \pm \omega_c \omega_p^2 / \omega$$

At a given frequency there are two modes of propagation with different wavevectors and different velocities. The two modes correspond to circularly polarized waves. Because a linearly polarized wave can be decomposed into two circularly polarized waves, it follows that the plane of polarization of a linearly polarized wave will be rotate by the magnetic field.