Problem 1

Kittel - Introduction to Solid State Physics - Chapter 9, Problem 7: De Haas-van Alphen period of potassium

a) Calculate the period $\Delta(\frac{1}{B})$ expected for potassium on the free electron model.

b) What is the area in real space of the extremal orbit, for $B = 10kG = 1T$? The same period applies to oscillations in the electrical resistivity, known as the Shubnikow-de Haas effect.

Solution

a) Potassium is arranged in a BCC crystal lattice structure. The electron concentration in a monovalent metal with a BCC lattice structure is $n = \frac{2}{a^3}$, as there are 2 electrons within the volume of a^3. Therefore, since the radius of the Fermi sphere is

$$k_{F,K} = (3\pi^2 n)^{\frac{1}{3}} = (\frac{6\pi^2}{a^3})^{\frac{1}{3}}$$

From Table 6.1, we have $k_{F,K} = 0.75 \times 10^8 cm^{-1}$. Therefore the Fermi surface area will be $S = \pi k_{F,K}^2$, which gives the following:

$$\Delta(\frac{1}{B}) = \frac{2\pi\frac{e}{\hbar c}}{S} = \frac{2\pi\frac{e}{\hbar c}}{\pi k_{F,K}^2}$$

We know from the example problem in Kittel on the Fermi surface of Gold that $2\pi\frac{e}{\hbar c} = 9.55 \times 10^7 gauss^{-1}cm^{-2}$, so using this relationship we find:

$$\boxed{\Delta(\frac{1}{B}) = \frac{9.55 \times 10^7 gauss^{-1}cm^{-2}}{\pi(0.75 \times 10^8 cm^{-1})^2} = 5.4 \times 10^{-9} gauss^{-1}}$$

b) In Chapter 6 of Kittel, the cyclotron frequency is defined under the *Motion in Magnetic Fields* subsection as $\omega_c = \frac{eB}{mc}$. We may use this and notice that $r = \frac{v_F}{\omega_c}$ and $v_F = \frac{\hbar k_F}{m}$. Therefore,

$$R = \frac{\frac{\hbar k_F}{m}}{\frac{eB}{mc}} = \frac{\hbar k_F}{eB}$$

$$\Rightarrow R = 5 \times 10^{-4} cm$$

$$\Rightarrow \boxed{S = \pi(5 \times 10^{-4} cm)^2 = 7.8 \times 10^{-7} cm^2}$$

Problem 2

Fermi Surface on Hypothetical Cubic Metal

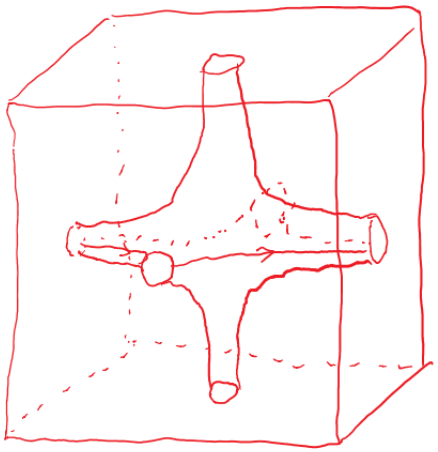
Consider a hypothetical monovalent monatomic simple cubic metal of lattice constant $4\mathring{A}$. Sketch the Fermi Surface you would expect treating the valence electron as nearly free, such that the neck and belly are in the ratio of $\frac{1}{5}$.

Suppose you able to use the de Haas-van-Alphen effect to investigate this Fermi Surface. Describe what you would expect to see for the H field in the [100], [110] and [111] directions with H $\sim$ 1 Tesla. Sketch the extremal orbits in each case.

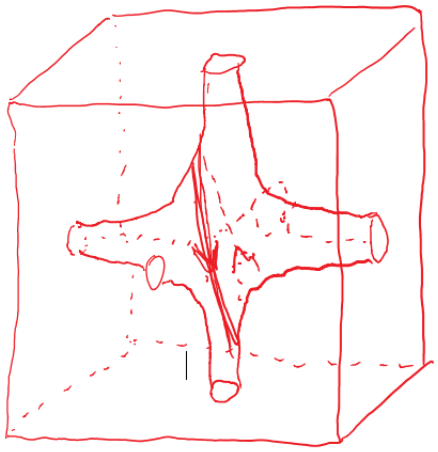
Solution

For a Simple Cubic of lattice constant a the reciprocol lattice is a simple cubic with side $\frac{2\pi}{a}$.

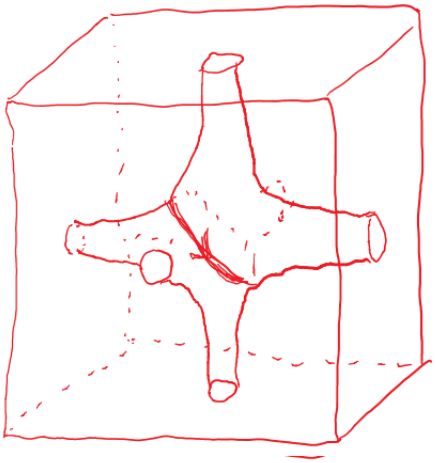
B in [100] direction:



B in [110] direction:



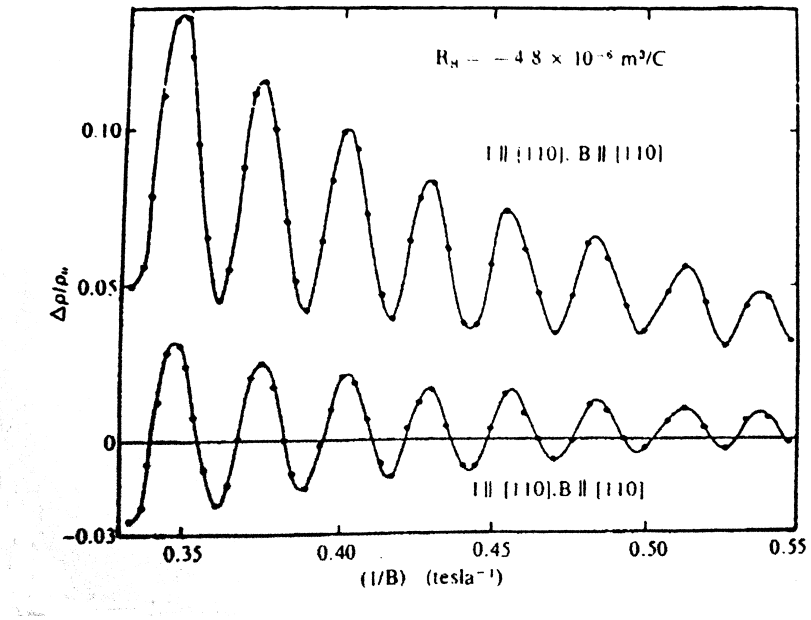
B in [111] direction:



Problem 3

GaSb experimental data

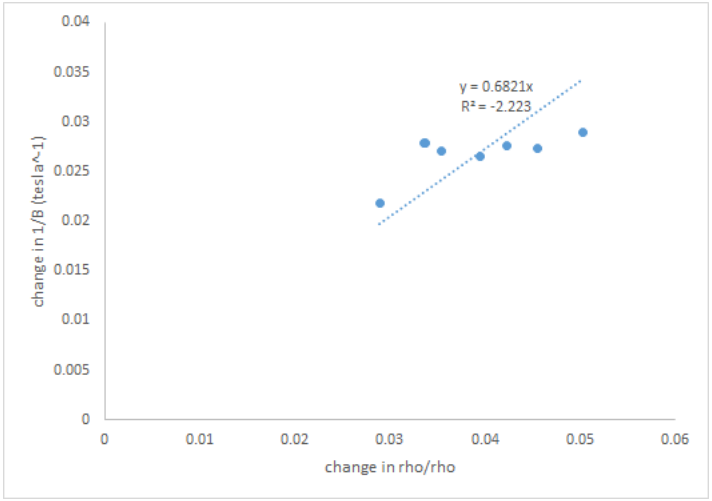
The figure below is experimental data for an n-type GaSb (semiconductor) at $T = 4.2K$. The Schubnikov-de Haas (SdH) trace (ρ vs. $\frac{1}{B}$) is shown. The resistance (ρ) displays oscillatory behavior associated with the Landau levels piercing the Fermi surface. The Hall resistance (R_H) is also provided.



Calculate the electron density from the SdH trace and see how it agrees with your calculation of the electron density from the Hall resistance.

Solution

The electron density is found by fitting the change in the magnetic field flux at the minima of each oscillation against the change in the resistance. The following graph is created from the provided experimental data:



Using the following relation:

$$n = \frac{2e}{\Delta(\frac{1}{\mathbf{B}})h}$$

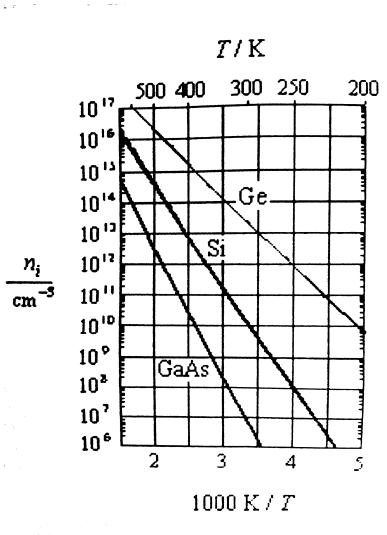
We obtain that the electron density is:

$$n = 700\mu m^{-}2$$

Problem 4

Determining the band gap from temperature and intrinsic carrier concentration

The graph below displays the intrinsic carrier concentration for Ge, Si, and GaAs as a function of inverse temperature.



Determine E_g for each of the semiconductors using this graph.

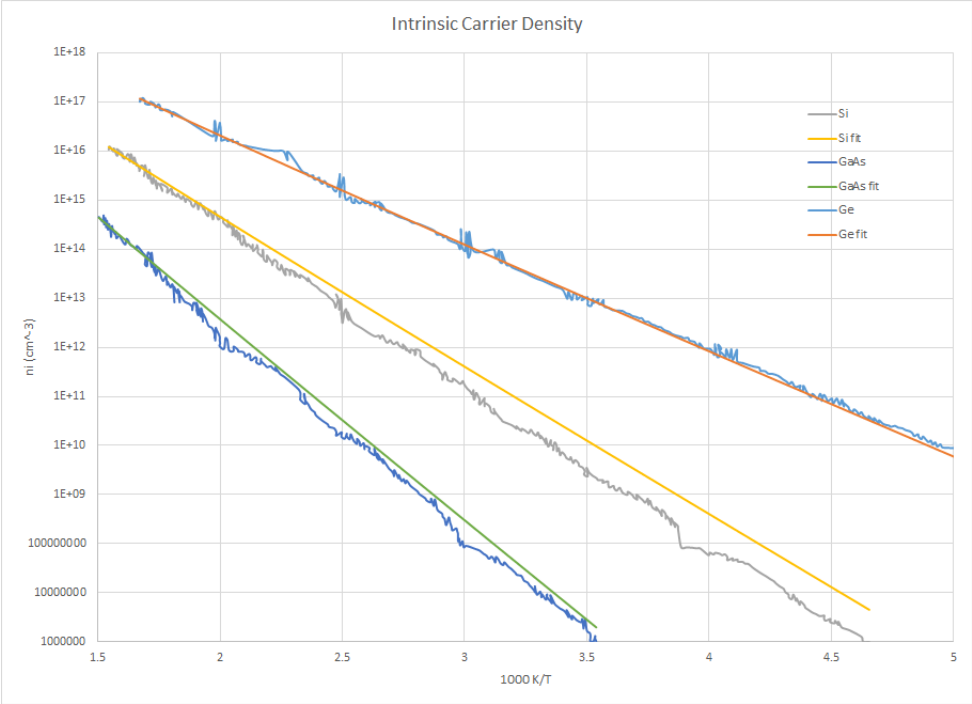
Solution

The intrinsic carrier density is given by Chapter 8, Equation 45 in Kittel as:

$$n_i = 2\Big(\frac{k_B T}{2\pi\hbar^2}\Big)^{\frac{3}{2}}(m_e m_h)^{\frac{3}{4}} e^{\frac{-E_g}{k_B T}}$$

The data in the graph can be fit using this equation and the band gap E_g can be extracted. Below is a graph of each fitting to the data.

Problem 4 continued on next page...



The extracted band gaps are as follows:

- Germanium: 0.8 eV
- Silicon: 1.1 eV
- Gallium Arsenide: 1.5 eV