

# PHYS 5243

## Solid State Physics

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## 2015-01-09: Chapter 1 - About Condensed Matter Physics

### Syllabus

Read Chapters 1 and 2 before next lecture

Graduate Student → 15% of the grade is HW.

2 Midterms: Wednesday nights (~ 4 hours are given to do them).

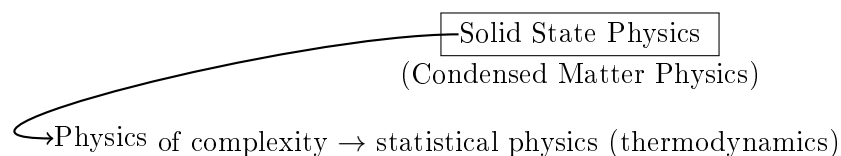
The Final counts for ~ 25% of grade for Graduate and Undergraduate Students.

Get the other books required for class → they are important!

Graduate Student difference → potentially a physics simulation will be required.

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### Class Notes



Collections of atoms

Somewhat under atomic physics field

Solids, liquids, and polymers

Hamiltonian:

$$\hat{H} = \underbrace{\frac{\mathbf{p}_n^2}{2M_n}}_{\substack{\text{momentum} \\ \text{of} \\ \text{ions}}} + \underbrace{\frac{\mathbf{p}_e^2}{2M_e}}_{\substack{\text{momentum} \\ \text{of} \\ \text{electrons}}} + \underbrace{\frac{e^2}{r_{i1} - r_{j1}}}_{\substack{\text{repulsion} \\ \text{between} \\ \text{ions}}} + \underbrace{\frac{e^2}{r_{i2} - r_{j2}}}_{\substack{\text{repulsion} \\ \text{between} \\ \text{electrons}}} - \underbrace{\frac{e^2}{r_{i1} - r_{j2}}}_{\substack{\text{attraction} \\ \text{between} \\ \text{electrons and ions}}}$$

At the moment only ~100 atoms can be solved (using supercomputer) → very difficult!

Emergent phenomenon is common

Superconductivity is emergent from collection of atoms

## 2015-02-20: Chapter 1 (Kittel) - Crystal Structure

Test on Everything but Crystal Structure. Closed book but will provide equations.

### Primitive Cells

Crystal Structure handout.

**(100)** plane of atoms.

**{100}** family of planes.

**[100]** direction.

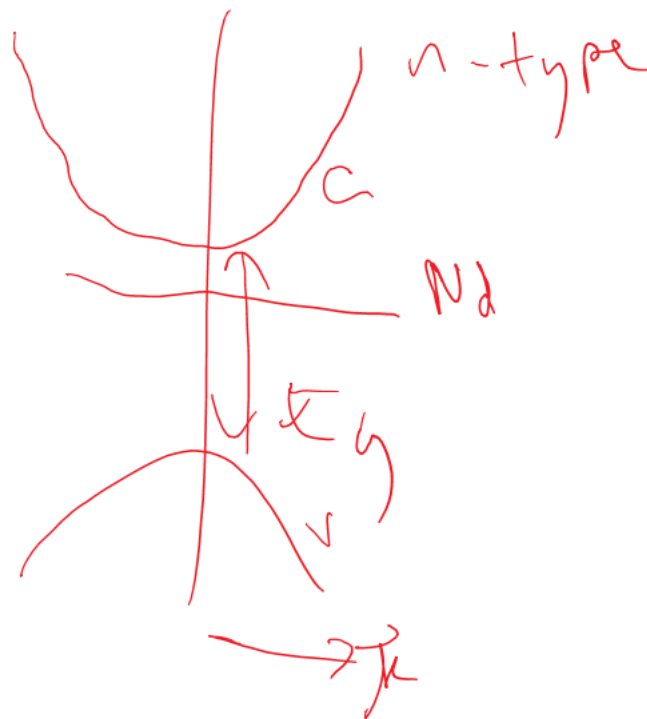
## 2015-04-06: Chapter 8 Kittel? - Semiconductor

### Exam postponed until Wednesday next week

All material will be covered up to semiconductors, no p-n junctions, but will have n and p semiconductors.

**n-type SC** donor level lives close to conduction band.

**p-type SC** donor level below valence band



p-type

## 2015-04-08: Optical Properties of Materials - Chapter 14 and 16 in Kittel

Will be doing:

Superconductivity - single lecture

Graphene - single lecture

Drude Model - fermi surface shifts in the reciprocal space due to electric field

Then we talked of SdH

What we haven't talked about how light interacts with material.

Guass's Law:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0} \quad (1)$$

$$\rho_{total} = \rho_{ext} + \rho_{int} \quad (2)$$

$\rho_{int}$  is materials dependent

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{p} = \epsilon \epsilon_0 \mathbf{E} \quad (3)$$

$\mathbf{D}$  is function of  $\omega$  and  $\mathbf{k}$ .

A  $\epsilon(\omega, 0) : k \rightarrow 0, x \rightarrow \infty$

collective excitations of Fermi sea

$\epsilon(0, k)$  : electro static response electron electron screening

A Long Wavelength Response

$$m \frac{d^2 x}{dt^2} = -e \mathbf{E} \quad (4)$$

$$\mathbf{E} \sim E_0 e^{-i\omega t} \quad (5)$$

$$x \sim x_0 e^{-i\omega t} \quad (6)$$

$$-m\omega^2 x(t) = -eE(t) \quad (7)$$

$$x(t) = +\frac{eE(t)}{m\omega^2} \quad (8)$$

$$x_0 \frac{eE_0}{m\omega^2} \quad (9)$$

dipole moment of one electron

$$\mathbf{p} = -eX_0 \quad (10)$$

$$\mathbf{p}_0 = \frac{\# \text{ of dipoles}}{\text{volume}} = -ex_0 n = \frac{-e^2 n E_0}{m\omega} \quad (11)$$

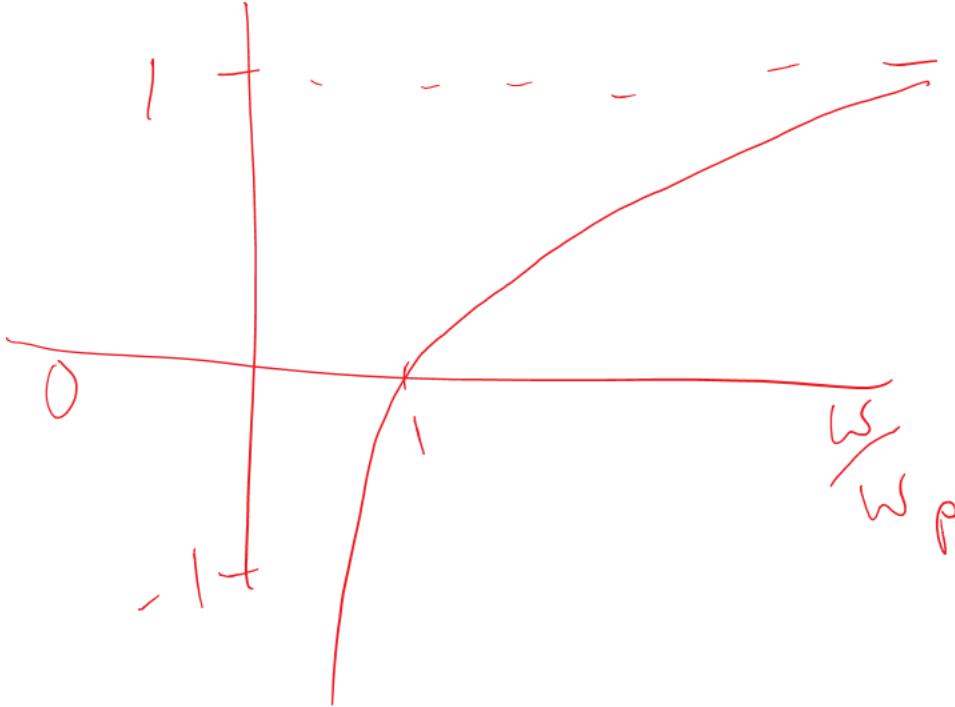
$\mathbf{p}_0$  is the polarization

$$\epsilon(\omega) = \frac{D(\omega)}{\epsilon_0 E(\omega)} = 1 + \frac{P(\omega)}{\epsilon E(\omega)} = 1 - \frac{2^2 n}{\epsilon_0 m \omega^2} \quad (12)$$

Plasmon frequency:

$$\omega_p^2 = \frac{n_e^2}{m\epsilon_0} \quad (13)$$

$$\epsilon(\omega) = 1 - \frac{\omega_p^2}{\omega^2} \quad (14)$$



Non magnetic:

$$\mu_0 \frac{d^2 \mathbf{D}}{dt^2} = \nabla^2 \mathbf{E} \quad (15)$$

$$\mathbf{E} \propto e^{i\omega t} e^{i\mathbf{k} \cdot \mathbf{r}} \quad (16)$$

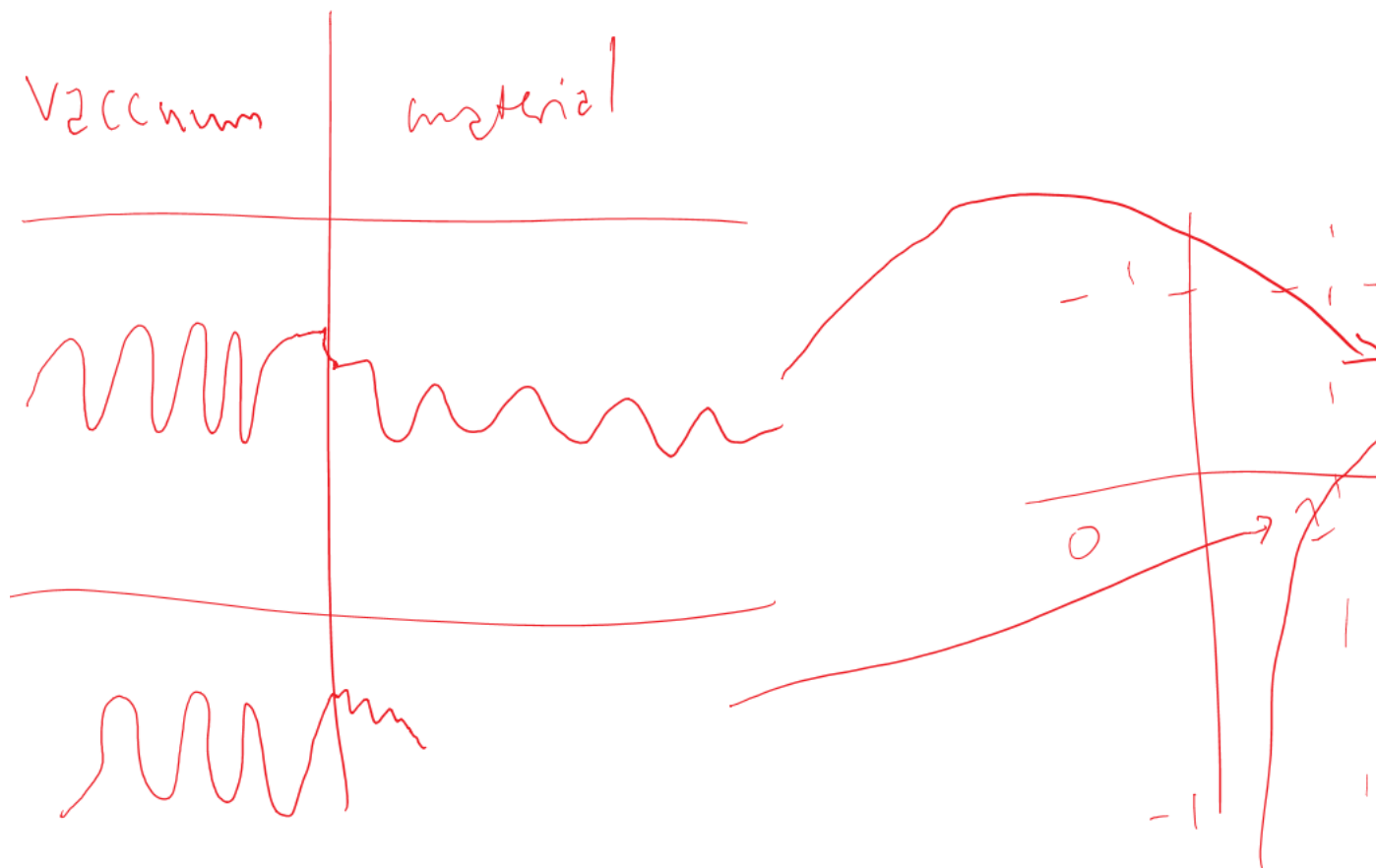
$$D = \epsilon(\omega, k) \mathbf{E} \quad (17)$$

$$\epsilon(\omega, \mathbf{k}) \epsilon_0 \mu_0 \omega^2 = k^2 \quad (18)$$

$\epsilon(\omega, \mathbf{k})$	yields
real $\epsilon > 0$	$\vec{k}$ is real propagation
real $\epsilon < 0$	$\vec{k}$ is imaginary damped
complex	$\vec{k}$ is complex

In general:





## 2015-04-22: Tight Binding - by someone not Murphy

**Tight binding** In a s system with translational symetry  
with  $i = 1 \dots N$  lattice sites.

If  $H$  is the Hamiltonian, we assume that:

$$\langle i|H|j\rangle = \begin{cases} -t & \text{for nearest neighbor} \\ 0 & \text{otherwise} \end{cases} \quad (19)$$

The eigenenergies are:

$$E_k = \langle k|H|k\rangle = \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \langle i|H|j\rangle \quad (20)$$

$$= \frac{1}{N} \sum_{\delta} \langle i|H|j\rangle e^{ik \cdot \delta} \quad (21)$$

$$= -t \sum_{\delta} e^{ik \cdot \delta} \quad (22)$$

where  $\delta$  are the NN vectors

In a square lattice,  $\delta = (\pm a, 0), (0, \pm a)$

$$e_k = -t \sum_{\delta} e^{ik \cdot \delta} = -2t[\cos(k_x a) + \cos(k_y a)] \quad (23)$$

**Graphene** Unlike a square in a triangle lattices, honeycomb lattice is not a Bravais Lattice since it cannot be generated by a single set of lattice generators The two sublattices require a component bases:

$$|i, S\rangle \quad (24)$$

where  $S = a, b$  for sublattice a or b.

$$\langle ks|H|ks'\rangle = \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^N \langle is|H|js\rangle e^{-ik \cdot r} e^{ik' \cdot r} \quad (25)$$

$$\langle ks|H|ks'\rangle = \frac{1}{N} \sum_{i=1}^N -t(1 - \delta_{s,s'})e^{ik \cdot \delta_{s,s'}} \quad (26)$$

$$\langle ka|H|kb\rangle = -t \sum_{\delta} e^{ik \cdot \delta_{ab}} \quad (27)$$

$$\langle ka|H|kb\rangle = -t \sum_{\delta} e^{ik \cdot \delta_{ba}} \quad (28)$$

Since

$$\delta_{ab}^1 = (a, 0) \quad (29)$$

$$\delta_{ab}^2 = (-a/2, \sqrt{3}/2a) \quad (30)$$

$$\delta_{ab}^3 = (-a/2, -\sqrt{3}/2a) \quad (31)$$

$$H_k^{ab} = \langle ka|H|kb\rangle = -t\phi_k \quad (32)$$

$$H_k^{ba} = \langle kb|H|ka\rangle = -t\phi_k^* \quad (33)$$

where  $\phi_k = \sum_{\delta} e^{ik \cdot \delta}$ .

The eigen values are

$$E_k = \pm t|\phi_k| \quad (34)$$

Expanding  $\phi_k$  around H:

$$\phi_{k+p} = (p_x - ip_y)\frac{3}{2}a \quad (35)$$

where p is the deviation from H

$$H_p = \nu p \cdot \sigma \quad (36)$$

(Dirac equaiton) which is why it's called a Dirac point

where  $\sigma = (\sigma_x, \sigma_y)$  are Pauli matrices and  $\nu = \frac{3}{2}ta$  is the Fermi velocity

Eigen vectors

$$\nu \sigma \cdot p \psi_{\pm} = \pm \nu a \psi_{\pm} \quad (37)$$

$$\psi_{\pm} = \frac{1}{\sqrt{2}} \pm e^{i\theta a} e^{i\delta \cdot r} \quad (38)$$

(Bloch wavefunction)

**Berry Phase**

$$\gamma = \oint_c da \psi_{\pm,a}^+ \nabla_a \psi_{\pm,a} \quad (39)$$