Gravitational Wave Parameter Estimation Using Machine Learning

Presented by -

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OzGrav Winter School - 2023

The University of Western Australia,

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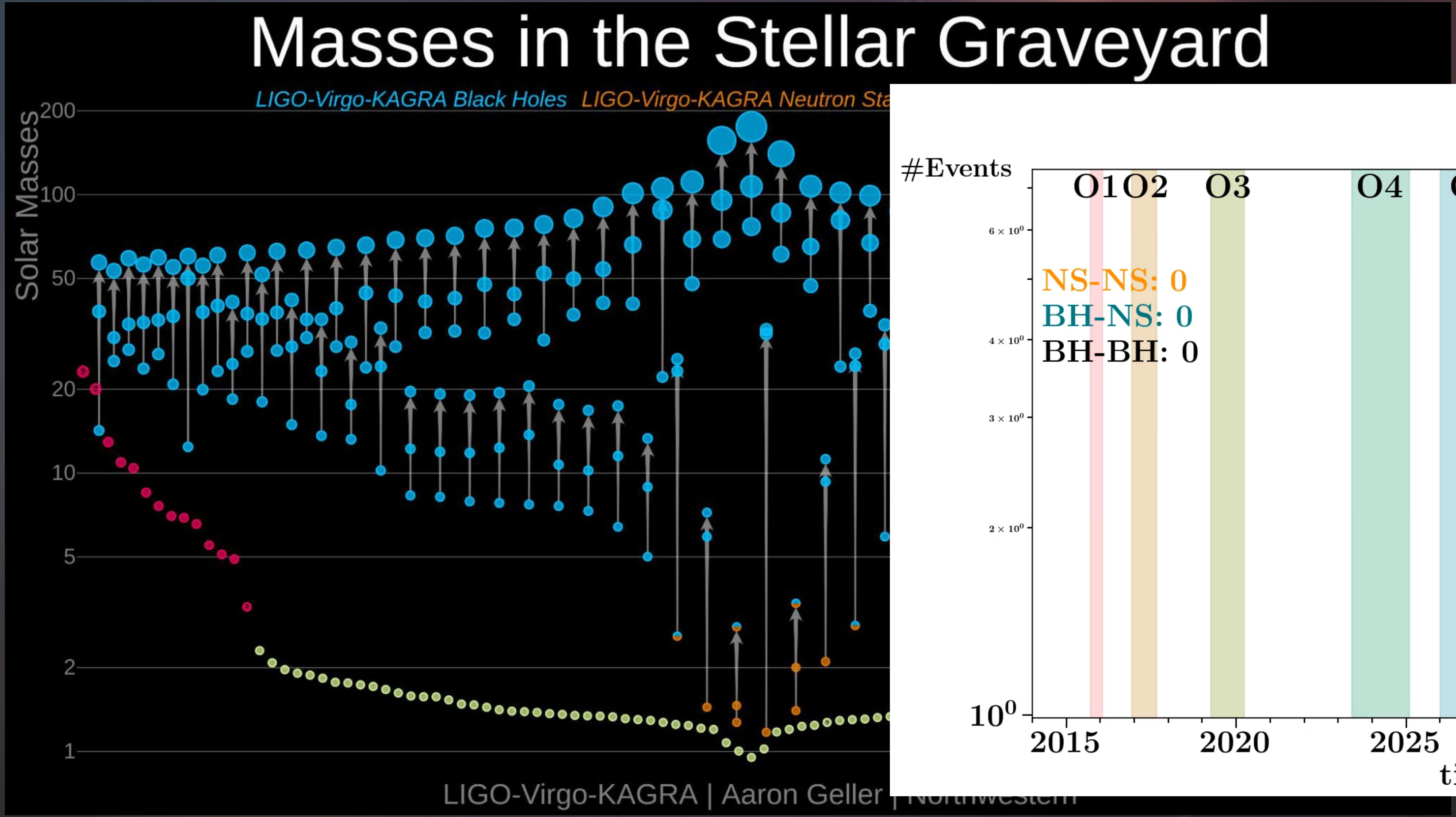
Current State of GW Astronomy



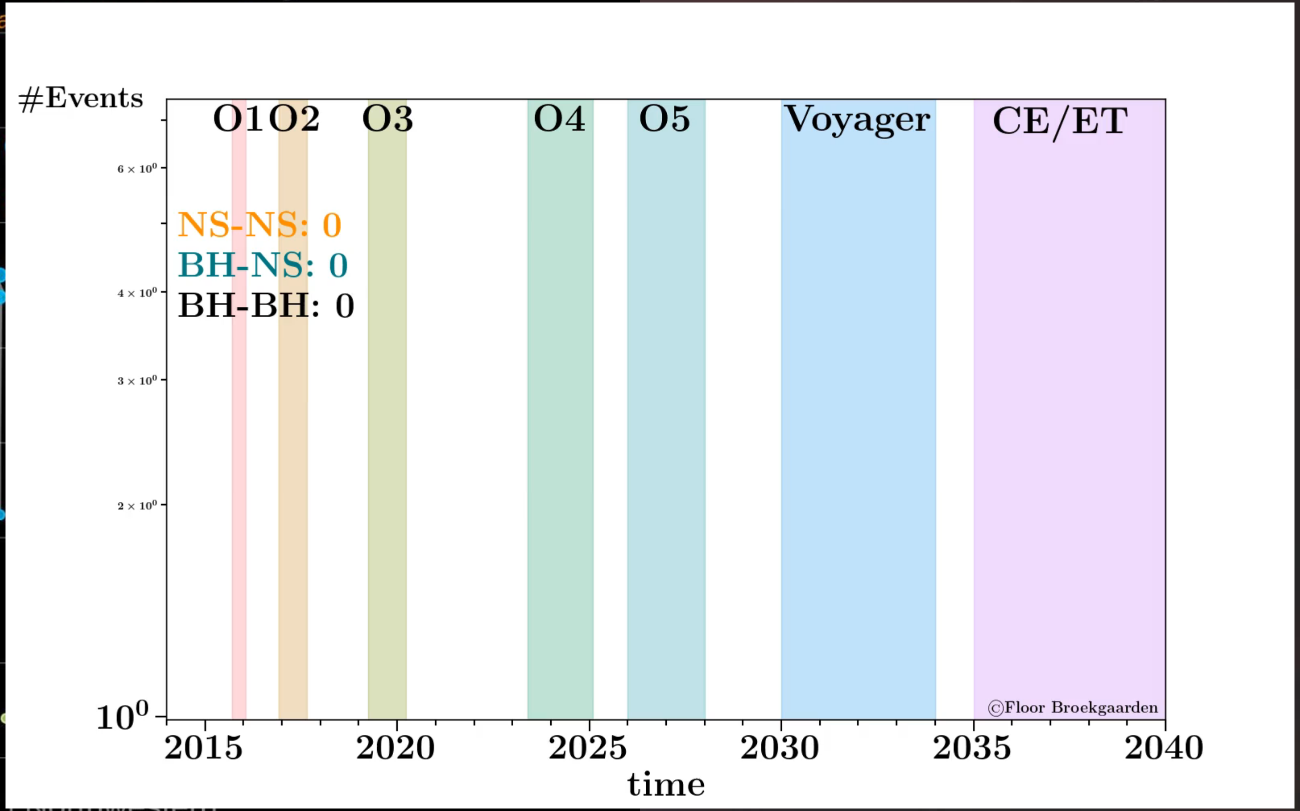
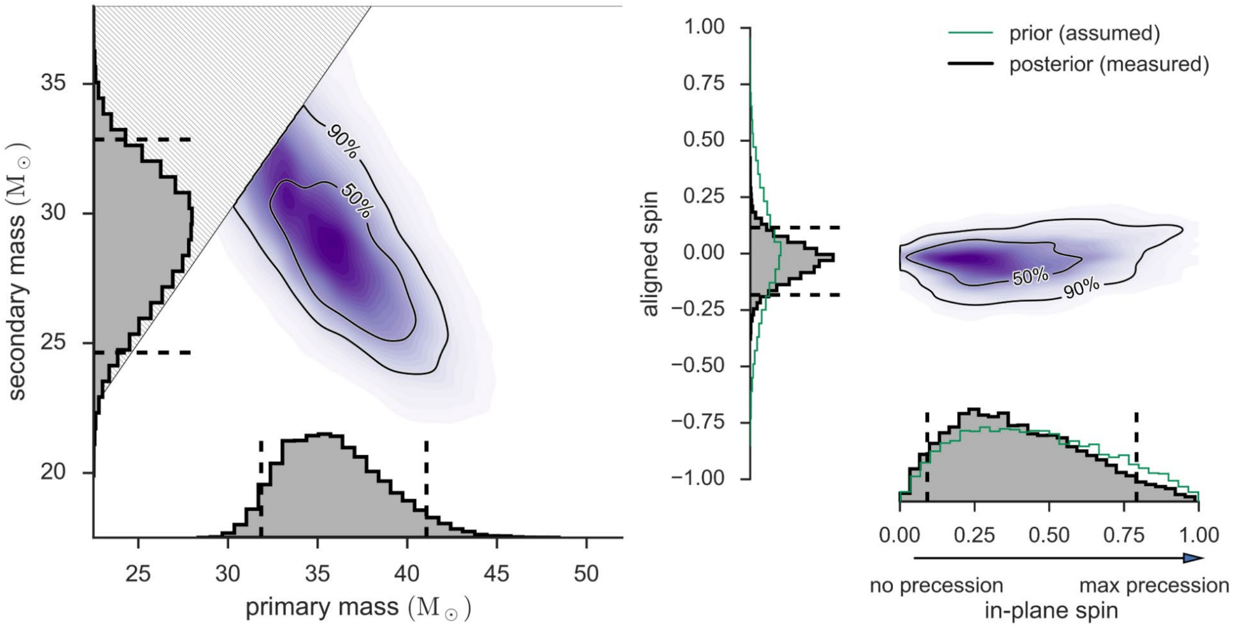


Illustration of observed black holes and neutron stars from gravitational and electromagnetic waves

Number of detected mergers expected from binary black holes, black hole-neutron stars, and binary neutron stars. Courtesy – Dr. Floor Broekgaarden

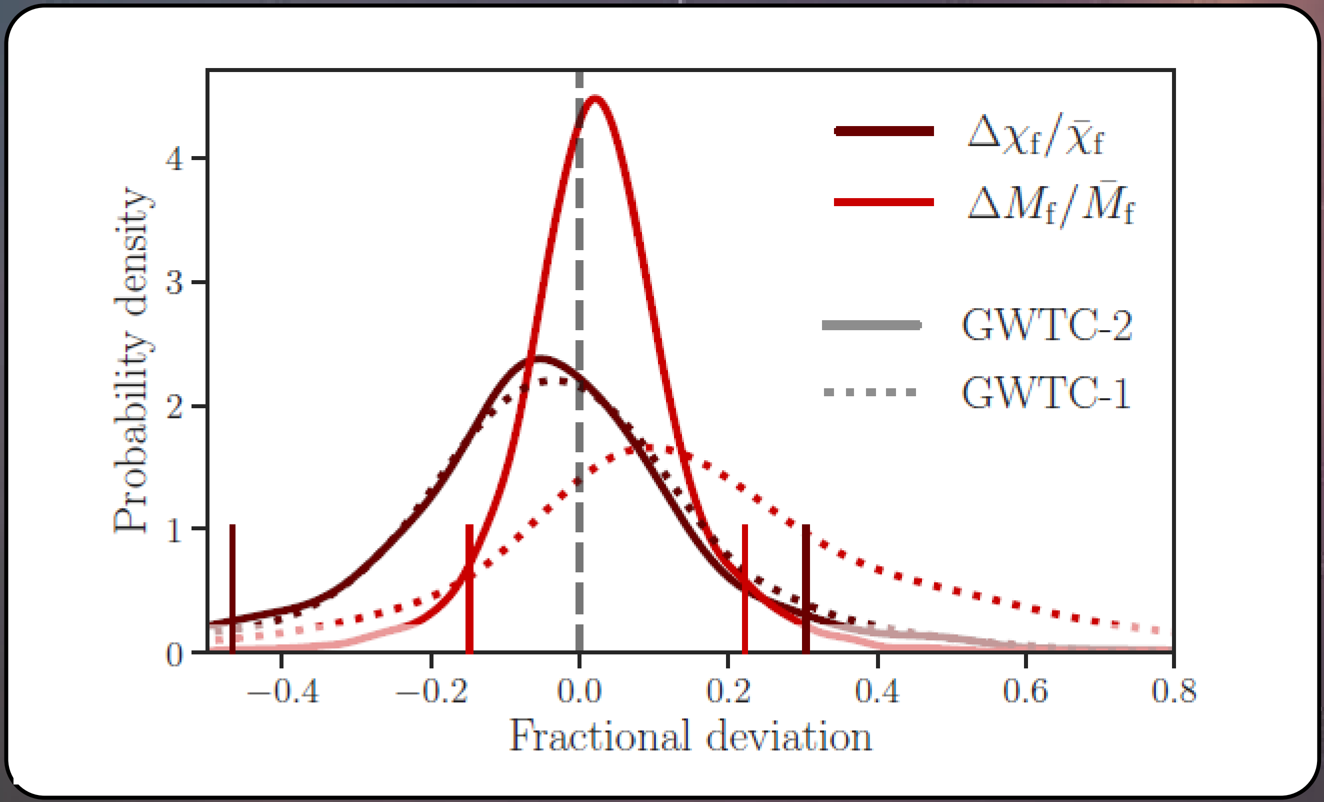
Beyond Detection



Ref: Abbott *et al.* (2016)

- Study single event source properties:
  - Component masses (m_1 , m_2).
  - Spins
  - Sky directions (ra, dec).
  - Luminosity distance.

Beyond Detection

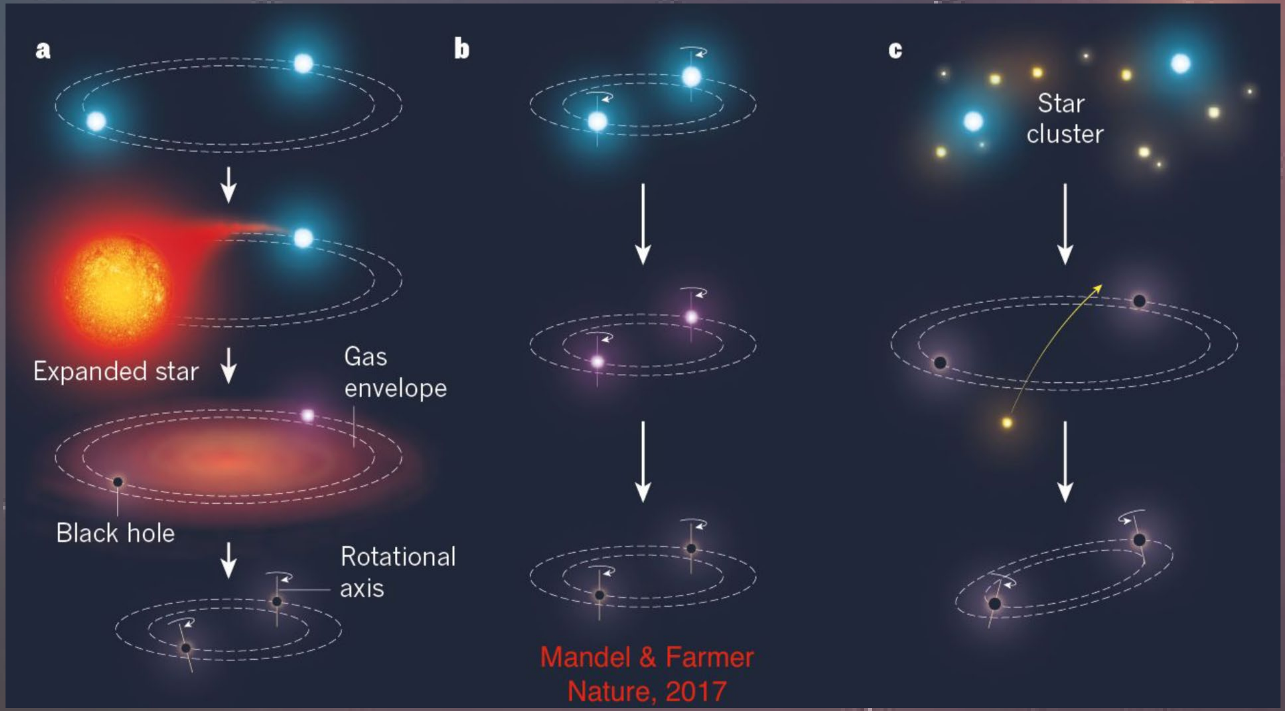


Posterior distributions on the mass-spin deviation parameters (black and purple curves) from GWTC-1 and GWTC-2.

Ref: Krishnendu and Ohme (2022)

- Study single event source properties:
  - Component masses (m_1, m_2).
  - Spins
  - Sky directions (ra, dec).
  - Luminosity distance.
- Model selection:
  - Test of presence of signal after merger (hypermassive NS or BH?).
  - Test of polarisation states (tensor/scalar/vector polarisations?)
  - Testing no-hair theorem.
  - Alternative theories of gravity.

Beyond Detection



Possible mechanisms for the merger of binary black holes (**Ref:** Mandel and Farmer, 2017):

a. Isolated evolution
b. Chemically homogenous evolution
c. Dynamical formation

- Study single event source properties:
  - Component masses (m_1 , m_2).
  - Spins
  - Sky directions (ra, dec).
  - Luminosity distance.
- Model selection:
  - Test of presence of signal after merger (hypermassive NS or BH?).
  - Test of polarisation states (tensor/scalar/vector polarisations?).
  - Testing no-hair theorem.
  - Alternative theories of gravity.
- Population studies:
  - Binary formation channels (isolated or dynamical?).

Gravitational Wave Parameter Estimation

Posterior distribution - for system parameters 'θ' (masses, spins, sky position, etc.) given detector strain data 's'

Likelihood – based on the assumption of stationary Gaussian noise.

$$p(\theta | s) = \frac{p(s | \theta) p(\theta)}{p(s)}$$

$$p(s | \theta) \propto \exp\left(-\frac{1}{2} \sum_{I} (s_I - h_I(\theta) | s_I - h_I(\theta)) \right)$$

$$(a | b) = 2 \int_0^\infty \mathrm{d}f \, \frac{\hat{a}(f)\hat{b}(f)^* + \hat{a}(f)^*\hat{b}(f)}{S_n(f)}$$

Prior – based on beliefs of system parameters before looking at data.

Gravitational Wave Parameter Estimation

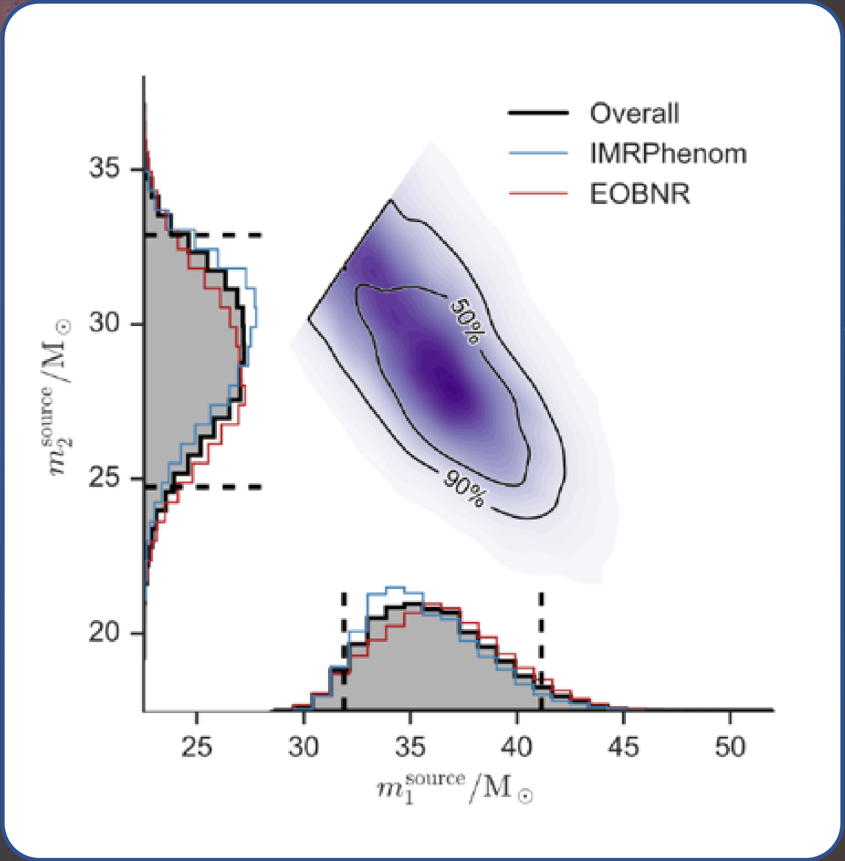
- **Bayesian Inference –** a way of computing probability density of things given data and model uncertainty.

- **Input:**
  - Experimental data
  - Model of gravitational wave signal.
  - Model of detector noise.

- **Output: Probability distribution**

  - Hypothesis: Signal or Noise? GR or not GR etc.

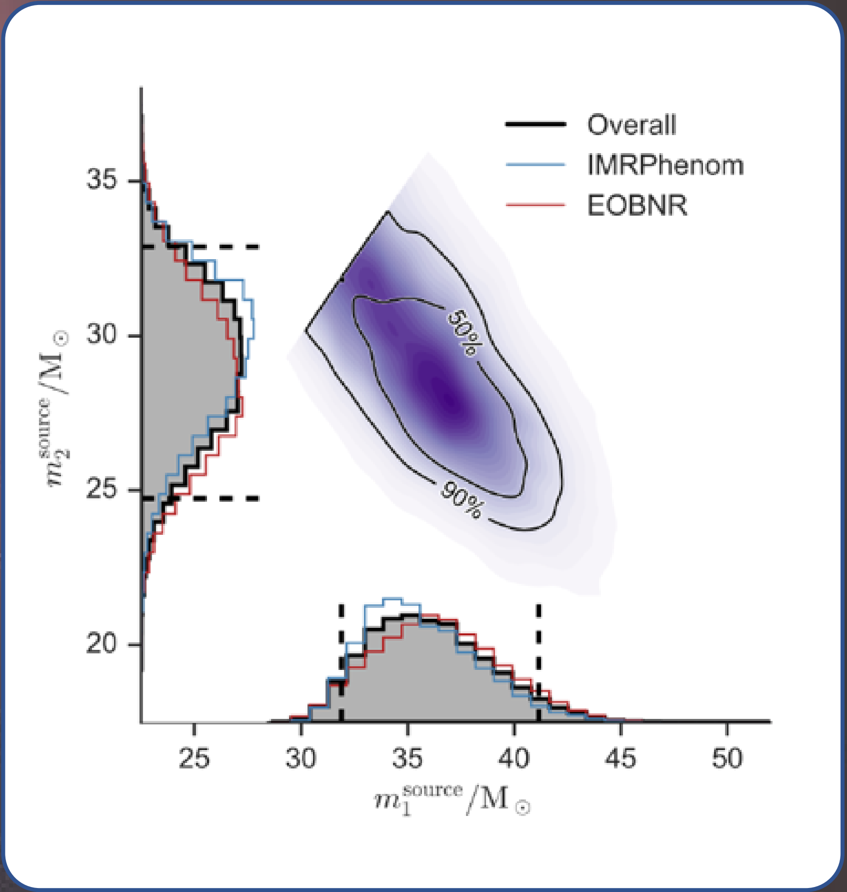
  - Source properties of compact binaries given noisy data and uncertainty in population model.



Ref: Abbott et al. (2016)

Gravitational Wave Parameter Estimation

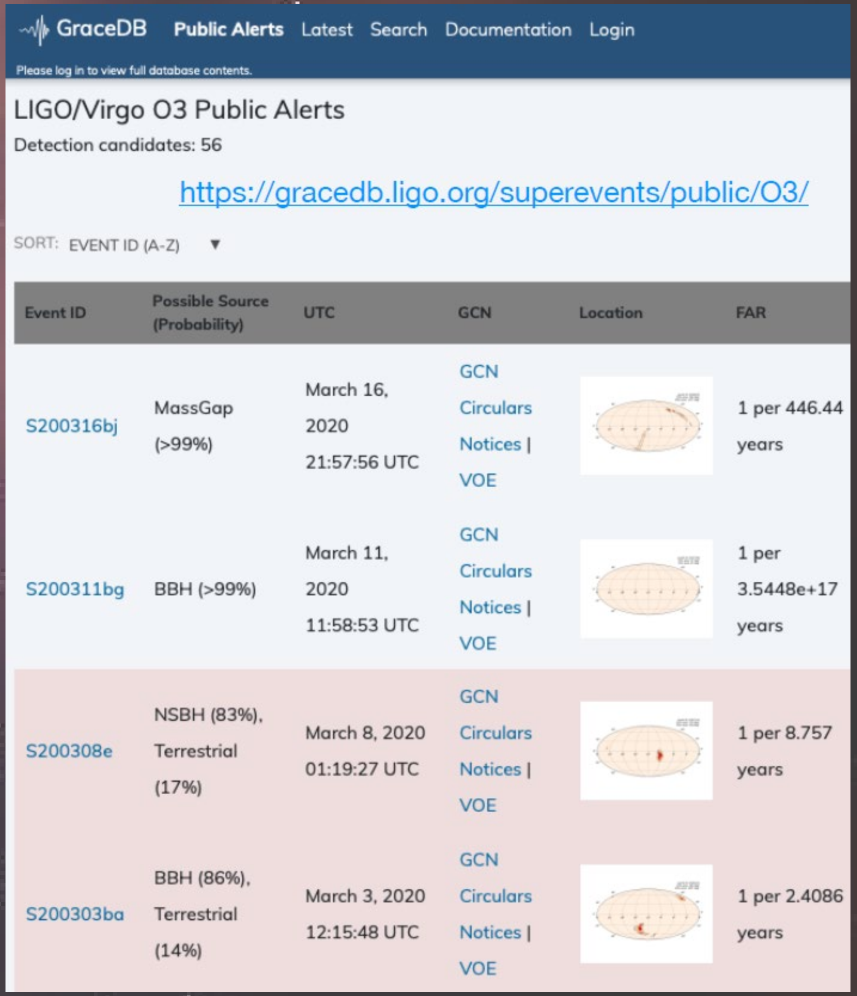
- We need to draw samples from the posterior.
- Iterative samplers like Markov Chain Monte Carlo (MCMC) and Nested Sampling are used to obtain posterior samples
- Move around in parameter space and compare strain data to waveform models.



Ref: Abbott et al. (2016)

Gravitational Wave Parameter Estimation

- Many likelihood evaluations are required for each independent sample.
- Likelihood is slow, requires a waveform to be generated.
- Typical analyses for O3 have taken between 6 hours and 5 days.
- Alternative – Machine Learning
  - Inference ~ milli-seconds.
  - Likelihood-free.
  - No posterior samples.
  - Only requires an ability to simulate training data (Simulation-based inference).



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LIGO/Virgo O3 Public Alerts

Detection candidates: 56

https://gracedb.ligo.org/superevents/public/O3/

SORT: EVENT ID (A-Z) ▼

Event ID	Possible Source (Probability)	UTC	GCN	Location	FAR
S200316bj	MassGap (>99%)	March 16, 2020 21:57:56 UTC	GCN Circulars Notices | VOE		1 per 446.44 years
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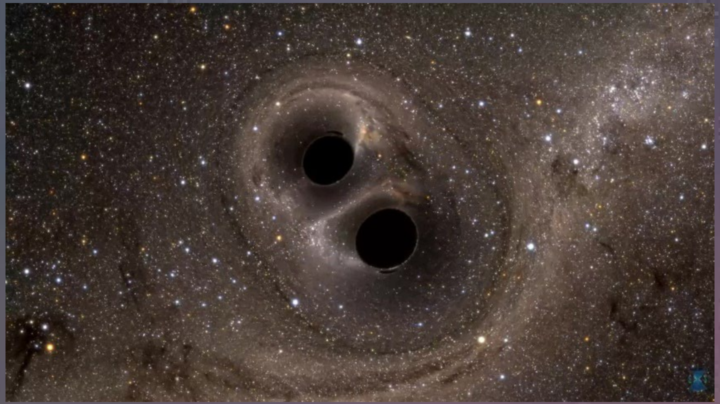
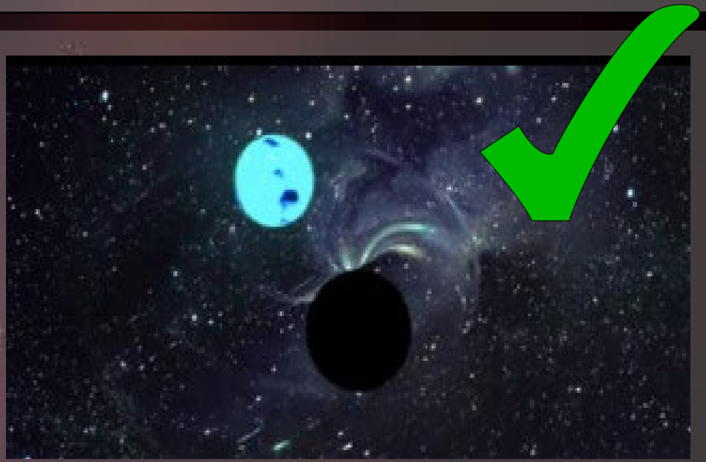
The Need for Rapid GW Discovery



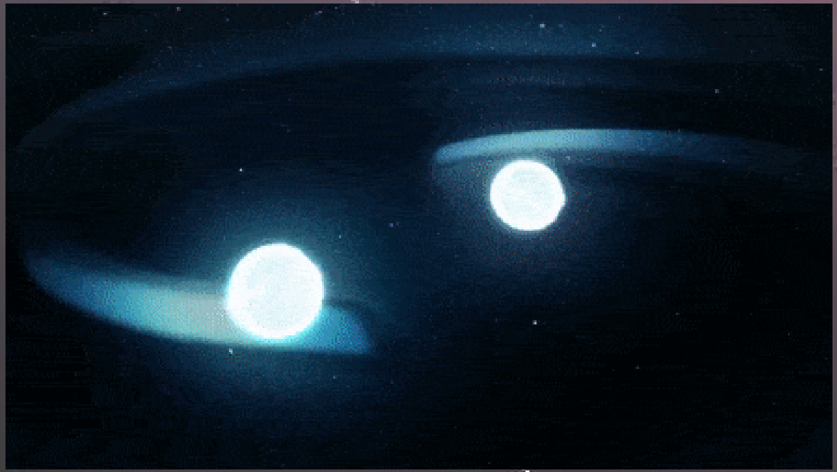
Binary Black Holes

Binary Neutron Stars



Neutron Star-Black Hole

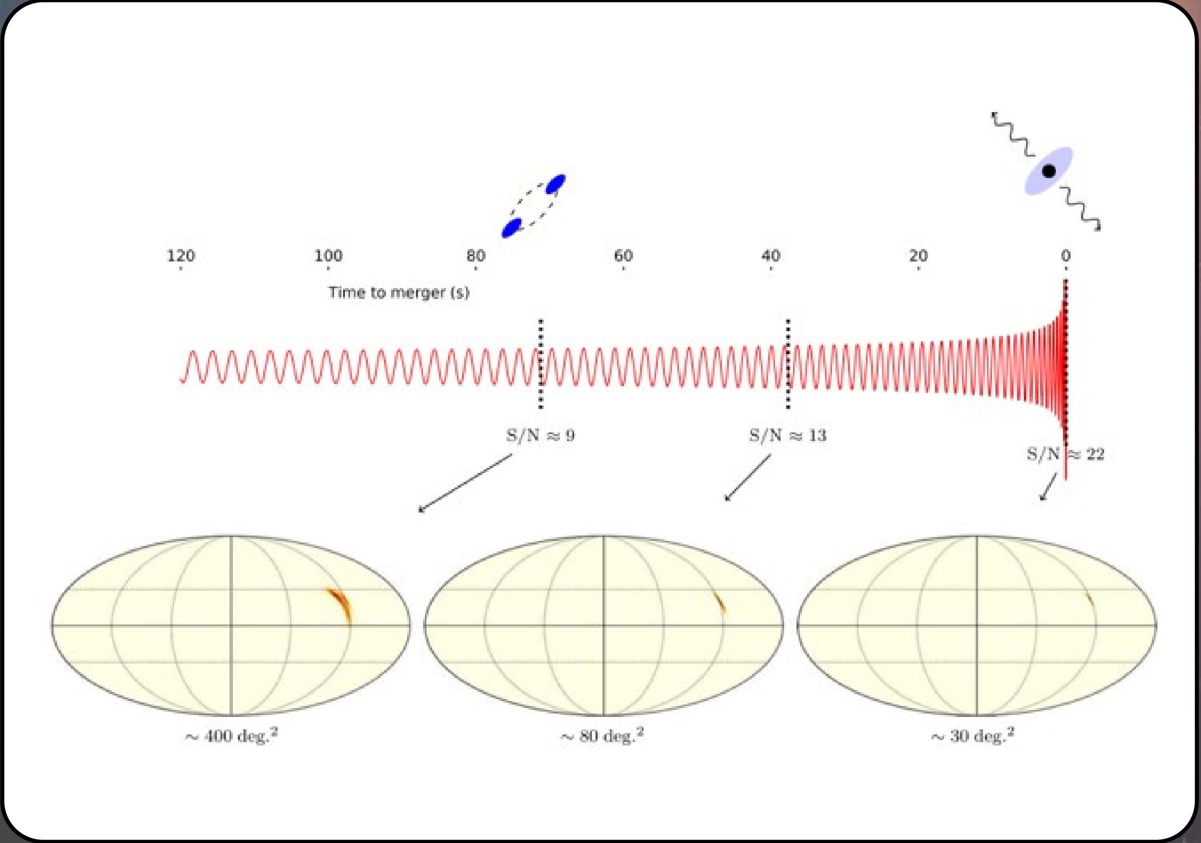
Compact Binary Coalescences (CBC)



GW + sGRB!

- Early electromagnetic observations are necessary for studying the following:
  - r-process nucleosynthesis and shock-heated ejecta (Optical & Ultraviolet).
  - Final state of the remnant (X-ray).
  - Premerger magnetosphere interactions (Radio).
  - Test models of BNS mergers as possible precursors of fast radio bursts (Radio).

Gravitational Wave 'Early Warning'

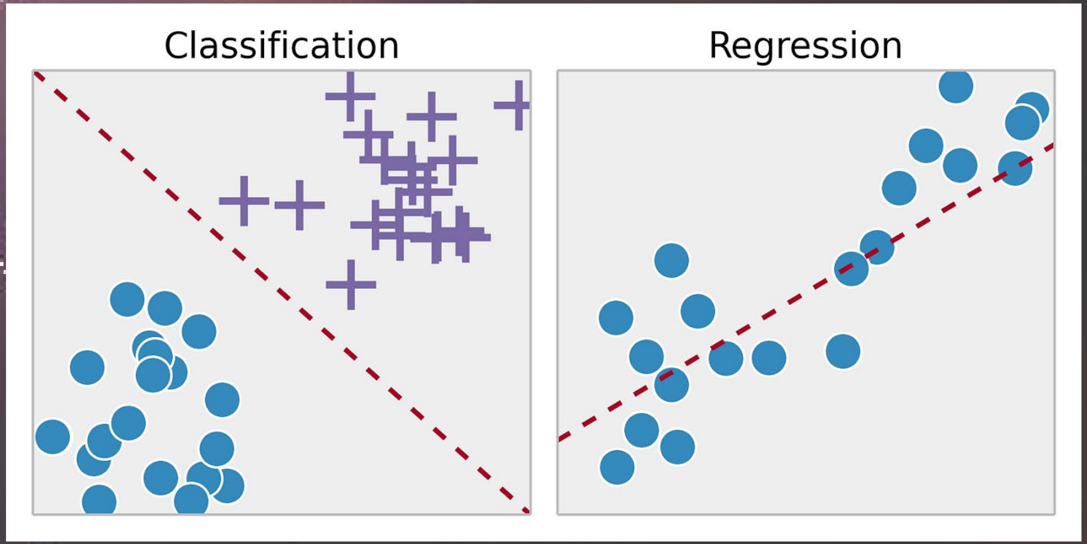


Ref: Magee *et al.* (2021), ApJL, 910, L21

- Binary neutron stars will spend 10–15 minutes in the band of Advanced LIGO and Virgo detectors at design sensitivity.
- ~ 7% (49%) of the total detectable BNS events will be detected 60 s (10 s) before the merger.
- About 2% will be detected before merger and localized to within 100 deg^2. – Sachdev *et al.* (2020), Kovalam *et al.* (2022).

Introduction to Machine Learning

- Machine Learning uses computers to learn patterns from data.
  - Typically used to solve problems that are hard to program in conventional ways.
- Typically we have a dataset $\{x^{(i)}\}$ consisting of many data points. The data points may or may not have associated labels $y^{(i)}$.
- Unsupervised learning: learn $p(x)$
  - Examples: Density estimation, sampling.
- Supervised learning: learn $p(y|x)$
  - Examples: Classification, regression



Machine Learning Recipe

- Build a dataset of training examples $(x^{(i)}, y^{(i)})$.
- Define a parametric probabilistic model of the data:

$$p(y|\boldsymbol{x};\boldsymbol{\theta}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{(y-\mu(\boldsymbol{x}))^2}{2\sigma^2}\right)$$

where $\mu(\boldsymbol{x}) = \boldsymbol{\theta}\cdot\boldsymbol{x}$; σ fixed.

- Choose a measure of performance (loss function):

$$J(\theta) = -\sum_{i=1}^{N} \log p(y^{(i)}|\boldsymbol{x}^{(i)};\boldsymbol{\theta})$$

$$= \frac{N}{2}\log 2\pi\sigma^2 + \sum_{i=1}^{N} \frac{(y^{(i)}-\mu(\boldsymbol{x}^{(i)}))^2}{2\sigma^2}$$

$\propto$ mean squared error

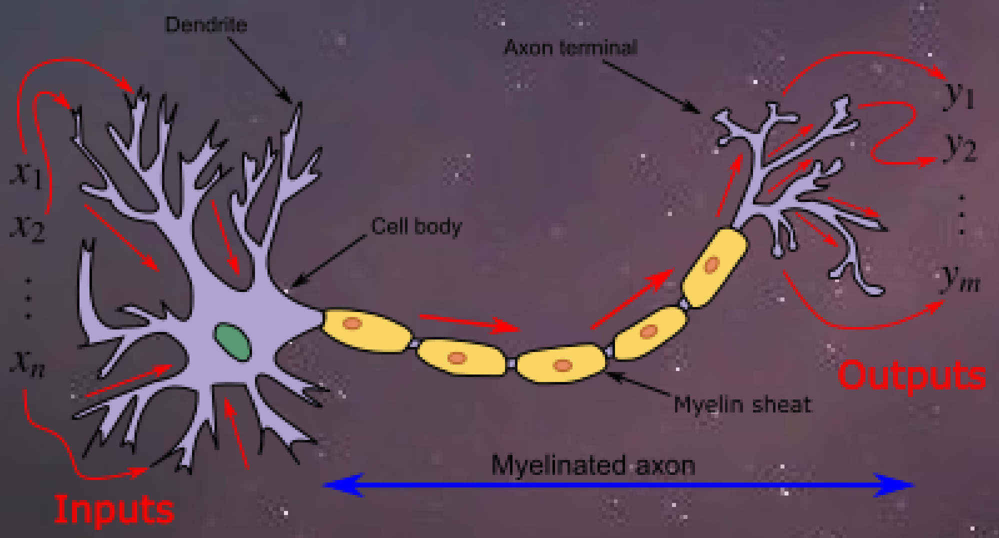
For greater flexibility, use a neural network

- Choose θ by fitting model to data according to loss function (maximum likelihood):

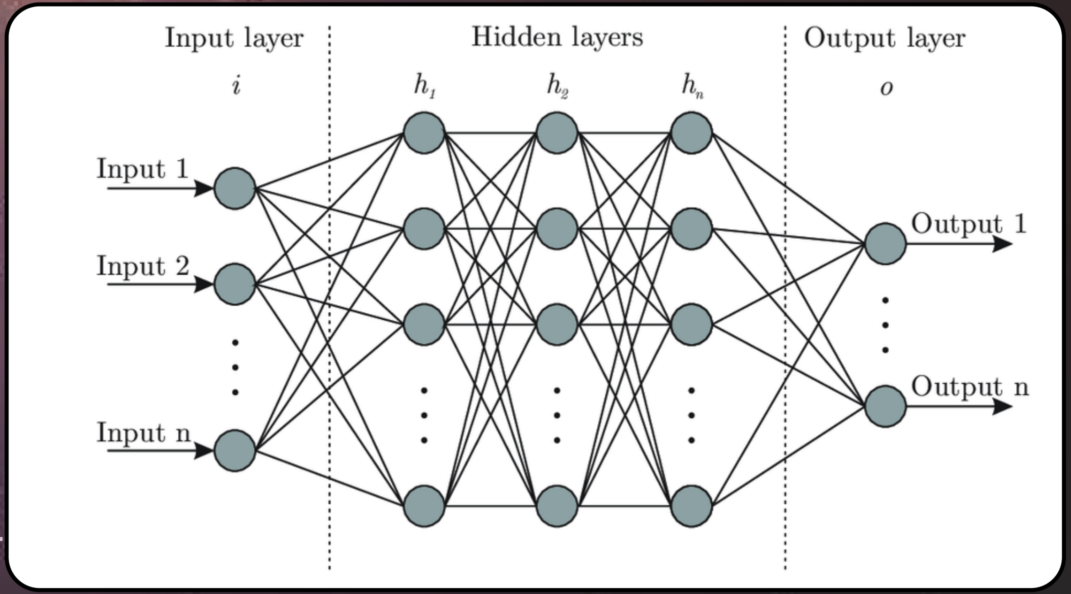
$$\nabla_{\boldsymbol{\theta}} J = 0 \quad \Longrightarrow \quad \boldsymbol{\theta}_{\text{ML}} = \left(X^\top X\right)^{-1} X^\top \boldsymbol{y}$$

Neural Networks

A Biological Neuron

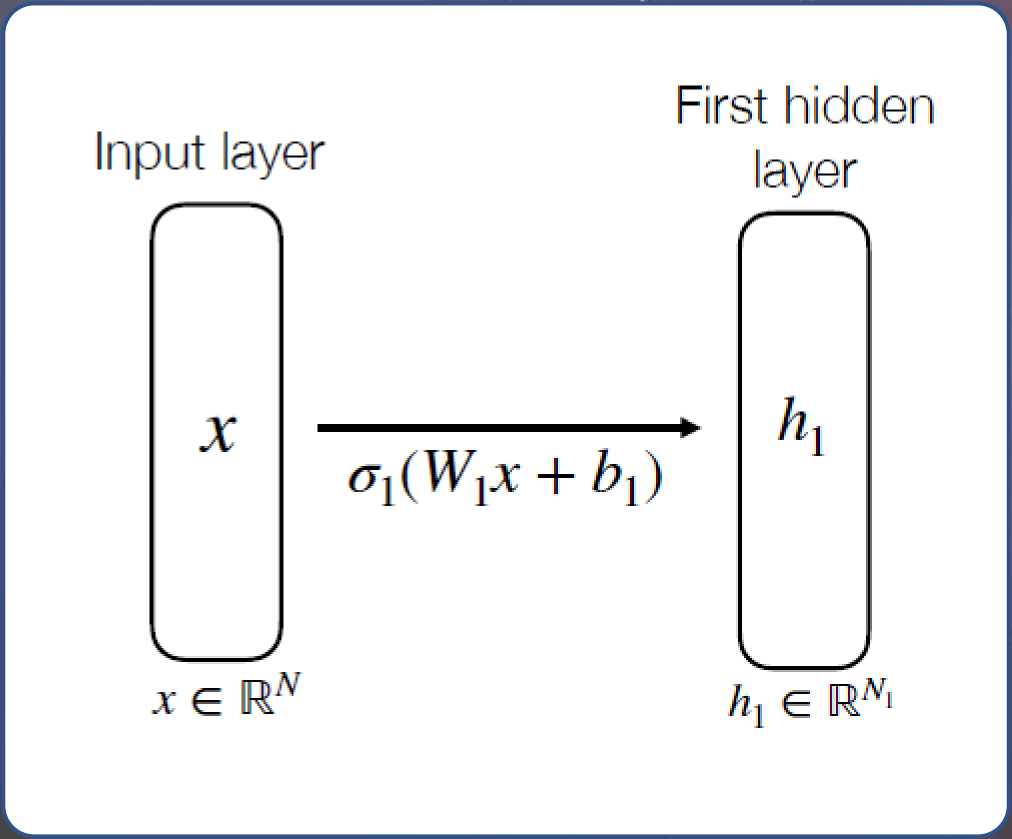


An Artificial Neural Network



Deep learning – a type of machine learning process that uses interconnected nodes or neurons in a layered structure that resembles the human brain.

Neural Networks



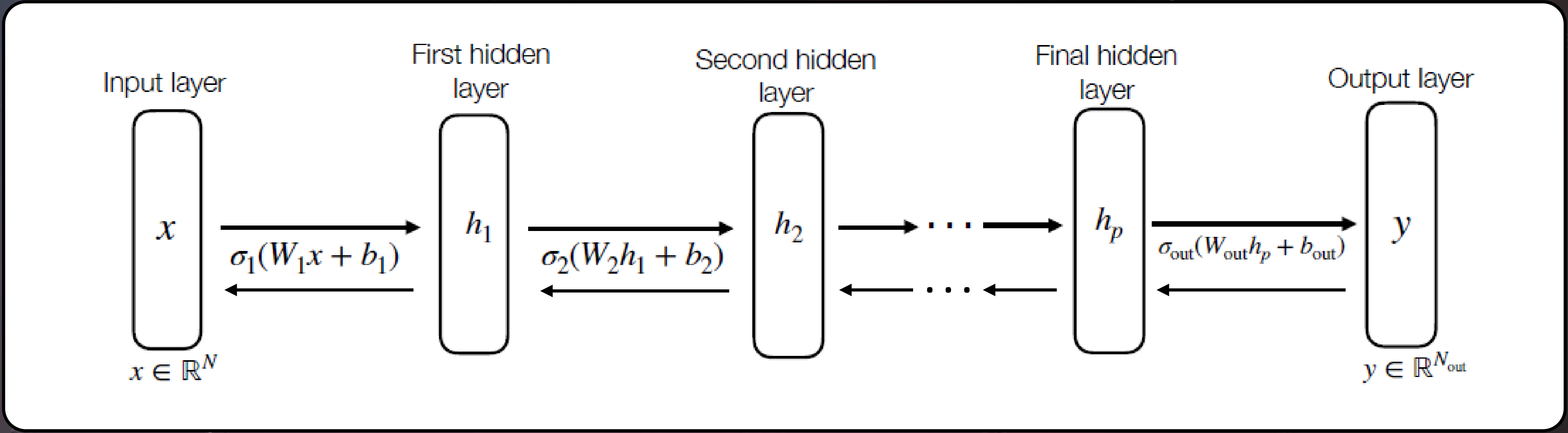
- Consists of:
  - A linear transformation –

$$Z_1 = W_1 x + b_1$$

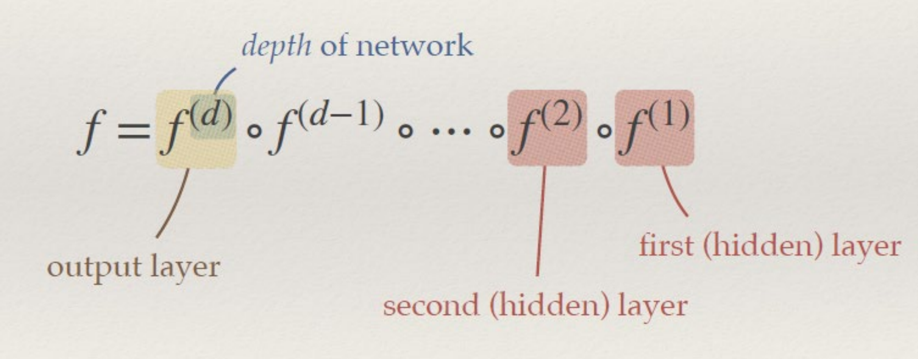
  - Simple element-wise non-linear mapping:

$$\sigma_1(x) = \begin{cases} x, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

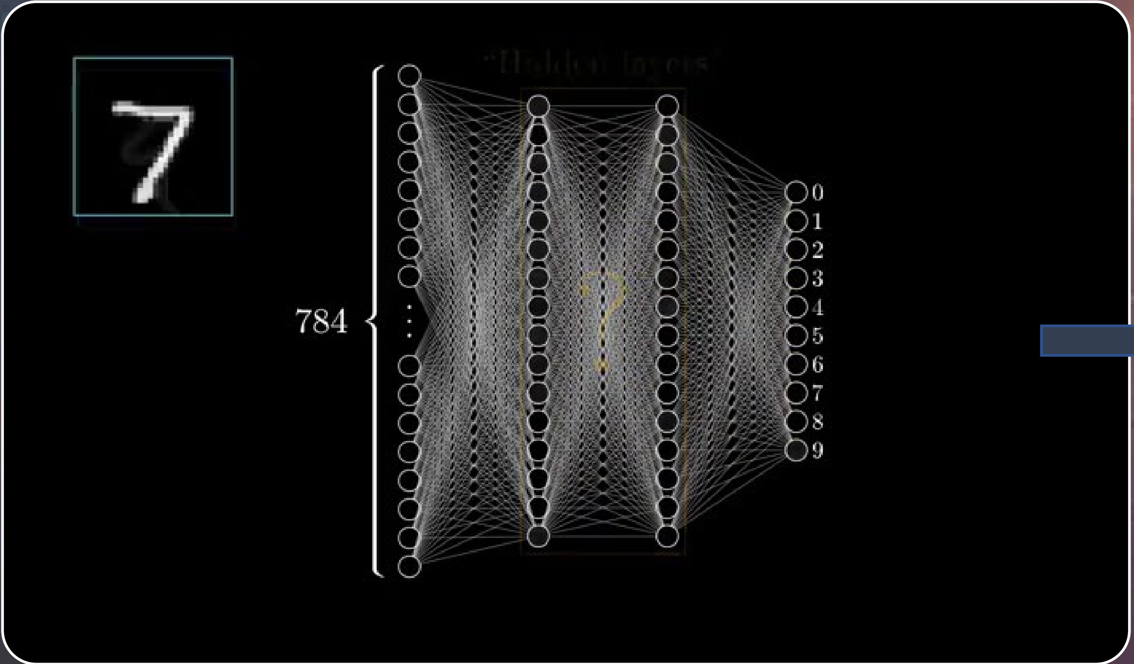
Neural Networks

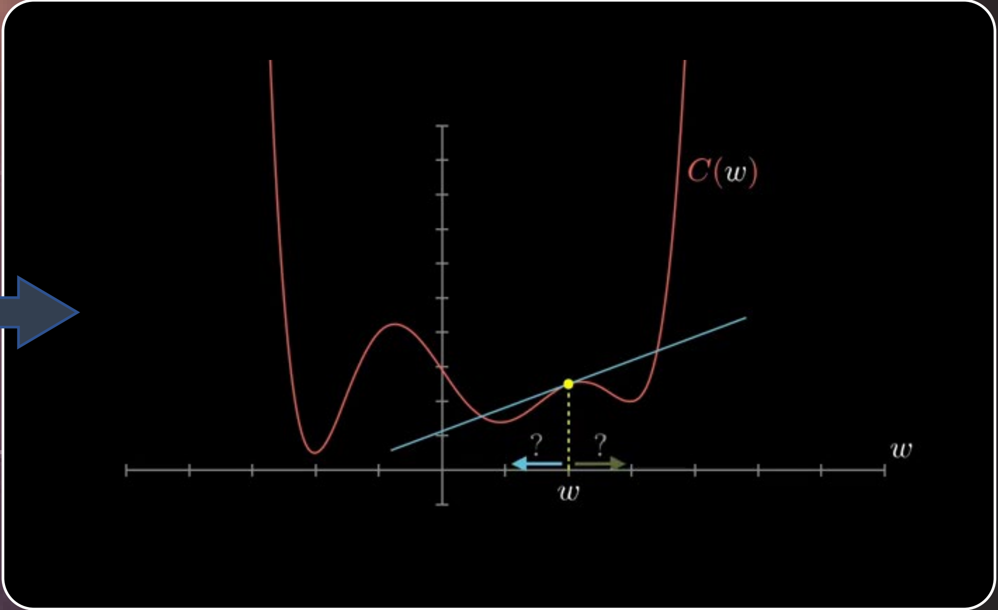


- Can be interpreted as a composition of mappings.
- Train network by tuning the weights W and biases b to minimize loss function.
- Stochastic Gradient descent combined with chain rule ("backpropagation") to adjust W and b.



Neural Networks – Optimizing a Loss Function





Neural Network –
Extracts features from input data

Stochastic Gradient Descent

$$\boldsymbol{\theta}_1 = \boldsymbol{\theta}_0 - \epsilon \nabla_{\boldsymbol{\theta}} J|_{\boldsymbol{\theta}_0}$$

Learning rate

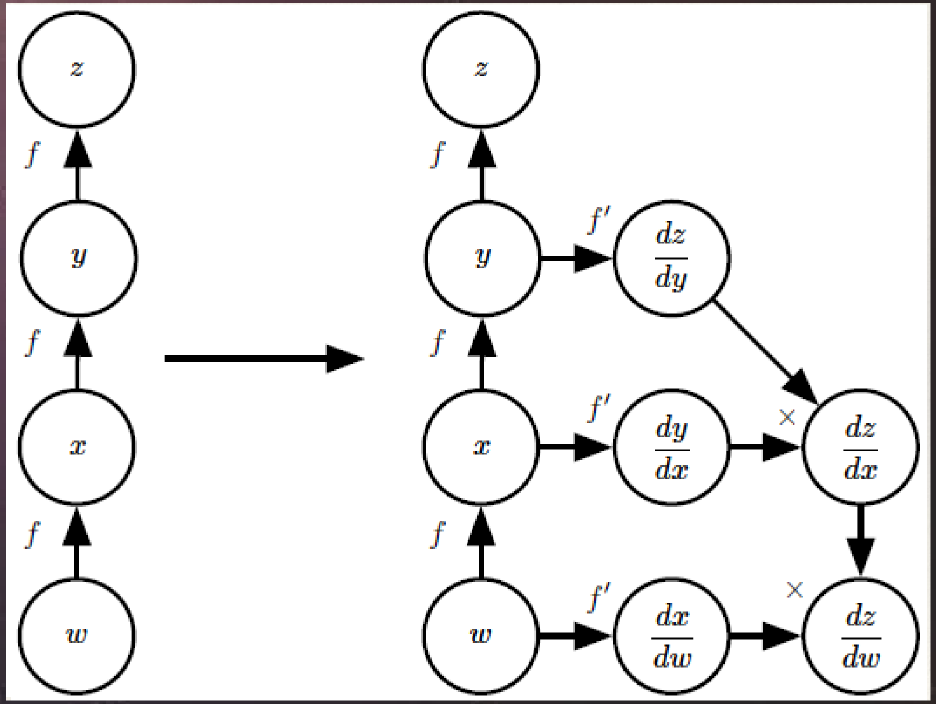
Backpropagation Algorithm

- To train the network, we need to efficiently compute gradients with respect to all of the network parameters. This is accomplished using a form of automatic differentiation called backpropagation.

- Relies on the compositional nature of neural networks, plus the chain rule of calculus and the differentiability of all operations.

$$
\begin{aligned}
\frac{\partial z}{\partial w} &= \frac{\partial z}{\partial y}\frac{\partial y}{\partial x}\frac{\partial x}{\partial w} \\
&= f'(y)f'(x)f'(z) \\
&= f'(f(f(w)))f'(f(w))f'(w)
\end{aligned}
$$

Efficient implementations in every deep-learning framework (PyTorch, TensorFlow, JAX, …)



Ian Goodfellow (2016)

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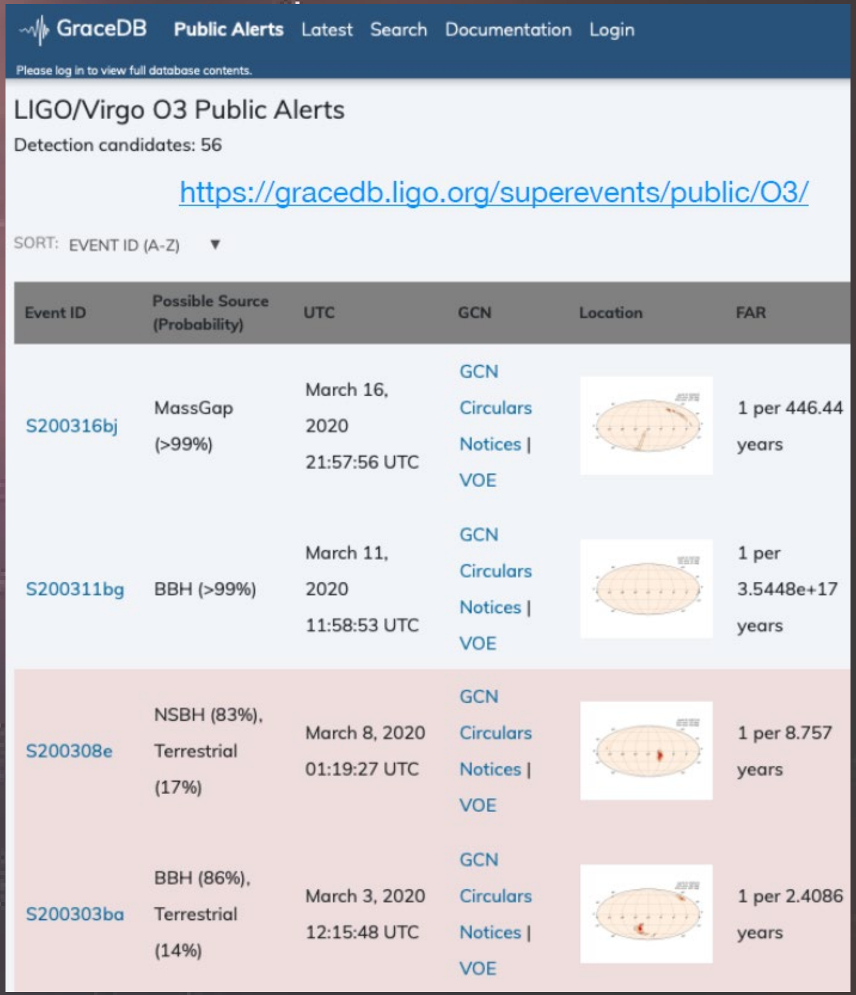

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Likelihood-free Inference with Neural Networks

- Objective:

$$p(\theta\,|\,s) \to p_{\text{true}}(\theta\,|\,s)$$

Intractable to evaluate posterior for each sample

- Cross entropy:

$$L = -\int \mathrm{d}s\, p_{\text{true}}(s) \int \mathrm{d}\theta\, p_{\text{true}}(\theta\,|\,s)\, \log p(\theta\,|\,s)$$

$p_{\text{true}}(s)\, p_{\text{true}}(\theta|s) =$
$p_{\text{true}}(\theta)\, p_{\text{true}}(s|\theta)$

- Use Baye's theorem and Monte Carlo approximation:

$$L = -\int \mathrm{d}\theta\, p_{\text{true}}(\theta) \int \mathrm{d}s\, p_{\text{true}}(s\,|\,\theta)\, \log p(\theta\,|\,s)$$

$$\approx -\frac{1}{N}\sum_{i=1}^{N} \log p(\theta^{(i)}|s^{(i)}),$$

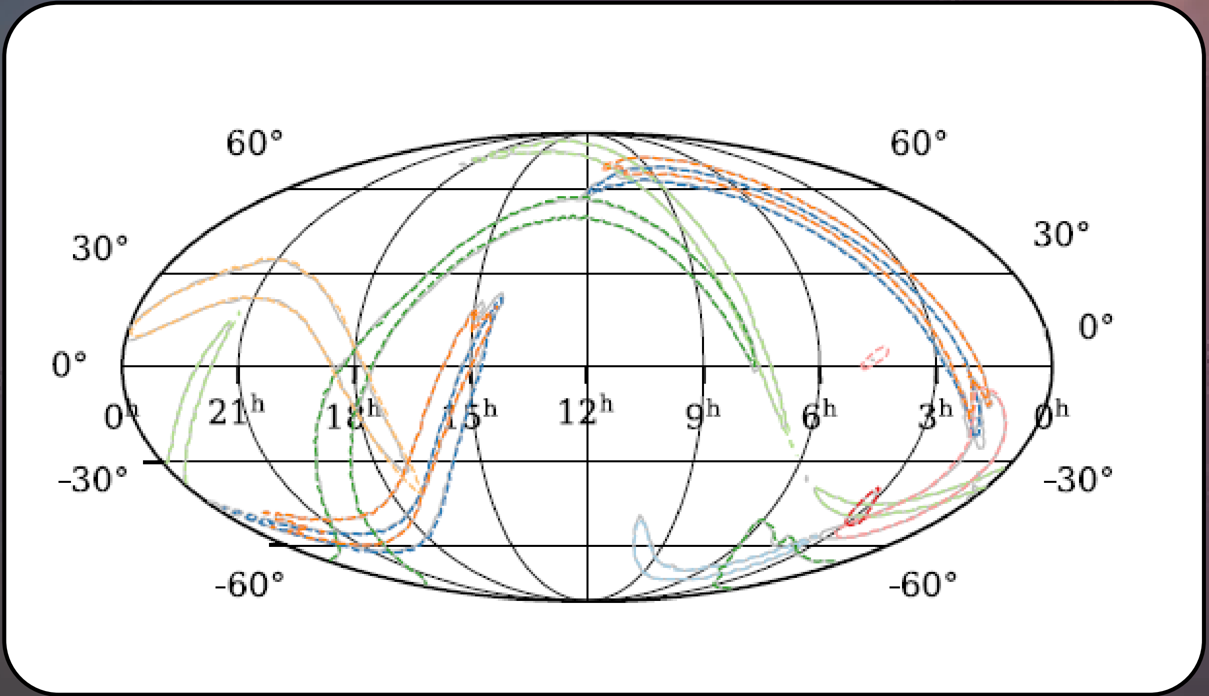
where $\theta^{(i)} \sim p_{\text{true}}(\theta)$,
$s^{(i)} \sim p_{\text{true}}(s|\theta^{(i)})$

No posterior samples.
No expensive likelihood evaluation.

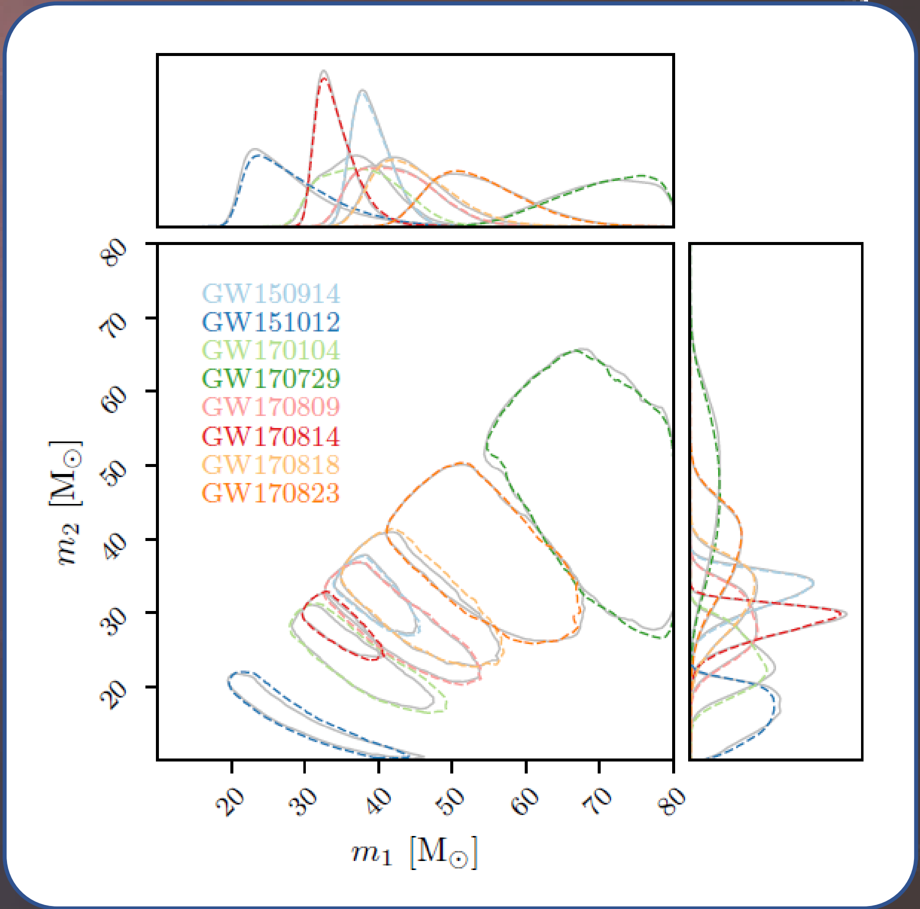
Deep Learning for BBH Parameter Estimation

Ref: Dax *et al.*, Phys. Rev. Lett. 127, 241103 (2021)

Sky Localization

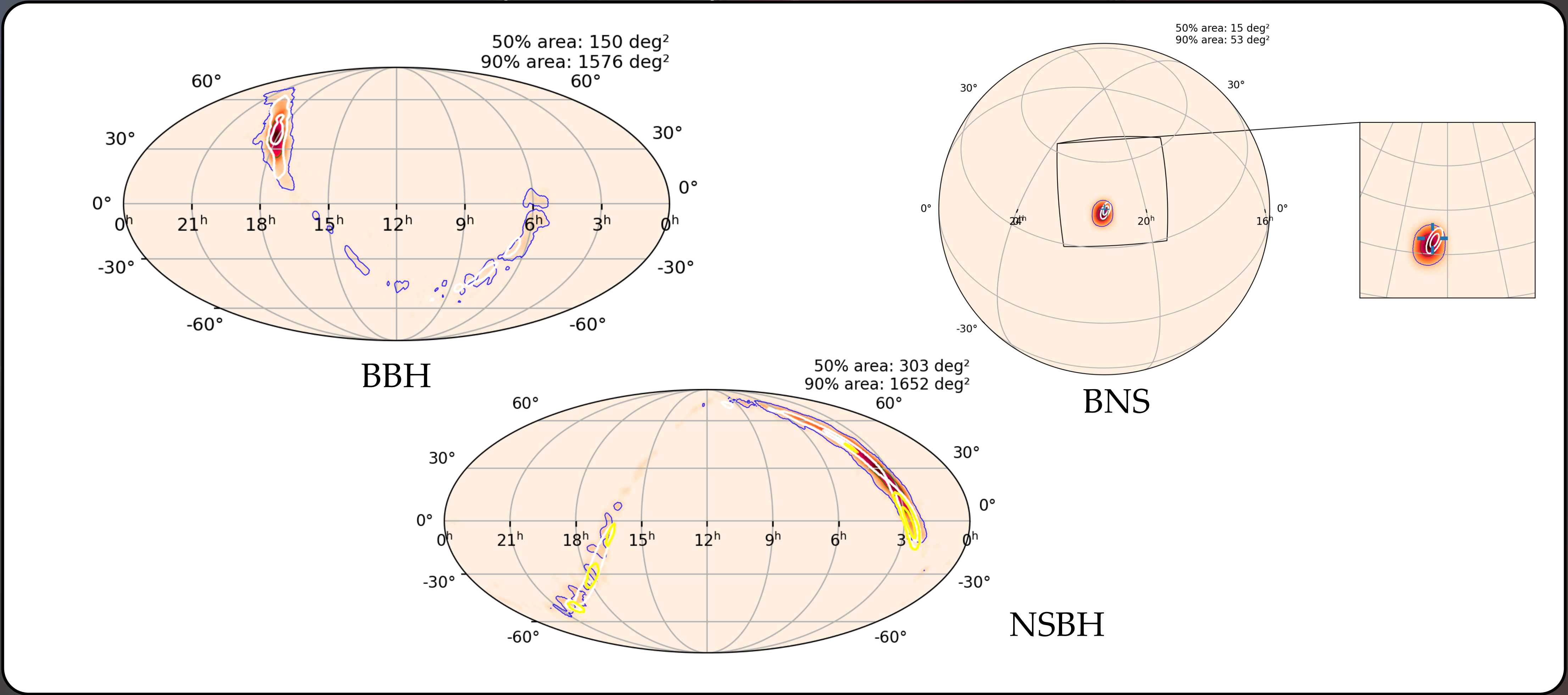


Mass Estimation



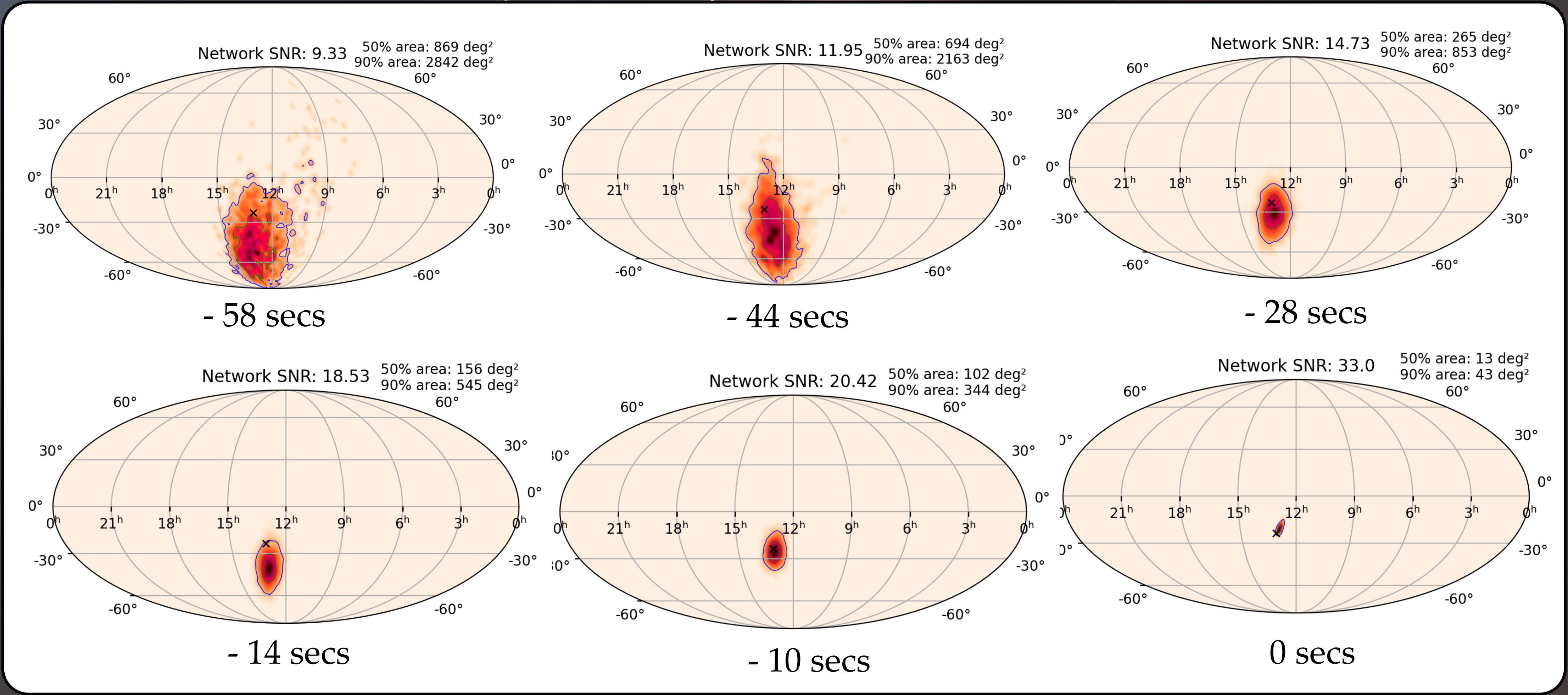
Analyzed all events consistent with prior $m_1, m_2 > 10$ solar mass

Deep Learning for GW Sky Localization



Ref: Chatterjee *et al.* , 2022 (Under review)

Deep Learning for Pre-merger Localization of GW170817



Ref: Chatterjee *et al.*, 2023 (accepted for publication in ApJ)