Class Exercise 4 (Least Squares & Bayes Theorem)
Introduction to Data Assimilation

a) Consider a state x that we are interested to estimate. We are given a first guess of x, i.e., $x_b = 3$, with $\sigma_b^2 = 3^2$ and one observation of x, i.e., $y_o = 0$, with $\sigma_o^2 = 1^2$. Find an optimal estimate of x and its error, i.e. ($\hat{x}$, $\hat{\sigma}$) using the least squares approach, assuming that the errors of the unbiased first guess and unbiased observation are independent (uncorrelated).

b) Solve problem a) using the Bayesian approach (hint: help normpdf). Assume here that the prior is normally distributed with mean, $x_b = 3$ and variance $\sigma_b^2 = 3^2$. Also assume that the observation is normally distributed with mean $y_o = 0$ and variance $\sigma_o^2 = 1^2$ and that both random variables are independent and identically distributed (iid). Compare your results with problem a). Also, plot the prior, obs likelihood, and posterior pdfs. What happens if $\sigma_o^2 = 0.5^2$ or $\sigma_b^2 = 2^2$?

c) Now, what if there are nobs = 4 independent observations rather than 1. Find $\hat{x}$ and $\hat{\sigma}$ using least squares approach. Use y_o_4.dat for this problem. What happens to your estimate and its error if there are nobs=1000 (use y_o_nobs.dat for this case)? Does it matter if we change the order of the observations? Why/Why not?

d) Solve problem c) using the Bayesian approach (for nobs=1000, plot only for obs {1,2,nobs-1,nobs}. Discuss your results.

e) Solve problem d for nobs=1000 and $\sigma_o^2 = 0.5^2$. Use y_o_nobs_05.dat for this case. What happens your estimate if you use $\sigma_o^2 = 1^2$ and y_o_nobs_05.dat?

f) Consider problem c) but using different obs error ($\sigma_{o,1}^2 = \sigma_{o,2}^2 = 1^2, \sigma_{o,3}^2 = \sigma_{o,4}^2 = 0.5^2$