Class Exercise 3 (Simple DA Examples)
Introduction to Data Assimilation

Problem 1:

a)   Consider our previous toy example on estimating room temperature. Here, we would like to estimate the temperature at room 0 given 10 realizations of first guesses of temperature at room 0 (Tb_0.dat), room 1 (Tb_1.dat), and room 2 (Tb_2.dat), as well as observations of temperature at room 1 (To_1.dat), and 2 (To_2.dat). Assume that the errors in our first guesses and observations are unbiased. Assume that errors in our observations are also uncorrelated. However, assume that errors in our first guesses are correlated. Determine the error variances and correlations using the 10 realizations given to you.

Plot your first guess (at room 0) and analysis for each realization. Calculate the mean of the first guess and analysis, as well as their error variances. Compare this with the root mean square error of your analysis (relative to To_0.dat) based on 10 realizations.

b)   Now, consider a case where we have observations of temperature at room 0 (To_0.dat). Find an analysis estimate for the temperature at room 0 given this observation and a first guess of temperature at room 0 (Tb_0.dat). Compare with our results above (in terms of mean analysis estimate and error variance).

Problem 2:

a)   Consider our previous toy example on estimating room temperature. Here, we would like to estimate the temperature at room 0 given 10 realizations of first guesses of temperature at room 0 (Tb_0.dat), room 1 (Tb_1.dat), as well as observations of temperature at room 1 (To_1.dat). Assume that the errors in our first guesses and observations are unbiased. Assume that errors in our observations are also uncorrelated. However, assume that errors in our first guesses are correlated. Determine the error variances and correlation using the 10 realizations given to you.

        Plot your first guess and analysis (at room 0) for each realization. Calculate the mean of the first guess and analysis, as well as their error variances. Compare this with the root mean square error of your analysis (relative to To_0.dat) based on 10 realizations.

b)   Now, consider a case where we have observations of temperature at room 0 (To_0.dat). Find an analysis for the temperature at room 0 given this observation and a first guess of temperature at room 0 (Tb_0.dat). Compare your analysis with your results above (in terms of mean analysis estimate and error variance).

Problem 3:

a)   Consider our previous toy example on estimating room temperature. Here, we would like to estimate the temperature at room 0 given 10 realizations of first guesses of temperature at room 0 (Tb_0.dat), room 1 (Tb_1.dat), and room 2 (Tb_2.dat), as well as 2 sets of observations of temperature at room 1 (To_1.dat, To_2.dat). Assume that the errors in our first guesses and observations are unbiased. Assume that errors in our observations are also uncorrelated. However, assume that errors in our first guesses (between room 0 and 1, and between room 0 and 2) are equally correlated while errors in our first guesses between room 1 and 2 are uncorrelated (i.e. observations in 1 and 2 are isolated). Determine the error variances and correlation using the 10 realizations given to you.

      Plot your first guess and analysis (at room 0) for each realization. Calculate the mean of the first guess and analysis, as well as their error variances. Compare this with the root mean square error of your analysis (relative to To_0.dat) based on 10 realizations.

b)   Now, consider a case where we have observations of temperature at room 0 (To_0.dat). Find an analysis for the temperature at room 0 given this observation and a first guess of temperature at room 0 (Tb_0.dat). Compare your analysis with your results above (in terms of mean analysis estimate and error variance).

Problem 4:

a)   Consider our previous toy example on estimating room temperature. Here, we would like to estimate the temperature at room 0 given 10 realizations of first guesses of temperature at room 0 (Tb_0.dat), room 1 (Tb_1.dat), as well as 2 sets of observations of temperature at room 1 (To_1.dat, To_11.dat). Assume that the errors in our first guesses and observations are unbiased. Assume that errors in our observations are also uncorrelated. However, assume that errors in our first guesses are correlated. Determine the error variances and correlation using the 10 realizations given to you.

Plot your first guess and analysis (at room 0) for each realization. Calculate the mean of the first guess and analysis, as well as their error variances. Compare this with the root mean square error of your analysis (relative to To_0.dat) based on 10 realizations.

b)   Now, consider a case where we have observations of temperature at room 0 (To_0.dat). Find an analysis for the temperature at room 0 given this observation and a first guess of temperature at room 0 (Tb_0.dat). Compare your analysis with your results above (in terms of mean analysis estimate and error variance).