Class Exercise 2 Introduction to Data Assimilation

Given:

Observation of room temperature at grid 1 and its error characteristics,

$$T_{o,1} = T_{t,1} + e_{o,1}$$
 where $E(e_{o,1}) = 0$, $E(e_{o,1}^2) = \sigma_{o,1}^2$

as well as, a first guess of the room temperature at grid $\mathbf{0}$ and $\mathbf{1}$ and their error characteristics,

$$T_{b,0} = T_{t,0} + e_{b,0}$$
 where $E(e_{b,0}) = 0$, $E(e_{b,0}^2) = \sigma_{b,0}^2$

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 where $E(e_{b,1}) = 0$, $E(e_{b,1}^2) = \sigma_{b,1}^2$

where

$$\sigma_{b,0}^2=\sigma_{b,1}^2$$
 and $Eig(e_{b,0}$, $e_{b,1}ig)=
ho_{0,1}\sigma_b^2$

Assume that the error in observation is uncorrelated with the errors in our first guess, i.e.,

$$E(e_{o,1}, e_{b,0}) = \sigma_{\{o,1\}\{b,0\}} = 0$$

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Question: Find an analysis estimate (and its error) of the temperature at grid 0 given observation in grid 1.