Class Exercise (Optimal Interpolation)
Introduction to Data Assimilation

Exercise I. (On the Weight Matrix)

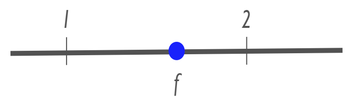


Figure 1. A network with 2 observations

Consider a 1D domain as suggested in Figure 1, where we would like to estimate the state at point f given an observation at two nearby stations. The normalized observation error variances at the two stations are equal ($\varepsilon_1^2 = \frac{\sigma_1^2}{\sigma_f^2} = \varepsilon_2^2 = \frac{\sigma_2^2}{\sigma_f^2} =$ 0.25). The background error correlation model is given by:

$$\rho_f(x) = \left(1 + \frac{|x|}{L}\right) exp^{\left(-\frac{|x|}{L}\right)}$$

and the observing station and analysis gridpoint locations are given in terms of $\frac{|x|}{L}$ where $|x|$ is the distance and L is length scale. The analysis gridpoint f is at $\frac{x_f}{L} = 0$, and the observation station 1 is at $\frac{x_1}{L} = -2.0$. The location of observation station 2 is allowed to vary with x; $-4.0 \le \frac{x_2}{L} \le 4.0$. Find the weights W_1 and W_2 as a function of observation station 2 location. The weights are given by the solution to:

$$\sum_{j=1}^{m} w_{jf}\left(b_{jk} + r_k\right) = b_{kf} \quad \text{where } k = 1, \ldots m$$

Hence, the weights are:

$$W_{1f} = \frac{\rho_{1f}(1 + \varepsilon_1^2) - \rho_{2f}\rho_{12}}{(1 + \varepsilon_1^2)(1 + \varepsilon_1^2) - \rho_{12}{}^2}$$

$$W_{2f} = \frac{\rho_{2f}(1 + \varepsilon_1^2) - \rho_{1f}\rho_{12}}{(1 + \varepsilon_1^2)(1 + \varepsilon_2^2) - \rho_{12}{}^2}$$

The normalized analysis error variance is calculated to be:

$$\varepsilon_a^2 = 1 - \frac{\rho_{1f}{}^2(1 + \varepsilon_2^2) + \rho_{2f}{}^2(1 + \varepsilon_1^2) - 2\rho_{1f}\rho_{2f}\rho_{12}}{(1 + \varepsilon_1^2)(1 + \varepsilon_2^2) - \rho_{12}{}^2}$$

Plot W_1, W_2 and ε_a^2 as a function of observation station 2 location.

Exercise II. (On Background/Observation Error Correlation)

Assume that we have 2 locations where we want to estimate the wind speed. At those two locations, the background value of the wind speed is 8 m/s and 12 m/s, while the observed wind speed is 9 m/s and 14 m/s respectively. What are the analyzed values of the wind speed and the expected standard deviation of the errors at the two locations for the following values of the background and observational errors:

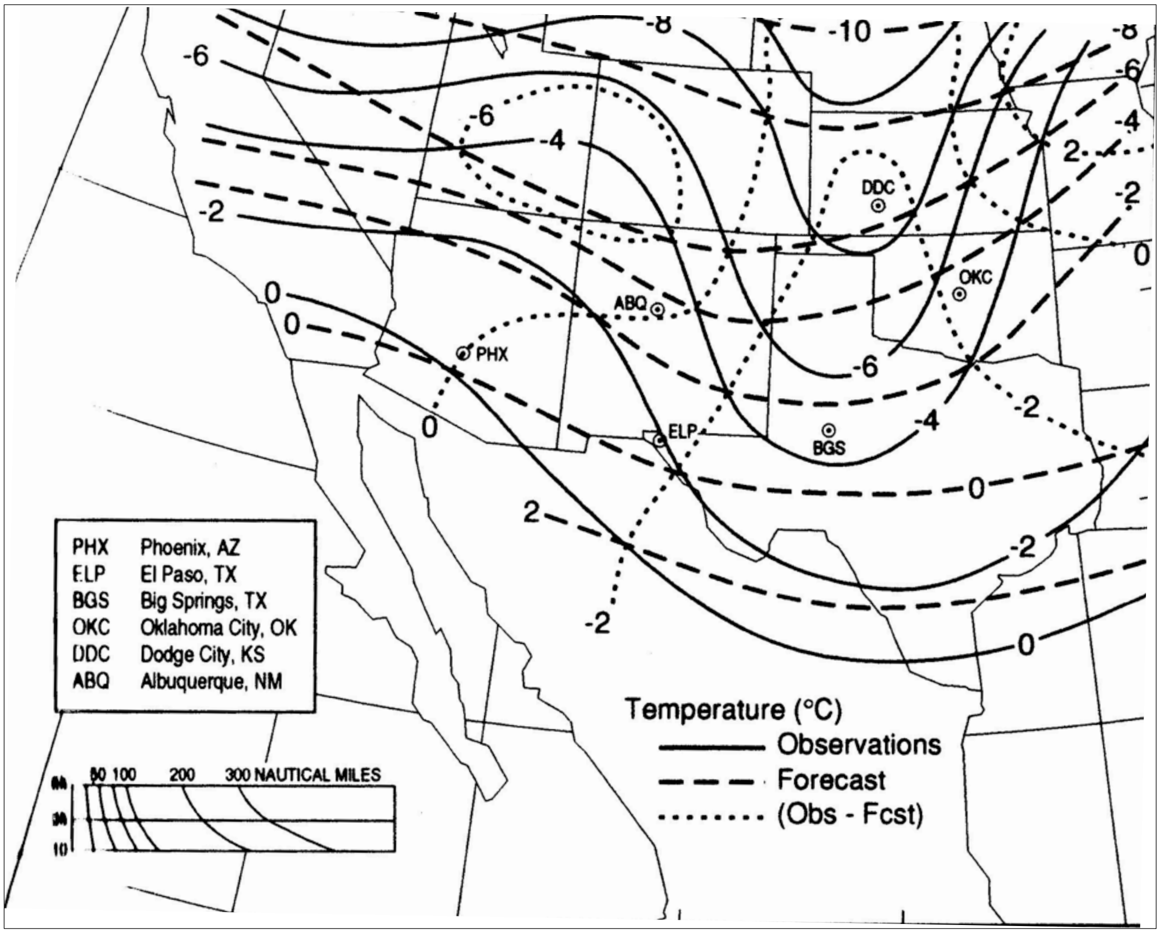
a) The standard deviation of the background error and the observation error is 1.5 m/s and 1.0 m/s (independent of the location with uncorrelated errors between the different locations).
b) Same as a) but the correlation between the background errors at the two locations is 0.5;
c) Same as a) but the correlation between the observation errors at the two locations is 0.5;
d) What happens when we increase the correlation to 0.9 in b) and c)?

Assume that the background is the same as above, but only one of the two observations is available, the one that measured 9 m/s. What are the analyzed values of the wind speed and the expected standard deviations of the errors at the two locations for the following values of the background and observation errors:

e) same as a)
f) same as b)

Exercise III. (On 2D OI)

Consider the synoptic map over the southwest and southcentral USA



Find the optimal estimate of temperature at BGS using the forecast and observations at BGS, ELP and OKC. Assume the following:

forecast error variance: $(1.0^oC)^2 = \sigma_f^2$
observation error variance: $(0.5^oC)^2 = \sigma_o^2$
spatial error covariance of forecast: $\sigma_f^2 exp\left(-0.5\left(\frac{d_{ij}}{L}\right)^2\right)$ where d_{ij} is the distance between stations and L = 556 km.

ELP: lat=31.8N, lon=106.4W
BGS: lat=32.39N, lon=101.48W
OKC: lat=35.40N, lon=97.60W

Use the following expression to calculate the distance (great-circle) between station 1 and 2:

$$d_{12} = 6378\ cos^{-1}[\cos(lat1)\cos(lat2)\cos(lon1 - lon2) + \sin(lat1)\sin(lat2)]$$

where the latitudes/longitudes are in radians.