Now, let's look at similar numerical example in Wikle and Berliner, 2007

2.1.1. Numerical examples

Case 1

Assume the prior distribution is $X \sim N(20, 3)$, and the data model is $Y_i|x \sim N(x, 1)$. We have two observations $\mathbf{y} = (19, 23)'$. The posterior mean is $20 + (6/7)(21 - 20) = 20.86$ and the posterior variance is $(1 - 6/7)3 = 0.43$. Fig. 1 shows these distributions graphically. Since the data are relatively precise compared to the prior, we see that the posterior distribution is "closer" to the likelihood than the prior. Another way to look at this is that the gain ($K = 6/7$) is close to one, so that the data model is weighted more than the prior.

Case 2

Next, assume the same observations and prior distribution, but change the data model to $Y_i|x \sim N(x, 10)$. The gain is $K = 6/16$ and the posterior distribution is $X|\mathbf{y} \sim N(20.375, 1.875)$. This is illustrated in Fig. 2. In this case, the gain is closer to zero (since the measurement error variance is relatively large compared to the prior variance) and thus the prior is given more weight.

Class Exercise 5

1. Find the posterior mean and variance using LS (case 1 & 2)

2. Do 1. but now use p(x|y) is proportional to p(y|x) p(x). That is, product of two Gaussian (case 1 & 2)

3. Reproduce Fig 1 and Fig 2.

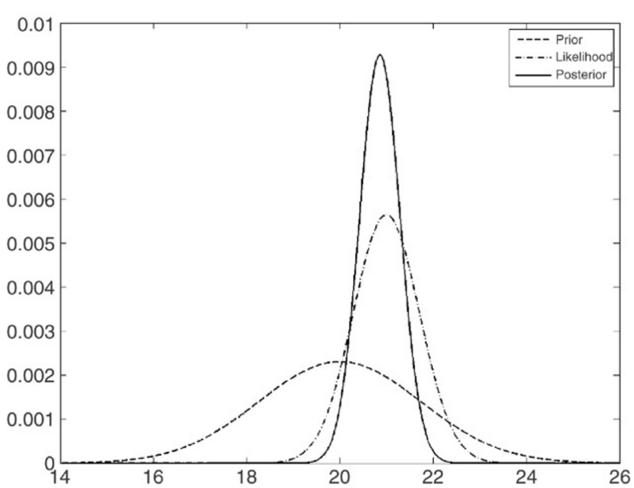

Fig. 1. Posterior distribution with normal prior and normal likelihood; relatively precise data.

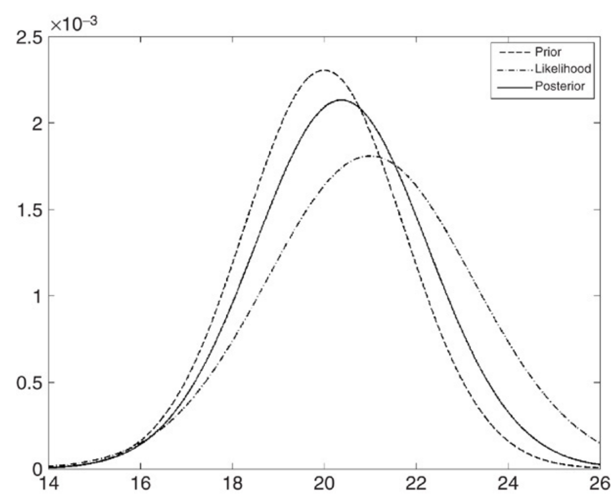

Fig. 2. Posterior distribution with normal prior and normal likelihood; relatively uncertain data.

hint: use normpdf if using matlab