IntroIE522

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Accuracy in pricing options is critical as inaccurate prices may lead to inefficiencies in the market. Inaccurate option prices can create arbitrage opportunities or deter investors, resulting in illiquid markets. Monte Carlo simulation plays a key roll in option pricing as it provides a method for prcing complex derivatives when an analytical solution is not available. Monte Carlo simulation estimates the value of a financial instrument by simulating an array of potential price trajectories. The standard Monte Carlo approach uses random sampling to better model uncertanties and then computes the expected payoff based on these modeled uncertanties. This expected payoff is then discounted to present time to obtain the present value of the price trajectory. This process is iteratively simulated many times, where the payoff of each price trajectory is calculated. In each iteration, the sample mean and updated and stored, reducing excess memory usage. The sample mean and squared sample mean are sufficient to calculate the final option price and its variance or standard error.

This report uses Monte Carlo simulation to price three types of options: European vanilla put options, Asian call options, and American put options. The European vanilla put option prices are calculated in the Black-Scholes-Merton model and using the standard Monte Carlo approach. In addition, the antithetic approach is also used to reduce variance and improve efficiency. This is completed by generating pairs of negatively correlated random variables, allowing for more accurate results with fewer simulations. The Quasi-Monte Carlo approach (QMC) is also implemented to improve efficiency with faster convergence. This is completed by using low-discrepancy sequences instead of random sampling for the generation of sampling points. In pricing the European vanilla put option, the sample size, option price, estimated standard error, 95% confidence interval, and the absolute pricing error are reported. Convergence between the different methods is also investigated.

To price the Asian call option in the Black-Scholes-Merton model, the standard Monte Carlo approach is used. In addition, to reduce variance, the control variate approach is used with geometric Asian call as a control. The geometric Asian call has a closed form solution with known expected value and is correlated with the price of the true Asian call option. The use of this additional control variate variable allows for final values to be more precise with smaller samples. In addition, the moment method approach is utilized to increase computational efficiency by using indirect simulation methods, reducing the need for direct simulation. In pricing the Asian call options, he sample size, option price, estimated standard error, 95% confidence interval, and computational time is reported for each Monte Carlo simulation. Also, the convergence of the Monte Carlo methods is investigated as the number of samples, N, increases.

To price the American vanilla option, the Longstaff-Schwartz method is employed using Monte Carlo simulation. This method uses regression to estimate early exercise strategy, with the value of the option being the average of the many simulated price trajectories. Different regressors, sample sizes, and time steps are investigated to see how they effect the option price. The Binomial Black-Scholes with Richardson Extrapolation (BBSR) method is also implemented and used as a high-accuracy benchmark for comparison. The BBSR method refines the binomial tree approach, improving accuracy with fewer time steps.