Numerical methods for first-passage times of branching processes

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Branching processes

- Beans can hop between pairs of points k and k' in a space, rate $r_{k \to k'}$
- Beans can split into two new beans, rate b_k

First passage times

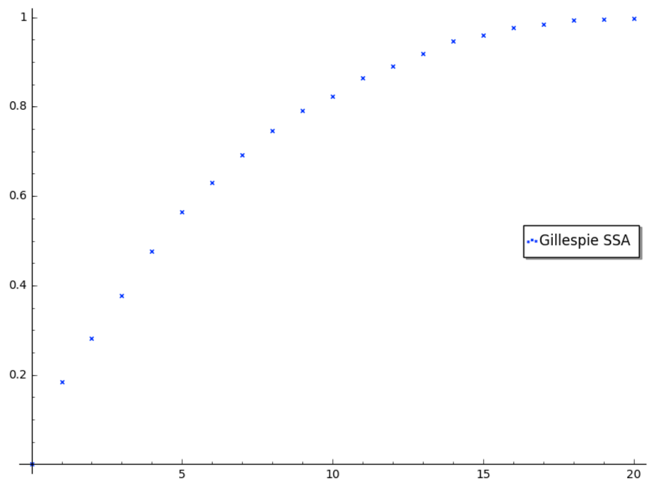
- What is the probability that a given region x has not been visited yet?
- This is the probability that the time t_x to reach x is at least t
- $P(t_x \geq t)$

Gillespie (SSA) algorithm

Monte Carlo.

- Calculate event rate Γ
- Generate random number $u = \exp(\Gamma)$
- Convert into specific event
- Generate time to next event $\Delta t = \exp(\Gamma^{-1})$
- $t+ = \Delta t$

Example of SSA



Conditioned means method

Deterministic, given initial conditions.

- For each point k, update the conditional expected population $\langle n_k \| n_x = 0 \rangle$.
- Update the probability of not being visited $P(n_x = 0)$.
- $P(t_x \geq t) = P(n_x = 0, t)$

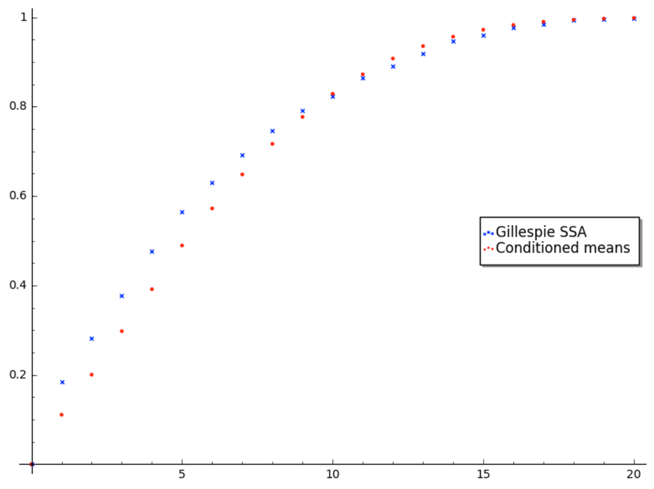
$$\langle \dot{n}_k \| n_x = 0 \rangle = \sum_{k'} \left[r_{k' \to k} \langle n_{k'} \| n_x = 0 \rangle - r_{k \to k'} \langle n_k \| n_x = 0 \rangle \right] + b_k \langle n_k \| n_x = 0 \rangle$$

$$P(n_x = 0) = \exp\left(- \int_0^t \sum_k r_{k \to x} \langle n_k \| n_x = 0 \rangle dt \right)$$

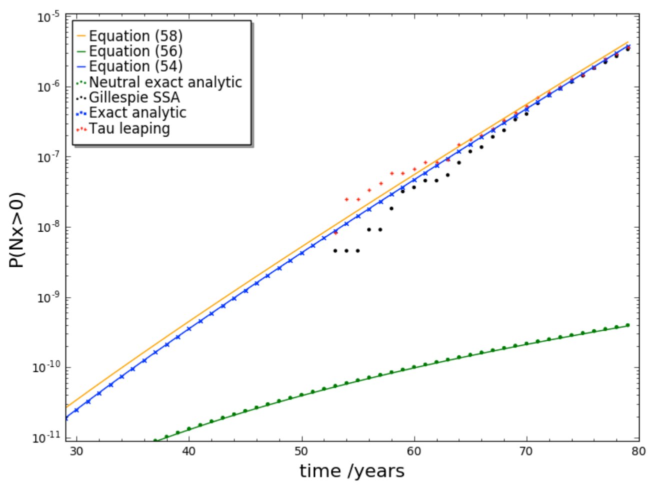
Conditioned means method

Why does this work?

Comparison



Comparison



Equation (58)
Equation (56)
Equation (54)
Neutral exact analytic
Gillespie SSA
Exact analytic
Tau leaping
P(Nx>0)
time /years
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
10^{-10}
10^{-11}
30
40
50
60
70
80

Thank you

Questions?