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# Time–frequency analysis for ultrasonic measurement of liquid-layer thickness

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ABSTRACT

Lubricant film thickness is one of the most important parameters to indicate the operating condition of machine elements, such as mechanical seals and hydrostatic slideway. When ultrasonic waves illuminate the interfaces between the substrates and a lubricant film, it will be reflected due to the change of the material properties at the interfaces. These reflected ultrasonic waves contain information about film thickness. In this paper, wavelet transform modulus maximum method was explored to extract the film thickness from its reflection ultrasonic signals. The performance of different wavelet functions within various scale factors was experimentally investigated, and the optimal wavelet function with the optimal scale factor was pointed out. It has been shown that the measurement error is less than 5% when the thickness of liquid layer is within a certain range.



## 1. Introduction

Durability of machine elements, such as mechanical seals and hydrostatic slideway, relies on the condition of lubricant film layer, which separates the contact surfaces and makes machine operate smoothly. Failure of this lubricant film is one of the most common causes for the failure of machine elements, which can lead to serious accidents for machine operation [1–2]. The thickness is one of the most important parameters to characterize the operating condition of the lubricant film, which can provide early warning of lubrication failure, and predict remaining life. The thickness of lubricant film depends on the lubricant properties, the geometry of the contact surfaces, and the operating condition [3]. Typically, the hydrostatic slideway is widely used in the field of CNC Machine Tools with films in the range of 0.1 μm to several hundred microns [4].

Ultrasonic technique has been widely used in gauging the thickness of lubricant film and a great deal of work has been done, for example, the pulse-echo method [3–5]. In these methods, an ultrasonic transducer sends pulses and receives reflected echoes from discontinuities in the sample, and time delays between echoes are used to assess the layer thickness. However, for thin layers the echoes reflected from two successive interfaces overlap in time, it is difficult to extract the time delay between echoes in time domain.

For thin layer measurement, an effective method in literatures is referred as the ultrasonic reflection coefficient method, in which the thickness of thin layer can be represented as resonant frequency or stiffness coefficient in frequency domain [3–6]. For example, Andson [2] have successfully applied the method of ultrasonic reflection coefficient to monitor the thickness of lubricant film in mechanical seal. However, for method of ultrasonic reflection coefficient measurement

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there is a requirement for a reference signal to deduce the amplitude of incident ultrasonic wave. This means that a referenced measurement must be conducted before the experimental measurement.

Another way to overcome the overlap of echoes would be the use of deconvolution techniques to reveal the sequence of reflections from the top and bottom of the layer [7]. In past decade, numerous signal processing methods were proposed for improving time resolution covering time [8], frequency [9], and wavelet [10–11] domains. The performances and computational complexities of these deconvolution techniques are analyzed and compared by some researchers [12–15]. For example, it has been shown that Wiener filtering followed by autoregressive spectral extrapolation can produce very good results at a reasonable computational cost. Therefore, a combination of these two signal processing techniques can be considered as a good candidate for ultrasonic testing applications. However, these traditional deconvolution methods are very sensitive to the accuracy of the convolution kernel. Therefore, a function of the convolution kernel which can accurately reflect system performance is needed, or else false results would be obtained. It is relatively difficult for ultrasonic practical testing to get an accurate convolution kernel through experimental methods in which ultrasonic signal has distortion in waveform during the propagation due to the frequency dependent attenuation. Therefore the requirement for an accurate convolution kernel makes the application of these signal processing techniques practically difficult.

The method of wavelet transform is widely used for processing transient and non-stationary signals [16–18]. It provides a good compromise between time and frequency resolution. Particularly, the wavelet transform decomposes a signal into overcomplete time–frequency representations which have greater robustness in the presence of noise, and have greater flexibility in matching echo components in ultrasonic signals [19–20]. Compare to the other deconvolution techniques, the wavelet transform has a unique advantage. It is easy to implement since it does not require an accurate convolution kernel corresponding to the practical signal. In this paper, wavelet transform was used to analyze the overlapped echoes to extract the transit time of ultrasonic waves in liquid layer.

## 2. Wavelet transform for sharp transient characteristic identification

### 2.1. Basic theories

For any signal$f(t)$, the wavelet transform of the signal is performed through the convolution of the signal with the scaled and translated kernel [12–13]:

$$Wf(t,s) = f(t)^*\varphi_s(t) = \int_{-\infty}^{+\infty} f(u)\varphi_s(t-u)du \tag{1}$$

where $\varphi_s(t)$ is the basic wavelet function. It is noted that the wavelet transform $Wf(t,s)$ is a function of time $t$ and scale $s$. The effect of scale in wavelet transform is similar to that of zooming in a picture. Coarse or large scales correspond to global features, and fine or small scales correspond to a detailed view. Therefore, irregular components or high frequency components of the signal can be seen at fine scales, and low frequency components of the signal show up as coarse scale features.

Formally, the identification of sharp transient characteristics is done by quantifying the local regularity of a signal. The local regularity of a function $f(t)$ at a particular point $t_0$ is often measured mathematically by the Lipschitz exponent [14–15]:

$$|f(t)-f(t_0)| \le A|t-t_0|^\alpha \tag{2}$$

where $A > 0, 0 \le \alpha \le 1$. The above function is called uniformly Lipschitz$\alpha$. The larger the value of $\alpha$, the smoother the function $f(t)$. Lipschitz 1 corresponds to a continuously differentiable function at point $t_0$; Lipschitz $\alpha(0 < \alpha < 1)$ means that function $f(t)$ is continuous at $t_0$ but the derivative of the function at that point is discontinuous; Lipschitz 0 indicates the function $f(t)$ being discontinuous but bounded in the neighborhood of $t_0$ and therefore the function is singular at $t_0$.

A classical tool for measuring the global regularity of a function $f(t)$ is to look at the asymptotic decay of its Fourier transform $\widehat{f}(\omega)$. Following the same analogy of global and local prosperities between the Fourier and wavelet transforms, the latter is sought to characterize the local regularity of function $f(t)$ at $t_0$. Therefore the proportional relationship between the wavelet transform and the Lipschitz exponent can be expresses as

$$|Wf(t,s)| \le A_\varepsilon s^\alpha \tag{3}$$

where $\varepsilon$ is a positive constant. It is important to note that at fine scales the condition characterizes the local regularity behavior of the signal in the neighborhood of $t_0$.

Actually it is not practical to detect sharp transient characteristics of a function by estimating the decay of $|Wf(t,s)|$ across all scales, since it requires intensive computations in the two-dimensional time–scale plane $(t,s)$. Mallat and Hwang proved that a more practical and popular way of applying condition (3) for sharp transient characteristics detection was through the modulus maxima:

$$Wf(t,s) = s\frac{\mathrm{d}}{\mathrm{d}x}(f^*\theta_s)(t) \tag{4}$$

where $\theta_s = (1/s)\theta(t/s)$. From Eq. (4), the wavelet transform is proportional to the derivative of $f$smoothed by $\theta_s$. The local maxima of $|Wf(t,s)|$ correspond to the inflection point with sharp variation on $f^*\theta_s$. As well, values of $|Wf(t,s)|$ at local maxima increase as scale $s$ increases. This is an important observation as it can be used to discriminate modulus maxima created by irrelevant noise from those by the signal.

In all, the Lipschitz exponent $\alpha$ reflects the sharp transient characteristics in signals, and gives the quantitative description of local features of the signals.

### 2.2. Proposed algorithm

In order to picking up the sharp transient characteristics for thickness measurement of liquid layer, an algorithm of wavelet transform is proposed to construct the maxima lines of overlapping echoes. The flow chart of signal process is shown in Fig. 1; it includes the following steps:

(1) Transform the signal by continuous wavelet at scales $s_i$, $i = 1,2,\ldots,S$. In principle, the coarsest scale $S$ should be large enough to visualize the important features. Therefore the continuous transform coefficients are located in the time–scale plane $(t,s)$.

(2) Calculate the local maximum of the modulus of wavelet transform at each scale. The point $(t_0,s_0)$is called local maximum, if

$$|Wf(t_0,s_0)| > |Wf(t_0 \pm k,s_0)| \tag{5}$$

where $k = 0,1,2,\ldots,K$, and$K$is typically a small integer possibly dependent on the scale.

(3) Link the two local maxima at successive scales if both of them are the closest maximum to each other. More precisely, let $M_{s_i}$ denote the set of local maxima at the scale $s_i$, then a local maximum $(t,s_i)$ at scale $s_i$ in linked to a local maximum $(\hat{t},s_{i-1})$ at scale $s_{i-1}$ if and only if

$$\begin{aligned} \hat{t} &= \arg\min_{v \in M_{s_{i-1}}} |t-v| \\ &\text{and} \\ t &= \arg\min_{v \in M_{s_i}} |v-\hat{t}| \end{aligned} \tag{6}$$

Then there exist a line $m = \{(t,s_n)\}$, i.e., a countable set of points in the time–scale plane that contains $(t,s_i)$ and $(\hat{t},s_{i-1})$.

(4) Merge lines with overlapping locations or gaps between scales into a single line. As a result, a set $\Gamma = \{m|m$ is a maxima

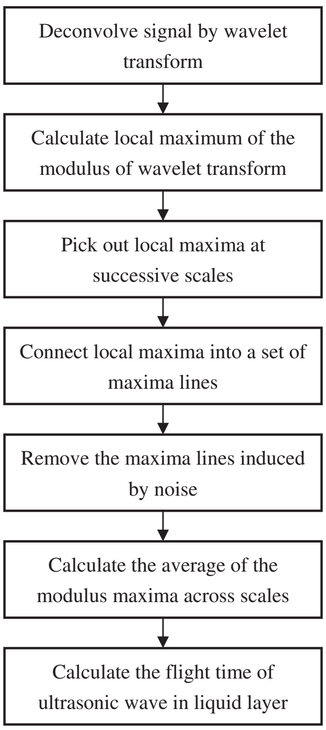


**Fig. 1.** Flow chart of wavelet transform for sharp transient characteristic identification.

line} is obtained where each maxima line is a sequence of wavelet maxima points.

(5) Let $\Gamma'$ be a set of $\Gamma$ where all the maxima lines are connected from coarsest to the finest scale, i.e.,

$$\Gamma' = \{m \in \Gamma | \exists t_1, t_2 : (t_1, s_1) \text{ and } (t_2, s_2) \in m\} \tag{7}$$

Therefore each maxima line converges $m \in \Gamma'$ towards sharp transient characteristics in the signal. In this way the information from all scales are taken into account in the detection of sharp transient characteristics.

(6) Remove the maxima lines caused by noise.
It is well known that the maxima lines caused by noise are mostly concentrated at fine scales, whereas maxima lines due to signal changes should be persistent across coarser scales. To distinguish sharp transient characteristics caused by noise from those that are generated from sharp signal transitions, only maxima lines that persist across all scales observed are considered as true signal transitions.

(7) Calculate the average of the modulus maxima across scales at each point in time.

$$AM(t) = \sum_{i=1}^{S} M_{s_i} \tag{8}$$

(8) According to the locations of the two prominent maxima of the averaged modulus maxima, $t_1^*$and $t_2^*$, the flight time of ultrasonic waves in liquid layer can be calculated:

$$\Delta t = (t_2^* - t_1^*)/2 \tag{9}$$

## 3. Experimental system and procedure for thickness measurement

The schematic diagram of the experimental configuration is shown in Fig. 2. The experimental setup is made up of pulse transmitter/receiver, transducer, digital oscilloscope, computer and mechanical apparatus with liquid layer for gauging. A commercial piezoelectricity transducer (V109, 5 MHz) was used to transmit and receive the longitudinal waves in pulse-echo mode. Panamemtrics5800PR pulse transmitter/receiver was used to excite the ultrasonic waves, receive and amplify the reflected signals. The mechanical apparatus consists of two uniform samples, two uniform spacing shims and a trough containing water. The two uniform samples are steel disk with dimension of 70 mm diameter and 5 mm thickness. The two uniform spacing shims are used as supports to the separate the two steel samples (as shown in Fig. 2), and the thickness of shims are preestablished and calibrated. The samples and shims are soaked in water. But the above steel disk is partly soaked in water to support the transducer. The space between the two steel disks is filled with water, which is considered as the liquid layer for gauging. Moreover the thickness of liquid layer can be controlled by replacing spacing shims with different thickness. Considered the minimum pulse width of 5800PR pulse transmitter/receiver (200 ns) and the velocity of longitudinal wave in water (1480 m/s), the minimum measurable layer thickness is about 150 μm. In this paper, four kinds of spacing shim are used to construct four liquid layers with different thickness, 100 μm, 150 μm, 200 μm, 250 μm.

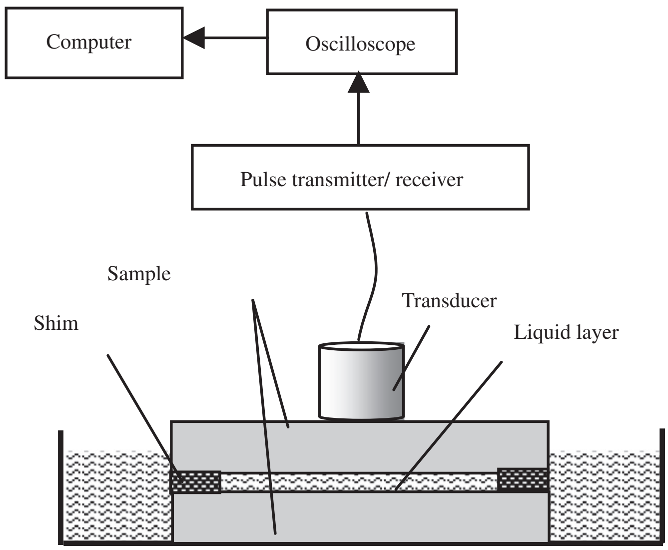


**Fig. 2.** Experimental system for thickness measurement for liquid layer.

Fig. 3 is a waveform of overlapping echoes reflected from the interfaces of liquid layer whose thickness is 200 μm. It is evident that the successive echoes become inseparable since the thickness of film is very thin. Five hundred sequential data points are chosen from the signal, and these points are transformed using the Mexican hat wavelet transform within scale $s=128$. The coefficients of wavelet transform are showed in color in Fig. 4. In Fig. 4, the brighter the color is, the bigger the coefficient is. At the same time, the modulus maxima of wavelet transform are picked out, and the distribution of modulus maxima across time–scale plane are shown in Fig. 5. It can be seen that the maxima lines connected from coarse to fine scales converge towards the characteristic points where the transmitted ultrasonic waves encounter the interfaces.

The averaged modulus maxima of wavelet transform is illustrated in Fig. 6. Based on interval of the interface-induced characteristic points and the velocity of longitudinal waves in liquid layer, the thickness of lubricant film can be calculated as following:

$$h=\frac{c\times\Delta t}{2} \tag{10}$$

where $h$ is the thickness of the liquid layer for gauging; $c$ is the velocity of longitudinal waves in liquid, and the velocity of longitudinal waves in water is about 1480 m/s; $\Delta t$ is the flight time of ultrasonic waves in liquid layer, which are described by Eq. (9). As shown in Figs. 4–6, the coefficients of wavelet transform are expressed in time–scale plane, and the variable

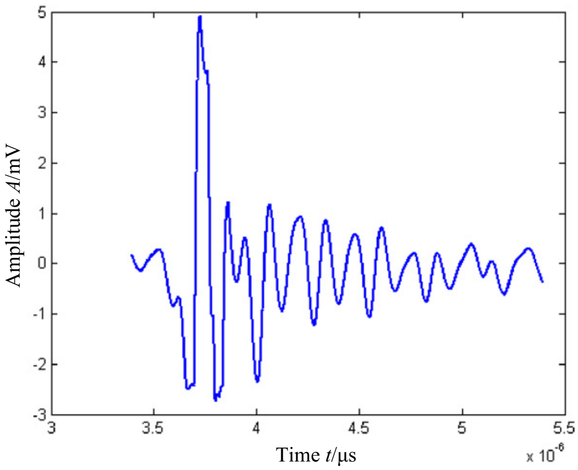


**Fig. 3.** Waveform of overlapping echoes reflected from interfaces of liquid layer.

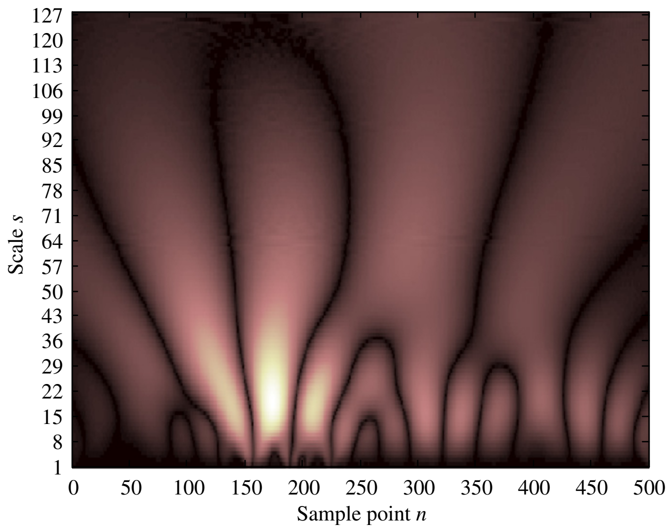


**Fig. 4.** Modulus of wavelet transform of overlapping echoes using Mexican hat wavelet at 128 scales. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

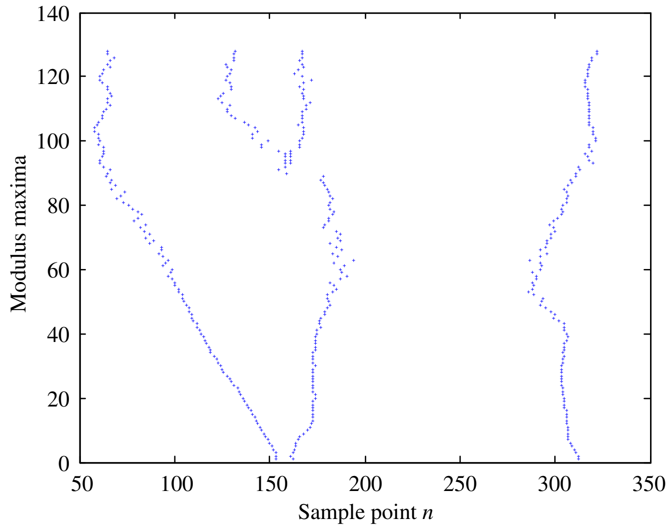


**Fig. 5.** Maxima lines of wavelet transform of overlapping echoes.

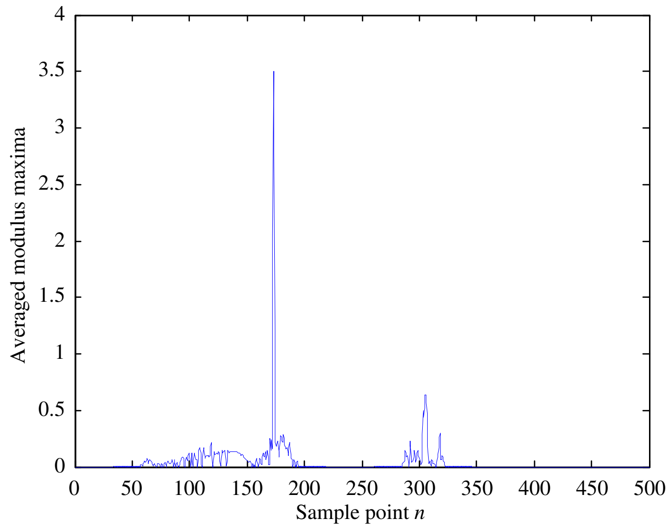


**Fig. 6.** Averaged modulus maxima of wavelet transform of overlapping echoes.

in time domain is generally sample point [18–20]. As a matter of fact, the flight time of ultrasonic waves in liquid layer is expressed as following:

$$h = (N_1 - N_2) \times \delta_t \tag{11}$$

where $N_1$ and$N_2$ are the sample points corresponding to the two prominent maxima in averaged modulus maxima, $\delta_t$ is the interval of adjacent sample points, in this paper $\delta_t = 0.002$ μs.

In Fig. 5, the numbers of sample points corresponding to the two prominent maxima are respectively 173 and 305. According to Eq. (11), the thickness of liquid layer can be determined as 195.36 μm. Compared with the thickness of space shims, 200 μm, the relative error is about 2.32%.

## 4. Experimental results and discussion

It is well known that the performance of wavelet transform is greatly influenced by the type of wavelet function and its scale. In this part, the performances of different wavelet function within various scales on thickness measurement of liquid layers were experimentally investigated. The optimal wavelet function with the optimal scale was then used to extract the interface-induced characteristic points in reflected echoes.

### 4.1. Performance of different wavelet functions

The performances of four types of wavelet functions, Mexican hat, Morlet, Sym4 and Meyer, for sharp transient characteristics picking-up were investigated. As typical results, the maxima lines transformed from echoes reflected from liquid layer whose thickness is 250 μm are shown in Fig. 7. It can be seen that there is great difference in the number of maxima lines when the identical ultrasonic signal is transformed with different wavelet functions. The number of maxima lines is the least for the case of Mexican hat wavelet (shown in Fig. 7(d)). For cases of Morlet, Sym4 and Meyer, there are too many pseudo-maxima lines as shown in Fig. 7(a)–(c), which would absolutely disturb the location of the characteristic points in ultrasonic signals.

The corresponding averaged modulus maxima are shown in Fig. 8, and the results of thickness measurement are shown in Table 1. It is obvious that the picking-up of interface-induced characteristic points using Mexican hat wavelet is almost not influenced by pseudo-maxima as shown in Fig. 8(d), and the results agree well with actual thickness of liquid layer as shown in Table 1. The possible reason is that Mexican hat wavelet is infinitely differentiable and not sensitive to noise. Mexican hat wavelet is therefore more suitable for location of the singular points in reflected echoes. For the other three cases, the picking-up of interface-induced singular points is greatly influenced by pseudo-maxima lines as shown in Fig. 8(a)–(c), and the results of thickness measurement for liquid layer are unreliable.

Further experiments under different thickness of liquid layer have been conducted. It is also shown that among the four wavelet functions, the Mexican hat wavelet is the optimal wavelet for picking-up of interface-induced characteristic points. In the following thickness measurement for liquid layer, the Mexican hat wavelet is used as the mother wavelet in transforming of the ultrasonic signals.

### 4.2. Performance of different scale factors

To investigate the performance of wavelet transform for transient points picking-up under different scale factors, an identical signal was transformed using Mexican hat wavelet function with maximum scale of 32, 64, 128 and 256. As typical results, the maxima lines transformed from echoes reflected at liquid layer whose thickness is 250 μm are shown in Fig. 9. The corresponding averaged modulus maxima are shown in Fig. 10. It can be seen that there is great difference in the number of maxima lines under different scale factors. The smaller the scale factor is, the more the number of maxima line is. For cases of small scale factors (32 and 64), there are so many pseudo-maxima lines(as shown in Fig. 9(a) and (b)) that the picking-up of interface-induced characteristic points are disturbed (as shown in Fig. 10(a) and (b)), and the thickness measurement results are unreliable as shown in Table 2. However, when the scale factor is too big (256),

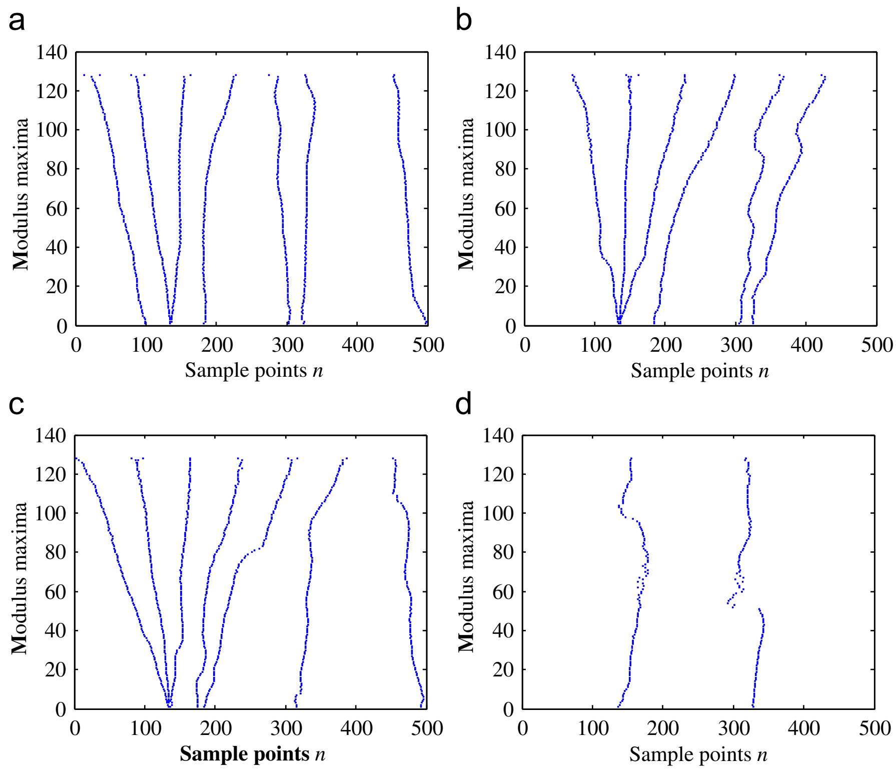


**Fig. 7.** Maxima lines of wavelet transform with different wavelet functions: (a) Sym4 (b) Meyer (c) Morlet (d) Mexican hat.

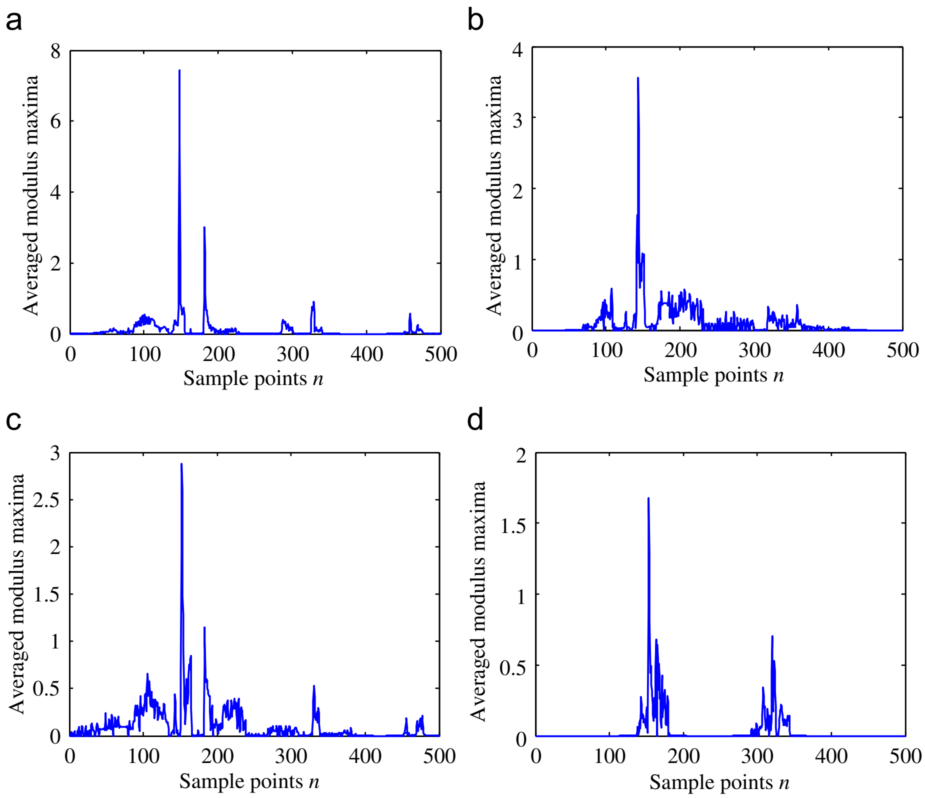


**Fig. 8.** Averaged modulus maxima of wavelet transform with different wavelet functions: (a) Sym4 (b) Meyer (c) Morlet (d) Mexican hat.

**Table 1**
Results of thickness measurement for liquid layer using wavelet transform with different wavelet transforms.

| Wavelet function | Maxima 1 | Maxima 2 | Difference of two points | Measurement results(μm) | Relative error(%) |
|---|---|---|---|---|---|
| Sym4 | 148 | 182 | 34 | 50.32 | 79.87 |
| Meyer | 144 | 358 | 214 | 316.72 | 26.69 |
| Morlet | 152 | 183 | 31 | 45.88 | 81.65 |
| Mexican hat | 153 | 320 | 168 | 248.64 | 0.54 |

the weak transient characteristic information induced by interface may be lost as shown in Fig. 9(d). As a result, the accuracy of thickness measurement for liquid-layer is influenced greatly by the scale factors as shown in Table 2. For cases the scale factors are too big (256) or too small (32 and 64), the results of thickness measurement for liquid layer are unreliable. On the contrary, the results of thickness measurement for liquid layer agree well with its actual value when the scale factor is 128.

In general, the scale factor of transform wavelet is mainly determined by the signal-to-noise ratio. In this paper, it is demonstrated by repeated experiments the results of thickness measurement agree well with the true value when the scale factor is 128. Therefore in the following thickness measurement for liquid layer, the ultrasonic signals are transformed within scale of 128.

### *4.3. Repeated results*

To verify the validity of the proposed method for characteristic points picking-up in thickness measurement of liquid layer, repeated experiments have been conducted on liquid layers with different thickness. The experimental results for water films with the thickness of 100 μm, 150 μm, 200 μm, 250 μm, are shown in Table 3.

It can be seen that, except the liquid layer with thickness of 100 um, the measurement results agreed well with their actual value. The experimental errors decrease with the increase of thickness of liquid layer, and the relative errors are generally less than 5%. It also can be seen that when the thickness for liquid layer is 100 um, the experimental results are unreliable. The possible reason is that the pulse width of the 5800PR pulse transmitter-receiver is 200 ns, while the flight

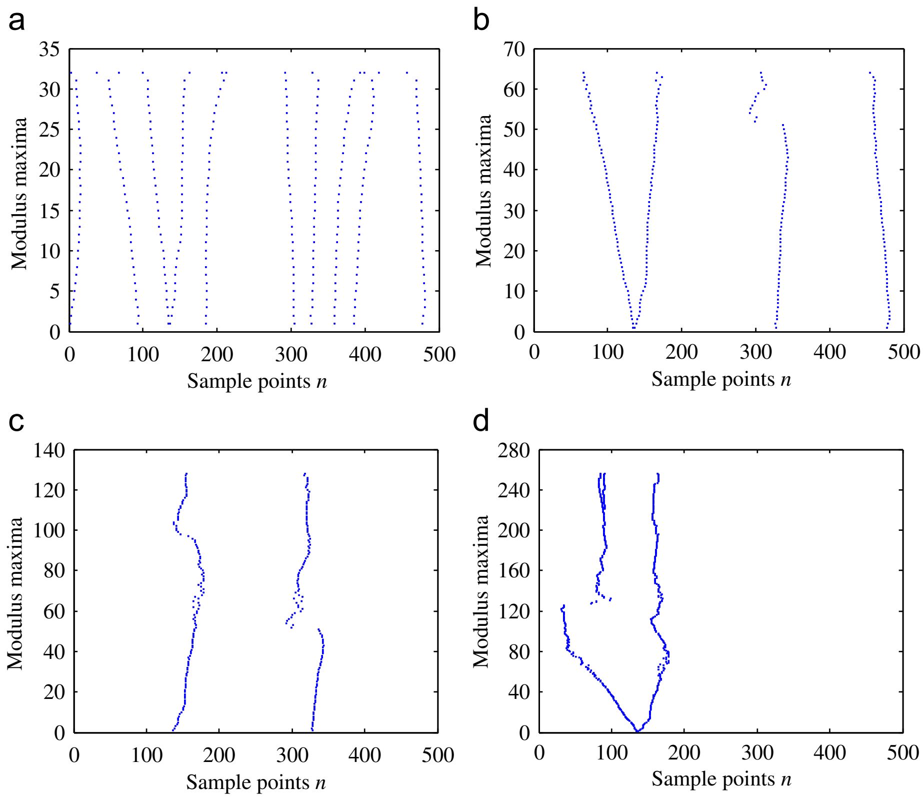


**Fig. 9.** Maxima lines of wavelet transform within different scale factors: (a) 32, (b) 64, (c) 128, (d) 256.

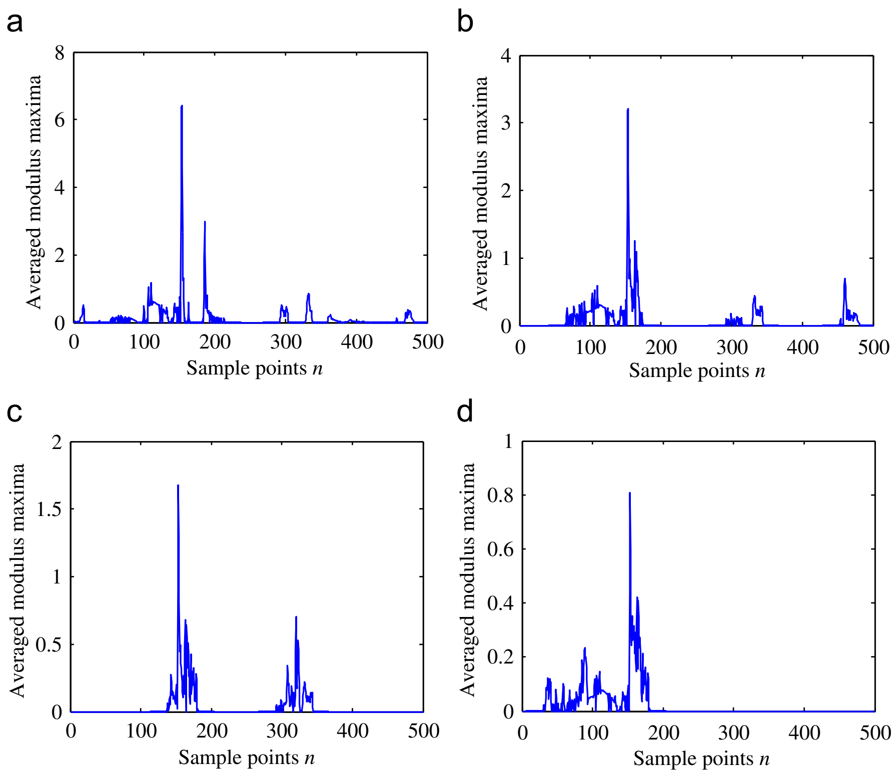


**Fig. 10.** Averaged modulus maxima of wavelet transform within different scale factors: (a) 32, (b) 64, (c) 128, (d) 256.

**Table 2**
Results of thickness measurement for liquid layer using wavelet transform within different scale factors.

| Scale extent | Maxima 1 | Maxima 2 | Difference of two points | Measurement results(μm) | Relative errors (%) |
|---|---|---|---|---|---|
| 32 | 153 | 186 | 33 | 48.84 | 80.46 |
| 64 | 153 | 460 | 307 | 454.36 | 81.74 |
| 128 | 153 | 320 | 167 | 247.16 | 1.14 |
| 256 | 89 | 153 | 64 | 94.72 | 62.11 |

**Table 3**
Results of thickness measurement for liquid layers.

| Actual thickness(μm) | Number | Maxima 1 | Maxima 2 | Difference of two points | Measurement thickness(μm) | Relative error (%) |
|---|---|---|---|---|---|---|
| 100 | 1 | 219 | 291 | 72 | 106.56 | 6.56 |
| | 2 | 172 | 205 | 33 | 48.84 | 51.16 |
| | 3 | 195 | 228 | 33 | 48.84 | 51.16 |
| | 4 | 174 | 206 | 32 | 47.36 | 52.64 |
| | 5 | 174 | 208 | 34 | 50.32 | 49.68 |
| | 6 | 173 | 205 | 32 | 47.36 | 52.64 |
| | 7 | 174 | 208 | 34 | 50.32 | 49.68 |
| | 8 | 180 | 214 | 34 | 50.32 | 49.68 |
| | 9 | 180 | 300 | 120 | 177.6 | 77.6 |
| 150 | 1 | 185 | 288 | 103 | 152.44 | 1.63 |
| | 2 | 171 | 273 | 102 | 150.96 | 0.64 |
| | 3 | 193 | 293 | 100 | 148 | 1.33 |
| | 4 | 173 | 273 | 100 | 148 | 1.33 |
| | 5 | 173 | 270 | 97 | 143.56 | 4.29 |
| | 6 | 173 | 272 | 99 | 146.52 | 2.32 |
| | 7 | 173 | 273 | 100 | 148 | 1.33 |
| | 8 | 179 | 278 | 99 | 146.52 | 2.32 |
| | 9 | 180 | 280 | 100 | 148 | 1.33 |
| 200 | 1 | 185 | 318 | 133 | 196.84 | 1.58 |
| | 2 | 171 | 303 | 132 | 195.36 | 2.32 |
| | 3 | 194 | 325 | 131 | 193.88 | 3.06 |
| | 4 | 174 | 306 | 132 | 195.36 | 2.32 |
| | 5 | 173 | 305 | 132 | 195.36 | 2.32 |
| | 6 | 173 | 304 | 131 | 193.88 | 3.06 |
| | 7 | 173 | 304 | 131 | 193.88 | 3.06 |
| | 8 | 180 | 310 | 130 | 192.4 | 3.8 |
| | 9 | 179 | 310 | 131 | 193.88 | 3.06 |
| 250 | 1 | 186 | 360 | 174 | 257.52 | 3.01 |
| | 2 | 171 | 341 | 170 | 251.6 | 0.64 |
| | 3 | 195 | 332 | 137 | 202.76 | 18.9 |
| | 4 | 173 | 342 | 169 | 250.12 | 0.048 |
| | 5 | 174 | 343 | 169 | 250.12 | 0.048 |
| | 6 | 173 | 343 | 170 | 251.6 | 0.64 |
| | 7 | 174 | 313 | 139 | 205.72 | 17.71 |
| | 8 | 179 | 346 | 167 | 247.16 | 1.14 |
| | 9 | 179 | 343 | 164 | 242.72 | 2.91 |

time of ultrasonic waves transmitted in water film with thickness of 100 um is 135 ns. Therefore, it is impossible for the experiment system to distinguish the echoes from two interfaces of the liquid-layer whose thickness is less than 150 um.

## 5. Conclusion

Wavelet transform modulus maximum method was used to pick up the interface-induced transient characteristic points in ultrasonic signals. The interval time between the characteristic points was then used to calculate the transmit time of ultrasonic waves in liquid layer. Therefore the thickness of liquid layer can be obtained by the wavelet transform modulus maximum of the reflected signals.

The performance of different wavelet functions and scale factors on transient characteristic points picking-up in thickness measurement of liquid layer are experimental investigated. It has been shown that the Mexican hat wavelet and 128 are the optimal wavelet function and scale factor for picking-up of interface-induced characteristic points.

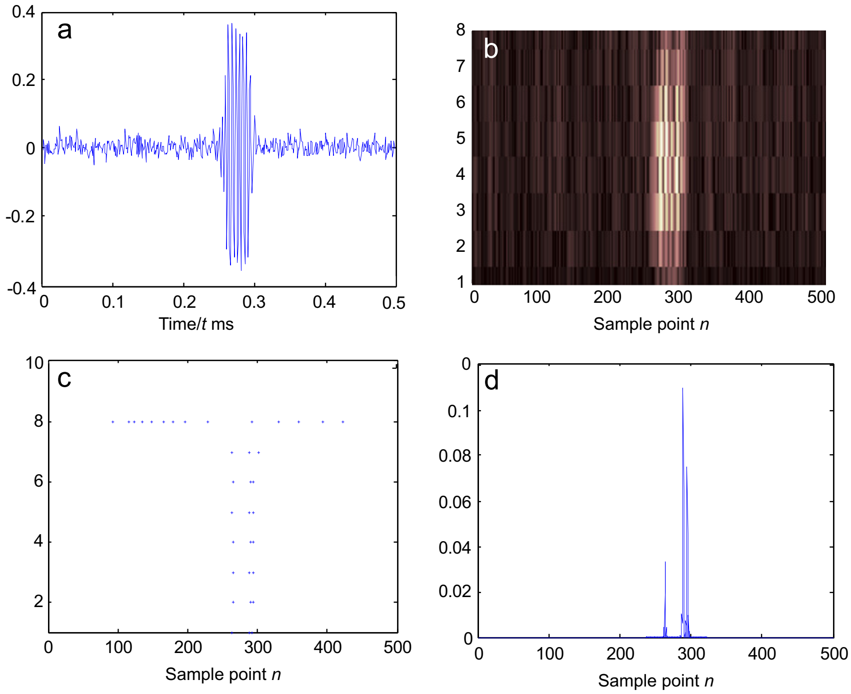


**Fig. A1.** Simulated signal and its wavelet transform when the scale factor is 8: (a) waveform; (b) modulus of WT; (c) maxima lines of WT; (d) averaged modulus maxima of WT.

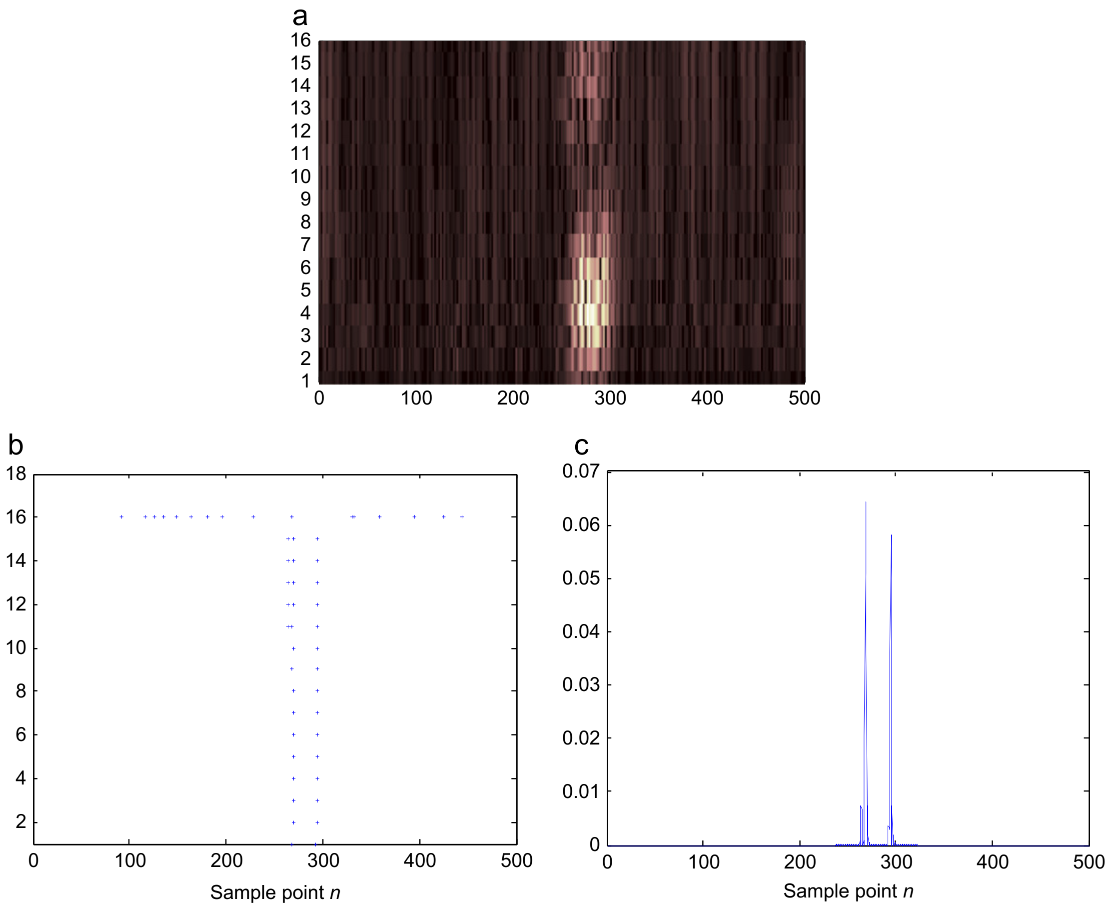


**Fig. A2.** Simulated signal and its wavelet transform when the scale factor is 16: Sample point $n$ (a) modulus of WT; (b) maxima lines of WT; (c) averaged modulus maxima of WT.

A series of experiments for thickness measurement has been conducted using the optimal wavelet and scale factor. It has been shown that the proposed method is effective on thickness measurement for liquid layer, and the relative errors are generally less than 5% when the thickness of liquid layer is within a certain range.

There is no need for a referenced measurement for the proposed method, but the range of measurement is limited by pulse width of the experimental system. Therefore the proposed method is an additional scheme for the thickness measurement of liquid layer.

## Acknowledgments

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## Appendix A

The performance of algorithm is investigated on processing of the simulated signals. The propagation of Lamb wave in one-dimensional plate is simulated. The excited signal is a five-cycle Hanning-windowed sine burst at 200 kHz. Suppose the pure A0 Lamb wave is excited in the plate, and the propagation characteristics of the A0 mode is known. It is assumed

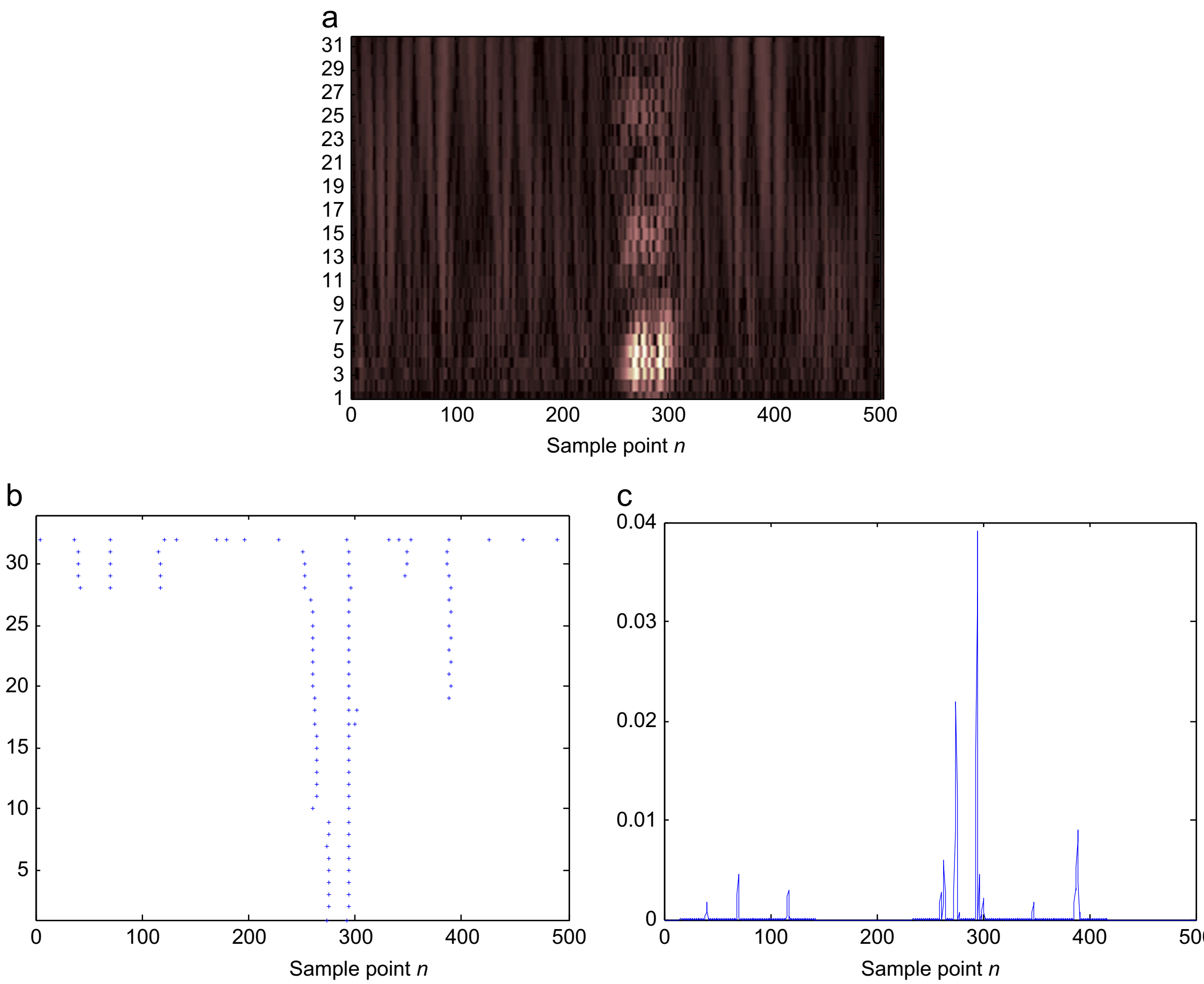


**Fig. A3.** Simulated signal and its wavelet transform when the scale factor is 32: (a) modulus of WT; (b) maxima lines of WT; (c) averaged modulus maxima of WT.

that there is no mode conversion when the Lamb wave is reflected at a feature. The received time-trace,$g(t)$, typically contains numerous overlapping signals corresponding to the reflections from features at different distances. The received time-trace can be predicted as follows [21]:

$$g(t) = \sum_j \int_{-\infty}^{\infty} A_j(\omega) F(\omega) e^{i(k(\omega)d_j - \omega t)} \tag{A.1}$$

where $A_j(\omega)$ is the reflection coefficient of each reflector, $F(\omega)$ is the Fourier transform of the excited signal $f(t)$, $k(\omega)$ is the wavenumber of the Lamb wave, $d_j$ is the distance between the source and the $j$th feature.

According to above equation, a time-trace is calculated as shown in Fig. A1(a), which contains two reflections. The distances of source and two features are respectively 600 mm and 640 mm, meanwhile stochastic noise is introduced in the simulated signal.

The same simulated signal (as shown in Fig. A1(a)) is transformed using Mexican hat wavelet functions within different scale factors. The typical results are shown in Figs. A1–A5, in which the scale factors are respectively 8, 16, 32, 64 and 128. It can be seen there is great difference in the number of maxima lines under different scale factors. When the scale factor is too bigger (64 and 128), there are so many pseudo-maxima lines (as shown in Figs. A4 and A5) that the picking-up of feature-induced characteristic points are disturbed. Selected from the five typical scale factors, the favorite scale factor for simulated signal is 16. It is different from the favorite scale factor described in paper. The main reasons include, the center frequency of the simulated signal is lower than the signals in paper, and its bandwidth is narrower, and its time range is longer.

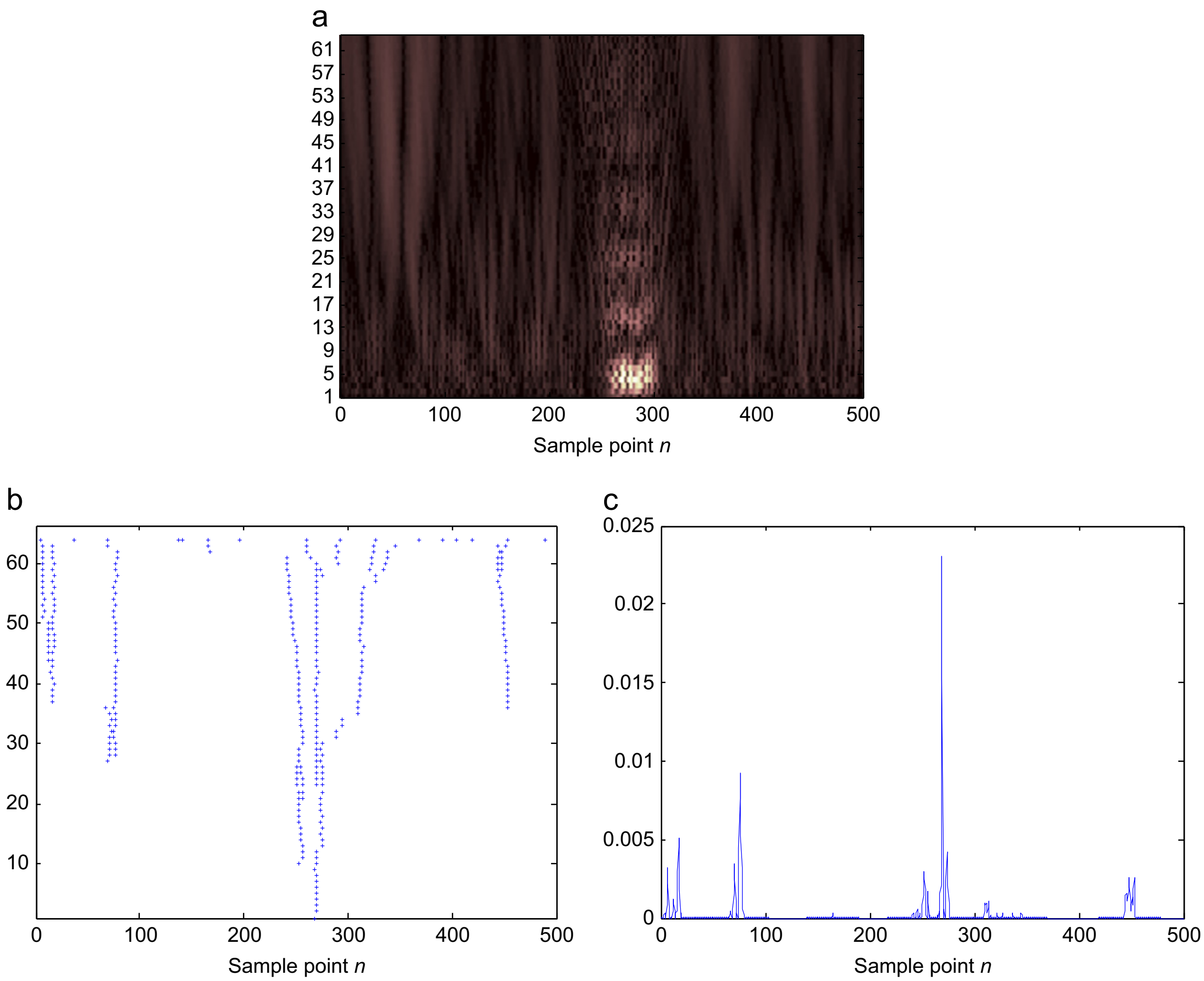


**Fig. A4.** Simulated signal and its wavelet transform when the scale factor is 64: (a) modulus of WT; (b) maxima lines of WT; (c) averaged modulus maxima of WT.

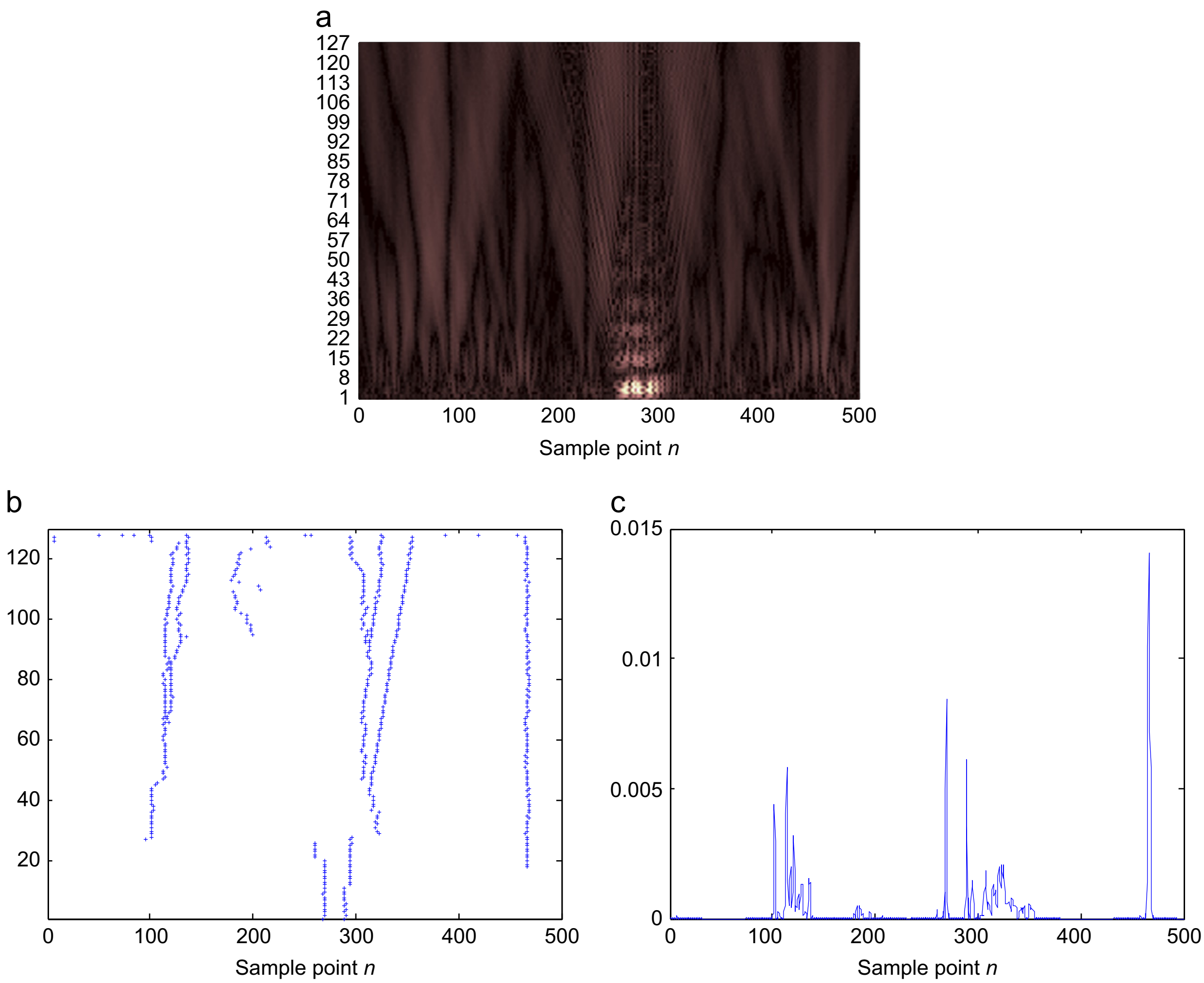


**Fig. A5.** Simulated signal and its wavelet transform when the scale factor is 128: (a) modulus of WT; (b) maxima lines of WT; (c) averaged modulus maxima of WT.

It can be seen that the performance of the proposed algorithm under different scale factors depends on frequency range and time range of the signals. Therefore the favorite scale may vary.

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