Homework

1. Does sensor fusion always reduce variance?

The optimal linear fusion of two, one-dimensional, Gaussian information sources is

$$\hat{x} = \frac{r}{q+r} x_p + \frac{q}{q+r} z$$

where $x_p \sim \mathcal{N}(0, q)$ and $z \sim \mathcal{N}(0, r)$. Prove

$$p \leq \min\{q, r\}$$

where p is the variance of $\hat{x}$.

2. Does a Kalman Filter always reduce variance?

Using our 1D Kalman Filter from Lecture 1, what is the relationship between the process noise q and the measurement noise r such that $p[t^+]$ is marginally stable (Hint: marginal stability implies $0 < \lim_{t\to\infty} p[t^+] < \infty$)? How is this different from Question 1?

3. Under what conditions is a Kalman Filter optimal?

a) List at least 6 assumptions made when using a Kalman Filter
b) How are the assumptions from (a) relaxed/changed for a EKF? UKF?

4. Creating an Extended Kalman Filter

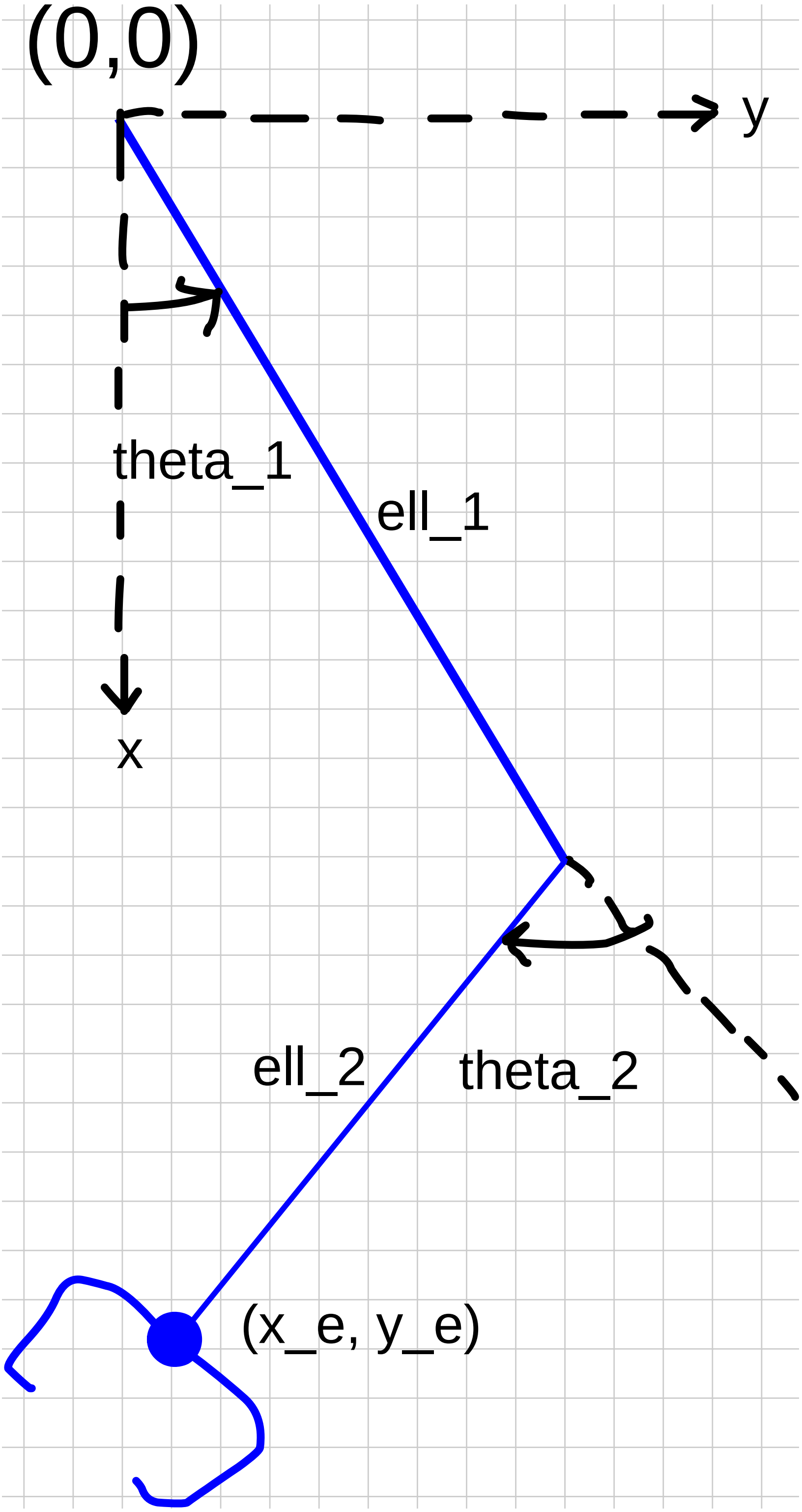
Design a discrete-time EKF to estimate the position of a robotic end effector using only measurements of θ_1 and θ_2 (e.g. Identify F_t, Q_t, H_t, and R_t). Assume a constant angular velocity model such that

$$\omega_1[t] = \omega_1[t-1] + \nu_1$$

and

$$\omega_2[t] = \omega_2[t-1] + \nu_2$$

where $\nu_1 \sim \mathcal{N}(0, \sigma^2_{\omega_1})$ and $\nu_2 \sim \mathcal{N}(0, \sigma^2_{\omega_2})$. **Positive angles are clockwise!**



a) Identify the continuous-time kinematic equations relating joint angles to the end effector position.

b) Discretize these equations using Euler Integration (also known as a first-order Taylor Series approximation).

c) Ensure these equations are **Markov** (i.e. a state at the next time step only depends on states at the current time step). By now, you should have a system of **6** equations describing the **process**. (Hint: You really only need to find 4 because two are given: $\omega_1[t]$ and $\omega_2[t]$)

d) List the states as a large vector $\mathbf{x}$. The system of equations should be expressed as a function $f : \mathbb{R}^6 \to \mathbb{R}^6$ that inputs the states and produces the state at the next time step.

e) You will now need to repeat steps (a)-(c) to identify the **observation or measurement** equations. These equations should be functions of the states you listed in (d) and return the desired measurement. These equations should be expressed as a function $h : \mathbb{R}^6 \rightarrow \mathbb{R}^n$ where n is the number of observations.

- A subtle assumption made by an EKF is all states are contained in the real numbers. However, the angles θ_1 and θ_2 wrap. This can cause problems when trying to perform measurement updates. For example, if my measurement is 0 and my current estimate is 359, the difference is 359 and not 1 (this differencing occurs when calculating the **residual**). *Be sure to select the observation equations such that this wrapping error does not occur*. (Hint: $e^{j\theta} = \sin\theta + j\cos\theta$)

f) The equations f and h should not be linear with respect to the states. To linearize them, calculate the Jacobians for both. (Hint:

$$F[i, j] = \frac{\partial f_i(\mathbf{x})}{\partial x_j}$$

where i and j denote the row and column of the Jacobian, respectively.)

g) Assuming ν_1 and ν_2 are uncorrelated (i.e. not related to each other), create the **process noise** matrix Q and the **observation noise** matrix R (Refer here for how these matrices are used). Also assume ν_1 and ν_2 are the only process noise sources and all observations have uncorrelated, Gaussian, zero-mean noise with $\sigma^2 = 0.1$.