a) $T(s) = \frac{C(s)}{X(s)} = \frac{5}{s+5}$ $\qquad X(s) = \frac{1}{s}$ $\qquad T(s) = K\frac{1/\tau}{s+1/\tau}$

$\Rightarrow C(s) = \frac{5}{s(s+5)}$ $\qquad$ ~~[illegible]~~ $1/\tau = 5$

$\Rightarrow \tau = 1/5$

$c(t) = \mathcal{L}^{-1}\{C(s)\} = 1 - e^{-5t}$

b) $\tau = 1/5$ $\qquad t_r \approx 2.2\tau = 0.44s$ $\qquad t_s \approx 4\tau = 0.8s$

c) t_r looks accurate t_s on graph looks a lot sooner than 20s

a) $T(s) = \frac{C(s)}{X(s)} = \frac{20}{s+20}$ $\qquad X(s) = \frac{1}{s}$

$C(s) = \frac{20}{s(s+20)}$

$\Rightarrow c(t) = \mathcal{L}^{-1}\{C(s)\} = 1 - e^{-20t}$

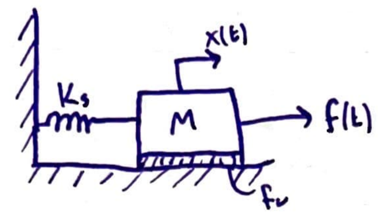
b) $T(s) = K\frac{1/\tau}{s+1/\tau}$ $\Rightarrow$ $1/\tau = 20$

$\tau = 1/20$

$t_r \approx 2.2\tau = 0.11s$

$t_s \approx 4\tau = 0.2$

c) Graph looks accurate



$M = 1\ Kg$
$K_s = 5\ N/m$
$f_v = 1\ Ns/m$
$F(t) = u(t)\ N$

Assume:
$x(0) = 0$
$\dot{x}(0) = 0$

$$\Sigma F = m\ddot{x} = f(t) - F_K - F_v$$

$$\Rightarrow m\ddot{x} + f_v\dot{x} + K_s x = f(t) = u(t)$$

$$\Rightarrow \ddot{x} + \dot{x} + 5x = u(t)$$

$$\mathcal{L} \Rightarrow s^2 X(s) + sX(s) + 5X(s) = \frac{1}{s}$$

$$\Rightarrow X(s)(s^2+s+5) = \frac{1}{s}$$

$$\Rightarrow T(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2+s+5}$$ where $F(s) = \frac{1}{s}$

$$X(s) = \frac{1}{s(s^2+s+5)}$$

a) $$X(t) = \mathcal{L}^{-1}\{X(s)\} = \frac{1}{5}\left(1 - e^{-t/2}\cos(2.18t) - 0.23 \cdot e^{-t/2}\sin(2.18t)\right)$$

$$T(s) = \frac{1}{s^2+s+5} = \frac{1}{5}\,\frac{5}{s^2+s+5} = K\frac{W_n^2}{s^2+2\zeta W_n s + W_n^2}$$

$W_n^2 = 5 \Rightarrow W_n = \sqrt{5}$

$2\zeta W_n = 1 \Rightarrow \zeta = \frac{1}{2\sqrt{5}} = 0.224$

b) $t_p \approx \frac{\pi}{W_n\sqrt{1-\zeta^2}} = 1.48s$

$t_s \approx \frac{4}{\zeta W_n} = 8s$

$\%OS = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \cdot 100\% = 48.6\%$

d) $\zeta = 0.224$

$0 < \zeta < 1$ $\therefore$ Underdamped

c) Graph has same %OS
t_s looks about the same
t_p looks accurate as well

$$T(s) = \frac{14.145}{(s^2+0.842s+2.829)(s+5)}$$

Poles: -5

$-0.421 \pm 1.628j$

Assume that pole at -5 is much faster than other two poles

approximate system as 2nd order, pole at -5 is negligible

$$\Rightarrow T(s) = \frac{14.145}{s^2+0.842s+2.829} = 5\,\frac{2.829}{s^2+0.842s+2.829}$$

$\omega_n^2 = 2.829 \Rightarrow \omega_n = 1.682$ $\quad K=5$

$2\zeta\omega_n = 0.842$

$$\Rightarrow \zeta = \frac{0.842}{2\omega_n} = \frac{0.842}{2(1.682)} = 0.25$$

$$t_r \approx \frac{1.8}{\omega_n} = \frac{1.8}{1.682} = 1.07\,s$$

$$t_s \approx \frac{4}{\zeta\omega_n} = \frac{4}{(0.25)(1.682)} = 9.51\,s$$

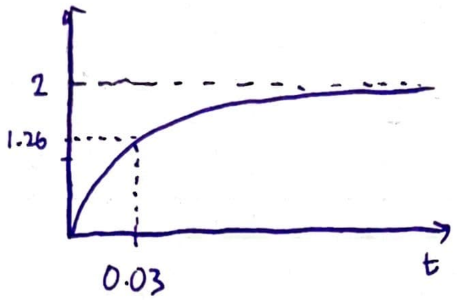
$$t_p \approx \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{(1.682)\sqrt{1-0.25^2}} = 1.93\,s$$

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} = 0.444$$

$\%OS = M_p \cdot 100 = 44.4\%$

Charlie Lambert | ~~Ch~~ HW 3- Problem 4 | 9/15/22 | 4

a)



$0.63 \cdot 2 = 1.26$

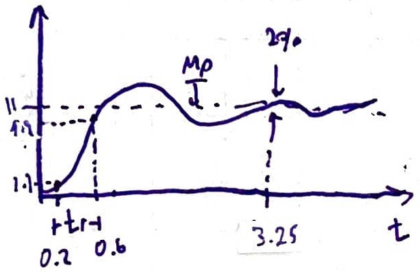
$t_r = 0.03 = 2.2\tau$

$\Rightarrow \tau = 0.0136$

$a = 73.33$

$$T(s) = K\frac{1/\tau}{s+1/\tau} = K\frac{a}{s+a} = 2\,\frac{73.33}{s+73.3}$$

b)



$0.1 \cdot 11 = 1.1$ $\qquad t_{0.1} = 0.2$

$0.9 \cdot 11 = 9.9$ $\qquad t_{0.9} = 0.6$

$t_r = 0.4 \approx \frac{1.8}{\omega_n}$

$\Rightarrow \omega_n = 4.5$

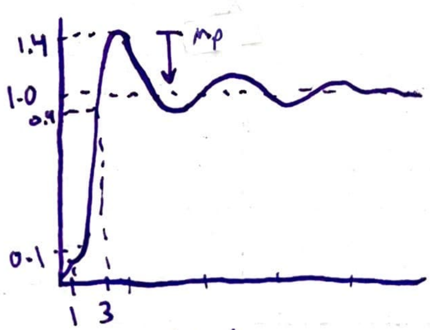
$\omega_n^2 = 20.25$

$$\%OS = \frac{14-11}{11}\cdot 100 = 27.3\%$$

$$\zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} = 0.38$$

$$T(s) = 11\left(\frac{(4.5)^2}{s^2 + 2(0.38)(4.5) + (4.5)^2}\right) = 11\left(\frac{20.25}{s^2 + 3.42s + 20.25}\right)$$

c)



$t_{0.1} = 1$, $t_{0.9} = 3$ $\Big\}\ t_r = 2 \approx \frac{1.8}{\omega_n}$

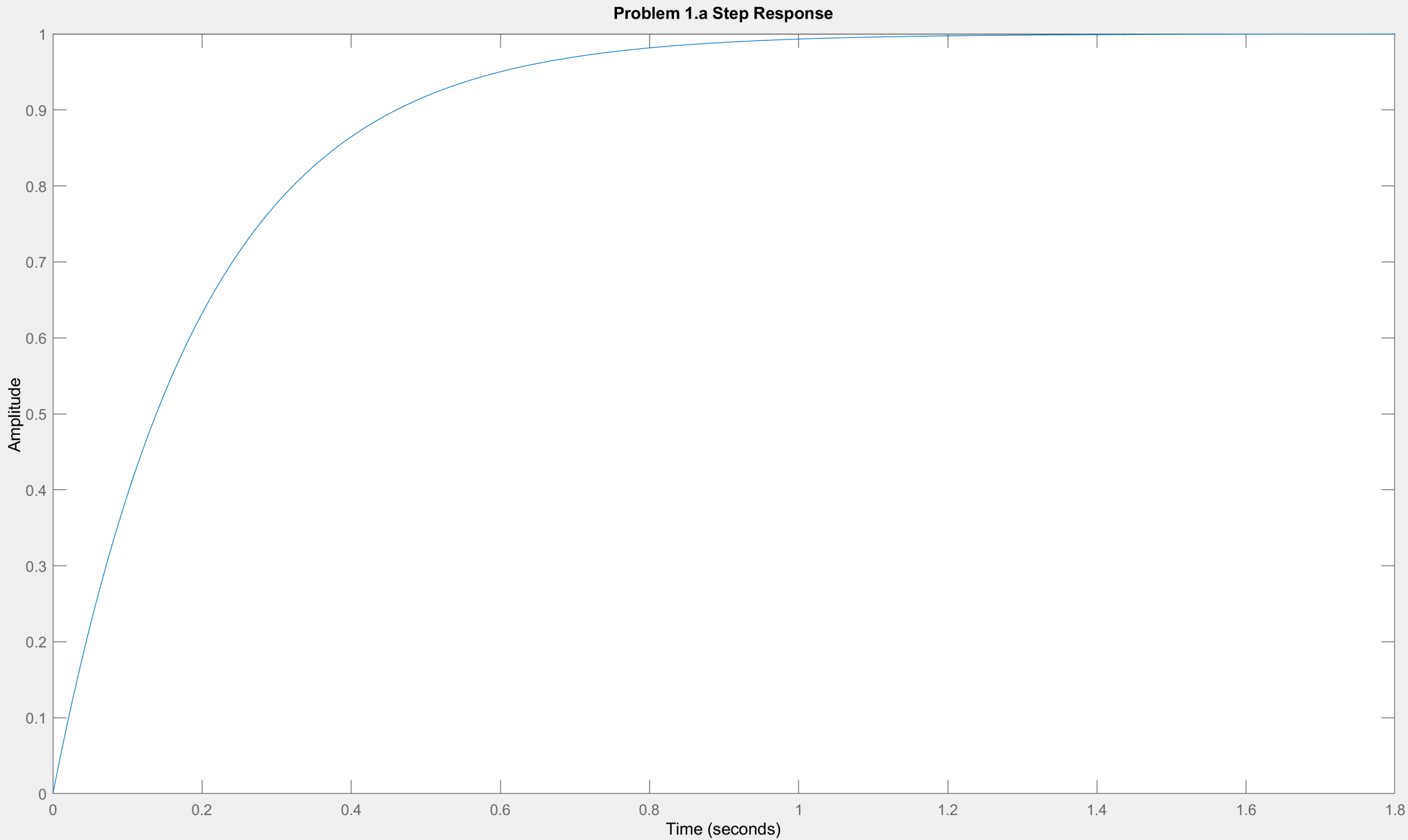
$\Rightarrow \omega_n = 1.111$

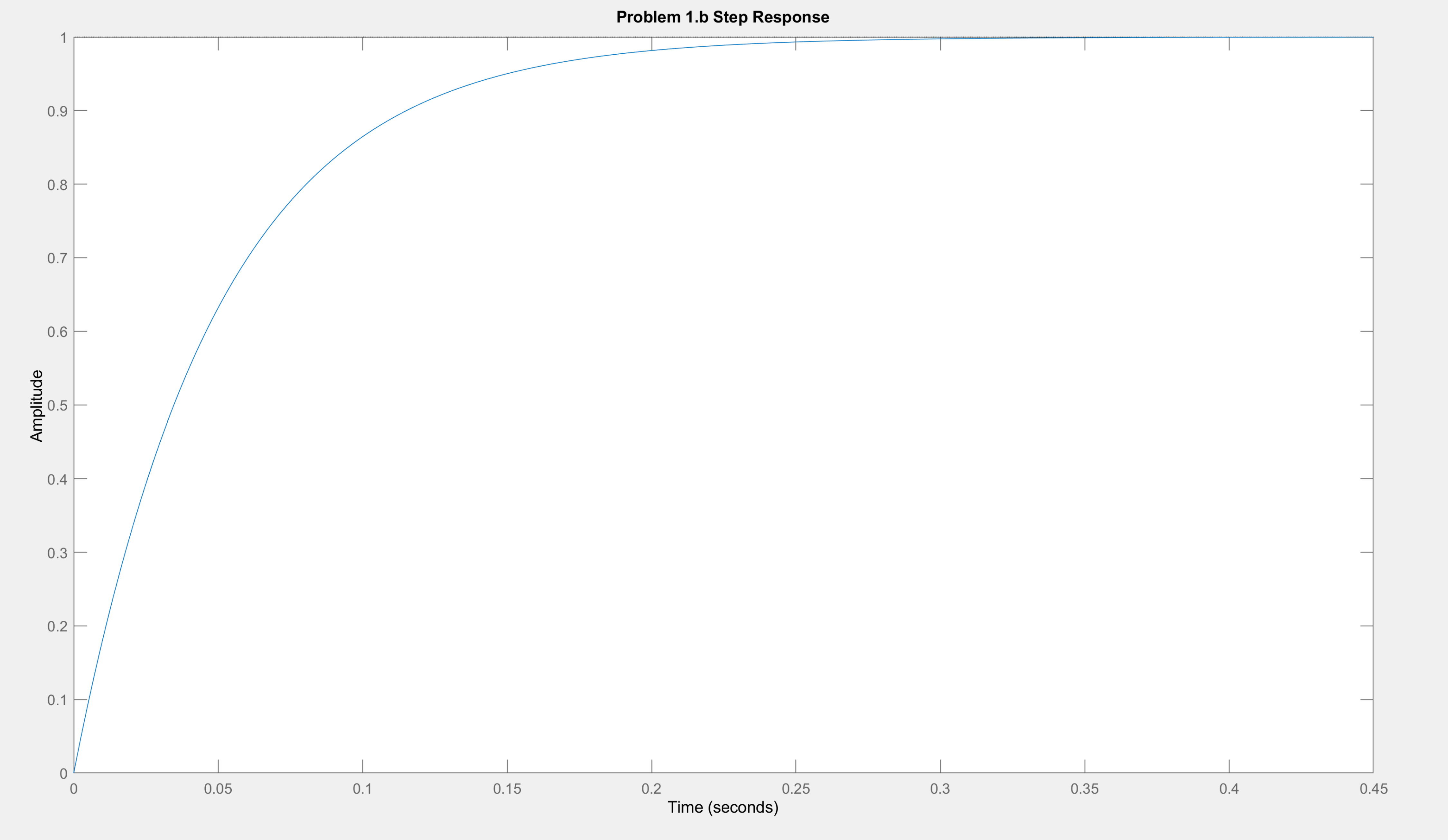
$\omega_n^2 = 1.234$

$$\%OS = \frac{1.4-1}{1}\cdot 100 = 40\% \qquad M_p = 0.4$$

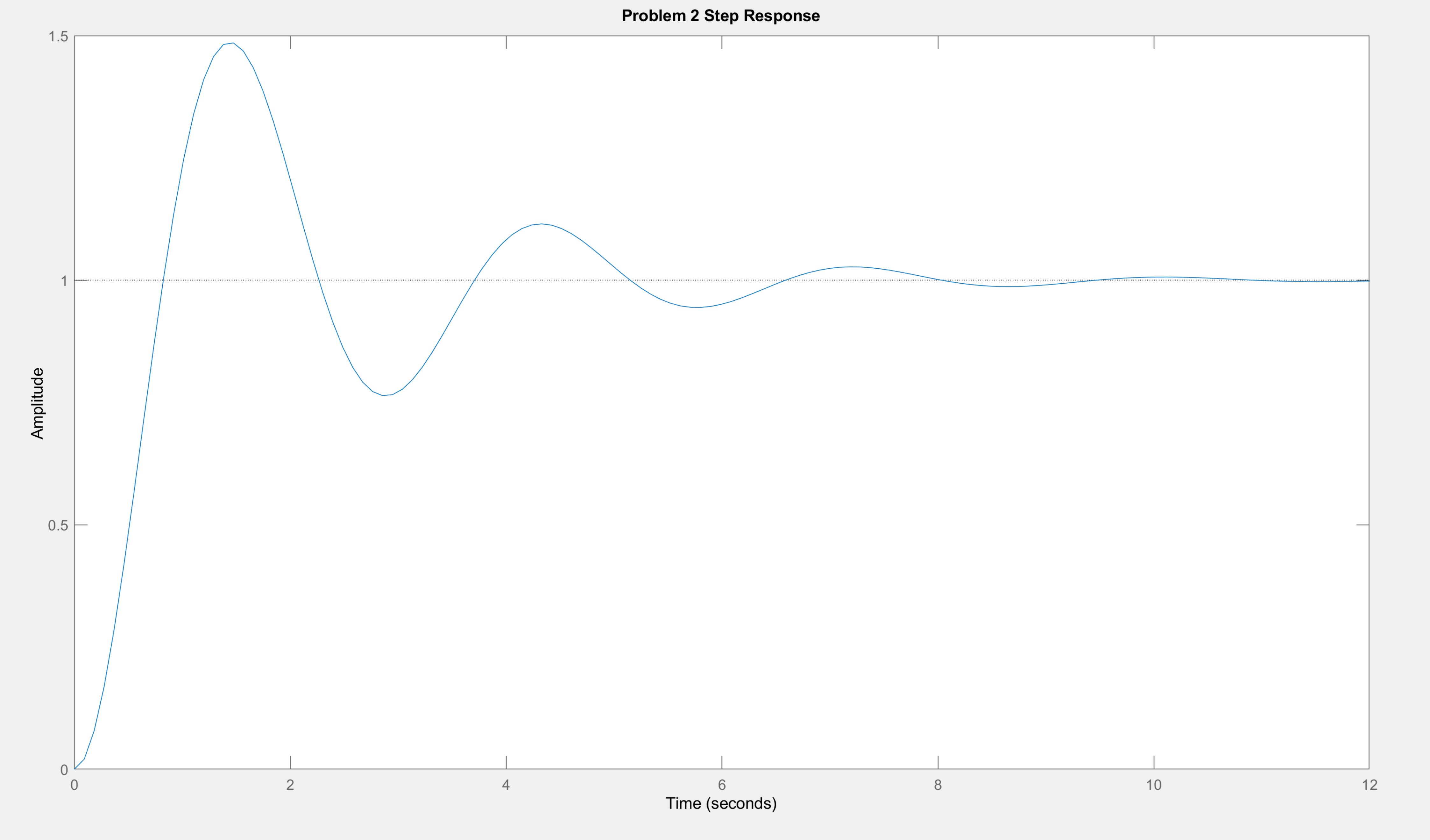
$$\zeta = \frac{-\ln(M_p)}{\sqrt{\pi^2 + \ln^2(M_p)}} = 0.28$$

$$T(s) = \frac{(1.11)^2}{s^2 + 2(0.28)(1.11)s + (1.11)^2} = \frac{1.23}{s^2 + 0.62s + 1.23}$$


Problem 1.a Step Response
Amplitude
Time (seconds)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8



Problem 1.b Step Response
Amplitude
Time (seconds)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45



Problem 2 Step Response
1.5
1
0.5
0
Amplitude
0
2
4
6
8
10
12
Time (seconds)

ClassicalControls_HW3.m

```
clc, clear all, close all

% Problem 1
% A)
sys1 = tf(5,[1,5]);
figure;
step(sys1);
title("Problem 1.a Step Response");

%B)
sys2 = tf(20,[1,20]);
figure;
step(sys2);
title("Problem 1.b Step Response");

%C)
sys3 = 5*tf(1,[1,1,5]);
figure;
step(sys3);
title("Problem 2 Step Response");
```