SGN-13000/SGN-13006 Introduction to Pattern Recognition and Machine Learning (5 cr)

Learning in Robotics - Reinforcement Learning

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Material

- Lecturer's (Mitchell's) slides and blackboard notes
- T.M. Mitchell. *Machine Learning*. McGraw-Hill, 1997: Chapter 13

Contents

Introduction

Learning in Dynamic Environments (AI Robotics)

Consider learning to choose actions, e.g.,

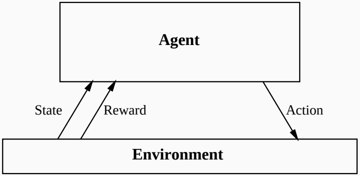
- Robot learning to dock on battery charger
- Learning to drive a car
- Learning to choose actions to optimize factory output

Note several problem characteristics:

- Delayed reward
- Opportunity for active exploration
- Possibility that states only partially observable
- Possible need to learn multiple tasks with same sensors/effectors
- Surprises happen

Reinforcement learning

Reinforcement Learning Problem



$$s_0 \xrightarrow[r_0]{a_0} s_1 \xrightarrow[r_1]{a_1} s_2 \xrightarrow[r_2]{a_2} \dots$$

Goal: Learn to choose actions that maximize

$$r_0 + \gamma r_1 + \gamma^2 r_2 + \dots \text{ , where } 0 \leq \gamma < 1$$

Reinforcement learning

Markov decision process

Markov Decision Processes

Assume

- finite set of states S
- set of actions A
- at each discrete time agent observes state $s_t \in S$ and chooses action $a_t \in A$
- then receives immediate reward r_t
- and state changes to s_{t+1}
- Markov assumption: $s_{t+1} = \delta(s_t, a_t)$ and $r_t = r(s_t, a_t)$
  - i.e., r_t and s_{t+1} depend only on *current* state and action
  - functions δ and r may be nondeterministic
  - functions δ and r not necessarily known to agent

Agent's Learning Task

Execute actions in environment, observe results, and

- learn action policy $\pi : S \rightarrow A$ that maximizes

$$E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots]$$

  from any starting state in S

- here $0 \leq \gamma < 1$ is the discount factor for future rewards

Note something new:

- Target function is $\pi : S \rightarrow A$
- but we have no training examples of form $\langle s, a \rangle$
- training examples are of form $\langle \langle s, a \rangle, r \rangle$

Value Function

To begin, consider deterministic worlds...

For each possible policy π the agent might adopt, we can define an evaluation function over states

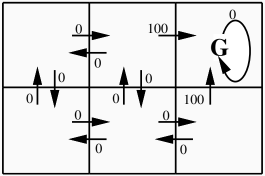
$$\begin{aligned} V^{\pi}(s) &\equiv r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + ... \\ &\equiv \sum_{i=0}^{\infty} \gamma^i r_{t+i} \end{aligned}$$

where $r_t, r_{t+1}, \ldots$ are generated by following policy π starting at state s

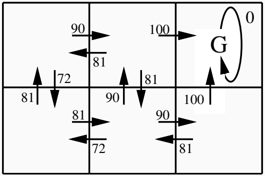
Restated, the task is to learn the optimal policy π^*

$$\pi^* \equiv argmax_{\pi} V^{\pi}(s), (\forall s)$$

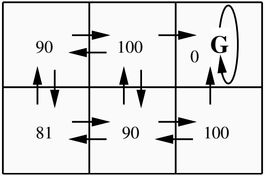
Robot Example



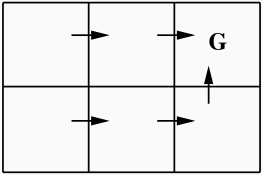
(a) $r(s, a)$ (immediate reward) values



(b) $Q(s, a)$ values



(c) $V^*(s)$ values



(d) One optimal policy

What to Learn

We might try to have agent learn the evaluation function V^{π^*} (which we write as V^*)

It could then do a lookahead search to choose best action from any state s because

$$\pi^*(s) = argmax_a[r(s, a) + \gamma V^*(\delta(s, a))]$$

A problem:

- This works well if agent knows $\delta : S \times A \rightarrow S$, and $r : S \times A \rightarrow \Re$
- But when it doesn't, it can't choose actions this way

Reinforcement learning

Q Learning

Q Function

Define new function very similar to V^*

$$Q(s,a) \equiv r(s,a) + \gamma V^*(\delta(s,a))$$

If agent learns Q, it can choose optimal action even without knowing δ!

$$\pi^*(s) = argmax_a[r(s,a) + \gamma V^*(\delta(s,a))]$$

$$\pi^*(s) = argmax_a Q(s,a)$$

Q is the evaluation function the agent will learn

Training Rule to Learn Q

Note Q and V^* closely related:

$$V^*(s) = \max_{a'} Q(s, a')$$

Which allows us to write Q recursively as

$$\begin{aligned} Q(s_t, a_t) &= r(s_t, a_t) + \gamma V^*(\delta(s_t, a_t))) \\ &= r(s_t, a_t) + \gamma \max_{a'} Q(s_{t+1}, a') \end{aligned}$$

Nice! Let $\hat{Q}$ denote learner's current approximation to Q. Consider training rule

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

where s' is the state resulting from applying action a in state s

Q Learning for Deterministic Worlds

For each s, a initialize table entry $\hat{Q}(s, a) \leftarrow 0$

Observe current state s

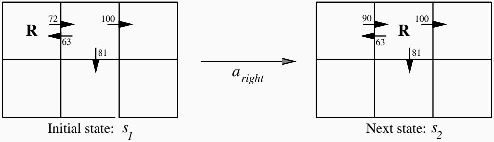
Do forever:

- Select an action a and execute it
- Receive immediate reward r
- Observe the new state s'
- Update the table entry for $\hat{Q}(s, a)$ as follows:

$$\hat{Q}(s, a) \leftarrow r + \gamma \max_{a'} \hat{Q}(s', a')$$

- $s \leftarrow s'$

Updating $\hat{Q}$



$$\begin{aligned}
\hat{Q}(s_1, a_{right}) &\leftarrow r + \gamma \max_{a'} \hat{Q}(s_2, a') \\
&\leftarrow 0 + 0.9 \ \max\{63, 81, 100\} \\
&\leftarrow 90
\end{aligned}$$

notice if rewards non-negative, then

$$(\forall s, a, n) \ \ \hat{Q}_{n+1}(s, a) \geq \hat{Q}_n(s, a)$$

and

$$(\forall s, a, n) \ \ 0 \leq \hat{Q}_n(s, a) \leq Q(s, a)$$

Updating $\hat{Q}$ - Convergence

$\hat{Q}$ converges to Q. Consider case of deterministic world where see each $\langle s, a \rangle$ visited infinitely often.

Proof: Define a full interval to be an interval during which each $\langle s, a \rangle$ is visited. During each full interval the largest error in $\hat{Q}$ table is reduced by factor of γ

...

- Yes, it converges, but the proof requires infinite number of samples
- How about boosting the learning by using more than one future prediction (will be covered later)?
- Better exploration strategies?

Nondeterministic Case

What if reward and next state are non-deterministic?

We redefine V, Q by taking expected values

$$\begin{aligned} V^{\pi}(s) &\equiv E[r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + \ldots] \\ &\equiv E[\sum_{i=0}^{\infty} \gamma^i r_{t+i}] \end{aligned}$$

$$Q(s, a) \equiv E[r(s, a) + \gamma V^*(\delta(s, a))]$$

Nondeterministic Case

Q learning generalizes to nondeterministic worlds

Alter training rule to

$$\hat{Q}_n(s,a) \leftarrow (1-\alpha_n)\hat{Q}_{n-1}(s,a) + \alpha_n[r + \max_{a'} \hat{Q}_{n-1}(s',a')]$$

where

$$\alpha_n = \frac{1}{1 + visits_n(s,a)}$$

Can still prove convergence of $\hat{Q}$ to Q [Watkins and Dayan, 1992]

Reinforcement learning

Temporal Difference Learning

Temporal Difference Learning

Q learning: reduce discrepancy between successive Q estimates

One step time difference:

$$Q^{(1)}(s_t, a_t) \equiv r_t + \gamma \max_a \hat{Q}(s_{t+1}, a)$$

Why not two steps?

$$Q^{(2)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \gamma^2 \max_a \hat{Q}(s_{t+2}, a)$$

Or n?

$$Q^{(n)}(s_t, a_t) \equiv r_t + \gamma r_{t+1} + \cdots + \gamma^{(n-1)} r_{t+n-1} + \gamma^n \max_a \hat{Q}(s_{t+n}, a)$$

Blend all of these:

$$Q^{\lambda}(s_t, a_t) \equiv (1-\lambda) \left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots \right]$$

Temporal Difference Learning

$$Q^{\lambda}(s_t, a_t) \equiv (1-\lambda)\left[Q^{(1)}(s_t, a_t) + \lambda Q^{(2)}(s_t, a_t) + \lambda^2 Q^{(3)}(s_t, a_t) + \cdots\right]$$

Equivalent expression:

$$\begin{aligned} Q^{\lambda}(s_t, a_t) \quad &= r_t + \gamma[\quad (1-\lambda)\max_a \hat{Q}(s_t, a_t) \\ &\qquad\qquad\quad +\lambda\ Q^{\lambda}(s_{t+1}, a_{t+1})] \end{aligned}$$

TD(λ) algorithm uses above training rule

- Sometimes converges faster than Q learning
- converges for learning V^* for any $0 \leq \lambda \leq 1$ (Dayan, 1992)
- Tesauro's TD-Gammon uses this algorithm

Subtleties and Ongoing Research

- Replace $\hat{Q}$ table with neural net or other generalizer (see, e.g., `http://deepdriving.cs.princeton.edu/`)
- Handle case where state only partially observable
- Design optimal exploration strategies
- Extend to continuous action, state
- Learn and use $\hat{\delta} : S \times A \rightarrow S$
- Relationship to dynamic programming

Summary

Summary

- Agents (robots) in open world need to learn by exploration
- All measurements are incomplete and uncertain
- World is full of surprises
- How to still learn something - Q Learning (understand the main principle)