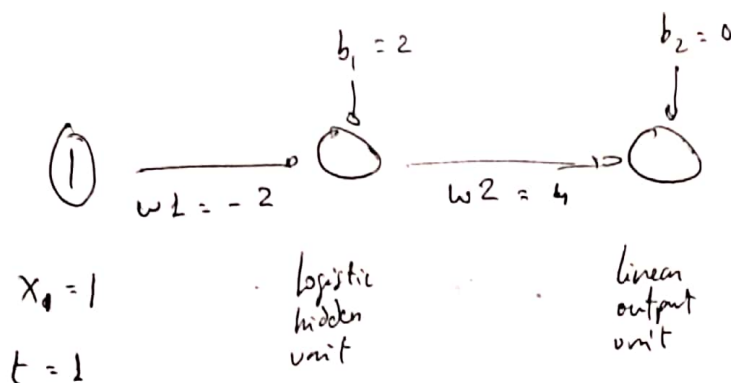


1)



$$\sigma(z) = \frac{1}{1 + e^{-z}}$$

$$a) \hat{y}_1 = \sigma(z_1) = \sigma(w_1 x_1 + b_1) = \frac{1}{1 + e^{-(w_1 x_1 + b_1)}}$$

$$= \frac{1}{1 + e^{-(-2 \cdot 1 + 2)}} = \frac{1}{1 + 1} = \frac{1}{2} \rightarrow x_2 = \frac{1}{2}$$

$$\hat{y}_2 = \sigma(z_2) = \sigma(w_2 x_2 + b_2) = \frac{1}{1 + e^{-(w_2 x_2 + b_2)}}$$

$$= \frac{1}{1 + e^{-(4 \cdot \frac{1}{2} + 0)}} = \frac{1}{1 + e^{-2}} = 0.8808$$

$$b) E = \frac{1}{2} (t - y)^2 = \frac{1}{2} (1 - 0.8808)^2 \approx 0.0071$$

$$c) \frac{\partial}{\partial w_2} E = \frac{\partial}{\partial w_2} \left(\frac{1}{2} \left(t - \frac{1}{1 + e^{-(w_2 x_2 + b_2)}} \right)^2 \right) = \frac{\partial}{\partial w_2} \left(\frac{1}{2} \left(t^2 + \frac{1}{(1 + e^{-(w_2 x_2 + b_2)})^2} - \frac{2t}{1 + e^{-(w_2 x_2 + b_2)}} \right) \right)$$

$$= \frac{\partial}{\partial w_2} \left(\frac{1}{2} \left(t^2 + \frac{1}{1 + e^{-2(w_2 x_2 + b_2)}} + \frac{2t}{1 + e^{-(w_2 x_2 + b_2)}} \right) \right)$$

$$= \frac{\partial}{\partial w_2} \left(\frac{1}{2} \left(t^2 + \frac{1}{1 + e^{-2(w_2 x_2 + b_2)}} + \frac{2t}{1 + e^{-(w_2 x_2 + b_2)}} \right) \right)$$

$$\frac{\partial}{\partial x} \sigma(x) = \sigma(x) (1 - \sigma(x))$$

$$= - \frac{\partial}{\partial \omega_2} \left(\frac{1}{1 + e^{-(\omega_2 x_2 + b_2)}} \right) = - \frac{1}{1 + e^{-(\omega_2 x_2 + b_2)}} \left(1 - \frac{1}{1 + e^{-(\omega_2 x_2 + b_2)}} \right)$$

$$= - \underbrace{\frac{1}{1 + e^{-(4 \cdot \frac{1}{2} + 0)}}}_{0.8808} \left(1 - \underbrace{\frac{1}{1 + e^{-(4 \cdot \frac{1}{2} + 0)}}}_{0.8808} \right) = -0.109$$

b)

~~$$\frac{\partial}{\partial \omega_1} E = \frac{\partial}{\partial \omega_1} \frac{1}{2} \left(t_1 - \frac{1}{1 + e^{-(\omega_2 (\omega_1 x_1 + b_1) + b_2)}} \right)^2 =$$

$$= - \frac{1}{1 + e^{-(\omega_2 (\omega_1 x_1 + b_1) + b_2)}} \left(1 - \frac{1}{1 + e^{-(\omega_2 (\omega_1 x_1 + b_1) + b_2)}} \right)$$~~

$$\frac{\partial}{\partial \omega_1} E = \frac{\partial}{\partial \omega_1} \frac{1}{2} \left(t_1 - \frac{1}{1 + e^{-(\omega_1 x_1 + b_1)}} \right)^2 =$$

$$= - \underbrace{\frac{1}{1 + e^{-(\omega_1 x_1 + b_1)}}}_{0.5} \left(1 - \underbrace{\frac{1}{1 + e^{-(\omega_1 x_1 + b_1)}}}_{0.5} \right) = -0.25$$