1.

$$s[n] = \begin{cases} -1 & \text{for } 0 \le n < 10 \\ 1 & \text{for } 10 \le n < 20 \end{cases}$$

$$\mathcal{H}_1 : \quad x[n] = s[n] + w[n]$$

$$\mathcal{H}_0 : \quad x[n] = w[n]$$

$$P(x|\mathcal{H}_1) = \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[- \frac{(x[n]-s[n])^2}{2\sigma^2} \right]$$

$$P(x|\mathcal{H}_0) = \prod_{n=0}^{N-1} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[- \frac{(x[n])^2}{2\sigma^2} \right]$$

$$\frac{P(x|\mathcal{H}_1)}{P(x|\mathcal{H}_0)} = \exp\left[- \frac{1}{2\sigma^2} \left(\sum_{n=0}^{N-1} (x[n]-s[n])^2 - \sum_{n=0}^{N-1} (x[n])^2 \right) \right] > \gamma$$

$$- \frac{1}{2\sigma^2} \left(\sum_{n=0}^{N-1} \underbrace{(x[n]-s[n])^2}_{x^2[n] + s^2[n] - 2x[n]s[n]} - \sum_{n=0}^{N-1} (x[n])^2 \right) > \ln\gamma$$

$$- \frac{1}{2\sigma^2} \sum_{n=0}^{N-1} (s[n])^2 + \frac{1}{\sigma^2} \sum_{n=0}^{N-1} x[n]s[n] > \ln\gamma$$

$$\sum_{n=0}^{N-1} x[n]s[n] > \sigma^2 \ln\gamma + \frac{1}{2} \sum_{n=0}^{N-1} (s[n])^2$$

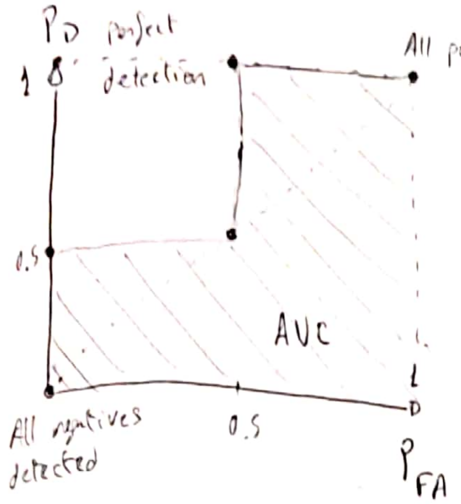
$$\begin{cases} \frac{1}{2}\sum_{n=0}^{N-1} x[n](-1) > \sigma^2 \ln\gamma + \frac{1}{2}\frac{1}{2}\underbrace{\sum_{n=0}^{N-1} (-1)^2}_{10} & 0 \le n < 10 \\ \frac{1}{2}\sum_{n=0}^{N-1} x[n]\cdot 1 > \sigma^2 \ln\gamma + \frac{1}{2}\frac{1}{2}\underbrace{\sum_{n=0}^{N-1} 1^2}_{10} & 10 \le n < 20 \end{cases}$$

$$\begin{cases} \frac{1}{2}\sum_{n=0}^{N-1} x[n] < \sigma^2 \ln\gamma + 5 & 0 \le n < 10 \\ \frac{1}{2}\sum_{n=0}^{N-1} x[n] > \sigma^2 \ln\gamma + 5 & 10 \le n < 20 \end{cases}$$

$$\begin{cases} \sum_{n=0}^{N-1} x[n] < 2\sigma^2 \ln\gamma + 10 & 0 \le n < 10 \\ \sum_{n=0}^{N-1} x[n] > 2\sigma^2 \ln\gamma + 10 & 10 \le n < 20 \end{cases}$$

2.

	Probability of class 1	True Label
Sample 1	0.8	1
" 2	0.3	1
" 3	0.4	0
" 4	0.2	0



T: threshold

$T < 0.2$: $P_D = 1.0$ $P_{FA} = 1.0$

$0.2 \leq T < 0.3$: $P_D = 1.0$ $P_{FA} : 0.5$

$0.3 \leq T < 0.4$: $P_D = \frac{2}{4} = 0.5$ $P_{FA} : 0.5$

$0.4 \leq T < 0.8$: $P_D = \frac{2}{4}\ 0.5$ $P_{FA} = 0.0$

$T > 0.8$: $P_D = 0.0$ $P_{FA} = 0.0$

Area Under Curve

$AUC = 0.5^2 + 0.5 = 0.75$