SGN-41006 Signal Interpretation Methods
Exam 1.3.2016
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- ▷ Use of calculator is allowed.
- ▷ Use of other materials is not allowed.
- ▷ The exam questions need not be returned after the exam.

1. Describe the following terms and concepts by a few sentences. (max. 6 p.)
   (a) Likelihood function
   (b) Receiver operating characteristics curve
   (c) Kernel Trick
   (d) Multilabel classifier
   (e) Rectified linear unit
   (f) Recursive feature elimination

2. Consider the model

$$x[n] = As[n] + w[n], \qquad n = 0, 1, \ldots, N-1,$$

   where $w[n] \sim \mathcal{N}(0, \sigma^2)$, $s[n]$ is a known signal, and A is the parameter to be estimated. Derive the maximum likelihood estimator of A.

3. (a) (4 pts) A dataset consists of two classes, whose distributions are assumed Gaussian, and whose sample covariances and means are the following:

$$\mu_0 = \begin{pmatrix} -5 \\ 5 \end{pmatrix} \qquad \mu_1 = \begin{pmatrix} 10 \\ -5 \end{pmatrix}$$
$$\mathbf{C}_0 = \begin{pmatrix} 20 & -10 \\ -10 & 20 \end{pmatrix} \qquad \mathbf{C}_1 = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

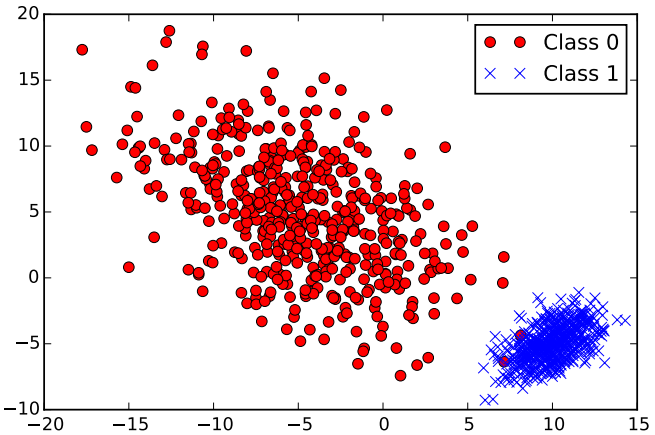

Figure 1: Training sample of question 3

A sample of data from these distributions is shown in Figure 1. Calculate the LDA projection vector $\mathbf{w}$. Hint: A 2×2 matrix is inverted using the rule

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}.$$

(b) (2 pts) The projected Gaussians are univariate normal: $\mathcal{N}(\mathbf{w}^T\boldsymbol{\mu}_1, \mathbf{w}^T\mathbf{C}_1\mathbf{w})$ and $\mathcal{N}(\mathbf{w}^T\boldsymbol{\mu}_2, \mathbf{w}^T\mathbf{C}_2\mathbf{w})$. Formulate the classification problem as a likelihood ratio test and choose the threshold based on that.

4. (a) (3 pts) Compute the gradient for L_2 penalized log-loss. Unregularized log-loss is defined as

$$\ell(\mathbf{w}) = \sum_{n=0}^{N-1} \ln(1 + \exp(y_n \mathbf{w}^T \mathbf{x}_n)). \tag{1}$$

(b) (3 pts) Consider the Keras model defined in Listing 1. Inputs are 32×32 RGB images from 10 categories. Compute the number of parameters for each layer, and their sum over all layers.

Listing 1: A CNN model defined in Keras

```
model = Sequential()

w, h = 3, 3
sh = (3, 32, 32)

model.add(Convolution2D(16, w, h, input_shape=sh, border_mode='same'))
model.add(MaxPooling2D(pool_size=(2, 2)))

model.add(Convolution2D(16, w, h, border_mode='same'))
model.add(MaxPooling2D(pool_size=(2, 2)))

model.add(Flatten())
model.add(Dense(100))
model.add(Dense(10, activation = 'softmax'))
```

5. In the lectures we saw that the kernel trick $\kappa(\mathbf{x}, \mathbf{y}) = (\mathbf{x} \cdot \mathbf{y})^2$ for $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$ corresponds to the mapping

$$\begin{pmatrix} u \\ v \end{pmatrix} \mapsto \begin{pmatrix} u^2 \\ v^2 \\ \sqrt{2}uv \end{pmatrix}$$

Find the explicit mapping corresponding to the inhomogeneous kernel $\kappa(\mathbf{x}, \mathbf{y}) = (\mathbf{x} \cdot \mathbf{y} + 1)^2$ with $\mathbf{x}, \mathbf{y} \in \mathbb{R}^2$.