Exercise group # 1 — Fundamentals of Robot Vision

1) Homogeneous coordinates

equation of a line in a plane $\quad x^T l = 0$

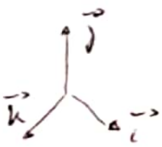
a) Convert cartesian to homogeneous format of the coordinates

$$x_1 = \begin{pmatrix}1\\-1\end{pmatrix} \longrightarrow \begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$x_2 = \begin{pmatrix}3\\-1\end{pmatrix} \longrightarrow \begin{pmatrix}3\\-1\\1\end{pmatrix}$$

$$x_3 = \begin{pmatrix}0\\1\end{pmatrix} \longrightarrow \begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$x_4 = \begin{pmatrix}-1\\0\end{pmatrix} \longrightarrow \begin{pmatrix}-1\\0\\1\end{pmatrix}$$



b) $l = x \times x'$ ← line through two points

$$l_1 = x_1 \times x_2 =$$

$$= (\vec i - \vec j + \vec k) \times (3\vec i - \vec j + \vec k) = -\vec k + \vec j + 3\vec k \cancel{- \vec i} - 3\vec j \cancel{+ \vec i} = 2\vec j + 2\vec k$$

$$= \begin{bmatrix}0\\2\\2\end{bmatrix}$$

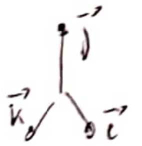
$$l_2 = x_2 \times x_3 = (\vec j + \vec k) \times (-\vec i + \vec k) = \vec k + \vec i - \vec j = \begin{bmatrix}1\\-1\\1\end{bmatrix}$$

$$a \times b = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} i - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_2 \end{vmatrix} j + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} k$$

c) $x = l \times l'$ – point intersection of two lines

$$x = l_1 \times l_2 = (2\vec{j} + 2\vec{k}) \times (\vec{i} - \vec{j} + \vec{k}) = -2\vec{k} + 2\vec{i} + 2\vec{j} + 2\vec{i} =$$

$$= 4\vec{i} + 2\vec{j} - 2\vec{k} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix} \longrightarrow \begin{bmatrix} 4/-2 \\ 2/-2 \end{bmatrix} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$



$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \longrightarrow \begin{bmatrix} x/w \\ y/w \end{bmatrix}$$

d) The intersection of two lines l and l' is the point $x = l \times l'$

Scalar triple product $(a \times b) c = 0$ if any two vectors are parallel or equal

$$\left.\begin{aligned} x^T l &= (l \times l') l = 0 \\ x^T l' &= (l \times l') l' = 0 \end{aligned}\right\}$$ both lines satisfy the equation so it must represent their intersection