Exercise 5

$x' = sRx + t \quad \begin{pmatrix} x' \\ y' \end{pmatrix} = s \begin{pmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} t_x \\ t_y \end{pmatrix} \quad \{x_1 \to x_1'\} \; \{x_2 \to x_2'\}$

a) $v' = X_2' - X_1' = (sRX_2 + t) - (sRX_1 + t) = sR(X_2 - X_1) =$

$\boxed{v = X_2 - X_1} \quad = sRv$

$\frac{\|v\|}{\|v'\|} = 1 \qquad \frac{v'}{\|v'\|} = sR\frac{v}{\|v\|} \to e' = 1 \cdot R \cdot e$

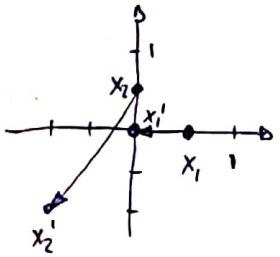
b) $\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \, \|\vec{v}\|}$

c) $t = x' - sRx \begin{cases} t_x = x_i' - (x_i\cos\theta - y_i\operatorname{sen}\theta) \\ t_y = y_i' - (x_i\operatorname{sen}\theta + y_i\cos\theta) \end{cases}$

$\left.\begin{aligned} x_1' &= x_1\cos\theta - y_1\operatorname{sen}\theta + t_{x_1} \\ y_1' &= x_1\operatorname{sen}\theta + y_1\cos\theta + t_{y_1} \end{aligned}\right\}$

d) $\{(0.5, 0) \to (0, 0)\}, \{(0, 0.5) \to (-1, -1)\}$

(x_1, x_1', x_2, x_2')



$s = 1 \quad \vec{u} = x_1' - x_1 = (0,0) - (0.5, 0) = (-0.5, 0) \quad \vec{v} = x_2' - x_2 = (-1, -1) - (0, 0.5) = (-1, -1.5)$

$\cos\theta = \frac{(-0.5, 0)\cdot(-1, -1.5)}{\sqrt{(-0.5)^2 + 0^2} \cdot \sqrt{(-1)^2 + (-1.5)^2}} = \frac{(-0.5)(-1) + 0 \cdot (-1.5)}{\sqrt{(-0.5)^2 + 0^2} \cdot \sqrt{(-1)^2 + (-1.5)^2}} =$

$= \frac{0.5}{0.5 \cdot \frac{\sqrt{13}}{2}} = \frac{2}{\sqrt{13}} \to \theta = \operatorname{acos}\frac{2}{\sqrt{13}} = 56.31°$

$t_{x_1} = 0 - (0.5 \cdot \cos(56.31°) - 0 \cdots)$

$t_{y_1} = 0 - (0.5\cos(56.31°) + 0 \cdots)$