Exercise Set 2

1.

n	0	1	2
$x(n)$	7	9	2
$y(n)$	11.6	14.8	3.5

$y(n) = a\,x(n) + b$

$$x = \begin{bmatrix} 7 \\ 9 \\ 2 \end{bmatrix} \quad y = \begin{bmatrix} 11.6 \\ 14.8 \\ 3.5 \end{bmatrix} \quad X = \begin{bmatrix} 7 & 1 \\ 9 & 1 \\ 2 & \underbrace{1}_{\text{last column all ones}} \end{bmatrix}$$

Vandermode matrix X

Least Squares Model: $\hat{\Theta} = \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (X^T X)^{-1} X^T \hat{y}$

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = \left(\begin{bmatrix} 7 & 9 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 7 & 1 \\ 9 & 1 \\ 2 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 7 & 9 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 11.6 \\ 14.8 \\ 3.5 \end{bmatrix} = \begin{bmatrix} 1.6154 \\ 0.2799 \end{bmatrix}$$

3.

$$P(x,\lambda) = \frac{e^{-\lambda} \lambda^{x}}{x!} \longrightarrow P(x,\lambda) = \sum_{n=0}^{N-1} \frac{e^{-\lambda} \lambda^{x(n)}}{x(n)!}$$

$$\ln(P(x,\lambda)) = \sum_{n=0}^{N-1} \ln e^{-\lambda} + \sum_{n=0}^{N-1} \ln \lambda^{x(n)} - \sum_{n=0}^{N-1} \ln(x(n)!) =$$

$$= \underbrace{\sum_{n=0}^{N-1} -\lambda}_{-N\lambda} + \sum_{n=0}^{N-1} x(n) \ln(\lambda) - \sum_{n=0}^{N-1} \ln(x(n)!)$$

$$\frac{\partial}{\partial \lambda} \ln(P(x,\lambda)) = -N + \sum_{n=0}^{N-1} x(n) \frac{1}{\lambda} \stackrel{!}{=} 0 \longrightarrow \lambda = \frac{1}{N} \sum_{n=0}^{N-1} x(n)$$

2.

$$x[n] = A\cos[n] + w[n] \qquad n = 0, \dots, N-1$$

$$w[n] \sim \mathcal{N}(0, \sigma^2)$$

$$P(x;A) = \frac{1}{(2\pi\sigma^2)^{N/2}} \exp\left[-\frac{1}{2\sigma^2}\sum_{n=0}^{N-1}(x[n] - A\cos[n])^2\right]$$

$$\ln P(X;A) = -\frac{N}{2}(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{n=0}^{N-1}\underbrace{(X[n] - A\cos[n])^2}_{X^2[n] + A^2\cos^2[n] - 2X[n]A\cos[n]}$$

$$\frac{\partial}{\partial A}\ln P(X;A) = -\frac{1}{\cancel{2}\sigma^2}\sum_{n=0}^{N-1}(\underbrace{\cancel{2}A\cos^2[n]}_{A\sum_{n=0}^{N-1}\cos^2[n]} - \cancel{2}X[n]\cos[n]) \equiv 0$$

$$A = \frac{\sum_{n=0}^{N-1} x[n]\cos[n]}{\sum_{n=0}^{N-1}\cos^2[n]}$$

$$A = \frac{\sum X(n)\cos(n)}{\sum \cos^2(n)}$$

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