SGN-13006 Introduction to Pattern Recognition and Machine Learning (5 cr)

Linear regression and classification

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Material

- Lecturer's slides and blackboard notes
- Very thorough treatment in C.M. Bishop. Pattern Recognition and Machine Learning. Springer, 2006: e.g. Chapter 4

Linear models are at the core of many powerful methods in PR and ML and therefore can be found from all ML & PR books - they provide intuitive mathematical introduction to the topic

Linear regression

Linear regression using high school maths

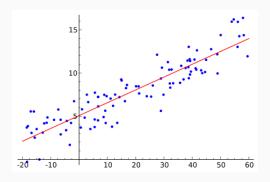


Figure 1: Linear regression (Wikipedia).

Linear classification

Linear classification

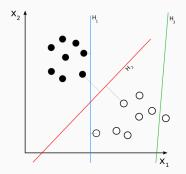


Figure 2: Linear classification (Wikipedia).

Linear regression and classification

in PR and ML

Linear regression and classification in PR and ML

- The two essential problems in PR and ML are *regression* and *classification*.
- We derived solutions to these problems by using high school geometry (fitting lines/hyperplanes to a set of points) and simple algebra → This magic is not that difficult after all!
- PR and ML are about different models of learning (e.g. y = ax + b) and different methods of learning using the models (e.g. LSQFit of a, b).
- However, He-Who-Must-Not-Be-Named is in the details and true mastery is to avoid His traps!

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- 3. What can make line fitting to fail?
 - Too few data points
 - Outliers
 - Division-by-zero (singularities)
 - Computational complexity (time/space)
 - Multiple solutions (local vs global optima)
 - Model selection (low vs. high order polynomial)
 - Background (prior) knowledge