ML PR

exercise 4

1. pen and paper

$$\mu_0 = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad C_0 = \begin{pmatrix} 2 & 0.1 \\ 0.1 & 0.2 \end{pmatrix}$$

$$\mu_1 = \begin{pmatrix} -5 \\ 2 \end{pmatrix} \qquad C_1 = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$$

$$\bar{J}(w) = \frac{(w^T \mu_1 - w^T \mu_0)^2}{w^T C_1 w + w^T C_0 w} = \frac{w^T S_B w}{w^T S_W w}$$

$$S_B = (\mu_1 - \mu_0)(\mu_1 - \mu_0)^T = \left[\begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right]\left[\begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right]^T =$$

$$= \underset{2\times 1}{\begin{pmatrix} -4 \\ 1 \end{pmatrix}} \underset{1\times 2}{\begin{pmatrix} -4 & 1 \end{pmatrix}} = \begin{pmatrix} 16 & -4 \\ -4 & 1 \end{pmatrix}$$

$$S_W = C_1 + C_0 = \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 0.1 \\ 0.1 & 0.2 \end{pmatrix} = \begin{pmatrix} 5 & -0.9 \\ -0.9 & 2.2 \end{pmatrix}$$

$$w = S_W^{-1}(\mu_1 - \mu_0) = \begin{pmatrix} 5 & -0.9 \\ -0.9 & 2.2 \end{pmatrix}^{-1} \left[\begin{pmatrix} -5 \\ 2 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \end{pmatrix}\right] =$$

$$= \begin{bmatrix} 0.2159 & 0.0883 \\ 0.0883 & 0.4907 \end{bmatrix} \begin{bmatrix} -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.7753 \\ 0.1374 \end{bmatrix}$$

2.

$$\mathcal{N}(w^T\mu_1, w^T C_1 w)$$

$$\mathcal{N}(w^T\mu_2, w^T C_2 w)$$

$$H_1 = P(x[0] \mid \mu = w^T\mu_1) = \frac{1}{\sqrt{2\pi\sigma_1^2}} \exp\left(-\frac{(x-\mu_1)^2}{2\sigma_1^2}\right)$$

$$H_2 = P(x[0] \mid \mu = w^T\mu_2) = \frac{1}{\sqrt{2\pi\sigma_2^2}} \exp\left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right)$$

$$H_1 > H_2 \rightarrow \frac{H_1}{H_2} > 1$$

$$\frac{\sqrt{\sigma_2^2}}{\sqrt{\sigma_1^2}} \exp\left[-\frac{(x-\mu_1)^2}{2\sigma_1^2} - \left(-\frac{(x-\mu_2)^2}{2\sigma_2^2}\right)\right] > 1$$

$$-\frac{(x-\mu_1)^2}{2\sigma_1^2} + \frac{(x-\mu_2)^2}{2\sigma_2^2} > \ln\sqrt{\frac{\sigma_1^2}{\sigma_2^2}}$$

$$2\sigma_1^2\sigma_2^2 \ln\frac{\sigma_2}{\sigma_1} + x^2(\sigma_1^2 - \sigma_2^2) - 2x(\mu_2\sigma_1^2 - \mu_1\sigma_2^2) +$$

$$+(\sigma_1^2\mu_2^2 - \sigma_2^2\mu_1^2) > 0$$

$$\mu_0 = w^T\mu_1 = \begin{pmatrix} -0.775 \\ 0.137 \end{pmatrix}^T \begin{pmatrix} -1 \\ 1 \end{pmatrix} = 0.912$$

$$\mu_1 = 4.143$$

$$\sigma_0 = w^T C_0 w = 1.16377$$

$$\sigma_1 = 2.05176$$

$$2.8x^2 + 3.98x - 20.6 - 6.47 < 0 \rightarrow -39 < x < 248$$

$$\text{if } x = (1,2) \rightarrow w^T x = \begin{pmatrix} -0.775 \\ 0.137 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = -0.501$$