(1) (1 point) Solve the following inequalities in $\mathbb{R}$:

$$\frac{x-2}{2x-8} \geq 1, \quad \log_{\frac{1}{3}}(x^2-3x+2) \geq 0, \quad \frac{x+2}{x+3} > \frac{2x+3}{x+6}.$$

SOLUTION:

$$\frac{x-2}{2x-8} \geq 1$$
$$\frac{x-2}{2x-8} - 1 \geq 0$$
$$\frac{x-2-2x+8}{2x-8} \geq 0$$
$$\frac{-x+6}{2x-8} \geq 0, \text{ either both part of the fraction have to be negative or positive.}$$

SOLUTION: $x \in (4, 6\rangle$

$$\log_{\frac{1}{3}}(x^2-3x+2) \geq 0$$
$$\log_{\frac{1}{3}}(x^2-3x+2) \geq \log_{\frac{1}{3}}(1)$$
$$x^2-3x+2 \leq 1, \text{ because } f(x) = \frac{1}{3^x} \text{ is decreasing function.}$$
$$x^2-3x+1 \leq 0$$
$$x_{1,2} = \frac{3 \pm \sqrt{5}}{2}, \text{ but also the argument of the logarithm has to be positive. Thus,}$$
$$x^2-3x+2 > 0$$
$$x_{1,2} = \frac{3 \pm \sqrt{1}}{2} = 1 \text{ or } 2$$

SOLUTION: $\langle \frac{1}{2}(3-\sqrt{5}), 1) \cup (2, \frac{1}{2}(3+\sqrt{5})\rangle$

$$\frac{x+2}{x+3} > \frac{2x+3}{x+6}$$
$$\frac{x+2}{x+3} - \frac{2x+3}{x+6} > 0$$
$$\frac{(x+2)(x+6)-(2x+3)(x+3)}{(x+3)(x+6)} > 0$$
$$\frac{x^2+8x+12-(2x^2+9x+9)}{(x+3)(x+6)} > 0$$
$$\frac{-x^2-x+3}{(x+3)(x+6)} > 0$$

SOLUTION: $(-6,-3) \cup \left(\frac{1}{2}(-1-\sqrt{13}), \frac{1}{2}(-1+\sqrt{13})\right)$

(2) (1 point) Solve the following in $\mathbb{R}$:

$$\sin 2x = \sin x, \quad 2xe^x = e^x, \quad 5x^2 - 8 = x^2 - x, \quad \log(x^2+1) = 2\log(3-x).$$

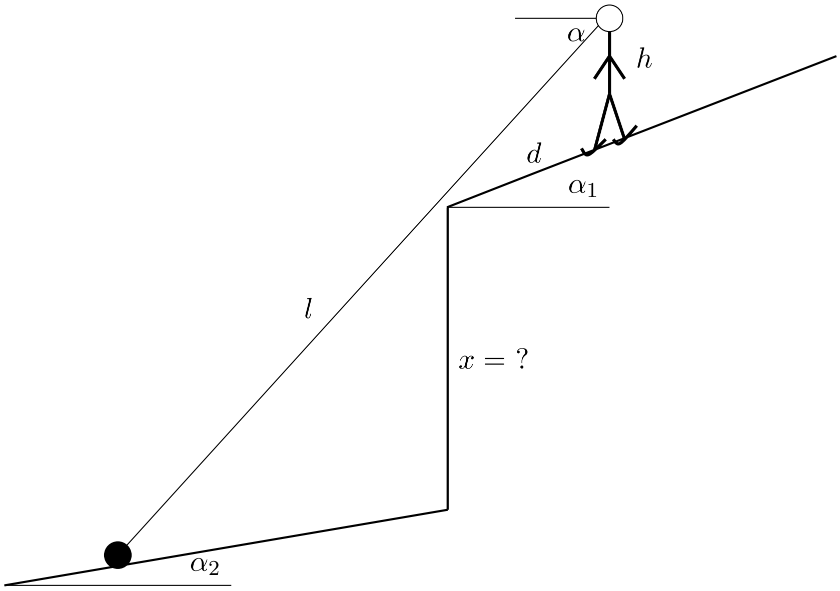
$$\begin{aligned}
\sin 2x &= \sin x \\
2\sin x \cos x &= \sin x \\
\sin x(2\cos x - 1) &= 0 \\
\text{SOLUTION: } x = k\pi \text{ or } x &= \frac{\pi}{3} + 2k\pi \text{ or } x = \frac{5\pi}{3} + 2k\pi, k \in \mathbb{Z}
\end{aligned}$$

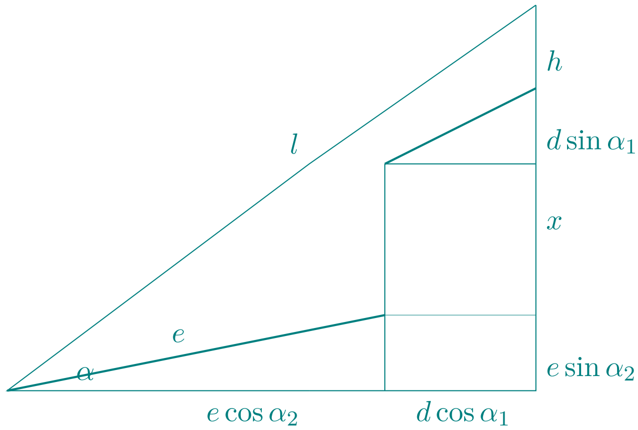
$$\begin{aligned}
2xe^x &= e^x \\
e^x(2x - 1) &= 0 \\
\text{SOLUTION: } x &= \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
5x^2 - 8 &= x^2 - x \\
5x^2 - 8 - x^2 + x &= 0 \\
4x^2 + x - 8 &= 0 \\
\text{SOLUTION: } x_{1,2} &= \frac{-1 \pm \sqrt{129}}{8}
\end{aligned}$$

$$\begin{aligned}
\log(x^2 + 1) &= 2\log(3 - x) \\
\log(x^2 + 1) &= \log(3 - x)^2 \\
x^2 + 1 &= (3 - x)^2 \\
x^2 + 1 &= x^2 - 6x + 9 \\
-8 &= -6x \\
\text{SOLUTION: } x &= \frac{4}{3}
\end{aligned}$$

(3) (3 points) Compute the height of a drop.
Imagine you are a skier/snowboarder and you want to jump a cliff. You are h meters tall. When you are d meters away from the cliff, the edge of the cliff lines up with a stone under the drop and you see the stone l meters away from you under an angle α. Moreover, the slope is α_1 and α_2 over and under the drop, respectively. (Some rounding applies.)
Check your solution for $\alpha = \frac{\pi}{4},\ \alpha_1 = 0,\ \alpha_2 = 0,\ l = 10\ m,\ h = 2\ m,\ d = 1\ m.$





SOLUTION: Let e be the distance of the stone to the bottom of the cliff. We also note, that the angle between the horizon and the line connecting stone and head of the skier is α. Then, we find two equations with use of the sine and cosine of α:

$$l \cdot \sin(\alpha) = e \cdot \sin(\alpha_2) + x + d \cdot \sin(\alpha_1) + h$$
$$l \cdot \cos(\alpha) = e \cdot \cos(\alpha_2) + d \cdot \cos(\alpha_1)$$

(4) (2 points) The following function are given:

$$f(x) = 3x^2 - x - 7 \tag{1}$$
$$f(x) = \left(\frac{7}{5}\right)^x - \frac{1}{2}x^3 \tag{2}$$
$$f(x) = 3\sin(x) + \cos(10x)\frac{1}{3}\sin(x) \tag{3}$$
$$f(x) = \Big|\Big|||x| - 1| - 1\Big| - 1\Big| \tag{4}$$
$$f(x) = \frac{1}{x} \tag{5}$$
$$f(x) = \log|x - 1| \tag{6}$$

Implement the method `plot_function(fct, x_min, x_max, step_size)` so that it can be used for visualization of the functions. Choose well the x-range. Determine the properties (domain, monotonicity, increasing, decreasing of the function, discontinuities,...) of the given functions.

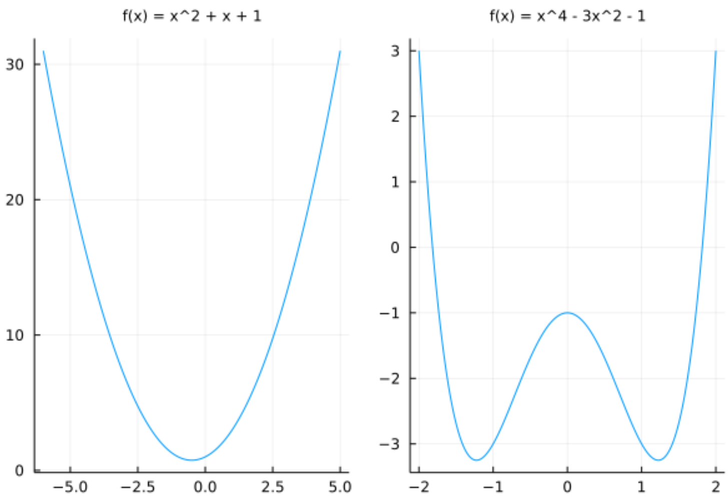


Figure 1: Plots of example functions.