MAT 396 Stochastic Processes

Instructor: Dr. Laurie Heyer

Students: Chadi Bsila and Allen Zhang

Term: Fall 2024

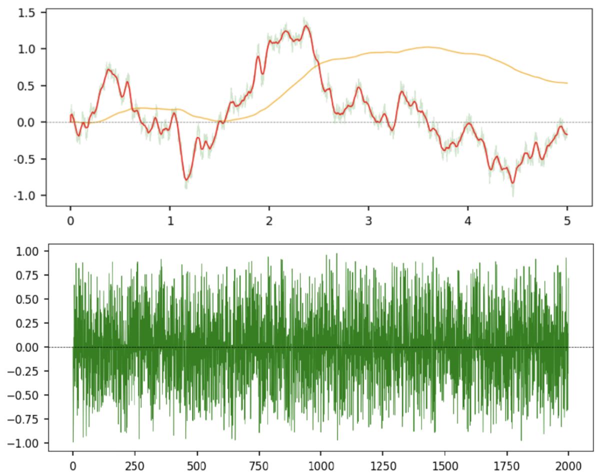
Class Location: Chambers 1015

Meeting Times: M 3:00pm – 4:30pm and TR 10am - 11:00am

(last updated September 6, 2024)

Course Description

Much of mathematics focuses on *deterministic* systems, where a given input consistently produces the same output. Examples include differential equations that model bacterial growth or optimization models with fixed constraints. However, in real-world applications, especially in engineering and the applied sciences, uncertainty and randomness are ubiquitous. Stochastic processes provide a robust framework for modeling systems that evolve over *time* in a probabilistic manner. In this course, students will establish the theoretical foundation needed to analyze stochastic systems and explore various types of stochastic processes like Markov chains, Poisson processes, and more. Additionally, students will develop coding proficiency in R to implement and analyze these models.



Top: Brownian (green), integrated (orange), Bottom: reflective random walk

Course Format

Students will independently read and carefully annotate the assigned **readings**. They will meet to explain concepts to one another and collaborate on **practice problems** to ensure they can apply the material to unfamiliar situations. On Mondays, they are to meet with the instructor to deliver formal and informal **presentations**, summarizing key concepts and discussing solutions to a subset of the assigned problems. Collaboration will also extend to occasional **coding assignments**, where the students will work together to implement and analyze stochastic models using R. At the end of the course, students will shift their focus toward a **final investigation**.

Prerequisites

Strong foundation in probability and linear algebra. Some background in analysis is recommended to grasp some derivations.

Textbook

Dobrow, Robert P. *Introduction to Stochastic Processes with R.* Wiley, 2016.

R

This course introduces R, a statistical computing language, to students with a prior background in programming. The emphasis will be on using R to simulate stochastic processes, providing hands-on experience that contrasts with the theory. R may also be incorporated into the final investigation. For help with learning R, students will mainly use:

> Hadley Wickham and Garrett Grolemund. *R for Data Science*. O'Reilly Media, 2017.

Learning Outcomes

Upon successful completion of this course, students should be able to:

- **Review Fundamental Probability Concepts:** Revisit conditional probability, conditional expectation, and discrete and continuous distributions. State and explain limit theorems.
- **Master Stochastic Processes:** Develop a comprehensive understanding of stochastic processes, including discrete-time and continuous-time Markov chains, Poisson processes, and Brownian motion.
- **Apply Stochastic Models to Real-World Scenarios:** Identify and model real-life situations using appropriate stochastic processes.
- **Achieve Proficiency in R:** Develop the ability to use R for simulating and analyzing stochastic models effectively.
- **Enhance Mathematical Reading Skills:** Improve the ability to read and understand dense mathematical texts and expositions.
- **Strengthen Communication Skills:** Hone skills in communicating unfamiliar and complex material through informal discussions, formal writing, and oral presentations.

Expectations and Grading Policy

Per Davidson College, students are expected to spend approximately 12 hours per week on the course. The final course grade will be computed as follows:

- 50% **Weekly Presentations**.
- 15% **Practice Problems**.
- 15% **Coding Problems**.
- 20% **Final Investigation**.

Tentative Schedule

The schedule is subject to change.

Week	Topics	Sections
1	Probability Review	§1, §B, §C
2	More Probability, Introduction to R	§1, §A, §B, §C
3	Discrete-time Markov Chains	§2
4	Discrete-time Markov Chains	§3
5	Branching Processes, Markov Chain Monte Carlo	§4, §5.1
6	More Markov Chain Monte Carlo	§5.2
7	Poisson Processes	§6.1, §6.2
8	More Poisson Processes	§6.4, §6.5, §6.8
9	Continuous-time Markov Chains	§7
10	Continuous-time Markov Chains	§7
11	Brownian Motion	§8
12	Brownian Motion	§8
13	Final Investigation	
14	Final Investigation	
15	Final Investigation Wrap-up	

Final Investigation

At the end of the course, students will undertake a **final investigation** to demonstrate their understanding and application of the course material. Possible project options (or a combination thereof) include, but are not limited to:

- **Computational Project**
  - Develop and implement a simulation of a stochastic system relevant to engineering or finance applications.
  - Analyze and interpret the results to draw meaningful conclusions about the system's behavior.
  - Present findings through well-documented code, visualizations, and a comprehensive report.
- **Advanced Topic Survey**
  - Conduct an in-depth review of an advanced topic related to the course that was not covered in class.
  - Present the topic in an undergraduate-friendly manner.
- **Research Proposal**
  - Identify a relevant (and feasible) research question.
  - Craft a detailed DRI[1] proposal.

[1] Davidson Research Initiative.

The investigation is due by the **end** of the finals period. **Note:** Students are encouraged to propose and pursue other project ideas that align with their interests and the course objectives, subject to time constraints and approval by the instructor.