The random limits lifetime model:

$$lnY = \alpha + \beta\, ln(x-V) + \varepsilon$$

, where $\beta < 0$, $0 < v_0 \leq V \leq v_1 < x$, $\varepsilon \sim N\left(0, \sigma^2\right)$, $V \sim U\left(v_0, v_1\right)$ [1].

Let $Z = lnY$,

$$Z = \alpha + \beta\, ln(x-V) + \varepsilon\,.$$

Therefore, we have

$$g(z,v) = \frac{1}{v_1 - v_0} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\alpha-\beta ln(x-v))^2}{2\sigma^2}}.$$

By the way, we can integrate $\int g(z,v)dv$ because V is a unknown variable. Hence, we have

$$g(z) = \int_{v_0}^{v_1} \frac{1}{v_1 - v_0} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(z-\alpha-\beta ln(x-v))^2}{2\sigma^2}} dv.$$

Let $U = \frac{\beta}{\sigma}\left[ln(x-v) - \frac{(z-\alpha)}{\beta}\right]$, we can rewrite g(z) as belown:

$$g(u) = \int_{k_0}^{k_1} \frac{1}{v_1 - v_0} \frac{1}{\beta\sqrt{2\pi}} e^{-\frac{u^2}{2}} du$$

, where $k_1 = \frac{\beta}{\sigma}\left[ln\left(x - v_1\right) - \frac{(z-\alpha)}{\beta}\right]$, $k_0 = \frac{\beta}{\sigma}\left[ln\left(x - v_0\right) - \frac{(z-\alpha)}{\beta}\right]$.

Using the approximation of cumulative normal density fucntion, we then have

$g(u) = \frac{1}{v_1 - v_0} \frac{1}{\beta}\left[\Phi\left(k_0\right) - \Phi\left(k_1\right)\right]$, where $\Phi(u) = 1 - \frac{exp\left\{-\frac{u^2}{2}\right\}}{\frac{44}{79} + \frac{8}{5}u + \frac{5}{6}\sqrt{u^2+3}}$ [2].

The likelihood fucntion can be obtained as follows:

$$L\left(\alpha, \beta, \sigma_0^2, v_0, v_1\right) = \prod_{i=1}^{n} g(u_i) = \prod_{i=1}^{n} \frac{1}{v_1 - v_0} \frac{1}{\beta}\left[\frac{exp\left\{-\frac{k_1^2}{2}\right\}}{\frac{44}{79} + \frac{8}{5}k_1 + \frac{5}{6}\sqrt{k_1^2+3}} - \frac{exp\left\{-\frac{k_0^2}{2}\right\}}{\frac{44}{79} + \frac{8}{5}k_0 + \frac{5}{6}\sqrt{k_0^2+3}}\right].$$

By dual annealing, we can obtained the parameters $\alpha, \beta, \sigma_0^2, v_0, v_1$ via maximum likellihood estimation.

Then, we have $\left(\alpha, \beta, \sigma_0^2, v_0, v_1\right)$ = (-9.82447, -8.95156, 0.498934, 0.497639, 0.513974).

For more extensions, you may refer to type 1, type 2 or hybrid censored schemes [3].

References

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