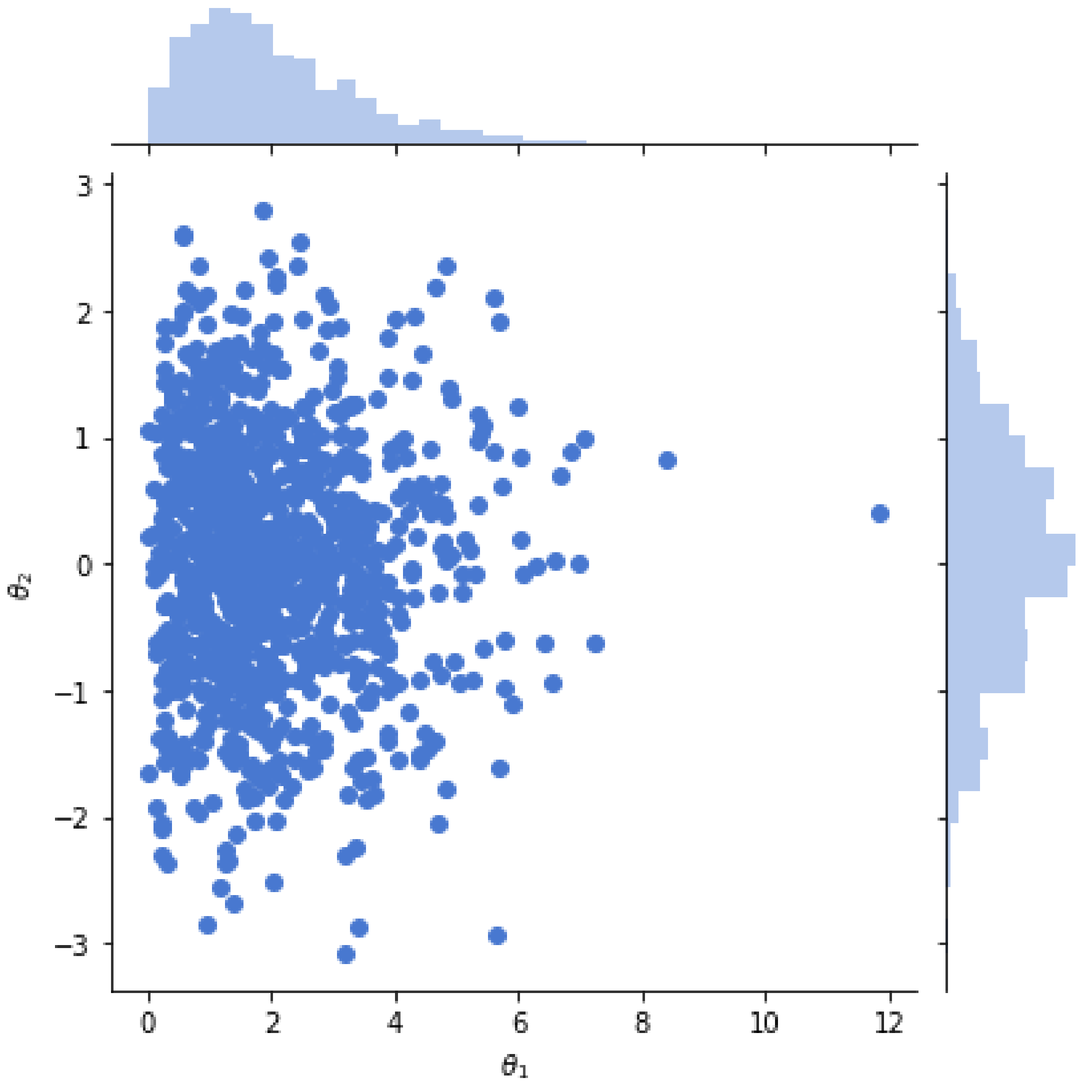
Bayesian Data Analysis Chapter 3

January 19, 2019

```
In [1]: %matplotlib inline
        import numpy as np
        import pymc3 as pm
        import pandas as pd
        from scipy import stats
        import matplotlib.pyplot as plt
        import seaborn as sns
        palette = 'muted'
        sns.set_palette(palette); sns.set_color_codes(palette)
        np.set_printoptions(precision=2)
        pd.set_option('display.precision', 2)
```

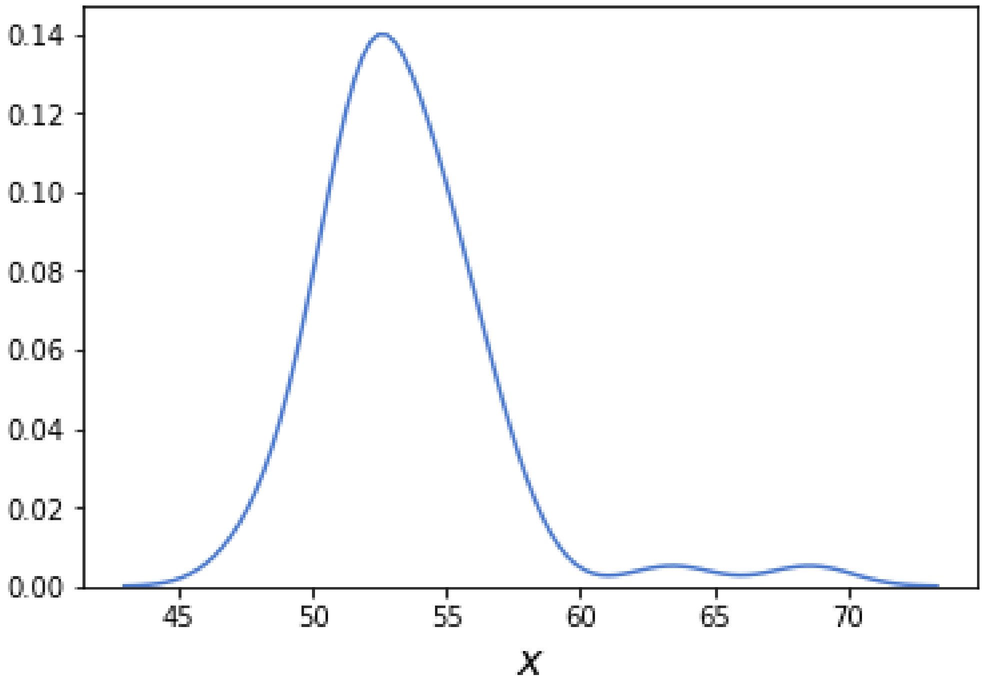
```
In [2]: np.random.seed(123)
        x = np.random.gamma(2, 1, 1000)
        y = np.random.normal(0, 1, 1000)
        data = pd.DataFrame(data=np.array([x, y]).T, columns=['$\\theta_1$', '$\\theta_2$'])
        sns.jointplot(x='$\\theta_1$', y='$\\theta_2$', data=data, stat_func=None);
```



1 Gaussian inferences

```
In [3]: data = np.array([51.06, 55.12, 53.73, 50.24, 52.05, 56.40, 48.45, 52.34, 55.65, 51.49,

        # remove outliers using the interquartile rule
        quant = np.percentile(data, [25, 75])
        iqr = quant[1] - quant[0]
        upper_b = quant[1] + iqr * 1.5
        lower_b = quant[0] - iqr * 1.5
        clean_data = data[(data > lower_b) & (data < upper_b)]

In [4]: sns.kdeplot(data)
        plt.xlabel('$x$', fontsize=16)
```

Out[4]: Text(0.5, 0, 'x')




```
In [5]: with pm.Model() as model_g:
            mu = pm.Uniform('mu', lower=40, upper=70)
            sigma = pm.HalfNormal('sigma', sd=10)
            y = pm.Normal('y', mu=mu, sd=sigma, observed=data)
            trace_g = pm.sample(1100, njobs=1)
```

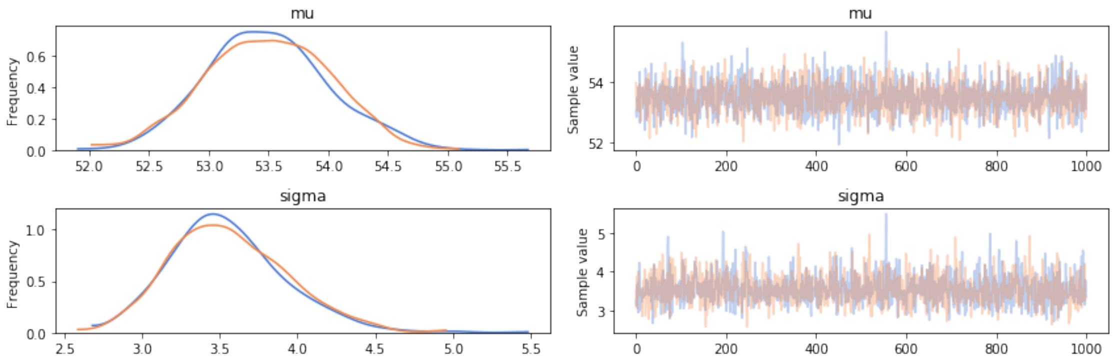
```
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Sequential sampling (2 chains in 1 job)
NUTS: [sigma, mu]
100%|| 1600/1600 [00:01<00:00, 1357.11it/s]
100%|| 1600/1600 [00:00<00:00, 2131.76it/s]
```

1.0.1 Note that the traces of each variable are the marginal distributions not the real posterior which is a 2-dimensionsl joint distribution

```
In [6]: chain_g = trace_g[100:]
        pm.traceplot(chain_g)
```

```
Out[6]: array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f909da45dd8>,
                <matplotlib.axes._subplots.AxesSubplot object at 0x7f909d9dd630>],
               [<matplotlib.axes._subplots.AxesSubplot object at 0x7f909da01860>,
```

```
<matplotlib.axes._subplots.AxesSubplot object at 0x7f909d9a4a90>]],
      dtype=object)
```



In [7]:
```python
pm.summary(chain_g)
```

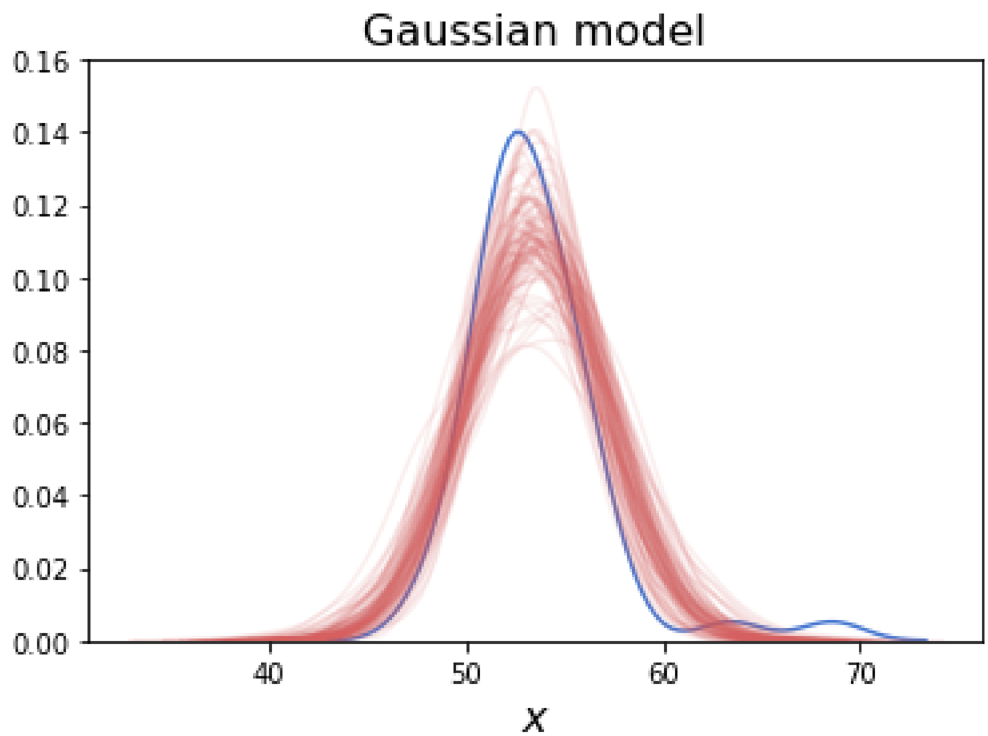
Out[7]:

	mean	sd	mc_error	hpd_2.5	hpd_97.5	n_eff	Rhat
mu	53.49	0.51	9.89e-03	52.51	54.47	2059.13	1.0
sigma	3.55	0.37	9.39e-03	2.87	4.28	1707.96	1.0

In [8]:
```python
y_pred = pm.sample_ppc(chain_g, 100, model_g, size=len(data))
sns.kdeplot(data, color='b')
for i in y_pred['y']:
    sns.kdeplot(i.flatten(), color='r', alpha=0.1)
plt.title('Gaussian model', fontsize=16)
plt.xlabel('$x$', fontsize=16)
```

```
/home/damianos/miniconda3/envs/pymc3/lib/python3.7/site-packages/ipykernel_launcher.py:1: Depre
  """Entry point for launching an IPython kernel.
100%|| 100/100 [00:00<00:00, 1442.89it/s]
```

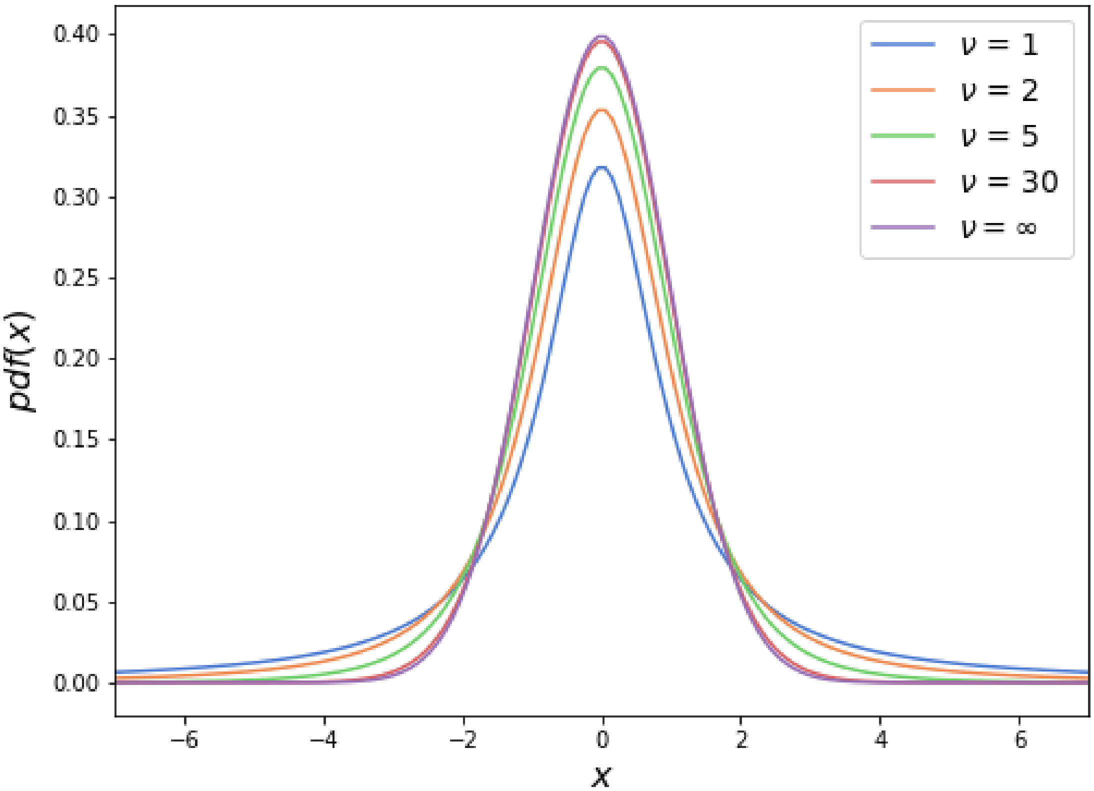
Out[8]:
```
Text(0.5, 0, '$x$')
```



1.1 Gaussian Robust inferences

Now we are going to learn how to make a robust model to outliers, that is a model that do not get too much excited by outliers. For that we are going to use a Student T distribution which can have heavier tails and allows for the presence of outliers

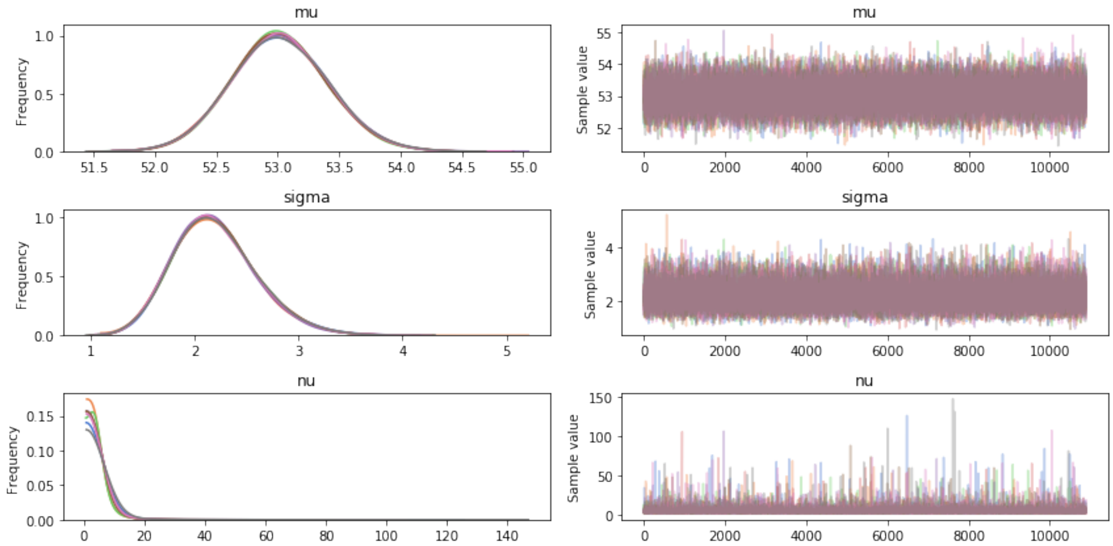
In [9]:
```
plt.figure(figsize=(8, 6))
x_values = np.linspace(-10, 10, 200)
for df in [1, 2, 5, 30]:
    distri = stats.t(df)
    x_pdf = distri.pdf(x_values)
    plt.plot(x_values, x_pdf, label=r'$\nu$ = {}'.format(df))

x_pdf = stats.norm.pdf(x_values)
plt.plot(x_values, x_pdf, label=r'$\nu = \infty$')
plt.xlabel('$x$', fontsize=16)
plt.ylabel('$pdf(x)$', fontsize=16, rotation=90)
plt.legend(loc=0, fontsize=14)
plt.xlim(-7, 7);
```




```
In [10]: with pm.Model() as model_t:
             mu = pm.Uniform('mu', 40, 75)
             sigma = pm.HalfNormal('sigma', sd=10)
             nu = pm.Exponential('nu', 1/30)
             y = pm.StudentT('y', mu=mu, sd=sigma, nu=nu, observed=data)
             trace_t = pm.sample(11000,njobs=8)
```

```
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (8 chains in 8 jobs)
NUTS: [nu, sigma, mu]
Sampling 8 chains: 100%|| 92000/92000 [00:21<00:00, 4206.86draws/s]
```

```
In [11]: chain_t = trace_t[100:]
         pm.traceplot(chain_t);
```




```
In [12]: pm.summary(chain_t)
```

Out[12]:

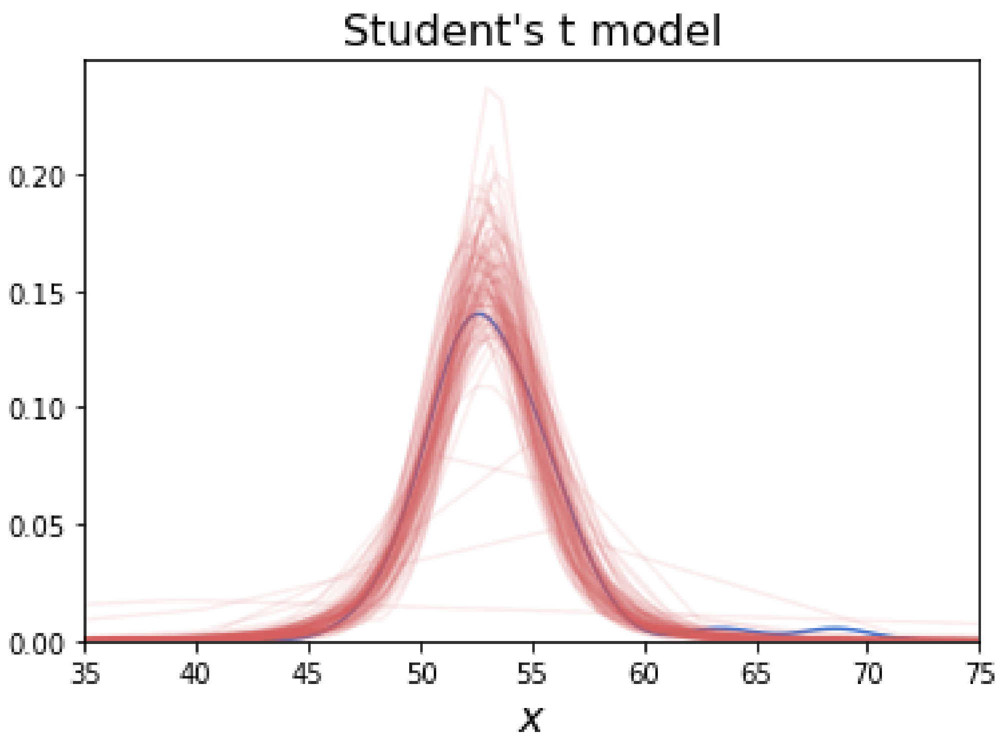
	mean	sd	mc_error	hpd_2.5	hpd_97.5	n_eff	Rhat
mu	53.01	0.39	1.71e-03	52.24	53.76	59098.57	1.0
sigma	2.19	0.40	2.07e-03	1.45	3.00	41200.08	1.0
nu	4.63	4.14	2.62e-02	1.15	10.18	28619.67	1.0

we can see that ν~4 which indicates heavier tails than the normal when ν->inf

```
In [13]: y_pred = pm.sample_ppc(chain_t, 100, model_t, size=len(data))
         sns.kdeplot(data, c='b')
         for i in y_pred['y']:
             sns.kdeplot(i.flatten(), c='r', alpha=0.1)
         plt.xlim(35, 75)
         plt.title("Student's t model", fontsize=16)
         plt.xlabel('$x$', fontsize=16)
```

```
/home/damianos/miniconda3/envs/pymc3/lib/python3.7/site-packages/ipykernel_launcher.py:1: Depre
  """Entry point for launching an IPython kernel.
100%|| 100/100 [00:00<00:00, 1081.86it/s]
/home/damianos/miniconda3/envs/pymc3/lib/python3.7/site-packages/matplotlib/cbook/__init__.py:
  seen=seen, canon=canonical, used=seen[-1]))
```

```
Out[13]: Text(0.5, 0, '$x$')
```



2 Tips example

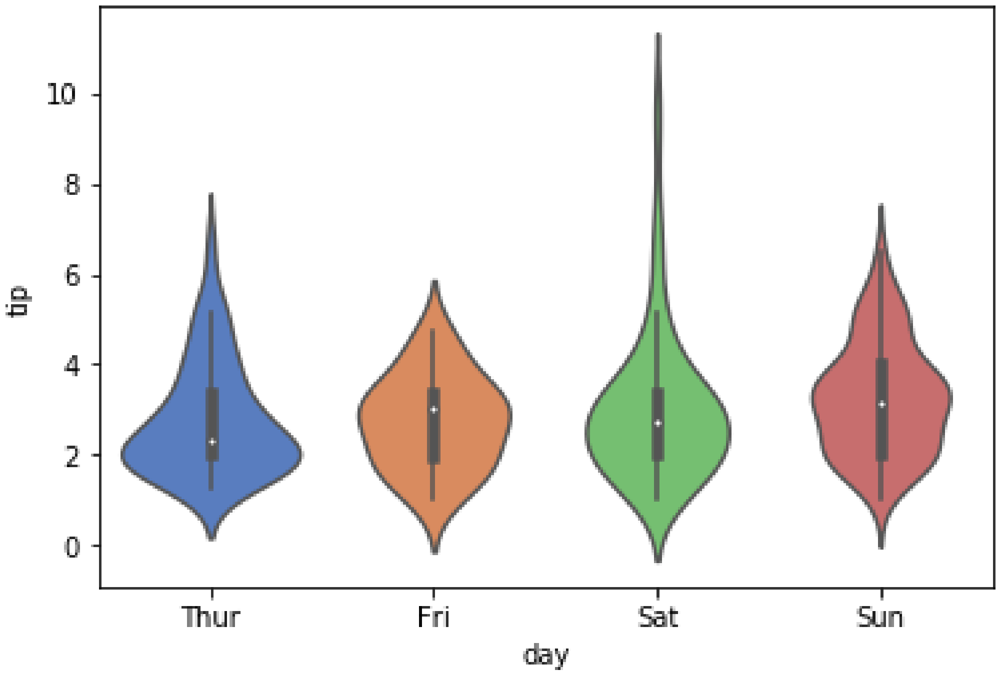
2.1 we want to investigate the effect the day of the week has on the tips

```
In [14]: tips = sns.load_dataset('tips')
         tips.tail()
```

Out[14]:

	total_bill	tip	sex	smoker	day	time	size
239	29.03	5.92	Male	No	Sat	Dinner	3
240	27.18	2.00	Female	Yes	Sat	Dinner	2
241	22.67	2.00	Male	Yes	Sat	Dinner	2
242	17.82	1.75	Male	No	Sat	Dinner	2
243	18.78	3.00	Female	No	Thur	Dinner	2

```
In [15]: sns.violinplot(x='day', y='tip', data=tips);
```



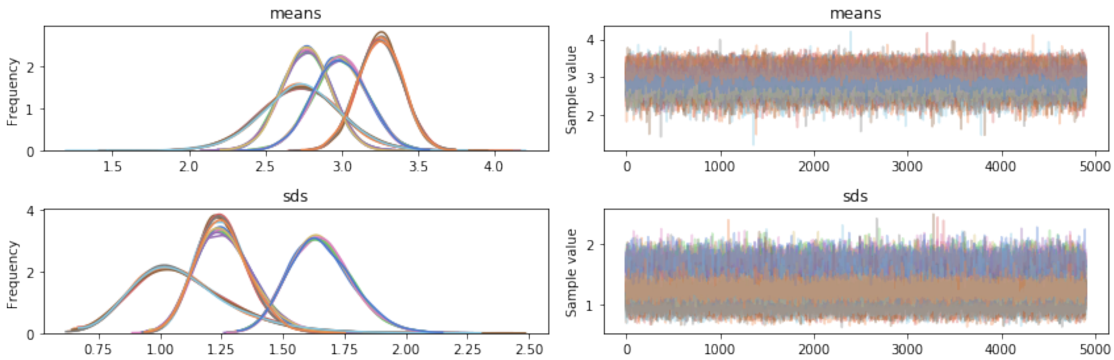

```
In [16]: y = tips['tip'].values
         x = pd.Categorical(tips['day']).codes
         categories = pd.Categorical(tips['day']).categories
         print('Tips sample: {}'.format(y[:5]))
         print('Encoded days sample: {}'.format(x[:5]))
         print('Decoded days sample: {}'.format(categories[x[:5]]))
```

```
Tips sample: [1.01 1.66 3.5  3.31 3.61]
Encoded days sample: [3 3 3 3 3]
Decoded days sample: Index(['Sun', 'Sun', 'Sun', 'Sun', 'Sun'], dtype='object')
```

```
In [17]: with pm.Model() as comparing_groups:
             # priors
             means = pm.Normal('means', mu=0, sd=10, shape=len(set(x)))
             sds = pm.HalfNormal('sds', sd=10, shape=len(set(x)))
             # a different trace for each weekday as encoded in `x`
             y_pred = pm.Normal('y', mu=means[x], sd=sds[x], observed=y)
             trace_cg = pm.sample(5000, chains=8, njobs=8)
```

```
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (8 chains in 8 jobs)
NUTS: [sds, means]
Sampling 8 chains: 100%|| 44000/44000 [00:21<00:00, 2066.64draws/s]
```

In [18]:
```python
chain_cg = trace_cg[100::]
pm.traceplot(chain_cg);
```



In [19]:
```python
pm.summary(chain_cg)
```

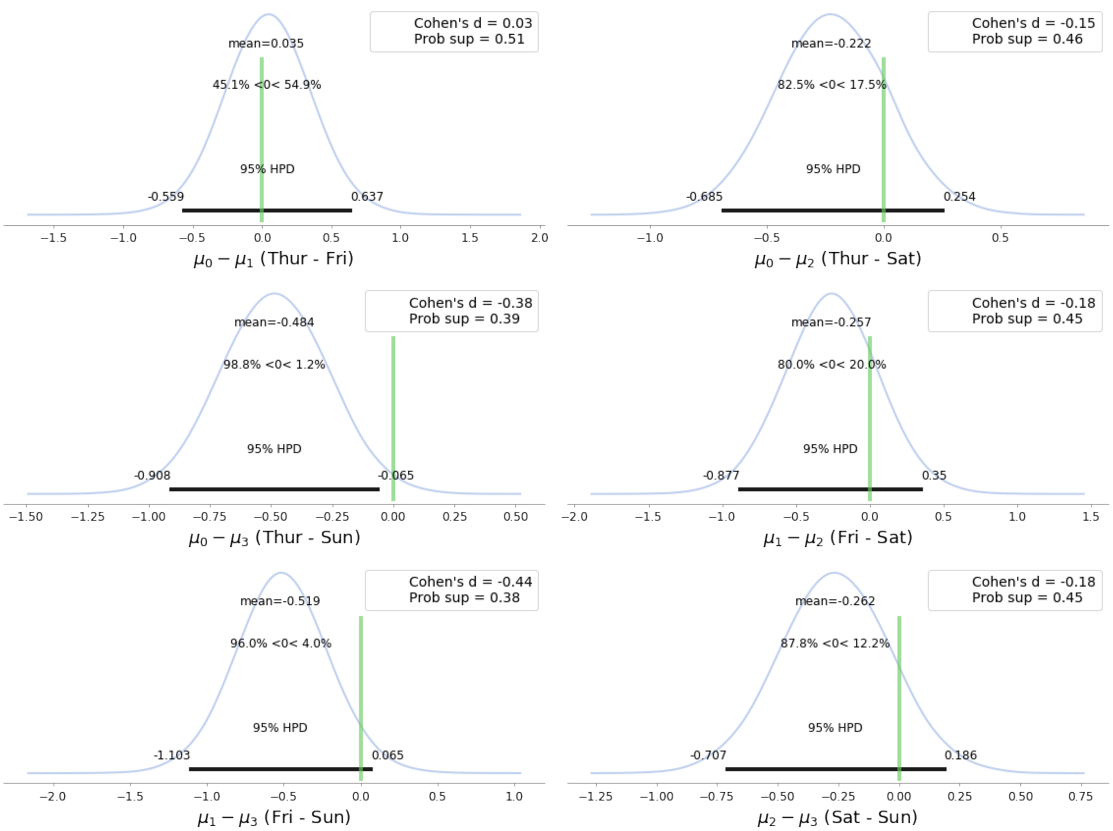
Out[19]:

	mean	sd	mc_error	hpd_2.5	hpd_97.5	n_eff	Rhat
means__0	2.77	0.16	7.14e-04	2.46	3.09	44085.42	1.0
means__1	2.73	0.26	1.22e-03	2.22	3.24	39073.35	1.0
means__2	2.99	0.18	7.78e-04	2.65	3.34	43876.68	1.0
means__3	3.25	0.14	6.19e-04	2.98	3.54	43603.40	1.0
sds__0	1.27	0.12	5.50e-04	1.05	1.50	40396.75	1.0
sds__1	1.10	0.20	1.01e-03	0.75	1.50	35760.93	1.0
sds__2	1.66	0.13	5.63e-04	1.41	1.91	44519.30	1.0
sds__3	1.26	0.11	4.91e-04	1.06	1.47	40481.08	1.0

In [20]:
```python
dist = dist = stats.norm()

_, ax = plt.subplots(3, 2, figsize=(16, 12))

comparisons = [(i,j) for i in range(4) for j in range(i+1, 4)]
pos = [(k,l) for k in range(3) for l in (0, 1)]

print('Comparison between the followin encoded days: {}'.format(comparisons))
print('Subplot indices: {}'.format(pos))
for (i, j), (k,l) in zip(comparisons, pos):
    # mean difference between days to be compared
    means_diff = chain_cg['means'][:,i]-chain_cg['means'][:,j]
    # effect size via Cohen's delta
    d_cohen = (means_diff / np.sqrt((chain_cg['sds'][:,i]**2 + chain_cg['sds'][:,j]**2
    # probability of superiority
    ps = dist.cdf(d_cohen/(2**0.5))

    pm.plot_posterior(means_diff, ref_val=0, ax=ax[k, l], kde_plot=True, lw=2)
    ax[k, l].plot(0, label="Cohen's d = {:.2f}\nProb sup = {:.2f}".format(d_cohen, ps
```

```
        ax[k, l].set_xlabel('$\mu_{}-\mu_{}$ ({} - {})'.format(i, j,categories[i],categor
        ax[k,l ].legend(loc=0, fontsize=14)


    plt.tight_layout()
```

```
Comparison between the followin encoded days: [(0, 1), (0, 2), (0, 3), (1, 2), (1, 3), (2, 3)]
Subplot indices: [(0, 0), (0, 1), (1, 0), (1, 1), (2, 0), (2, 1)]
```



2.1.1 **We can see that the only comparison where HPD95% excludes the reference values of zero is Thursday and Sunday. For the rest we cannot rule out a difference of zero. But the mean difference for Thursday - Sundsay is ~0.5, is this worth working weekends? Statisticians cannot answer these kind of questions**

2.1.2 **For Cohen's delta see https://en.wikipedia.org/wiki/Effect_size#Cohen's_d**

2.1.3 **Probability of superiority $ps = \Phi(\frac{\delta}{\sqrt(2)})$ if we assume the group disrtibutions are Normal (not really used much)**

3 Hierarchical Models

In [21]:
```
N_samples =  [30, 30, 30]
# Number of good samples, i.e samples with lead content below WHO recommendations
G_samples =  [18, 3, 3] # [13, 3, 3] [18, 3, 3] [18, 18, 18]

group_idx = np.repeat(np.arange(len(N_samples)), N_samples)
data = []
for i in range(0, len(N_samples)):
    data.extend(np.repeat([1, 0], [G_samples[i], N_samples[i]-G_samples[i]]))
print('Group id of each sample: {}'.format(group_idx))
print('Samples : (0->Bad, 1->Good) {}'.format(data))
print('Total number of samples = {}'.format(len(data)))
```

```
Group id of each sample: [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 
 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2
 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2]
Samples : (0->Bad, 1->Good) [1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 0, 0, 0, 0,
Total number of samples = 90
```

$$\alpha \sim HalfCauchy(\beta_\alpha)$$

$$\beta \sim HalfCauchy(\beta_\beta)$$

$$\theta \sim Beta(\alpha, \beta)$$

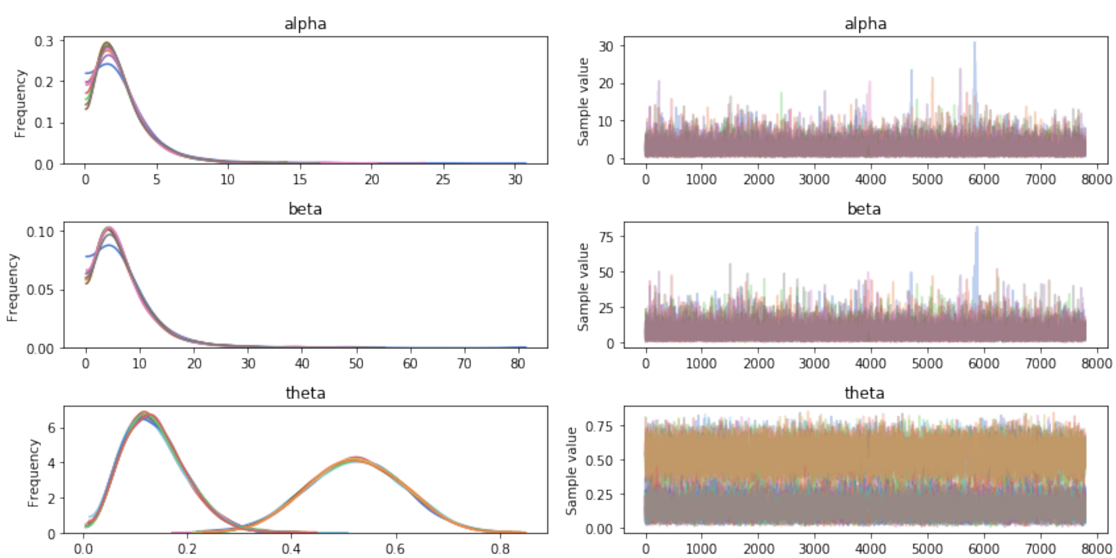
$$y \sim Bern(\theta)$$

In [22]:
```
with pm.Model() as model_h:
    # hyper-priors
    alpha = pm.HalfCauchy('alpha', beta=10)
    beta = pm.HalfCauchy('beta', beta=10)
    # prior
    theta = pm.Beta('theta', alpha, beta, shape=len(N_samples))
    # likelihood
    y = pm.Bernoulli('y', p=theta[group_idx], observed=data)

    trace_h = pm.sample(8000,njobs=8,chains=8)
```

```
Auto-assigning NUTS sampler...
Initializing NUTS using jitter+adapt_diag...
Multiprocess sampling (8 chains in 8 jobs)
NUTS: [theta, beta, alpha]
Sampling 8 chains: 100%|| 68000/68000 [00:24<00:00, 2787.89draws/s]
There were 71 divergences after tuning. Increase `target_accept` or reparameterize.
There were 14 divergences after tuning. Increase `target_accept` or reparameterize.
There were 31 divergences after tuning. Increase `target_accept` or reparameterize.
There were 46 divergences after tuning. Increase `target_accept` or reparameterize.
There were 33 divergences after tuning. Increase `target_accept` or reparameterize.
There were 36 divergences after tuning. Increase `target_accept` or reparameterize.
There were 104 divergences after tuning. Increase `target_accept` or reparameterize.
There were 54 divergences after tuning. Increase `target_accept` or reparameterize.
The number of effective samples is smaller than 10% for some parameters.
```

In [23]:
```python
chain_h = trace_h[200:]
pm.traceplot(chain_h)
```

Out[23]:
```
array([[<matplotlib.axes._subplots.AxesSubplot object at 0x7f9083733208>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7f9083758e10>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x7f90836fa080>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7f90837142b0>],
       [<matplotlib.axes._subplots.AxesSubplot object at 0x7f90836b04e0>,
        <matplotlib.axes._subplots.AxesSubplot object at 0x7f90836cc710>]],
      dtype=object)
```



In [24]:
```python
pm.summary(chain_h)
```

Out[24]:

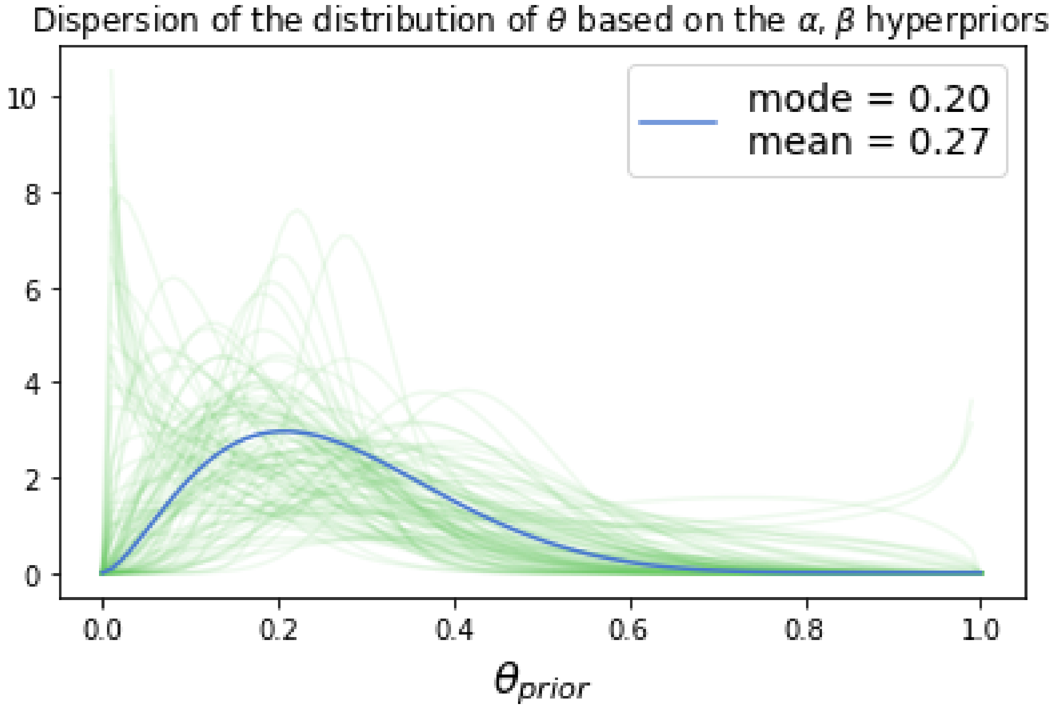
	mean	sd	mc_error	hpd_2.5	hpd_97.5	n_eff	Rhat
alpha	2.66	2.08	2.95e-02	0.16	6.41	4278.79	1.0
beta	7.34	5.86	8.26e-02	0.31	17.83	4362.81	1.0
theta__0	0.52	0.09	5.41e-04	0.35	0.70	28992.11	1.0
theta__1	0.14	0.06	3.47e-04	0.03	0.26	32777.08	1.0
theta__2	0.14	0.06	3.03e-04	0.03	0.26	32472.88	1.0

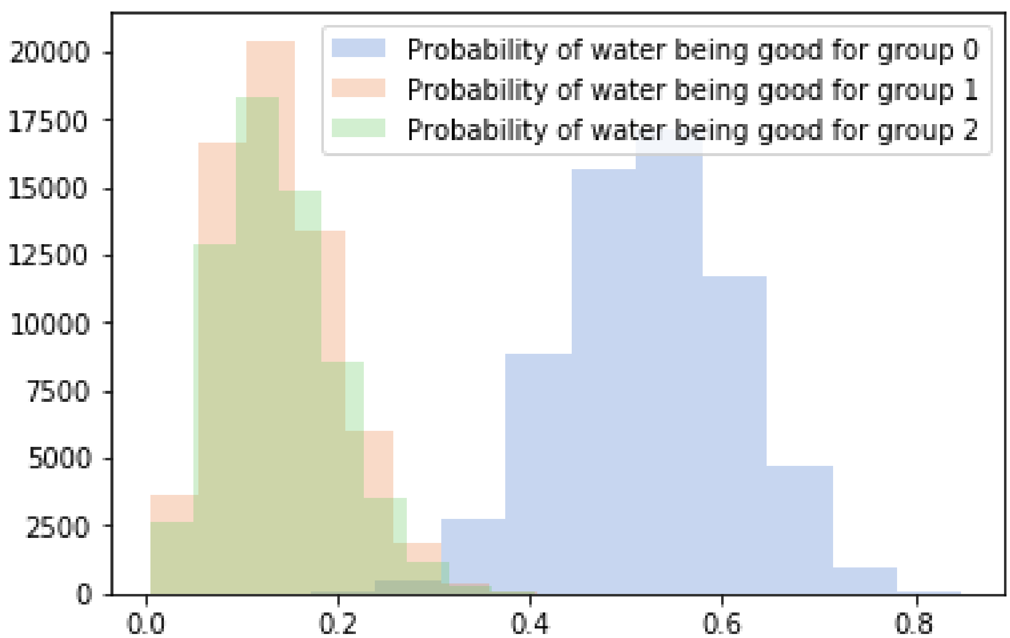
In [25]:

```
x = np.linspace(0, 1, 100)
print(chain_h['theta'].shape)

for i in np.random.randint(0, len(chain_h), size=100):
    pdf = stats.beta(chain_h['alpha'][i], chain_h['beta'][i]).pdf(x)
    plt.plot(x, pdf,  'g', alpha=0.1)

dist = stats.beta(chain_h['alpha'].mean(), chain_h['beta'].mean())
pdf = dist.pdf(x)
mode = x[np.argmax(pdf)]
mean = dist.moment(1)
plt.plot(x, pdf, label='mode = {:.2f}\nmean = {:.2f}'.format(mode, mean))
plt.title(r'Dispersion of the distribution of $\theta$ based on the $\alpha, \beta$ hy

plt.legend(fontsize=14)
plt.xlabel('$\\theta_{prior}$', fontsize=16)
plt.tight_layout()

plt.figure()
for i, thetaChain in enumerate(chain_h['theta'].T):
    plt.hist(thetaChain,alpha=0.3, label='Probability of water being good for group {
plt.legend()
```

```
(62400, 3)
```

Out[25]: <matplotlib.legend.Legend at 0x7f908371eba8>






```
In [26]: import sys, IPython, scipy, matplotlib, platform
         print("This notebook was created on a %s computer running %s and using:\nPython %s\nI
```

```
This notebook was created on a x86_64 computer running debian buster/sid and using:
Python 3.7.2
IPython 7.2.0
PyMC3 3.6
NumPy 1.16.0
SciPy 1.2.0
Matplotlib 3.0.2
Seaborn 0.9.0
Pandas 0.23.4
```

```
/home/damianos/miniconda3/envs/pymc3/lib/python3.7/site-packages/ipykernel_launcher.py:2: Depre
```