



- Cheating/Overwriting will result in a zero score.
- No exchange of items is permitted.
- Total time: 10 minutes.

Q-1: Mark appropriate answer for following. (Objective)**(CLO-4.1)**

Marry is a software engineer at TechGenius Inc., working on optimizing algorithms for their new data processing tool. Help Marry solve these algorithmic challenges by selecting the best answer.

1	<p>Marry writes a recursive function:</p> <pre>int f(int n) { if (n <= 1) return 1; return f(n - 1) + f(n - 2); }</pre> <p>What is the space complexity for above recursive function?</p> <p>A) O(1) B) O(n) C) O(n^2) D) O(2^n)</p>
2	<p>Marry has $T(n) = 2T(n/2) + n$. What is the time complexity?</p> <p>A) O(n) B) O(n log n) C) O(n^2) D) O(log n)</p>
3	<p>Marry replaces a recursive algorithm with an iterative one. What is likely improved?</p> <p>A) Time complexity B) Space complexity C) Readability D) Input size</p>
4	<p>Marry compares recursive and iterative factorial implementations. Which is true?</p> <p>A) Recursive has better time complexity. B) Iterative has better space complexity (O(1)). C) Recursive is always faster. D) Iterative requires a stack.</p>
5	<p>Marry has $T(n) = T(n-1) + n$. Can the Master Theorem be used?</p> <p>A) Yes, Case 1 applies B) Yes, Case 3 applies C) No, not in standard form D) Only for $n \leq 1$</p>



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- **Total time: 80 minutes.**
- **Time management tip:** Allocate approximately 20 minutes per question.

(Subjective Part)

Q-2: Design an algorithm which takes input of population of 15 countries and sort them in ascending order. Discuss its complexity. (5 marks)

(CLO 5.1)

If we're using insertion sort,

1. Algorithm Name:

SortPopulationOfCountries

2. Input:

A list of populations for 15 countries: $P[1..15]$

3. Output:

The list P sorted in ascending order.

4. Algorithm Steps (Using Insertion Sort for clarity and simplicity):

Algorithm SortPopulationOfCountries ($P[1..15]$)

// Input: An array P of population values for 15 countries

// Output: Sorted array P in ascending order

1. for $i \leftarrow 2$ to 15 do
2. key $\leftarrow P[i]$
3. $j \leftarrow i - 1$
4. while $j > 0$ and $P[j] > key$ do
5. $P[j + 1] \leftarrow P[j]$
6. $j \leftarrow j - 1$
7. end while
8. $P[j + 1] \leftarrow key$



9. end for

10. return P

Case Time Complexity

Best Case O(n)

Average Case O(n^2)

Worst Case O(n^2)

If use built in function of sorting in python:

1. Algorithm Name:

SortPopulationUsingBuiltIn

2. Input:

A list $P[1..15]$ of population values.

3. Output:

List P sorted in ascending order.

4. Algorithm Steps:

```
Algorithm SortPopulationUsingBuiltIn(P[1..15])
// Input: A list P of 15 population values
// Output: Sorted list in ascending order
```

1. $P.sort()$
2. return P

Python code:

```
def sort_population(populations):
    populations.sort() # in-place sorting
    return populations
```

Example usage:

```
populations = [10234, 5000, 20450, 11000, 6000, 8700, 7600, 9200, 13400, 3000, 4500, 2500,
19000, 14000, 1000]
```



```
sorted_pop = sort_population(populations)
print("Sorted populations:", sorted_pop)
```

Python's `sort()` uses **Timsort**, with:

- **Best Case:** $O(n)$
- **Average Case:** $O(n \log n)$
- **Worst Case:** $O(n \log n)$

Q-3: Solve the following recurrence relation using substitution method and verify it using master theorem. (10 marks)

$$T(n) = \begin{cases} 2T\left(\frac{7n}{14}\right) + c, & \text{if } n > 1 \\ 1 & \text{if } n = 1 \end{cases}$$

' $T(n) = 2T\left(\frac{7n}{14}\right) + c$ ' can be written as:

$$T(n) = 2T\left(\frac{7(n)}{7(2)}\right) + c$$

$$T(n) = 2T\left(\frac{n}{2}\right) + c$$

$$\begin{aligned} T(n) &= T(n/2) + c \quad \text{--- (1)} \\ T(n/2) &= T\left(\frac{n/2}{2}\right) + c \quad \text{putting } n=n/2 \\ T(n/4) &= T\left(\frac{n/4}{2}\right) + c \quad \text{--- (2)} \\ T(n/8) &= T\left(\frac{n/8}{2}\right) + c \quad \text{--- (3)} \\ \text{Substitute (2) in (1)} \\ T(n) &= T(n/4) + c + c \Rightarrow T(n/4) + 2c \\ T(n) &= T\left(\frac{n}{4^2}\right) + 2c \Rightarrow T\left(\frac{n}{2^2}\right) + c + 2c \\ T(n) &= T\left(\frac{n}{2^3}\right) + 3c \Rightarrow T\left(\frac{n}{2^4}\right) + 3c \\ T(n) &= \left\{T\left(\frac{n}{2^4}\right) + c\right\} + 3c = T\left(\frac{n}{16}\right) + 4c \\ T(n) &= T\left(\frac{n}{2^4}\right) + 4c \end{aligned}$$

for 'k' number of steps,

$$T(n) = T\left(\frac{n}{2^k}\right) + kc \quad \text{--- (A)}$$

Base cond. is $T(1) = 1$

substitute $n = 2^k$ in (A)

(A) becomes,

$$\begin{aligned} T(n) &= T\left(\frac{n}{n}\right) + kc \\ &= T(1) + kc \end{aligned}$$

$$T(n) = 1 + kc \quad \text{--- (B)}$$

$n = 2^k \Leftrightarrow$ Taking log on both sides

$$\begin{aligned} \log(n) &= \log(2^k) \\ \Rightarrow \log_2(n) &= k \log_2 2 \Rightarrow k = \log_2(n)/1 = \log_2(n) \\ T(n) &= 1 + \{\log_2(n)\} \cdot c \Leftrightarrow \text{from B} \\ T(n) &= O(\log_2(n)) \end{aligned}$$

Q-4: Explain the difference between Pseudocode and Algorithm. Explain with examples. (5 marks) **(CLO 2.1)**



Difference Between Algorithm and Pseudocode

Feature	Algorithm	Pseudocode
Definition	A step-by-step procedure to solve a problem.	A way of expressing the algorithm using code-like structure (language-agnostic).
Formality	More general and descriptive.	Closer to actual code structure (but not real code).
Language	Written in plain English or structured text.	Uses code-like syntax but is not tied to any specific programming language.
Purpose	To describe <i>what</i> the solution is.	To describe <i>how</i> the solution can be implemented.
Audience	Suitable for beginners or explaining logic.	Suitable for developers or technical audiences who will implement the logic.

Example Problem: Find the Maximum of 3 Numbers

Algorithm

Name: FindMaximumOfThree

Input: Three numbers a, b, c

Output: The largest of the three numbers

Steps:

1. Start
2. Read three numbers: a, b, c
3. If $a \geq b$ and $a \geq c$, then $\text{max} = a$
4. Else if $b \geq a$ and $b \geq c$, then $\text{max} = b$
5. Else, $\text{max} = c$
6. Output max
7. End

Pseudocode

```

Algorithm FindMaximum(a, b, c)
    if a >= b and a >= c then
        max ← a
    else if b >= a and b >= c then
        max ← b
    else
        max ← c
    end if
    return max
End Algorithm
  
```