Fault tolerant architecture for quantum computation using electrically controlled semiconductor quantum dots

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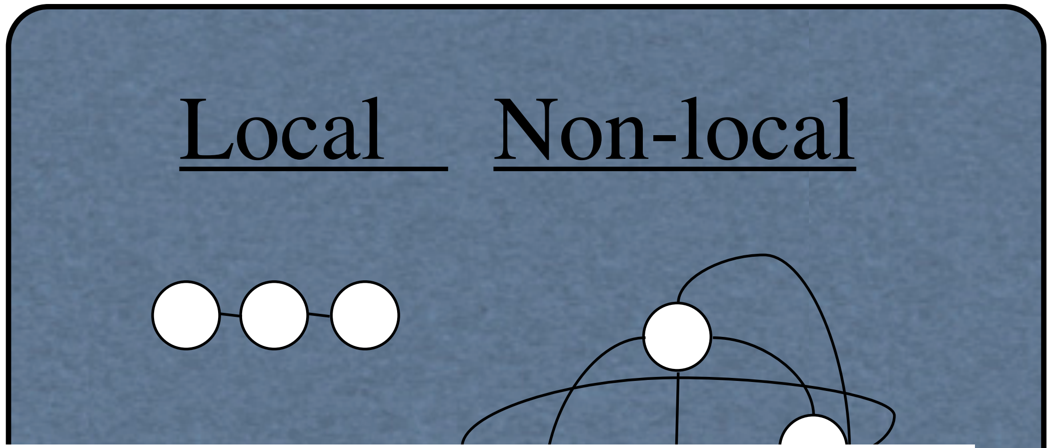
Focus of this talk

- Coherent spin manipulation in dots
  - overview
  - hyperfine-related spin dephasing
- Recent advances
  - electrical control of a two-spin system
  - realization of a dynamical DFS
- Elements of a scalable architecture
  - gates within the DFS
  - non-local coupling
  - integration
- Fault tolerance: a threshold analysis

Why solid-state?

- The good: large scale integration, long term stability



- The bad: most qubit

If we can make 2 excellent qubits AND a high fidelity quantum "bus": scalability(???)

- The ugly: strongly coupled local environment

Our general approach to fault tolerant computation

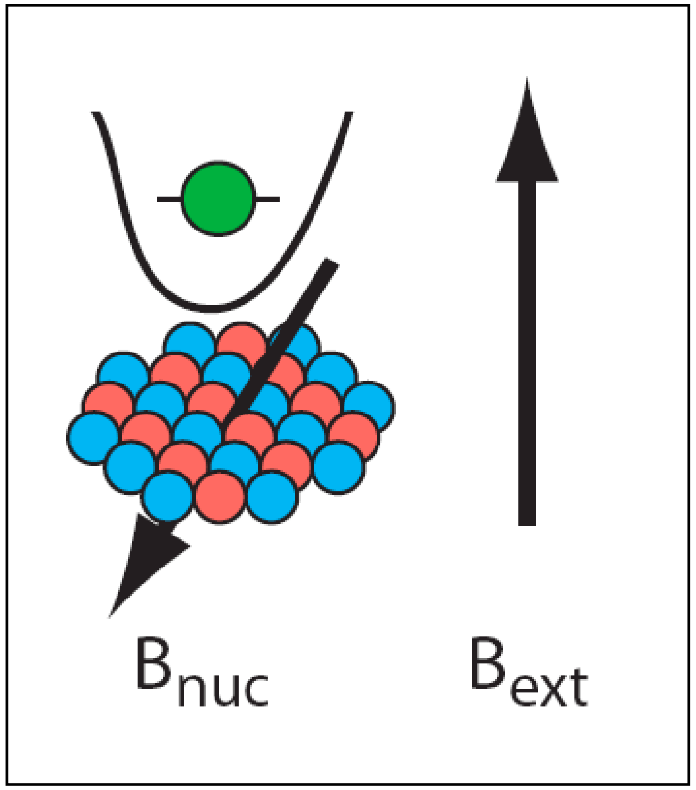
- Identify dominant noise mechanisms
  - Quantum dots: hyperfine interactions
    [Merkulov PRB 2002, Khaetskii PRL 2002, Erlingsson PRB 2002]
- Choose qubit basis to protect against this; engineer devices to minimize this
- Develop long-distance transport, teleportation, or connection mechanism
- Use modular devices for parallelism

Coherent spin manipulation in quantum dots

- Single electron spin in a quantum dot
  - Achieved:
    Control electron number from 0,1,2...
    Single-charge measurement using QPC or SET
    State selective "ionization" for spin measmt.
  - Missing: coherent spin rotations by, e.g., ESR
    - Why? Dominant noise: hyperfine (nuclei)
      High power, low frequency
      phase noise: $T2^* = 10$ ns!

Hyperfine interaction in quantum dots

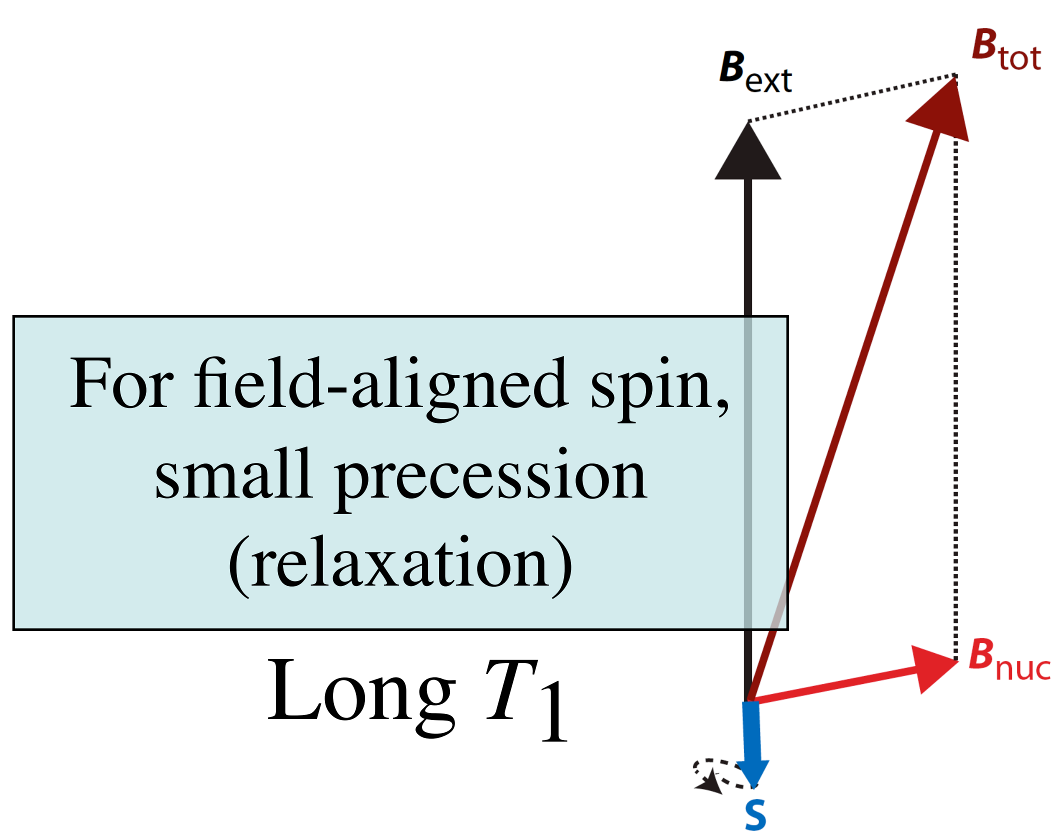
- Contact interaction with conduction band e⁻
  - Large orbital level spacing, confinement suppresses spin-orbit terms
- Effective magnetic field
  - Overhauser shift
  - Randomly oriented
  - Slowly varying
    - Low frequency "noise"
    - Quasi-static approximation

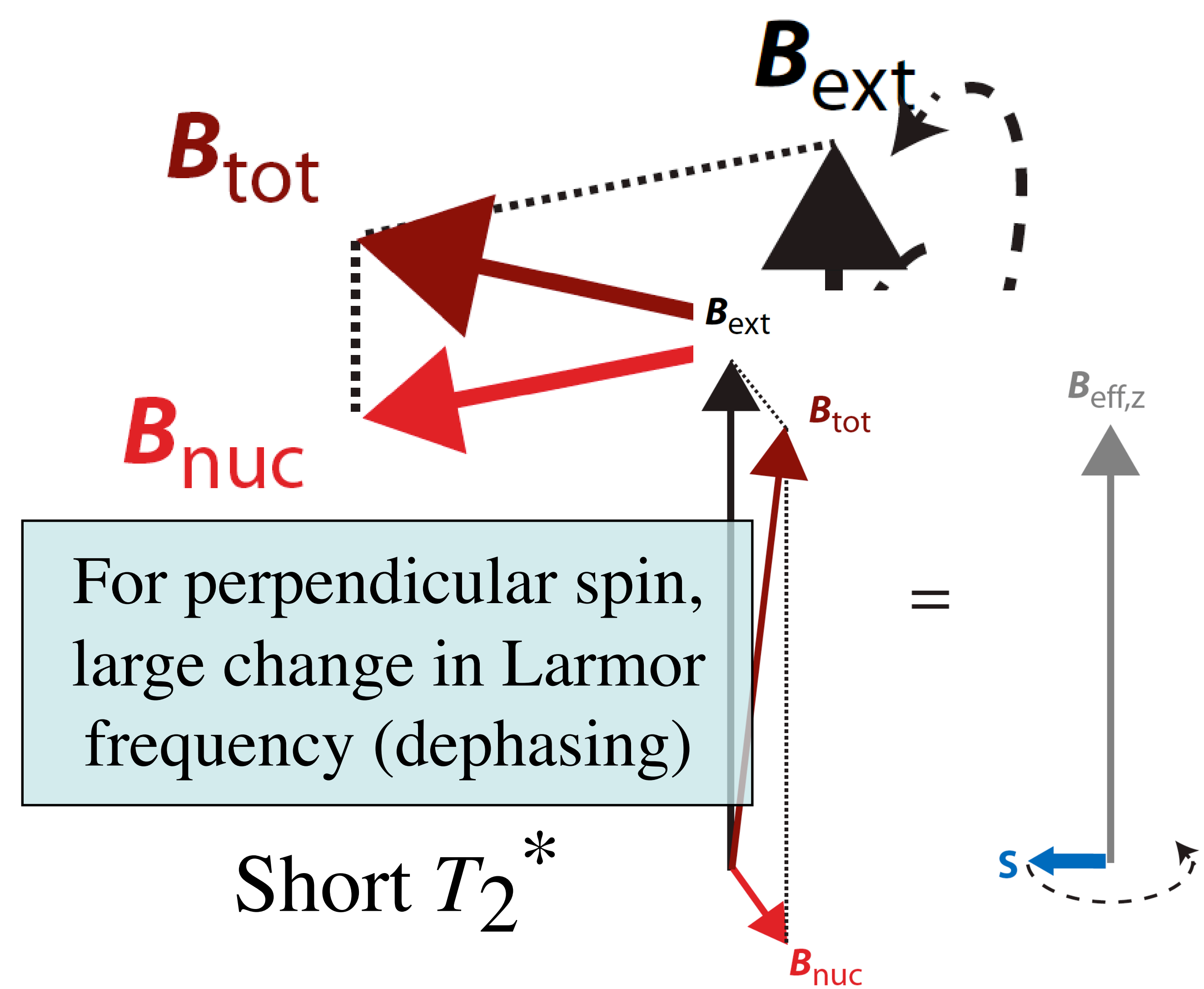


$$H = g^* \mu_B B_{ext} S_z + A \vec{S} \cdot \sum \vec{I}^{(k)}$$

Nuclear spin-driven relaxation and dephasing

- Effective field picture: $$H_{hf} = g^* \mu_B (\underbrace{\mathbf{B}_{ext} + \mathbf{B}_{nuc}}_{\mathbf{B}_{eff}})\hat{\mathbf{S}}$$
  - Effective nuclear magnetic field $$\langle \vec{B}_{nuc} \rangle = \frac{A}{g^* \mu_B} \sum \langle \vec{I}^{(k)} \rangle$$
- $|B_{ext}| >> |B_{nuc}|$

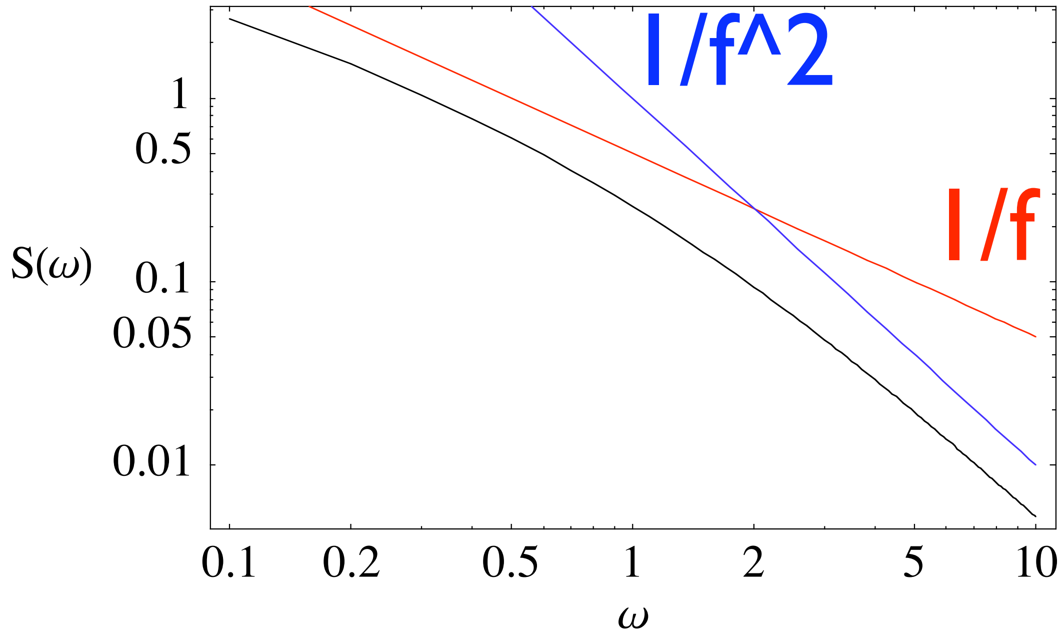




Fast dephasing, long correlation time

- Interaction with electron spin fast: $T_2^* = N^{1/2}/A \sim 10$ ns!
- Nuclear spin evolution slow ($\gg T_2^*$): dipole-dipole and Knight-shift terms weak
- Correlation times approach seconds. Use a spectral function approach in RWA
- $H_{\text{eff}} = [B_{\text{ext}} + \omega(t)]\hat{S}_z$

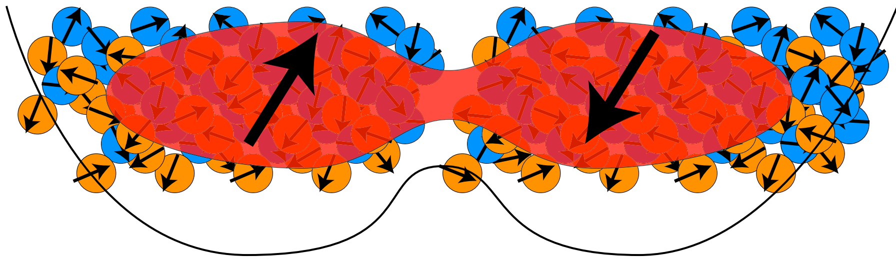
$$\langle \omega(t)\omega(t')\rangle = \int d\nu\ S(\nu) e^{i\nu(t-t')}$$



Recent advances

A new system: two spins as a qubit

- Work with two electrons, double-well potential
- Access three possible regimes:
  - Separated (no tunnel coupling)
  - Exchanging (finite tunnel coupling, but charges in separate dots)
  - Biased (charges in same dot allowed)

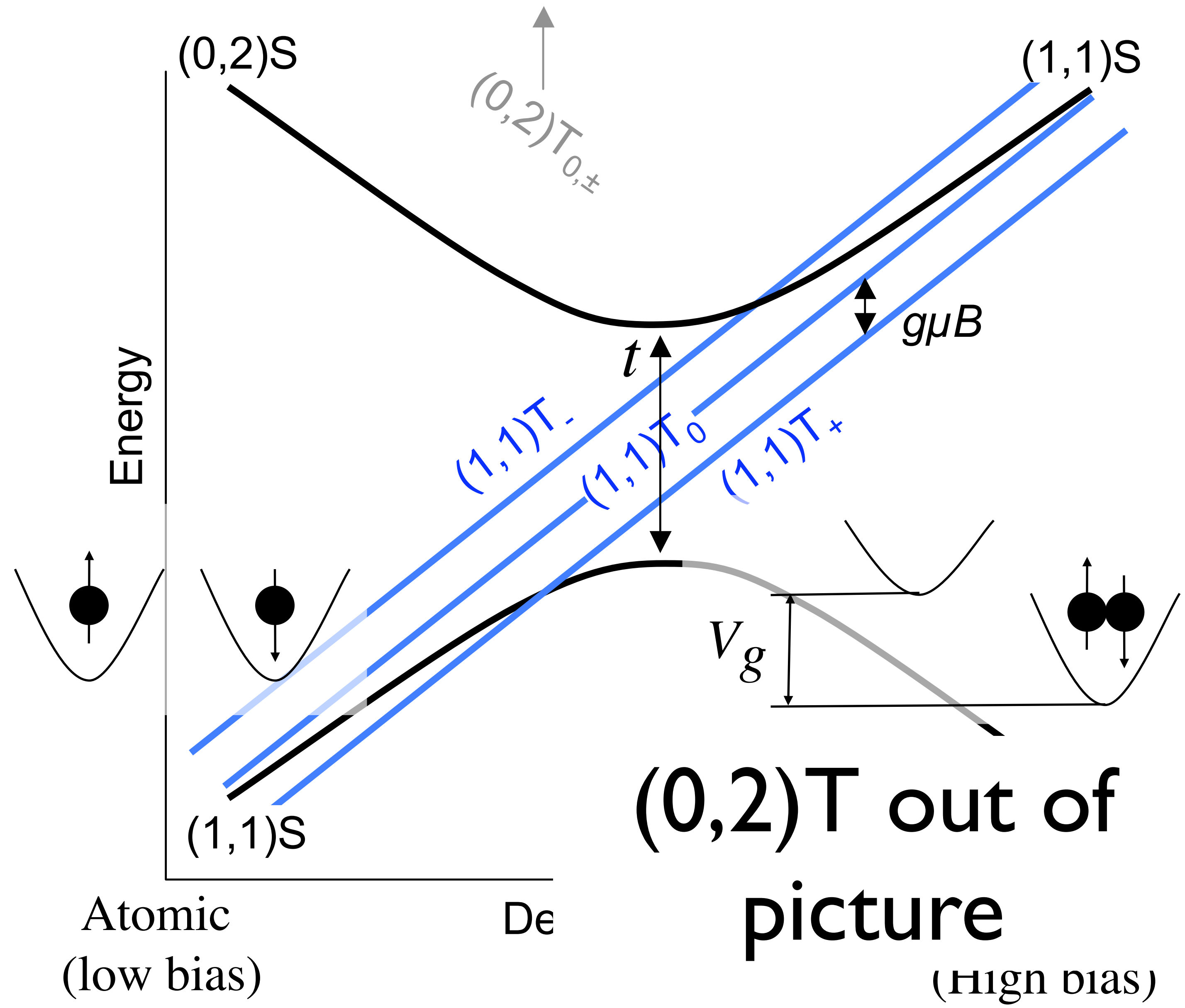


$$V = J\vec{S}_1 \cdot \vec{S}_2$$

- Exchange interaction: fast (sub-ns), controlled by changing potential of the double-dot: electrical!

Relevant states for two-spin system

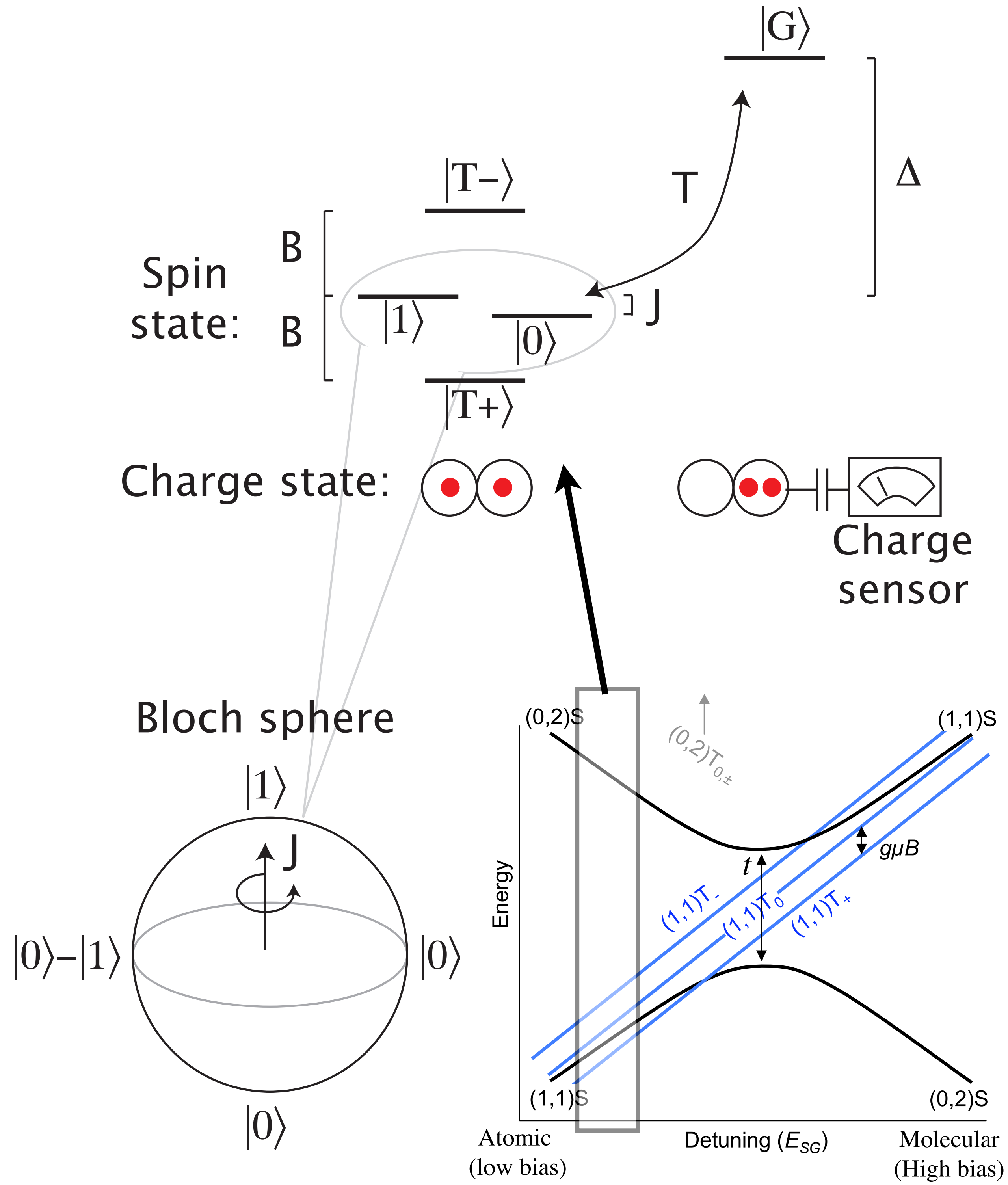
- Control the charges by changing the bias between wells
- Access atomic (two separated) and molecular (same potential) regimes
- In between: finite exchange, can make a gate by bias control



(0,2)T out of picture

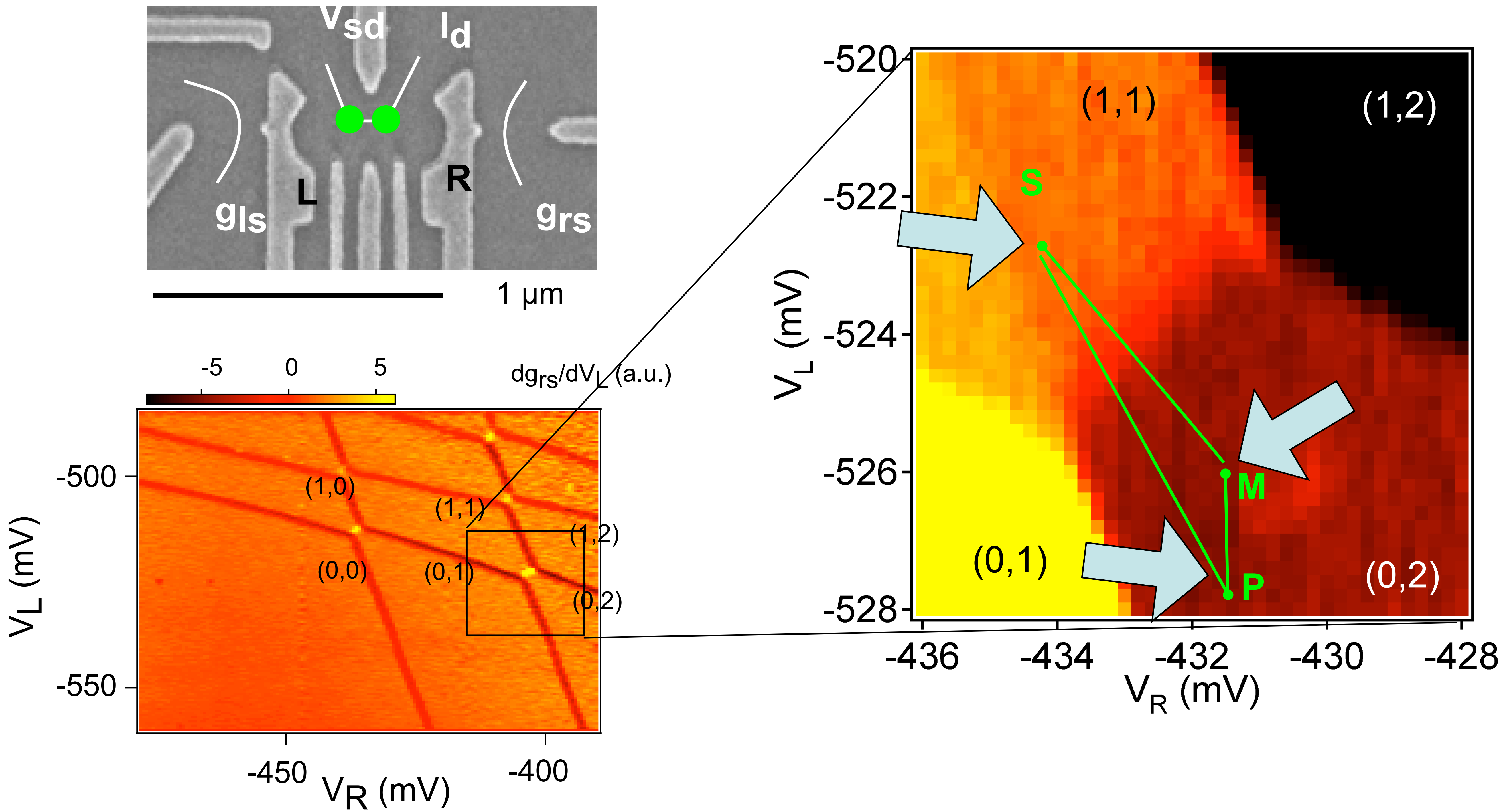
Working within the DFS

- Load from leads to produce a singlet: kT < J
- "Rapid adiabatic" transfer to (1,1) produces logical zero
- Reverse: spin-to-charge conversion
- Exchange gates for Z rotations
- SWAP to protect against nuclei. *Dynamical* DFS



[Levy PRL 2002, Wu & Lidar PRL 2002, Mohseni & Lidar PRL 2005]

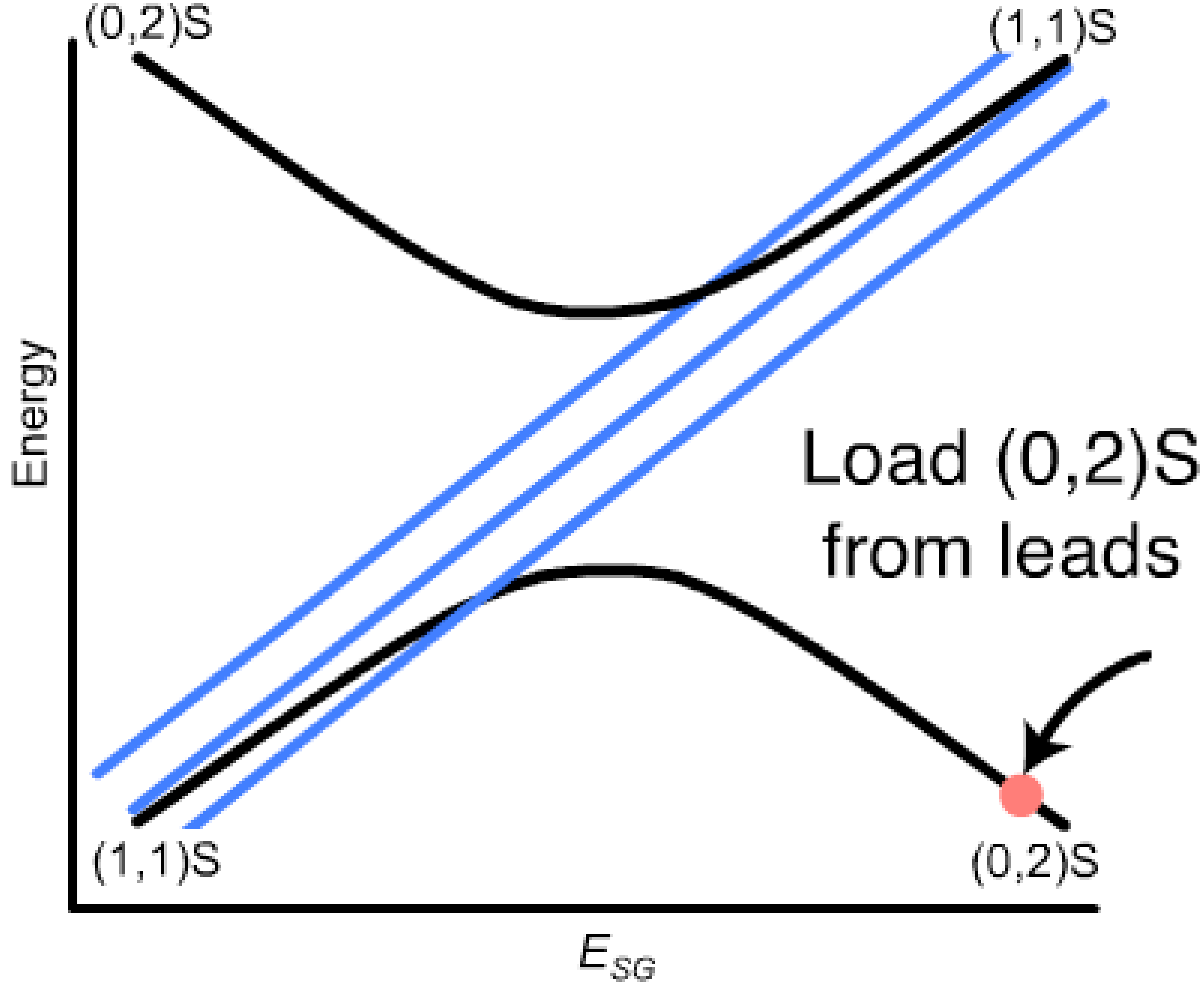
Experimental setup



Double dot in the two-electron regime
Two nearby QPCs provide charge sensing
Fast pulses (<1 ns rise time) applied to gates

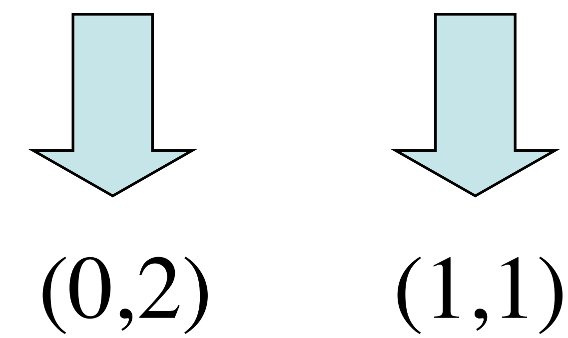
Preparation, measurement, and T_2^*

- Adiabatic transfer from atomic to molecular configuration, and back



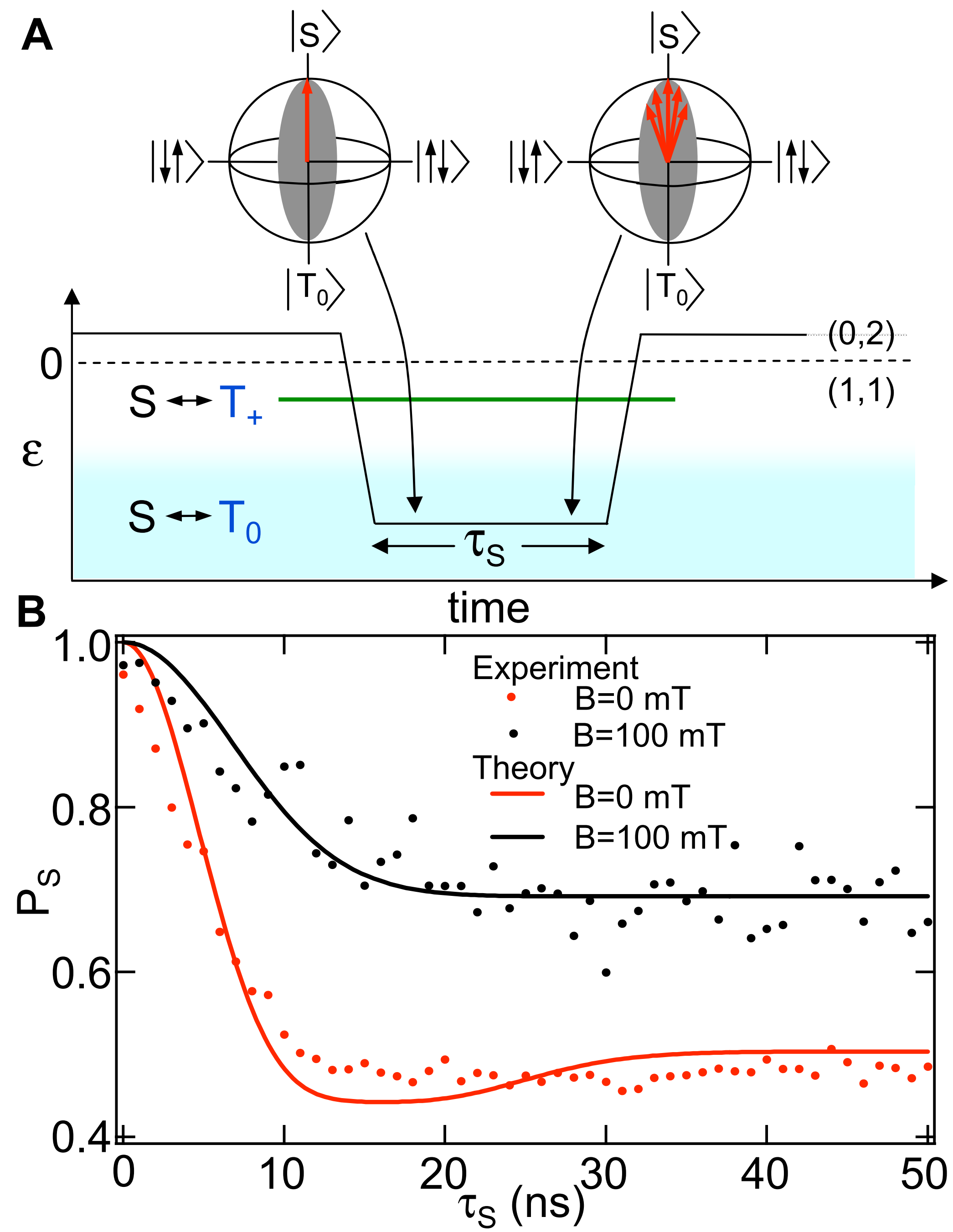
$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \overset{1/T_2}{\Leftrightarrow} |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$



Example: experimental results

- Load a singlet
- Separate electrons for a controlled time t_s
- Recombine: measure charge state
  - Singlet dephases due to hyperfine interactions: $T_2^*=10$ ns

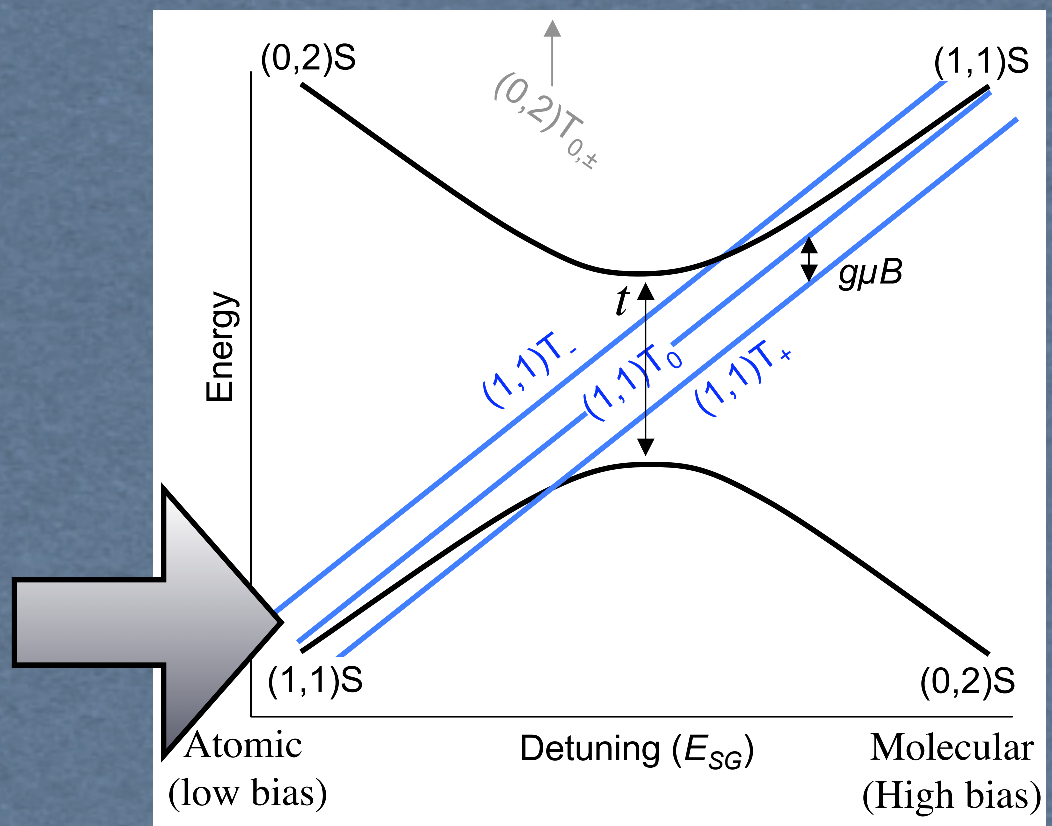


[Petta *et al.* Science (in press)]

Proof-of-principle exchange gate

How do we probe the exchange interaction?

Answer: use *slow* adiabatic loading.
prepares eigenstate of the hyperfine field



DDFS: how does it protect?

$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \,, |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

Magnetic field gradient:
rotates singlet to triplet

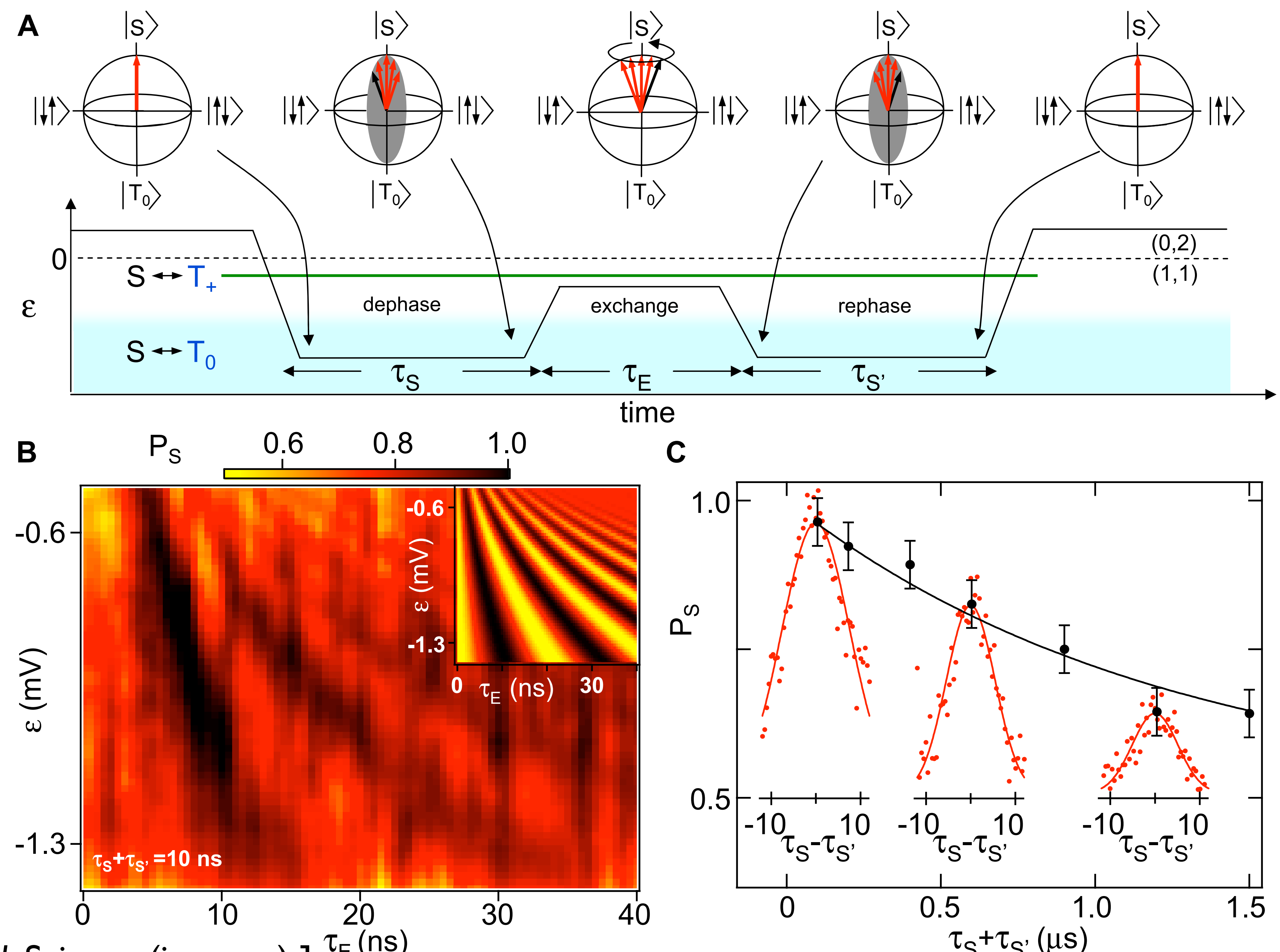
Exchange (SWAP) undoes rotation

$$S_{DDFS}(\omega) = S(\omega)\frac{256}{\tau^2\omega^2}\cos^2(\frac{\tau\omega}{8})\sin^6(\frac{\tau\omega}{8})$$

$$\approx \left(\frac{\tau}{T_2^*}\right)^2 (\tau\gamma)^4 / 1000$$

[Taylor *et al.* PRL (2005)]

Proof of principle: spin echo using SWAP

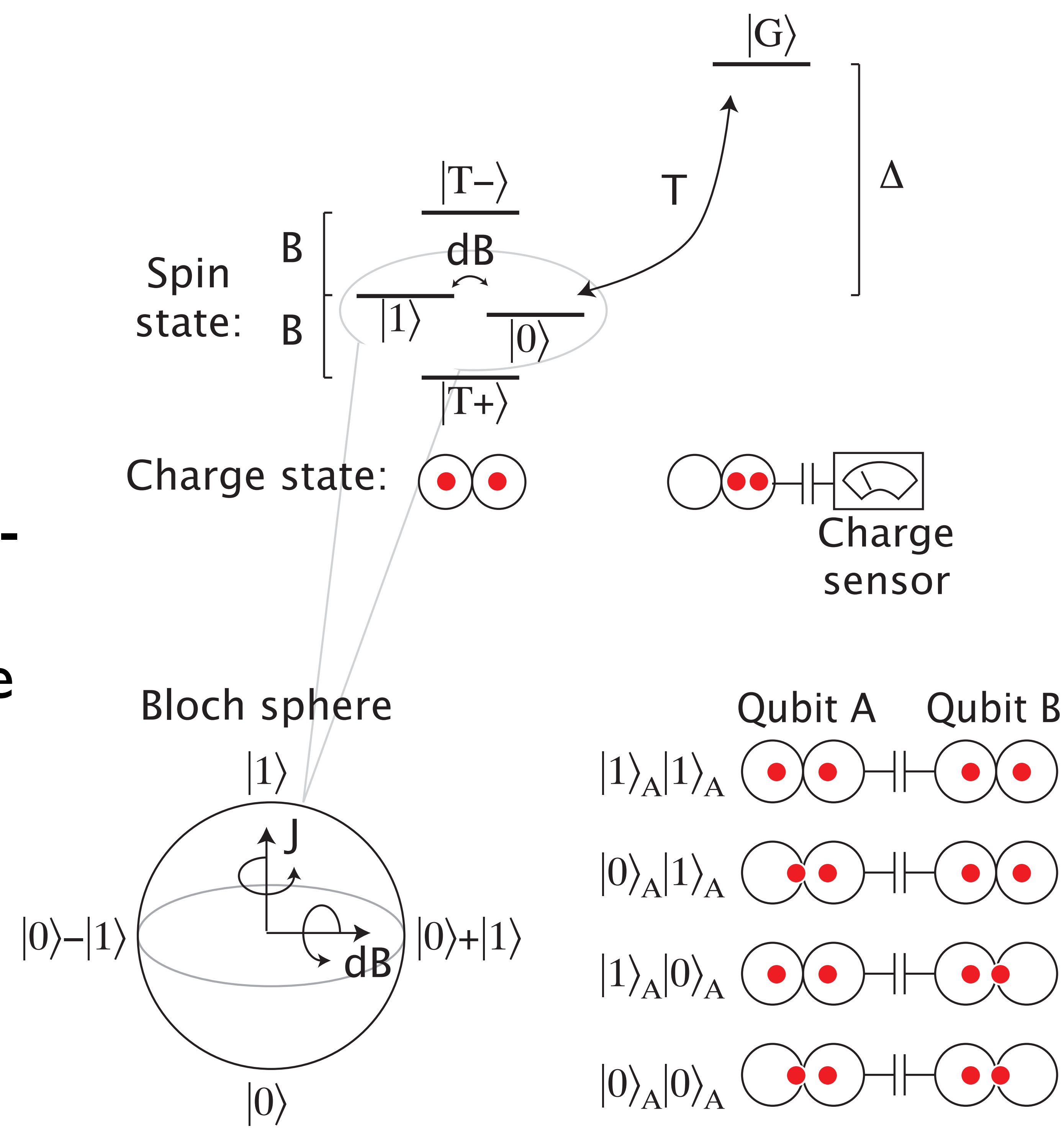


[Petta *et al.* Science (in press)]

Elements of a fault tolerant architecture

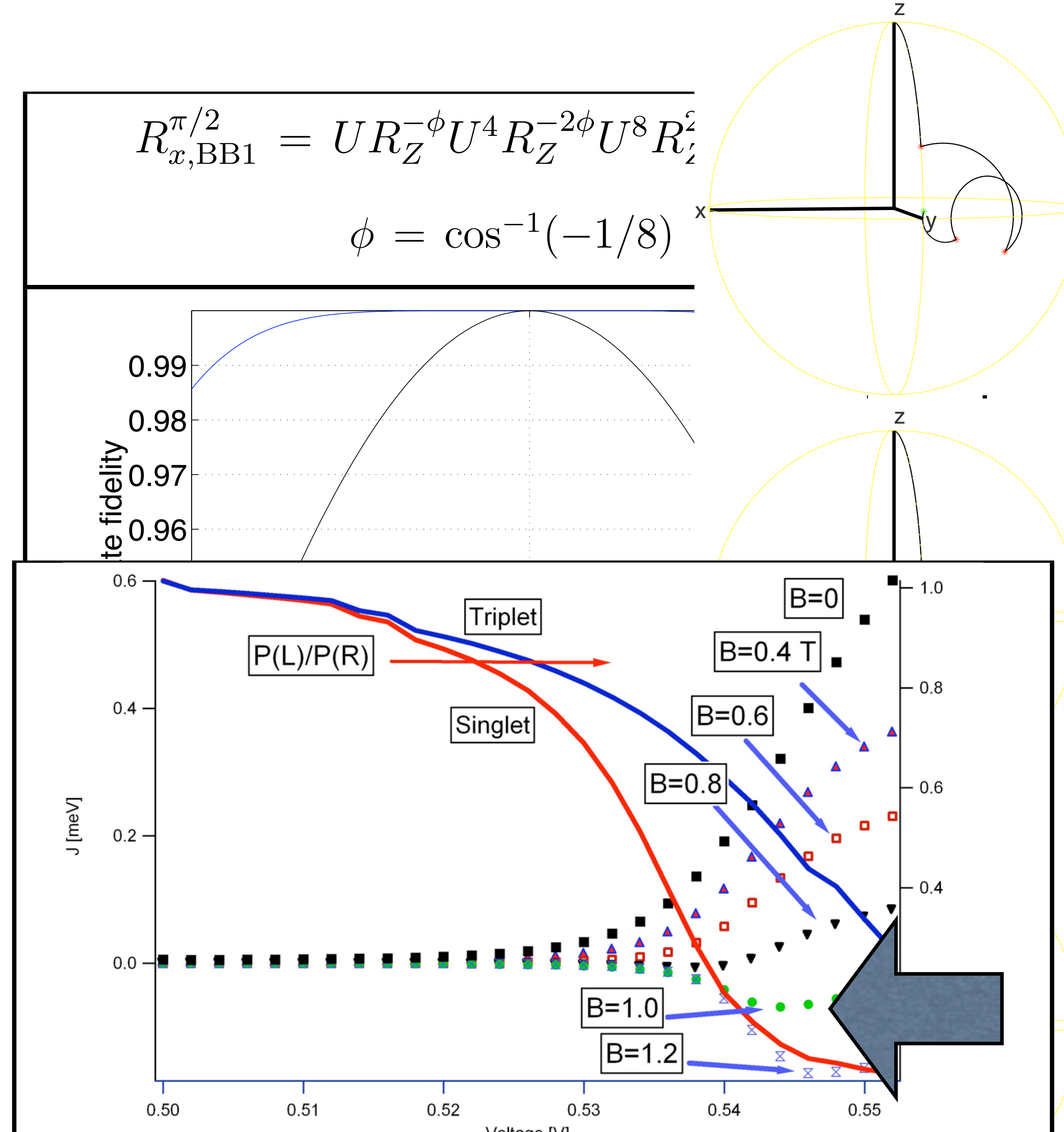
Completing the set of quantum operations

- Field gradient for X rotations: nuclei, g-factor
- Capacitive interaction for 2-qubit gate
  - use partial spin-to-charge conv. to limit dephasing



Improving quantum gates

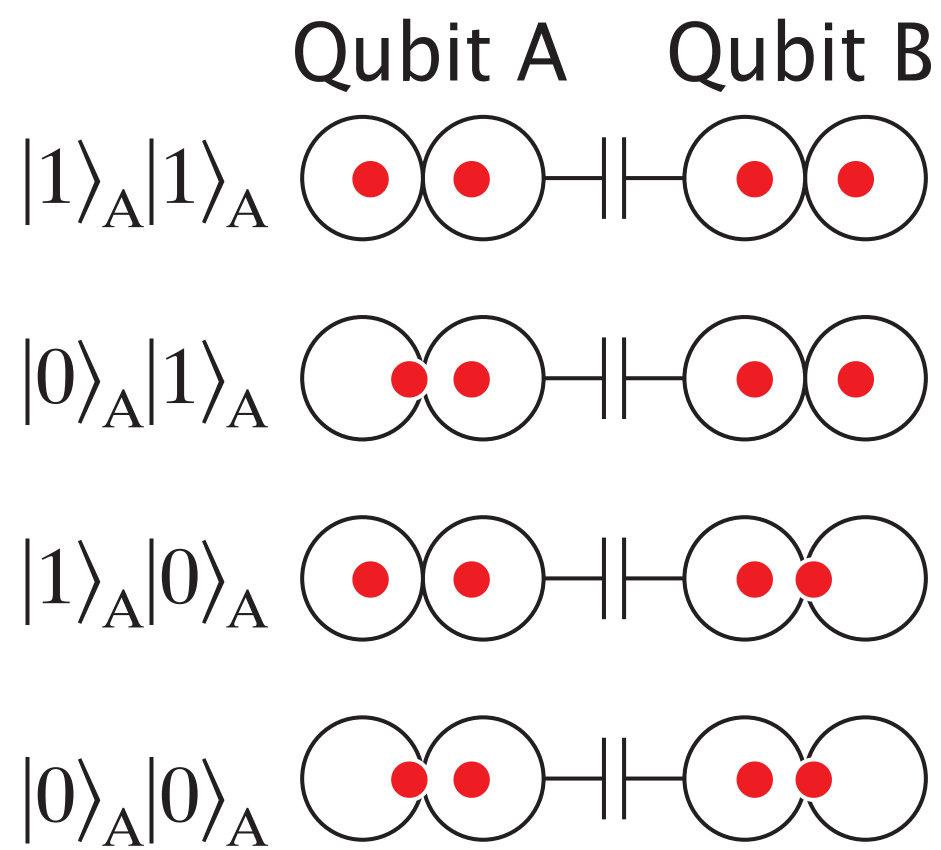
- For X rotations, use a composite pulse sequence (BB1)
    - eliminates errors up to 3rd order, allowing a gradient of 30 ± 3 mT for high fidelity X
- For Z rotations, use a low-noise exchange point.
    - dJ/dV=0 point



Courtesy of M. Stopa

A 2-qubit (4 electron) controlled phase gate

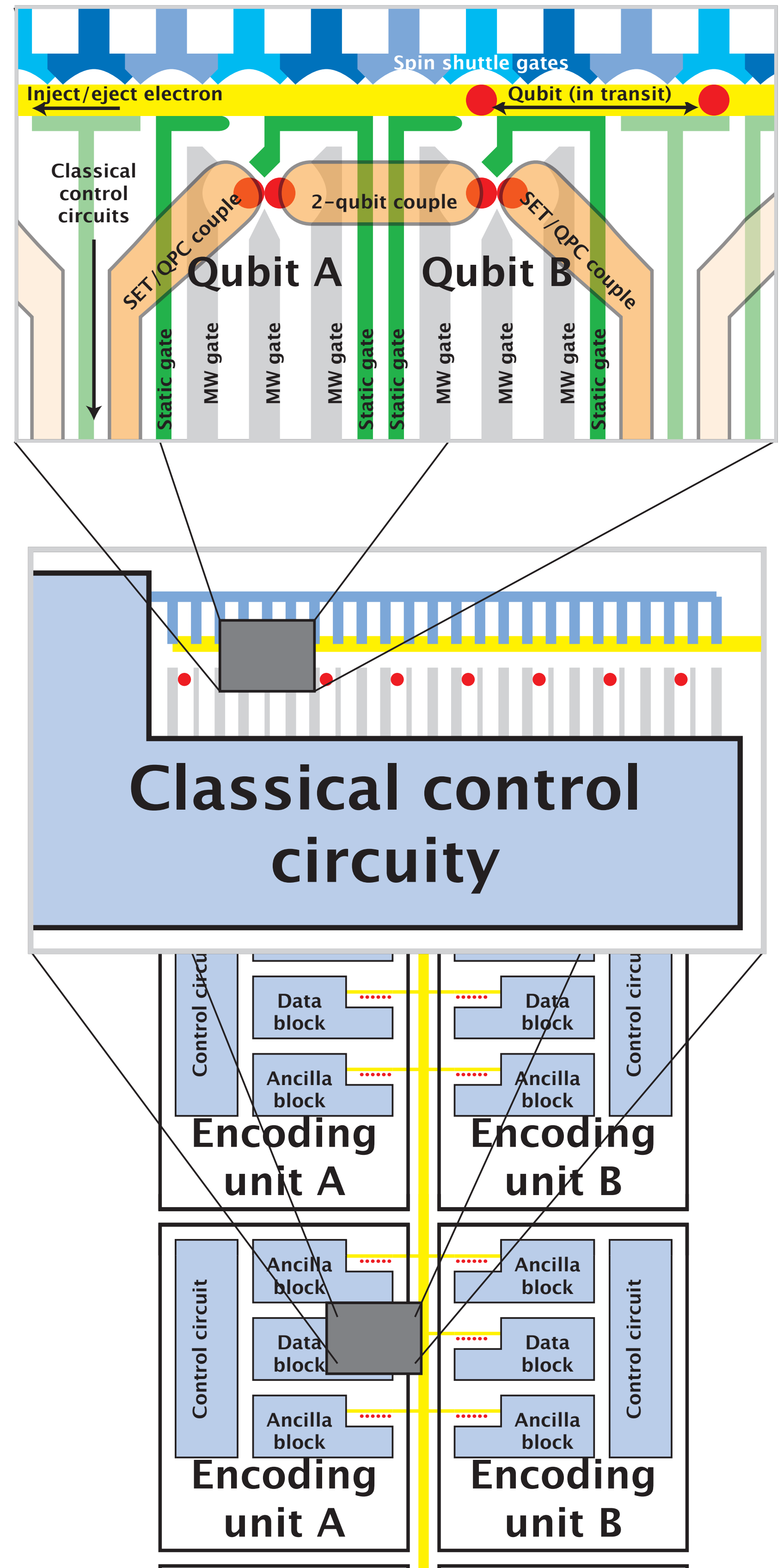
- Use adjacent double-dots; cross-capacitance leads to phase on (0,2) relative to (1,1) for each
- Singlet-singlet accumulates additional phase—a fast, dipole-dipole, gate
- Minimize admixture of doubly occupied state



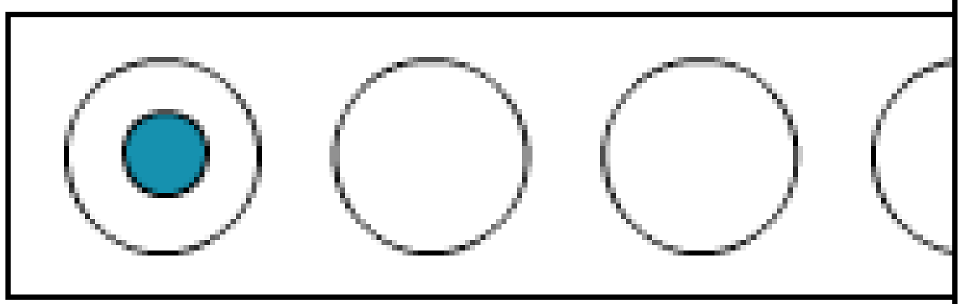
Spin-based architecture

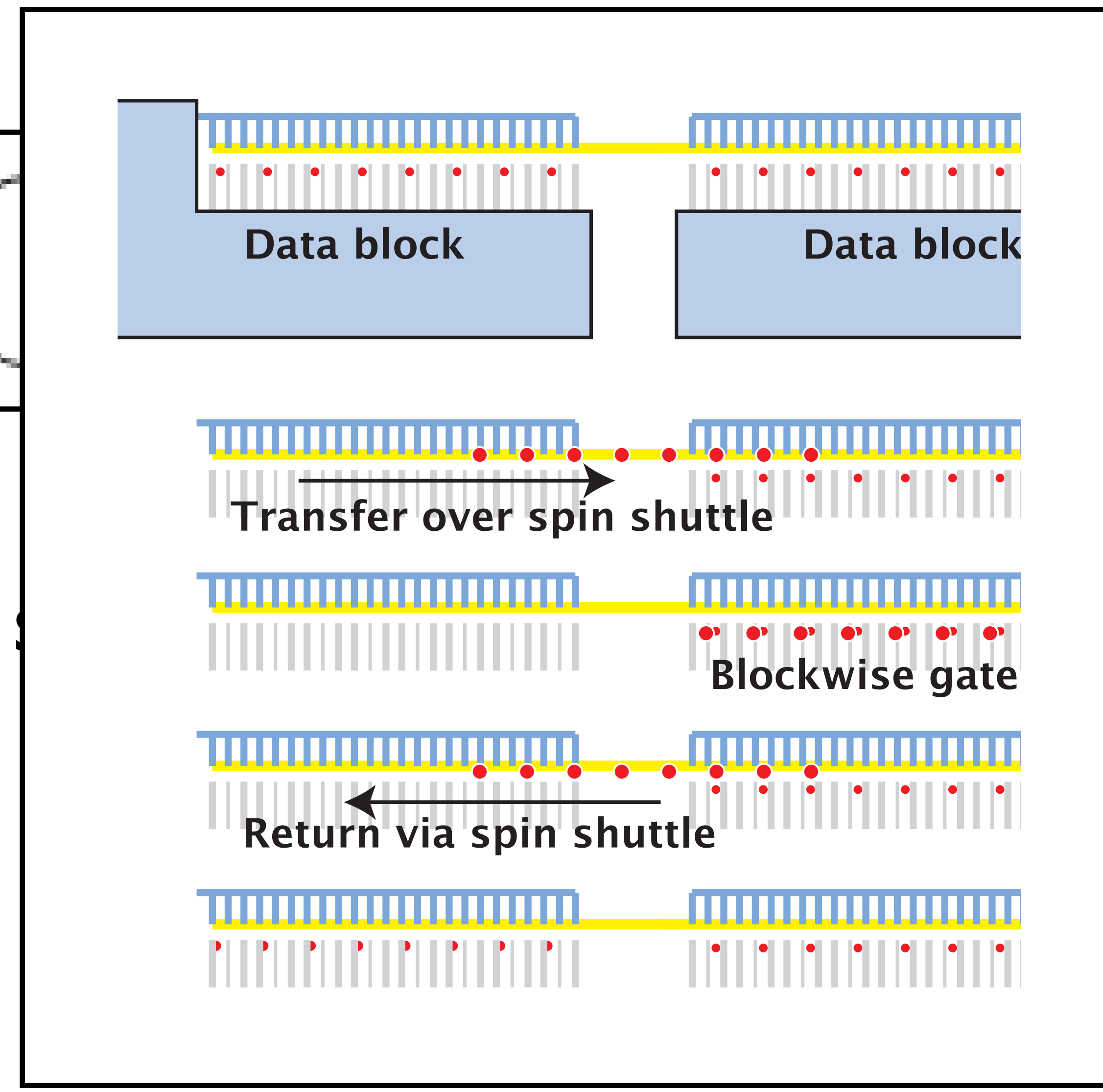
$$|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle, |\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$$

- Use two spins:
  - good quantum memory
- Transport of electrons:
  - long-range, parallel coupling
- All-electrical control:
  - modular for integration



Non-local coupling?
A spin shuttle





- Single electron CCD or a
  (electron pump)
- Adiabaticity condition
  easily satisfied at GHz
- Can be highly parallel,
  a la ion trap schemes

Error properties of transport

- External field produces Zeeman splitting: reduces spin-flip processes, e.g., spin-orbit
- Transporting encoded pairs reduces phase noise (static phase rotations reversed by the 2nd spin)

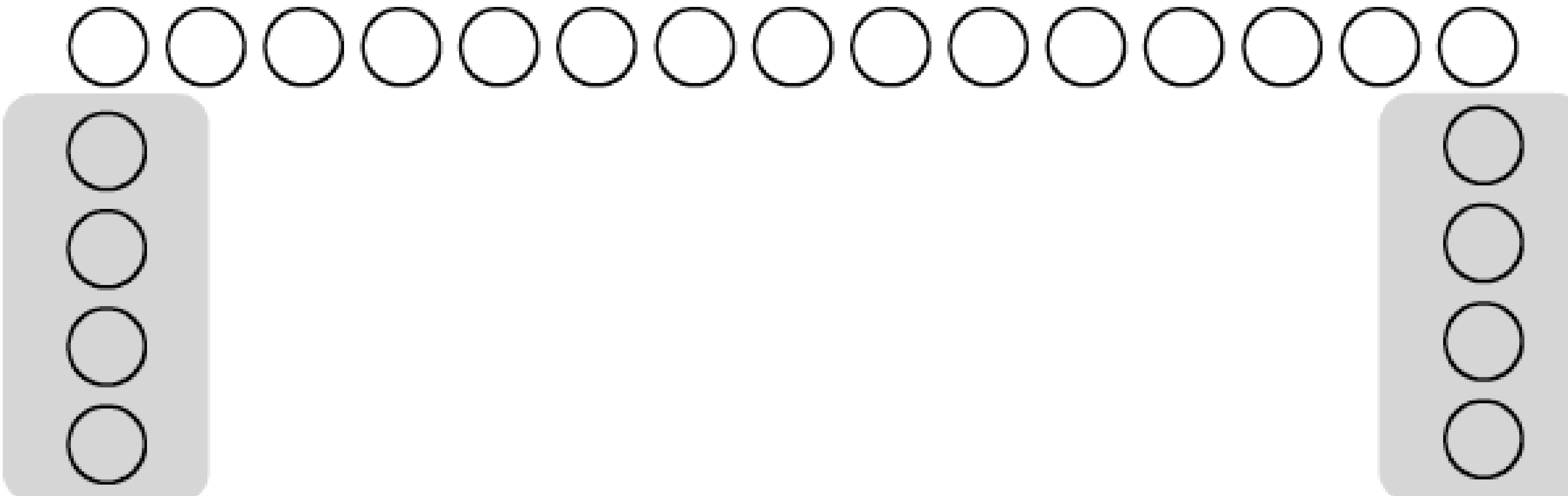
$$H_{\rm eff} = [B_{\rm ext} + \omega(t, x_1)]\hat{S}_z^1 + [B_{\rm ext} + \omega(t, x_2)]\hat{S}_z^2$$

- Effective spectral function:

$$S_T(\omega) = \int_{-\infty}^{\infty} S(\nu) \sin^2[\frac{\nu\tau_T}{2}] \frac{e^{-(\frac{\tau_T}{4})^2(\nu-\omega)^2/2}}{\sqrt{2\pi(4/\tau_T)^2}}\, d\nu \, .$$

- Transport channel phase noise < 1 in 10^7

Preparing long-distance entanglement through a spin shuttle



- Use generation of entangled singlet pairs of electron pairs: exchange only approach feasible
- Adiabatically pump electron charge through a series of dots
  - Averages over fluctuating fields
  - Work entirely in singlet-triplet dynamical DFS to further reduce errors
- Local operations purify fidelity of final pair (and remove leakage)
- Teleportation-based non-local gates implemented with purified pair

[Taylor *et al.* PRL (2005)]

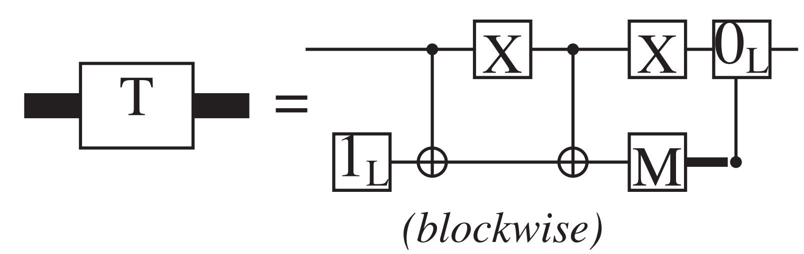
Fault tolerance: a threshold analysis

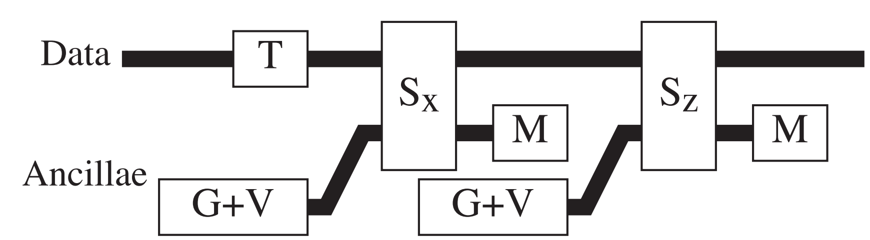
Scaling analysis

- Use a probability flow method: keep track of errors. Failure when an error-corrected gate develops more errors than the error correcting code can correct
- Recursively implement the same protocol at each stage.
- Add additional steps for first encoding to correct "leakage" out of the logical space

Fault tolerance analysis

- Steane [[7,1,3]] code in logical space: two step correction of X and Z errors
- "T" network: correct spin flip errors
- Consider probabilities of 1 error, 2 errors, 0 errors
- Repeat to generate a set of errors

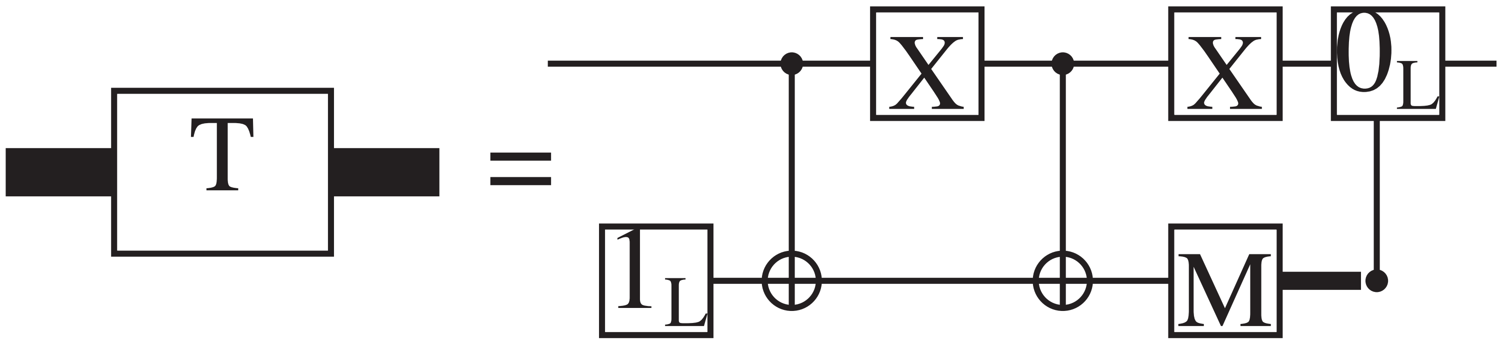




$$\begin{pmatrix} 1-P(2|0)-P(1|0) & 1-P(2|1)-P(1|1) \\ P(1|0) & P(1|1) \end{pmatrix}$$

T network

- Recall: 2-qubit gate is singlet-triplet dependent...
- But, X rotations only mix m_s=0 subspace
- Solution: parity check using ancilla
- Overhead? Teleportation based approach?

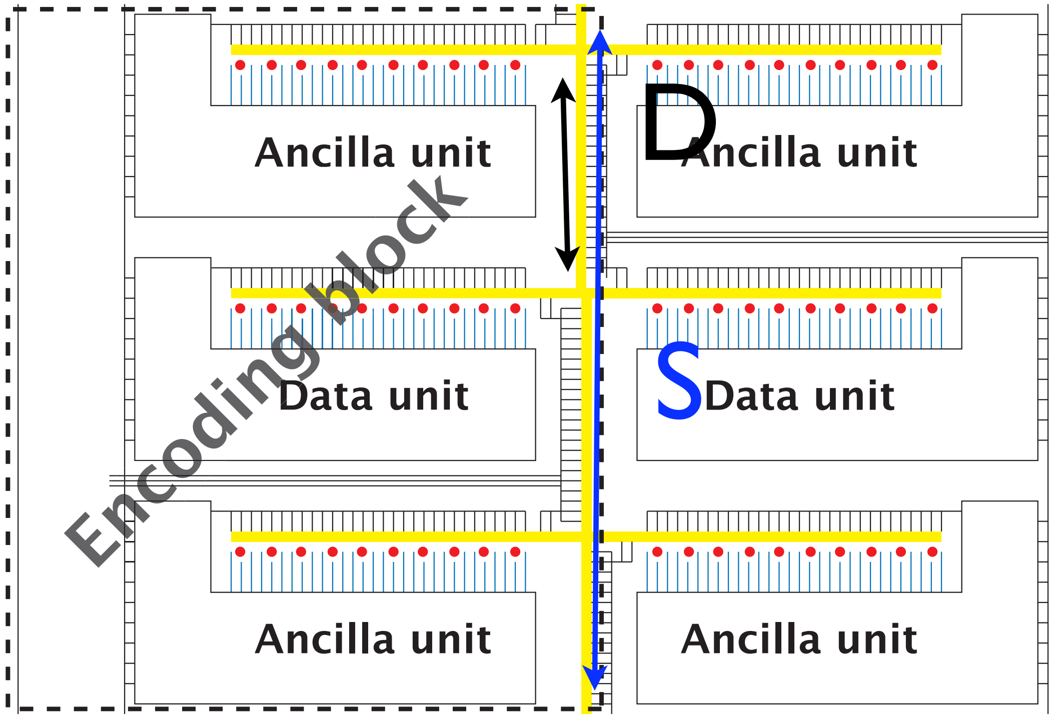


(blockwise)

Fault tolerant threshold

- Consider error model for each level of recursion: p1-p4, m1-m4
- Find a map from previous level to next level (increasing distance, increasing memory time)
- Consider memory error, transport error less than gate errors

	error prob	memory err prob
1-qubit unitaries	p_1	m_1
2-qubit unitaries	p_2	m_2
Measurement of single spin	p_3	m_3
Shuttle one qubit l quantum dots	lp_4	lm_4



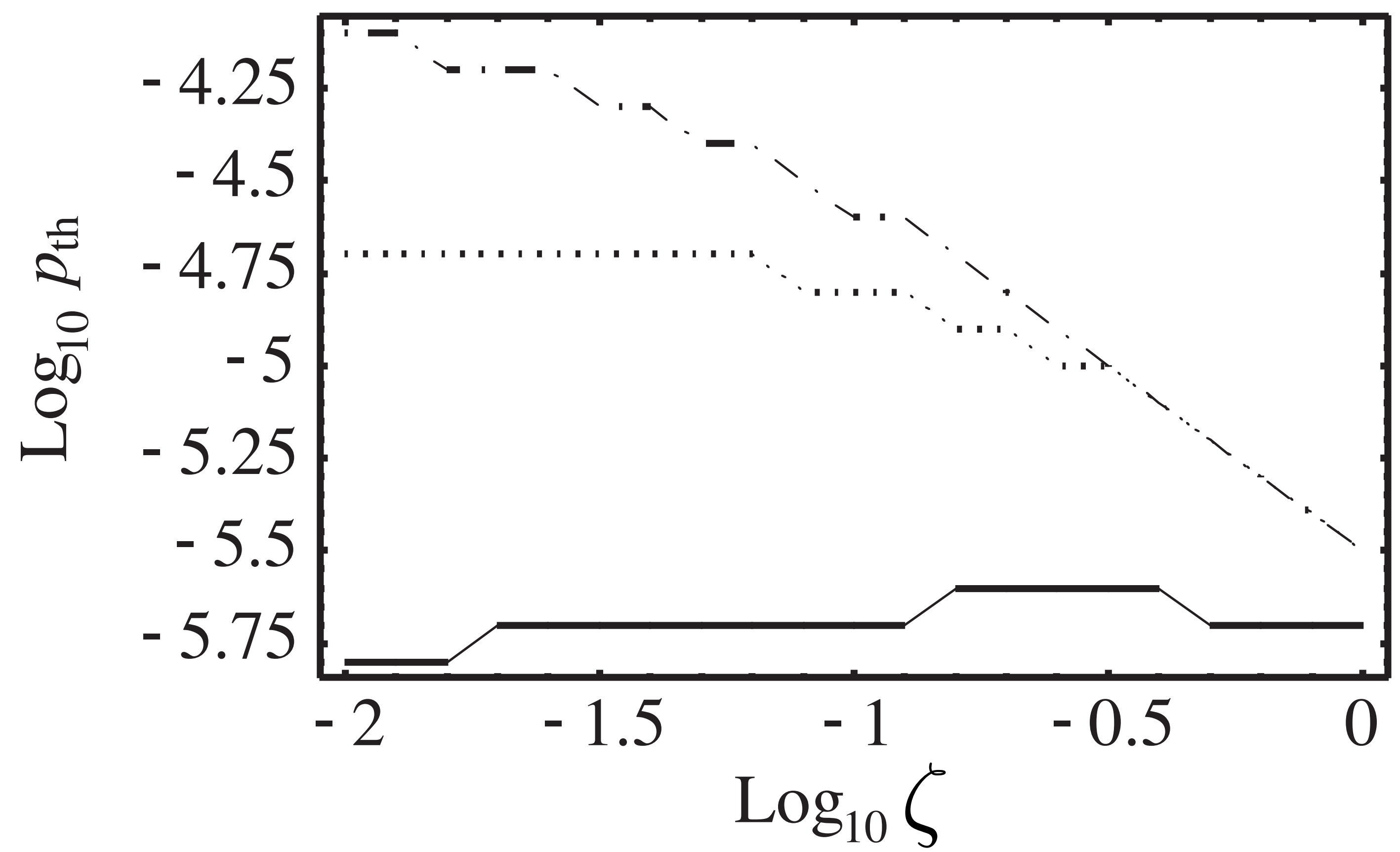
Threshold results

- Choose initial error vectors for variety of memory and transport errors.

$$\vec{p}^{0} = p\{1,1,1,\zeta\}$$

$$\vec{m}^{q} = \epsilon p\{1,1,10,1\}$$

- Solve flow matrix equations for each element of the error vector
- Use new set of errors for next encoding level
- Iterate to double precision limits



Perspective for 1 GHz FTQC

	Current	FTQC
Prep/meas	>80%	>99.9%
1-qubit gate	~15	>1,000
2-qubit gate	--	>1,000
Memory	>1,000	>100,000
Transport	--	>100,000

Outlook

- Accomplished: preparation, measurement, Z rotations, quantum memory
- Experimental program:
  - complete set of gates (X rotations, CPHASE)
  - spin shuttle for coherent spin transport
- Long term outlook:
  - high speed measurement
  - on-chip control circuitry
  - modular error correcting register
  - long-range entanglement generation