Models of Computation

Tutorial Exercises 4

1. Show that the following grammars are ambiguous by exhibiting distinct leftmost derivations:

   (a) $(\{S\}, \{a, b\}, \mathcal{R}, S)$ with $\mathcal{R} = \{\ S \ \rightarrow\ a\,S\,b\,S \mid b\,S\,a\,S \mid \epsilon\ \ \}$

   (b) $(\{S, A, B\}, \{a, b\}, \mathcal{R}, S)$ with $\mathcal{R} = \{\ \begin{array}{lcl} S & \rightarrow & A\,B \mid a\,a\,B, \\ A & \rightarrow & a \mid A\,a, \\ B & \rightarrow & b\ \ \} \end{array}$

2. Let L be the set of binary strings w such that every postfix of w has at least as many 0's as 1's. A *postfix* of a string w is any v such that $w = u\,v$ for some u. For example $01100, 1010 \in L$ but $110 \notin L$.

   (a) Find a CFG for L, and show that it is correct. Is your grammar ambiguous?

   (b) [*Harder*] If not done as answer for (a) find an *unambiguous* grammar for L, and justify your answer fully.

3. (optional) Give an implementation-level NPDA that accepts the language generated by the CFG $(\{S, A\}, \{a, b\}, \mathcal{R}, S)$ with $\mathcal{R} = \{\ \begin{array}{lcl} S & \rightarrow & A\,A \mid a, \\ A & \rightarrow & S\,A \mid b\ \ \} \end{array}$

4. (optional) An *atomic* TM is one in which every transition consists of a change of state and one other action. The transition may write on the tape or move the tape head, but not both. Prove that atomic TMs accept precisely the r.e. languages.

5. Design a Turing machine, informally and at implementation level, that decides the language

$$\{a^n b^m \mid n \text{ divides } m\}$$

6. Prove that the collection of decidable languages is closed under the operations of

   (a) union

   (b) concatenation

   (c) star

   (d) complementation

   (e) intersection.

   Is the collection of r.e. languages closed under all the above operations?

7. (optional) Prove that a language C is r.e. iff a decidable language D exists such that

$$C \;=\; \{\, x : \exists y \,.\, \langle x, y \rangle \in D \,\}.$$

8. Determine whether the following problems are decidable or undecidable, and give proof:

   (a) whether a given TM runs for at least 127^{127} steps on input a^{127}

(b) whether a given TM ever reenters its start state on any input

9. Is it decidable whether a given Turing machine accepts every string?

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