Models of Computation

Tutorial Exercises 3

1. Use the Pumping Lemma to prove that the following languages over the alphabet $\Sigma = \{0, 1\}$ are not regular:

   (i) $L_1 = \{ 0^n 1^m 0^n : m, n \geq 0 \}$.

   (ii) $L_2 = \Sigma^* \setminus \{ 0^n 1^n : n \geq 0 \}$.

   (iii) $L_3 = \{ 0^m 1^n : m \neq n \}$.

2. Consider $L = \{ w \in \{0, 1\} : w \text{ is not a palindrome} \}$. A *palindrome* is a string that reads the same forward or backward. Show that L is not regular in three different ways:

   (i) using the fact that the set of palindroms is not regular;

   (ii) using the Myhill-Nerode Theorem;

   (iii) using the Pumping Lemma.

3. Let $L_1 = \{ 01^i 01^j 01^j \mid i, j > 0 \}$ and $L_2 = \{ w \in \{0, 1\}^* \mid w \text{ contains } 00 \}$.

   (i) Show that $L = L_1 \cup L_2$ satisfies the Pumping Lemma with pumping length 3.

   (ii) Prove that $L = L_1 \cup L_2$ is not regular using the Myhill-Nerode theorem.

   (iii) Does (ii) contradict (i)?

4. The alphabet is $\Sigma = \{0, 1\}$. Give context-free grammars that generate the following languages.

   (i) The empty set.

   (ii) The language of palindromes.

   (iii) The set of strings containing at least three 1's.

   (iv) The set of odd-length strings whose middle symbol is 0.

5. Find context-free grammars for the following languages (where $m, n \geq 0$):

   (a) $L = \{ a^n b^m : n \leq m + 3 \}$

   (b) $L = \{ a^n b^m : n \neq 2m \}$

6. (a) Describe what is meant by *well-bracketing* in the case where there are two kinds of parentheses, namely, ( ) and [ ]. For example ([ ]) and ([ ] (( ))) [ ] are well-bracketed, but ([ )] is not.
   Give a CFG over $\Sigma = \{ [,], (,) \}$ that generates well-bracketed parentheses.

   (b) Say that (and] are positive and) and [are negative. A string over Σ is said to be *alternating* if any two adjacent symbols have opposite polarities.
   Give a CFG that generates the set L of alternating and well-bracketed strings over Σ. For example [( )] [ ] and ([([ ])]) ( ) are in L but [ ] ( ) and [[ ]] are not.

Tom Melham, Trinity 2009
Adapted from materials by Hanno Nickau and Luke Ong