Models of Computation

Tutorial Exercises 2

1. Give regular expressions for each of the following subsets of $\{ a, b \}^*$.

   (i) $\{ x : x$ contains an even number of a's $\}$

   (ii) $\{ x : x$ contains an odd number of b's $\}$

   (iii) $\{ x : x$ contains an even number of a's or an odd number of b's $\}$

2. Give NFAs accepting the sets of strings denoted by the following regular expressions:

   (i) $(000^* + 111^*)^*$

   (ii) $(01 + 10)(01 + 10)(01 + 10)$

   (iii) $(0 + 1(01^*0)^*1)^*$

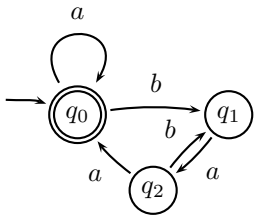
   Try to simplify as much as possible.

3. Use the procedure given in the lectures to convert the following NFA

   $$N \;=\; (\{ q_0, q_1, q_2 \}, \{ a, b \}, \delta, \{ q_0 \}, \{ q_0 \})$$

   to a regular expression where δ is given by:

δ	a	b
q_0	$\{ q_0 \}$	$\{ q_1 \}$
q_1	$\{ q_2 \}$	$\emptyset$
q_2	$\{ q_0 \}$	$\{ q_1 \}$



4. Prove or disprove each of the following:

   (i) $(E + F)^* \equiv E^* + F^*$

   (ii) $(EF + E)^*E \equiv E(FE + E)^*$

   (iii) $E(FE + E)^*F \equiv EE^*F(EE^*F)^*$

5. Prove that if L is a regular language, so is $L^R = \{ w \mid$ the reversal of w is in $L \}$.

6. Show that the regular languages are closed under the following operations:

   (i) $min(L) = \{ w \mid w$ is in L but no proper prefix of w is in $L \}$.

   (ii) $max(L) = \{ w \mid w$ is in L and for no x other than ϵ is wx in $L \}$

   (iii) $init(L) = \{ w \mid$ for some x, wx is in $L \}$.

   [*Hint*: Start with a DFA for L and perform an appropriate construction.]

Tom Melham, Trinity 2009
Adapted from materials by Hanno Nickau and Luke Ong