Homework #1   J, Olaoyia

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Group:- Sec02, HW7, Group H

1   Given Kernel density function

$$\hat{f}(a) = \frac{1}{n} \sum \frac{1}{h} K\left(\frac{a - x_i}{h}\right), \quad -\infty < a < \infty \quad -(1)$$

where (1) $K(u) \geq 0 \quad \forall\, u_i$

(2) $\int_{-\infty}^{\infty} K(u)\, du = 1$

(3) $h > 0$

(a) From the properties of a density function

(1) $f(a) > 0$

(2) $\int_{-\infty}^{\infty} \hat{f}_n(a) = 1$

Proving the above

$$\int_{-\infty}^{\infty} \hat{f}(a) = \int_{-\infty}^{\infty} \frac{1}{n} \sum_{i=1}^{n} K\left(\frac{a - x_i}{h}\right) dx \Rightarrow$$

$$\Rightarrow \frac{1}{n} \sum \int_{-\infty}^{\infty} \frac{1}{h} K\left(\frac{a - x_i}{h}\right) dx \Rightarrow \frac{1}{n} \sum_{i=1}^{n} 1 = \frac{n}{n} = 1$$

as claimed $f(a)$ is a valid density function when $K(u) > 0$.

So we can say, If $K > 0$ and $\int_{-\infty}^{\infty} K(u)\, du = 1$, then f is Prob. density function

Significance: As K is cont, f will be continuous and differentiable each obs/value, because of this property it helps us to underst the distribution of the r.v's and the probability assumption of the to lie in any interval is simply equal to the area under the curve under the given interval.

(b) $m_1 = \int_{-\infty}^{\infty} u \hat{f}_h(u)\, du$

$$x = \frac{u - x_i}{h}$$

$$\int_{-\infty}^{\infty} u \hat{f}(u)\, du \Rightarrow \frac{1}{n} \sum_{i=1}^{n} \int_{-\infty}^{\infty} u \frac{1}{h} K\left(\frac{u - x_i}{h}\right) du$$

$$= \frac{1}{n} \sum_{i=1}^{n} \int_{-\infty}^{\infty} (x_i + xh) K(x)\, dx$$

$$\Rightarrow \frac{1}{n} \sum x_i \int_{-\infty}^{\infty} K(x)\, dx + \frac{1}{n} \sum h \int_{-\infty}^{\infty} x K(x) \quad -(1)$$

we know that $\int_{-\infty}^{\infty} K(x)\, dx = 1 \quad -(a)$

and 1^{st} moment $\int_{-\infty}^{\infty} x K(x)\, dx = 0 \quad -(b)$

Substituting (a) and (b) in (1)

$$= \frac{1}{n} \sum_{i=1}^{n} x_i + \frac{1}{n}(0) = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$\int_{-\infty}^{\infty} u \hat{f}(u)\, du \Rightarrow \frac{1}{n} \sum_{i=1}^{n} x_i$$

= mean of the sample

x_i		$u = \left\lvert \frac{x-2.25}{0.8} \right\rvert$	$1_{[0,1]}\left\lvert \frac{x_i-2.25}{0.8} \right\rvert$	$3/4(1-u^2)$
2. x_1	1	$\left\lvert \frac{1-2.25}{0.8} \right\rvert = 1.5625 > 1$	0	1.0810
x_2	1.2	$\left\lvert \frac{1.2-2.25}{0.8} \right\rvert = 1.3125 > 1$	0	0.5419
x_3	1.5	$\left\lvert \frac{1.5-2.25}{0.8} \right\rvert = 0.9375 < 1$	1	0.6445
x_4	2.8	$\left\lvert \frac{2.8-2.25}{0.8} \right\rvert = 0.6875 < 1$	1	0.3955
x_5	3	$\left\lvert \frac{3-2.25}{0.8} \right\rvert = 0.9375 < 1$	1	0.09082

$$\hat{f}_h(a) = \frac{1}{nh} \sum_{i=1}^{n} K\left(\frac{x_i - a}{h}\right) \qquad -\infty < a < \infty$$

$n \Rightarrow 5 \qquad h = 0.8$

$$= \frac{1}{5 * 0.8}\left(0 + 0 + 0.09082 + 0.3955 + 0.09\right.$$

$\hat{f}_h(a) \Rightarrow 0.145285$