# Uber driver strategic behavior

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## 1 Basic Model

Consider a set of $n$ nodes as vertices of a graph $G = (V, E)$ defined over a geographical area. We are interested in creating an optimal contingency plan of trips, for a particular driver who starts his day at node $v_0 \in V$. We assume that the driver decides the maximum units of service time $T$ that he would spend ferrying passengers at the beginning of the day. Let time $t$ denote the number of time units left for a driver before the end of his pre-decided service time.

- For $i, j \in V, d_{ij}$ is the Euclidean distance between the two nodes. The costs to the driver in terms of gas and vehicle depreciation value in traversing this edge is $c_{ij}$ and his earnings in ferrying a passenger from $i$ to $j$ is represented by $p_{ij}$. Time required to traverse the edge is denoted by $\tau_{ij}$. The costs, the earnings and the time required to make this ride are all proportional to the distance travelled.
- Let $S^{(t)}_{n\times 1}$ be the surge multiplier vector over the nodes of the graph. Each entry $s^{(t)}_i$ denotes the surge multiplier in effect at node $i$ at time $t$. When $s^{(t)}_i > 1$, the driver earning from ferrying a passenger from node $i$ to $j$ of the graph is $s^{(t)}_i \times p_{ij}$.
- The arrival process of passengers at node $i$ at time $t$ is a Poisson process with rate $\lambda^{(t)}_i$. Let vector $\Lambda^{(t)}_{n\times 1}$ denote the passenger arrival rates across all the vertices of $G$ at time $t$. While some passengers may arrive simultaneously (compound Poisson process), we assume that this effect is negligible. Furthermore, we assume that if the driver is present at node $i$ when a passenger arrives, he picks up the passenger with no further delay. As the inter-arrival times of passengers are exponentially distributed, the expected value of idle time for a driver waiting at node $i$ is $1/\lambda^{(t)}_i$. We implicitly assume that the passenger arrival process is stationary for a finite time slice around $t$.
- The matrix $M^{(t)}_{n\times n}$ represents the transition probabilites between the nodes of the graph such that the fraction of passengers at node $i$ whose destination is node $j$ at time $t$ is represented by entry $m^{(t)}_{ij}$, where $\forall t, m^{(t)}_{ij} \in \mathbb{R}_{\geq 0}, m^{(t)}_{ii} = 0$ and $\sum_j m^{(t)}_{ij} = 1$.

We formulate a dynamic program to find the contingency plan to maximise the expected earnings of a driver in his service time $T$. Each entry $H(t, i)$ of the matrix $H$ represents the maximum expected earnings in the remaining $t$ units of service time when the driver is stationed at node $i$. The maximum expected revenue of the driver at the beginning of the day is thus $H(T, v_0)$.

- Initialization $\forall i, H(0, i) = 0$.

- While calculating each $H(t,i)$, our algorithm is faced with two possible choices.
    1. Keep waiting at node $i$ for a passenger.
    2. Traverse to some other node $j$ in search of a passenger.
- It chooses whichever choice maximises the expected earnings in the remaining time units $t$.

$$H(t,i) = \max \begin{cases} \sum_{j \neq i} m_{ij}^{(t)} \left[ \left( s_i^{(t)} p_{ij} - c_{ij} \right) + H(t - \tau_{ij}, j) \right], & \text{if } \frac{1}{\lambda_i^{(t)}} + \mathbf{E}[\tau_i | M] < t, \\ \max_j \left[ H(t - \tau_{ij}, j) - c_{ij} \right], & \text{if } \tau_{ij} < t \end{cases}$$