Definite Integral

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Definition

Definition

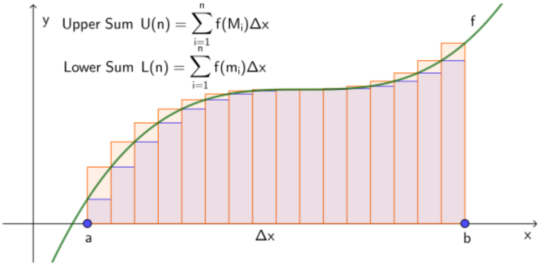
If $f(x)$ is a function defined on an interval $[a, b]$, the definite integral of f from a to b is given by

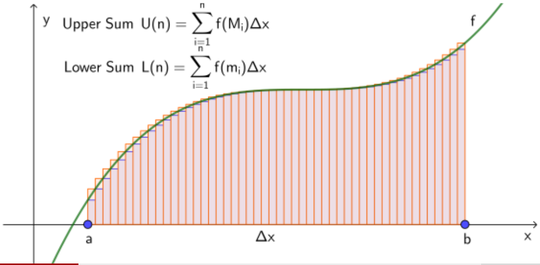
$$\int_a^b f(x)dx = \lim_{n \to +\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i$$

provided the limit exists.

- If this limit exists, the function $f(x)$ is said to be integrable on $[a, b]$, or an integrable function.

Lower Sum and Upper Sum





Reimann Sum

Definition

If f is defined on the closed interval $[a, b]$ and the limit of Riemann sums exists, then

$$\lim_{x \to +\infty} \sum_{i=1}^{n} f(c_i)\Delta x_i = \int_a^b f(x)dx$$

. The limit is called the definite integral of f from a to b.

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Fundamental Theorem of Integration

Theorem

Let f be the continuous function on $[a, b]$ and F be a primitive of f. Then

$$\int_a^b f(x)dx = [F(x)]_a^b = F(b) - F(a)$$

Properties

Theorem

1. $\displaystyle\int_a^b f(x)dx = -\int_b^a f(x)dx$
2. $\displaystyle\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
3. $\displaystyle\int_a^a f(x)dx = 0$
4. $\displaystyle\int_a^b (f(x) \pm g(x))dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$
5. $\displaystyle\int_a^b cf(x)dx = c\int_a^b f(x)dx, c \in \mathbb{R}$

Example

Compute the following Integral

1. $\displaystyle\int_{-3}^{1} 6x^2 - 5x + 2dx$
2. $\displaystyle\int_{4}^{0} \sqrt{t}(t-2)dt$
3. $\displaystyle\int_{1}^{2} \frac{2y^2 - y + 3}{y^2}dy$
4. $\displaystyle\int_{0}^{\pi/3} 2\sin(\theta) - 5\cos(\theta)d\theta$

Substitution Method for Definite Integral

Theorem

If $u = g(x)$ is continue and differentiable on $[a, b]$ then

$$\int_a^b f[(x)]g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

Example: Compute

1. $I = \int_0^2 x(x^2+1)^3 dx$
2. $I = \int_1^2 \frac{3x^2}{(x^3+1)} dx$

Definite Integral by Part

Theorem

If f and g are continuous on $[a, b]$ and differentiable on (a, b) then

$$\int_a^b f(x)g'(x)dx = [f(x).g(x)]_a^b - \int_a^b g(x).f'(x)$$

or

$$\int_a^b udv = [uv]_a^b - \int_a^b vdu$$

Example: Compute

1. $I = \int_0^{\pi/2} x\cos(x)dx$
2. $I = \int_0^4 xe^x dx$

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Area Under the Curve

Theorem

Let f be a continuous function on $[a, b]$. The area of region bounded by the curve $y - f(x)$, the x-axis and two vertical lines $x = a$ and $x = b$ is given by

$$A = \int_a^b f(x)dx$$

Example: Find the area of the region bounded by the graph of $f(x) = x^2 - 5x + 6$ the x-axis and the lines $x = 0$ and $x = 3$

Area Under the Curve

Note

If $f(x) \leq 0$ and continue on$[a, b]$ then for all $x \in [a, b], -f(x) \geq 0$, so the area of the region on interval $[a, b]$ is

$$A = -\int_a^b f(x)dx$$

Example: Find the area of region of function $y = 4 - x^2$ and x-axis on interval $[-2, 3]$

Area Under the Curve

Theorem

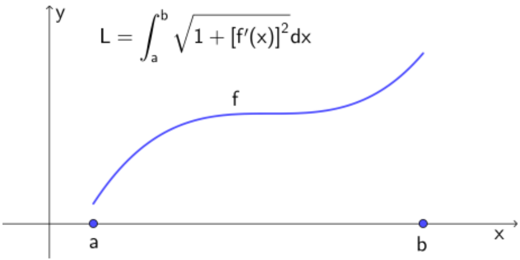
if f and g are continuous function on $[a, b]$ and $f(x) \geq g(x)$ for all x in $[a, b]$, then the area of the region bounded by the graphs of f and g and the vertical lines $x = a$ and $x = b$ is

$$A = \int_a^b [f(x) - g(x)]$$

Example: Find the area of the region between the graphs of

$$f(x) = 3x^3 - x^2 - 10x \text{ and } g(x) = -x^2 + 2x$$

Length of the Arc



Theorem

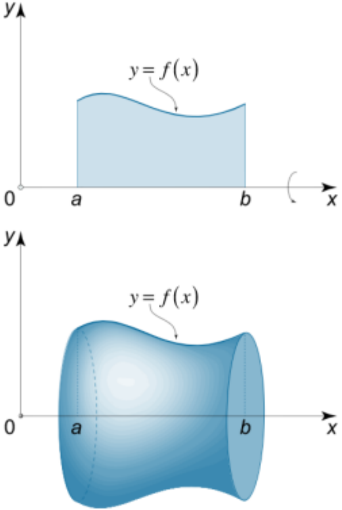
Let f be a continuous function on $[a, b]$. The length of the arc of the curve $y = f(x)$ between two abscissae are $x = a$ and $x = b$ is given by

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

Volume of Solide

The volume of the solide obtained by the revolving about x-axis of the arc of the curve $y = f(x)$ from the points $x = a$ and $x = b$ is given by

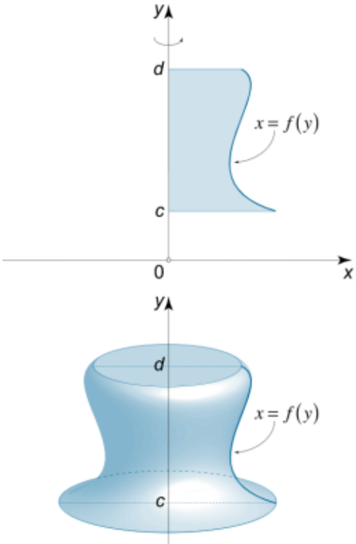
$$V = \int_a^b \pi[f^2(x)]dx$$



Volume of Solide

The volume of solide obtained by revolving about $y - axis$ of the arc of the curve $x = f(y)$ from the points $y = c$ and $y = d$ is given by

$$V = \int_c^d \pi[f^2(y)]dy$$



Volume of Solide

Example:

1. Show that the volume of spherical is $V = \dfrac{4}{3}\pi r^3$
2. Calculate the volume of solide obtained by the revolving about x-axis on interval $[-2, 1]$ from the function $y = 2 - x$
3. Find the volume of solide obtained from $y = \sqrt{3 - x}$ and revolved about y-axis that is $0 \leq y \leq \sqrt{3}$