# Conics

Mr. CHEA Makara
4th Year Engineering in Majoring Data Science,
Department of Applied Mathematics and Statistics, ITC

May 14, 2024

# Table of Contents


1. Conic Section
2. Parabola
3. Ellipse
4. Hyperbola

## Definition

A **conics** is the curve obtained as the intersection of a plane with the surface of a cone.

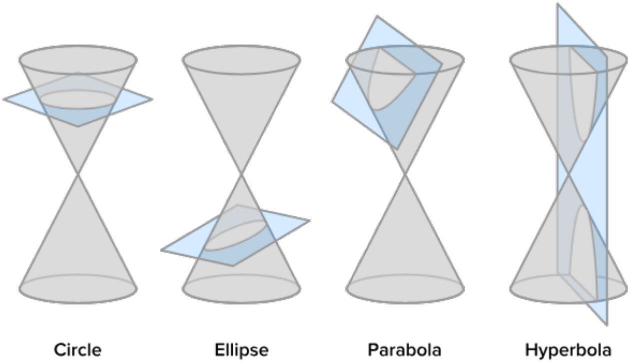



Figure: Non-degenerate conics

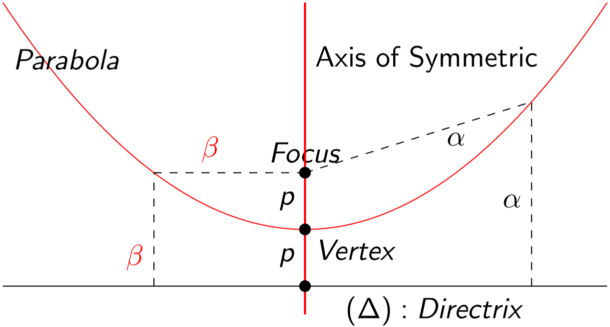
# Table of Contents


1. Conic Section
2. Parabola
3. Ellipse
4. Hyperbola

# Parabola

## Definition

A parabola is the set of all points in a plane that are equidistant from a fixed point (called the focus) and a fixed line (called the directrix)

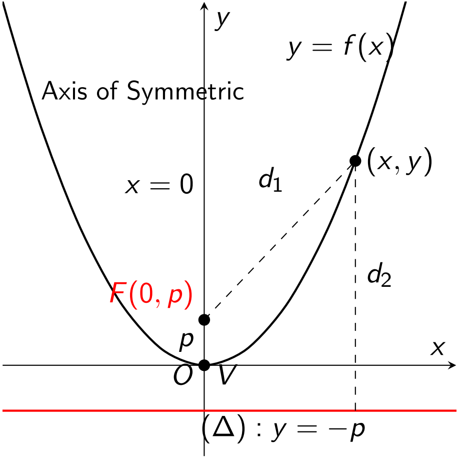


# Parabola's Properties

## Properties of Parabola

- The midpoint between the focus and the directrix is the **vertex**
- The line passing through the focus and the vertex is the **axis of symmetric**
- $p > 0$ is the distance between the vertex and the focus.

**Standard Equation of Parabola that has Vertex at Origin and Horizontal Directrix or Vertical Axis of Symmetric**




we have vertex $V(0,0)$ (origin) and focus $F = (0, 0+p) = F(0,p)$ and directrix $(\Delta) : y = 0 - p = -p$

we know that $d_1 = d_2$

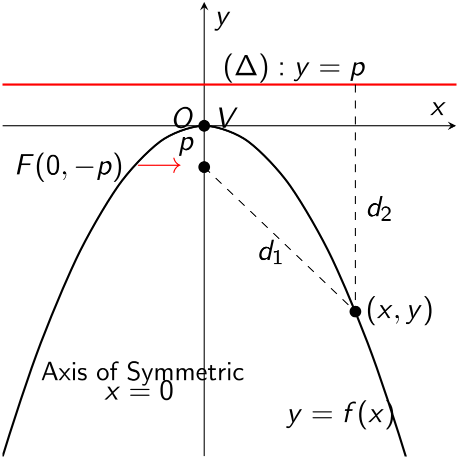
$$\iff \sqrt{(x-0)^2 + (y-p)^2} = y - (-p)$$

$$\iff x^2 + (y-p)^2 = (y+p)^2$$

$$\iff x^2 = 4py$$

Therefore the Standard Form of Parabola at Origin is given By $\boxed{x^2 = 4py}$

**Standard Equation of Parabola that has Vertex at Origin and Horizontal Directrix or Vertical Axis of Symmetric**




we have vertex $V(0,0)$ (origin) and focus $F = (0, 0-p) = F(0,-p)$ and directrix $(\Delta): y = 0-(-p) = p$

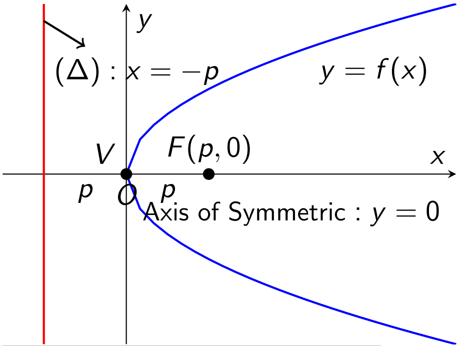
we know that $d_1 = d_2$

$$\Longleftrightarrow \sqrt{(x-0)^2 + (y+p)^2} = p - y$$
$$\Longleftrightarrow x^2 + (y+p)^2 = (y-p)^2$$
$$\Longleftrightarrow x^2 = -4py$$

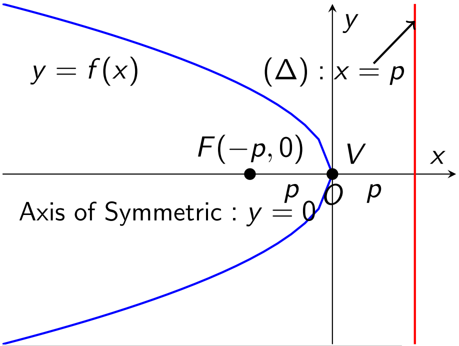
Therefore the Standard Form of Parabola at Origin is given By $\boxed{x^2 = -4py}$

# Standard Equation of Parabola

**Parabola has vertex at origin and directrix parallels to y-axis**

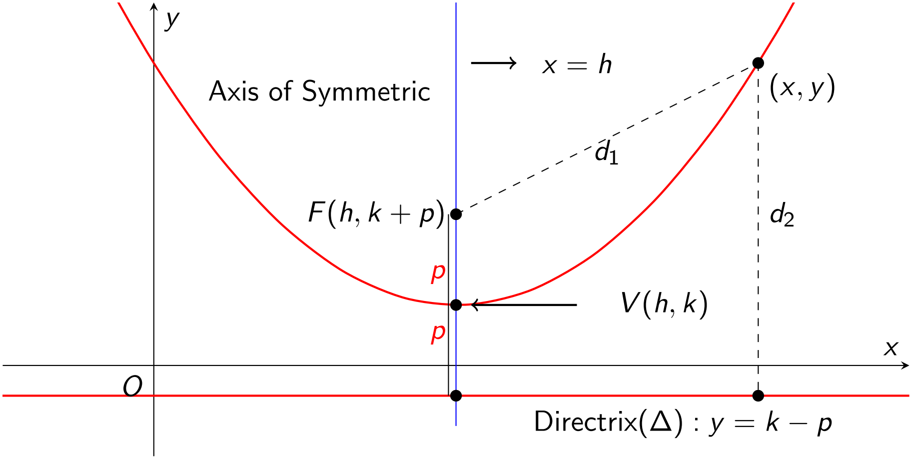



Standard Equation: $y^2 = 4px$




Standard Equation $y^2 = -4px$

**Parabola has Vertex don't belong at origin and Directrix Parallels with x-axis**



# Standard Equation of Parabola

**Proof.**

We have $d_1 = d_2$, hence

$$\iff \sqrt{(x-h)^2 + (y-(k+p))^2} = y - (k-p)$$

$$\implies (x-h)^2 + y^2 - 2y(k+p) + (k+p)^2 = (y-(k-p))^2$$

$$\implies (x-h)^2 + y^2 - 2y(k+p) + (k+p)^2 = y^2 - 2y(k-p) + (k-p)^2$$

$$\implies (x-h)^2 = 2y\left[(k+p)-(k-p)\right] + (k-p)^2 - (k+p)^2$$

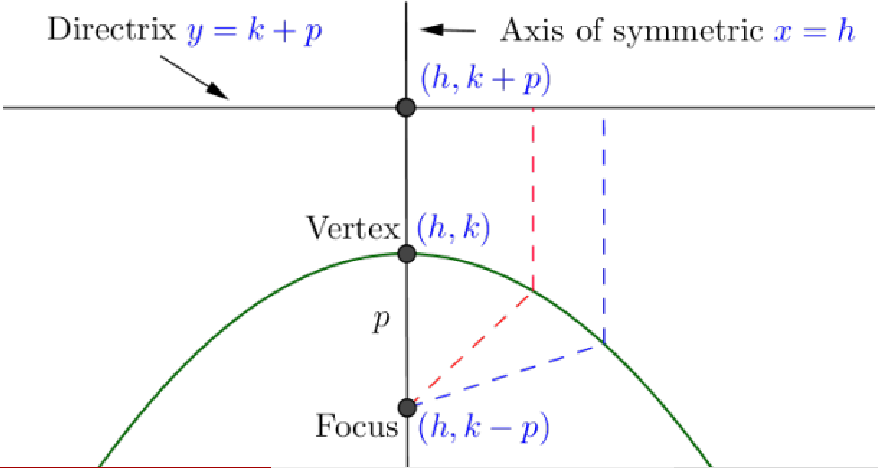
$$\implies (x-h)^2 = 4py + (2k)(-2p) = 4py - 4pk$$

$$\iff (x-h)^2 = 4p(y-k) \qquad \square$$

$\boxed{\text{Therefore, the Standard equation is } (x-h)^2 = 4p(y-k)}$

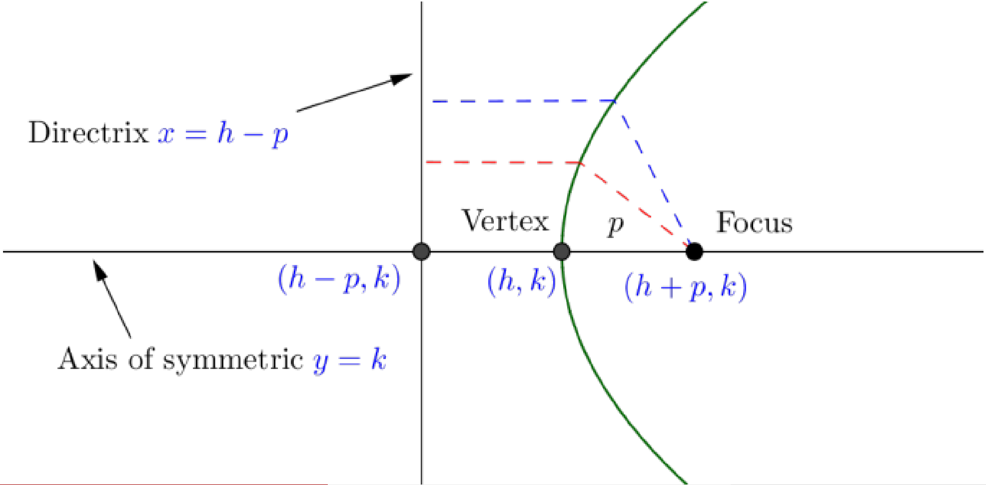
# Standard Equation of Parabola

Standard Equation of Parabola : $\boxed{(x-h)^2 = -4p(y-k)}$



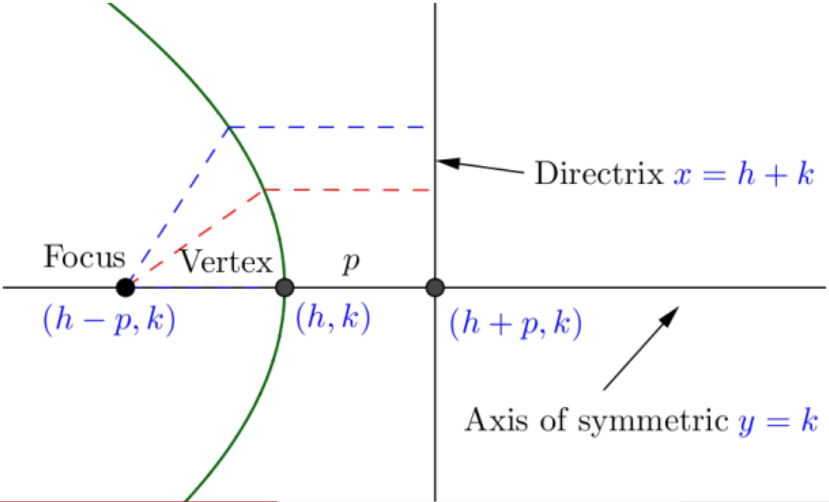
# Standard Equation of Parabola

Standard Equation of Parabola : $\boxed{(y-k)^2 = 4p(x-h)}$



# Standard Equation of Parabola

Standard Equation of Parabola : $\boxed{(y-k)^2 = -4p(x-h)}$



# Table of Contents

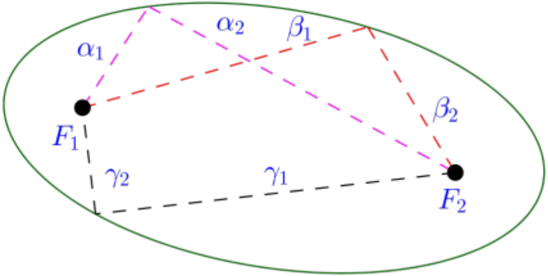

1 Conic Section

2 Parabola

3 Ellipse

4 Hyperbola

# Ellipse

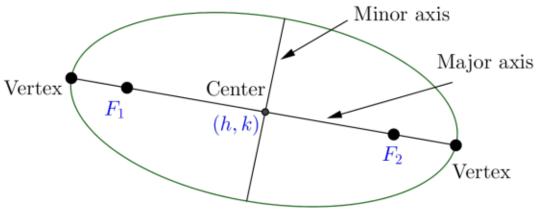



$$\alpha_1 + \alpha_2 = \beta_1 + \beta_2 = \gamma_1 + \gamma_2$$

## Definition

Ellipse An ellipse is the set of points in a plane whose distances from two fixed points ($F_1$ and $F_2$) in the plane have a constant sum.
The two fixed points are the **foci** of the ellipse. In general, $\alpha_1 + \alpha_2 = 2a$

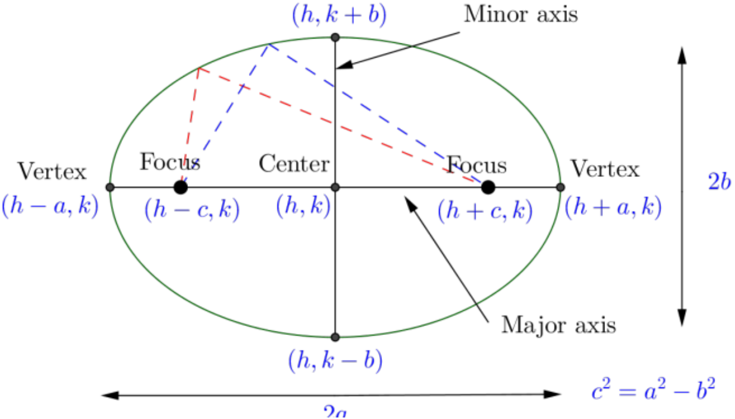
# Ellipse




- The line through the foci, $F_1$ and $F_2$ of an ellipse is the ellipse's focal axis.
- The point on the axis halfway between the foci is the center.
- The points where the focal axis and ellipse cross are the vertices
- The chord joining the vertices is the major axis
- The chord perpendicular to the major axis at the center is the minor axis.

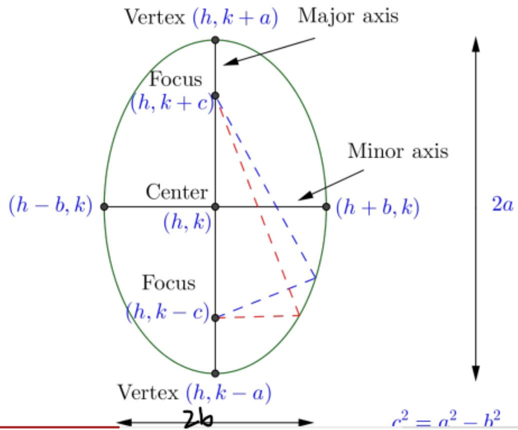
# Standard Equation of Ellipse

Standard Equation of Ellipse: $\boxed{\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1}$



# Standard Equation of

Standard Equation of Ellipse: $\boxed{\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1}$




$c^2 = a^2 - b^2$

# Eccentricity of Ellipse

**Definition**

The ratio of the distance of the focus from the center of the ellipse $(c)$, and the distance of one end of the ellipse from the center of the ellipse $a$(a haft of major axis), that is $e = \dfrac{c}{a}$

We have $0 < c < a \Longleftrightarrow 0 < e < 1$

- If $e \to 0$, then shape of ellipse is almost a circular
- If $e \to 1$ then the shape of ellipse is elongated.

# Table of Contents


1 Conic Section

2 Parabola

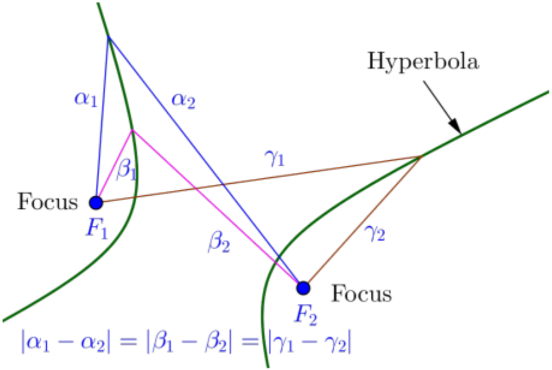
3 Ellipse

4 Hyperbola

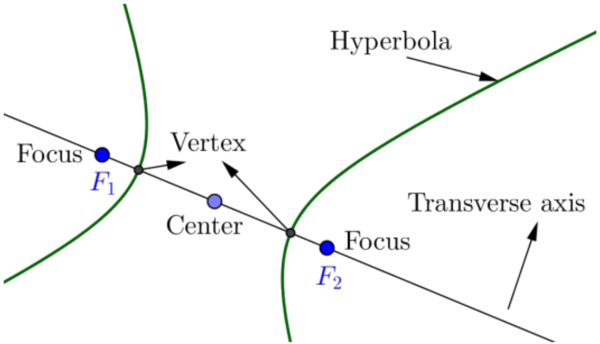
# Hyperbola

## Definition

A hyperbola is the set of points in a plane whose distance from two fixed points $F_1$ and $F_2$ in the plane have a constant difference. The two fixed points are the foci of the hyperbola. In general, $|\alpha_1 - \alpha_2| = 2a$



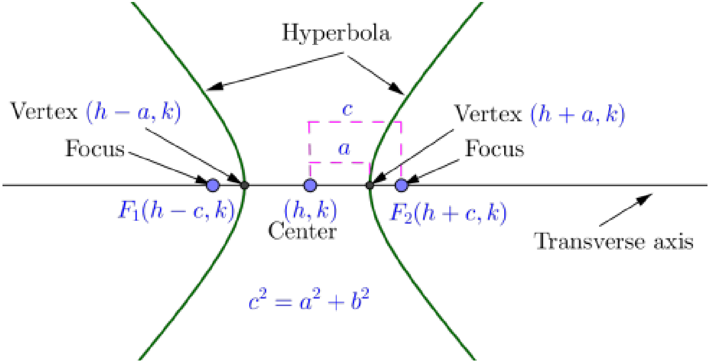
# Hyperbola




- The line through the two foci intersects a hyperbola at two points called the vertices.
- The line segment connecting the vertices is the transverse axis.
- The midpoint of the traverse axis is the center of hyperbola

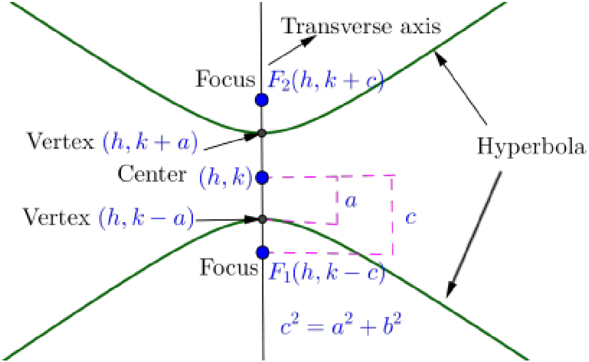
# Stnadard Equation of Hyperbola

Standard Equation of Hyperbola: $\boxed{\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1}$



# Standard Equation of Hyperbola

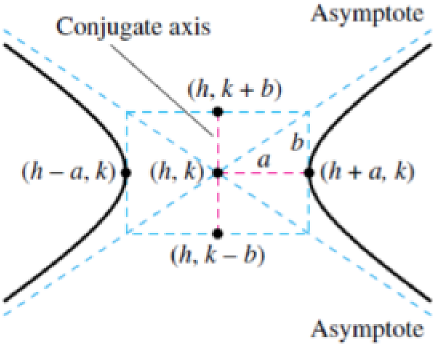
Standard Equation of Hyperbola: $\boxed{\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1}$



# Asymptote of Hyperbola

For a horizontal tranverse axis, the equation of the astymptote are

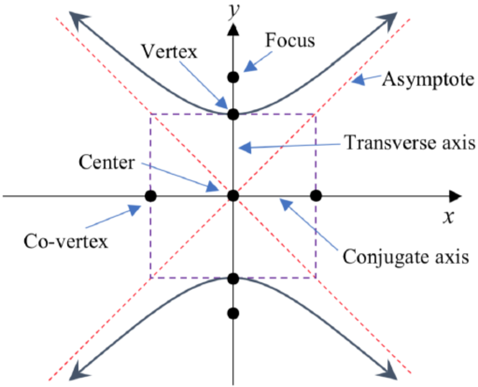
$$y = \frac{b}{a}(x-h)+k \quad \textbf{and} \quad y = -\frac{b}{a}(x-h)+k$$



# Asymptote of Hyperbola

For a vertical traverse axis, the equation of asymptotes are given by

$$y = \frac{a}{b}(x - h) + k \quad \text{and} \quad y = -\frac{a}{b}(x - h) + k$$



# Eccentricity

For every conic $e = \frac{c}{a}$, where $c$ is the distance from center to focus and $a$ is the distance from center to vertex.

1. For parabola has eccentricity $e = 1$
2. For ellipse has eccentricity $0 < e < 1$
3. For hyperbola has eccentricity $e > 1$