Lesson Two

02/16/24

Operations with Functions

Addition $(f+g)(x) = f(x) + g(x)$

example: $f(x) = 2x+3 \quad g(x) = x^2$

$(2x+3) + x^2 =$ $x^2 + 2x + 3$

Subtraction $(f-g)(x) = f(x) - g(x)$

example: $f(x) = 2x+3 \quad g(x) = 2x + x^2$

$(2x+3) - (2x+x^2)$

$2x + 3 - 2x + x^2$ $\quad 3 - x^2$

Multiplication $(f \cdot g)(x) = f(x) \cdot g(x)$

example: $f(x) = 2x+3 \quad g(x) = x^2$

$(2x+3)(x^2)$ $\quad 2x^3 + 3x^2$

Division $(f/g)(x) = f(x) / g(x)$

example: $f(x) = 4x^2+2x \quad g(x) = 2x$

$\frac{4x^2+2x}{2x} \quad \frac{4x^2}{2x} + \frac{2x}{2x} \quad 2x+1$

Composition of Functions

$g \circ f \rightarrow$ the $\circ$ is called **composition**

↳ **function composition** is applying one function to the results of another → written as $(g \circ f)(x) \rightarrow g(f(x))$

example: $f(x) = 2x + 3$ $\quad g(x) = x^2$

$(g \circ f)(x) = (2x+3)^2$

$x^2 = (2x+3)^2$

$= (2x+3)(2x+3)$

$= 4x^2 + 6x + 6x + 4$

$= 4x^2 + 12x + 4$

example: $(f \circ g)(x) = f(g(x))$

$= 2x^2 + 3$ not equal

$(g \circ f) \neq (f \circ g)$

example: $f(n) = n^2$ $\quad g(n) = n + 1$ $\quad h(n) = n - 1$

1 $(g \circ f) = n^2 + 1$

2 $(h \circ g) = (n+1) - 1 = n$

3 $(f \circ n) = n^2 = (n-1)^2$

$= (n-1)(n-1)$

$= n^2 - n - n + 1$

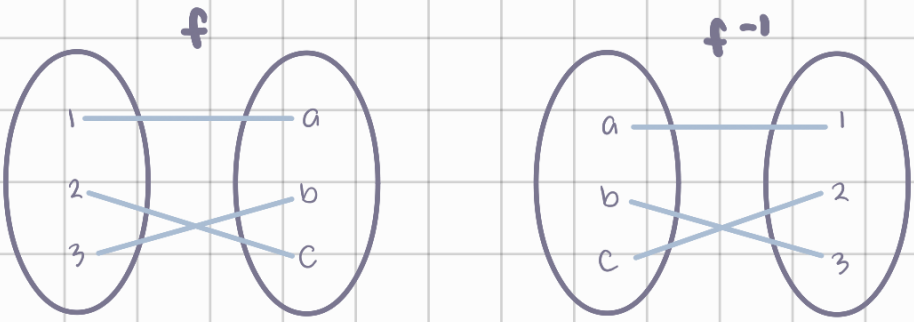
$= n^2 - 2n + 1$

Inverse function

↳ denoted by f^{-1}, & it exists when f is both one to one and onto function

$$f = \begin{pmatrix} (1, 2) \\ (3, 5) \\ (6, 7) \end{pmatrix} \qquad f^{-1} = \begin{pmatrix} (2, 1) \\ (5, 3) \\ (7, 6) \end{pmatrix}$$

they swap the domain & codomain



STEPS for finding inverse

- replace the f(x) by y in the equation
- interchange x and y
- solve for y
- replace y by $f^{-1}(x)$

example: $f(x) = 6 - x/2$

step 1 : $y = 6 - x/2$

step 2 : $x = 6 - y/2$

step 3: $x = 6 - y/2$

$y/2 = 6 - x$

$y = 12 - 2x$

step 4: $f^{-1}(x) = 12 - 2x$

example: $f(x) = x^3 + 2$

step 1 : $y = x^3 + 2$

step 2: $x = y^3 + 2$

step 3: $x - 2 = y^3$

$(x-2)^{1/3} = y$

step 4:

$f^{-1}(x) = (x-2)^{1/3}$