Sparse System Identification :

It is known that the conventional adaptive filtering algorithms can have good performance for non-sparse systems identification, but unsatisfactory performance for sparse systems identification. The normalized least mean absolute third (NLMAT) algorithm which is based on the high-order error power criterion has a strong anti-jamming capability against the impulsive noise, but reduced estimation performance in case of sparse systems

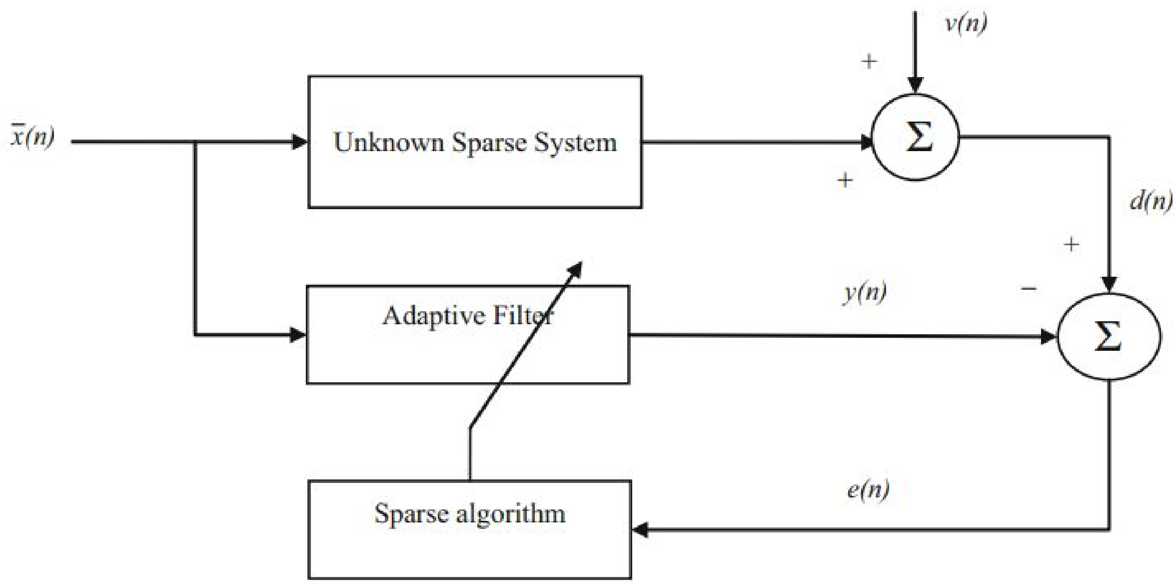



**Fig. 1** Block diagram of sparse system identification

Exploiting sparsity :

Zero-Attracting NLMAT
Reweighted zero-attracting NLMAT
Reweighted l1-nprm NLMAT
Non-Uniform Norm Constraint NLMAT
Correntropy0Induced Metric CIM-NLMAT

I implemented NNC-NLMAT with following additions :

Proportionality Constraint
Set-Membership Constraint

NLMAT :

The objective function of LMAT algorithm is

$$\begin{aligned} J_{\mathrm{LMAT}}(n) &= |e(n)|^3 \\ &= |d(n) - y(n)|^3 \end{aligned}$$

The weight update rule of LMAT is given as

$$\bar{W}(n+1) = \bar{W}(n) + \mu e^2(n)\mathrm{sgn}[e(n)]\bar{x}(n)$$

where the positive constant $\mu$ is the step-size parameter.
$\mathrm{sgn}(x)$ denotes the sign function of $x$ which is defined as

$$\begin{aligned} \mathrm{sgn}(x) &= \frac{x}{|x|}, \quad x \neq 0 \\ &= 0, \quad x = 0 \end{aligned}$$

The drawback of the LMAT algorithm is that its convergence performance is highly dependent on the power of the input signal.

To avoid the limitation of the LMAT algorithm, the NLMAT algorithm [43] is derived by considering the following minimization problem [34]:

$$\min_{\bar{W}(n+1)} \left\{ \frac{1}{3} \left| d(n) - \bar{W}^T(n+1)\bar{x}(n) \right|^3 + \frac{1}{2} \|\bar{x}(n)\|^2 \left\| \bar{W}(n+1) - \bar{W}(n) \right\|^2 \right\} \quad (5)$$

The weight update equation for the NLMAT algorithm is given by

$$\bar{W}(n+1) = \bar{W}(n) + \mu \frac{e^2(n)\mathrm{sgn}[e(n)]\bar{x}(n)}{\bar{x}^T(n)\bar{x}(n) + \delta} \quad (7)$$

where $\mu$ is a step-size parameter, and $\delta$ is a small positive constant to prevent division by zero when $\bar{x}^T(n)\bar{x}(n)$ vanishes.

Non-Uniform Norm Constraint :

In order to further improve the performance of sparse system identification, the non-uniform p-norm-like constraint is incorporated into NLMAT algorithm.

$$||\mathbf{w}(n)||_{p,L}^{p} = \sum_{i=1}^{L} |w_i(n)|_i^p, \quad 0 \le p_i \le 1.$$

the p-norm-like constraint will cause
an estimation error for the desired sparsity exploitation. To solve this problem, the non-uniform p-norm-like definition which uses a different value of p for each of the L entries in W¯ (n) is provided,

$$w_i(n) > v(n) \text{ or } w_i(n) < v(n)$$

$$\begin{aligned} w_i(n+1) =& w_i(n) + \mu e(n) x(n-i) \\ & - \kappa f \operatorname{sgn}[w_i(n)], \quad \forall 0 \le i < L \end{aligned}$$

$$f = \frac{\operatorname{sgn}[v(n) - |w_i(n)|] + 1}{2}, \quad \forall 0 \le i < L$$

$$\begin{aligned} w_i(n+1) =& w_i(n) + \mu e(n) x(n-i) \\ & - \frac{\kappa f \operatorname{sgn}[w_i(n)]}{1 + \varepsilon |w_i(n)|}, \quad \forall 0 \le i < L \end{aligned}$$

Proportionality Constraint :

The updating equation of the PNLMS algorithm can be
described as ;-

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \frac{\mu \mathbf{x}(n) \mathbf{Q}(n-1) e(n)}{\mathbf{x}^T(n) \mathbf{Q}(n-1) \mathbf{x}(n) + \varepsilon},$$

where μ is an overall step size, and ε is a small regularization parameter. Q ( n − 1 ) is the step size
assignment matrix, which is diagonal and can be described as

$$\mathbf{Q}(n-1) = \operatorname{diag}\{q_0(n-1), q_1(n-1), \dots, q_{N-1}(n-1)\} .$$

The elements in Q ( n − 1 ) are calculated by

$$q_j(n-1) = \frac{\alpha_j(n-1)}{\sum_{i=0}^{N-1} \alpha_i(n-1)}, 0 \le j \le N-1,$$

Set-Membership Constraint :

The updating equation of the SM-PNLMS is

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \frac{\mu_{\mathrm{SM}} \mathbf{x}(n) \mathbf{Q}(n-1) e(n)}{\mathbf{x}^T(n) \mathbf{Q}(n-1) \mathbf{x}(n) + \varepsilon_{\mathrm{SM}}},$$

$$\mu_{\mathrm{SM}} = \begin{cases} 1 - \frac{\gamma}{|e(n)|}, \text{if } |e(n)| > \gamma \\ 0, \ \text{otherwise} \end{cases} .$$

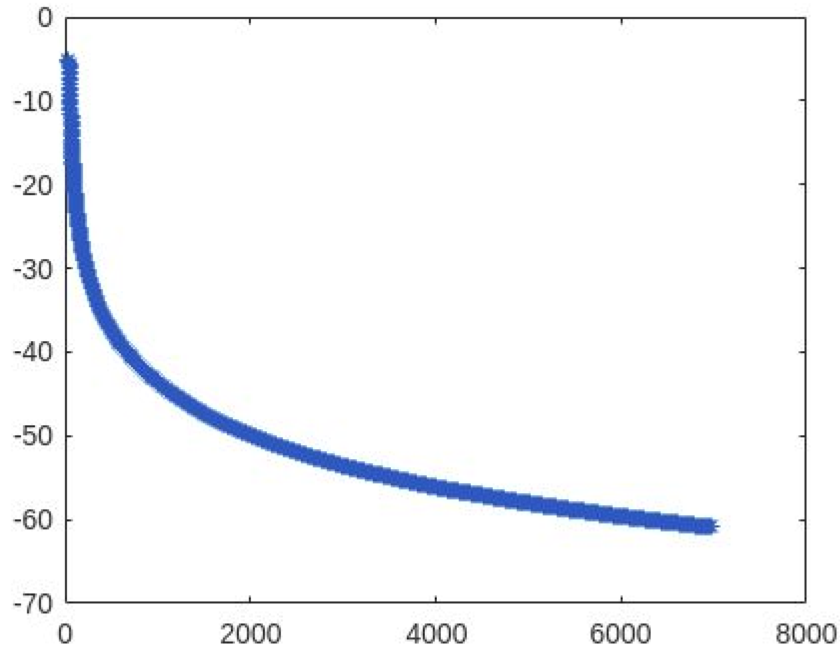
The Algorithm :

**Table 1** Pseudocodes

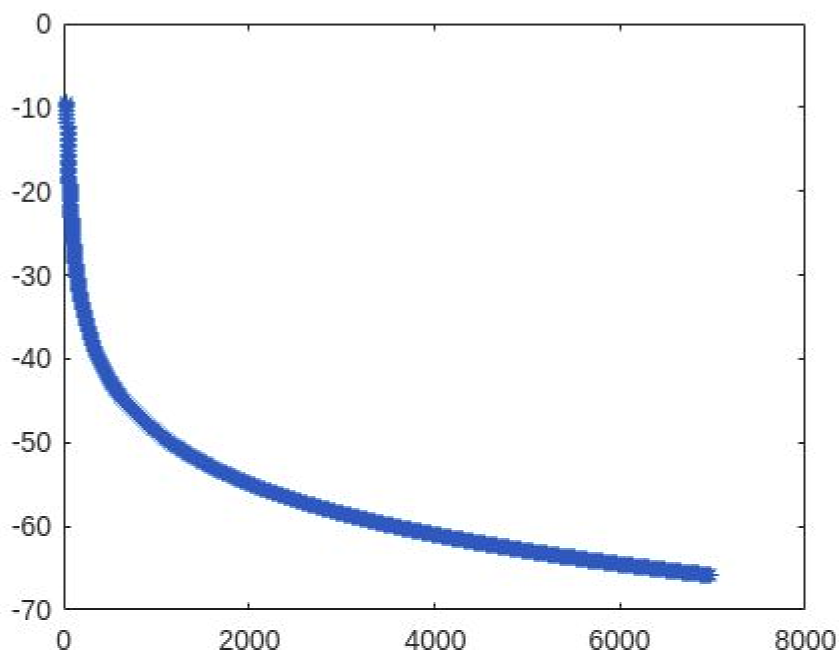
| | |
|---|---|
| Initialization | $\bar{W}(0) = 0_{\mathrm{Lx1}}, \sigma_e(0) = 0, N_w = L$ |
| Parameters | $\mu, \delta, \rho_{\mathrm{ZA}}, \rho_{\mathrm{RZA}}, \varepsilon_{\mathrm{RZA}}, \rho_{\mathrm{RL1}}, \delta_{\mathrm{RL1}}, \rho_{\mathrm{NNC}}, \varepsilon_{\mathrm{NNC}}, \rho_{\mathrm{CIM}}, \sigma$ |
| Loop | For $n = 1, 2, 3 \ldots$<br>$y(n) = \bar{W}^T(n)\bar{x}(n)$<br>$e(n) = d(n) - y(n)$<br>The proposed algorithms can be written in a unifying form as<br>$\bar{W}(n+1) = \bar{W}(n) + \mu f(e(n))\bar{x}(n) + \rho g(\bar{W}(n))$<br>where<br>$f(e(n)) = \frac{\mathrm{sgn}[e(n)]}{\bar{x}^T(n)\bar{x}(n)+\delta}\min\left\{e^2(n), e_{\mathrm{up}}\right\}$<br>$e_{\mathrm{up}} = \frac{\sqrt{2\pi}\sigma_e(n)}{\mu}, \sigma_e(n) = \sqrt{\frac{O^T(n)T_w O(n)}{N_w - K}}$<br>and for $\mathrm{ZA-NLMAT}: g(\bar{W}(n) = -\mathrm{sgn}(\bar{W}(n))$<br>$\mathrm{RZA-NLMAT}: g(\bar{W}(n) = -\frac{\mathrm{sgn}(\bar{W}(n))}{1+\varepsilon_{\mathrm{RZA}}\lvert\bar{W}(n)\rvert}$<br>$\mathrm{RL1-NLMAT}: g(\bar{W}(n) = -\frac{\mathrm{sgn}(\bar{W}(n))}{\delta_{\mathrm{RL1}}+\lvert\bar{W}(n-1)\rvert}$<br>$\mathrm{NNC-NLMAT}: g(\bar{W}(n) = -\frac{F\mathrm{sgn}(\bar{W}(n))}{1+\varepsilon_{\mathrm{NNC}}\lvert\bar{W}(n)\rvert}$<br>$\mathrm{CIM-NLMAT}: g(\bar{W}(n) = -\frac{1}{L\sigma^3\sqrt{2\pi}}\bar{W}(n)\exp\left(-\frac{(\bar{W}(n))^2}{2\sigma^2}\right)$ |

Correlated Results :
Input and noise correlated
Results look similar to the ones in the paper.A proper comparison is yet to be done.

Uncorrelated Results :
Input and noise uncorrelated
Results look similar to the ones in paper.A proper comparison is yet to be done.

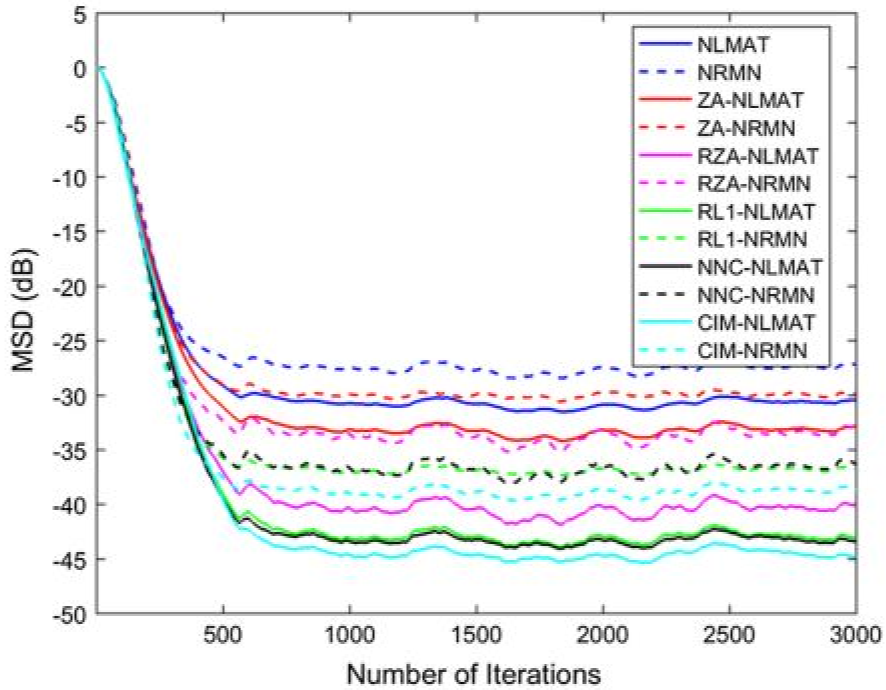



Correlated




Uncorrelated

Their results :




Remainder :

- Comparing the results for various noise models among themselves and that of the paper;
- Understanding the algorithms
- Understanding why they perform better than LMS ;