Sparse System Identification

Final Presentation

20161205

System Identification :

Cited from Wiki

System identification is a methodology for building mathematical models of dynamic systems using measurements of the system's input and output signals.

The process of system identification requires that you:

- Measure the input and output signals from your system in time or frequency domain.
- Select a model structure.
- Apply an estimation method to estimate value for the adjustable parameters in the candidate model structure.
- Evaluate the estimated model to see if the model is adequate for your application needs.

Sparse System Identification :

It is known that the conventional adaptive filtering algorithms can have good performance for non-sparse systems identification, but unsatisfactory performance for sparse systems identification. The normalized least mean absolute third (NLMAT) algorithm which is based on the high-order error power criterion has a strong anti-jamming capability against the impulsive noise, but reduced estimation performance in case of sparse systems

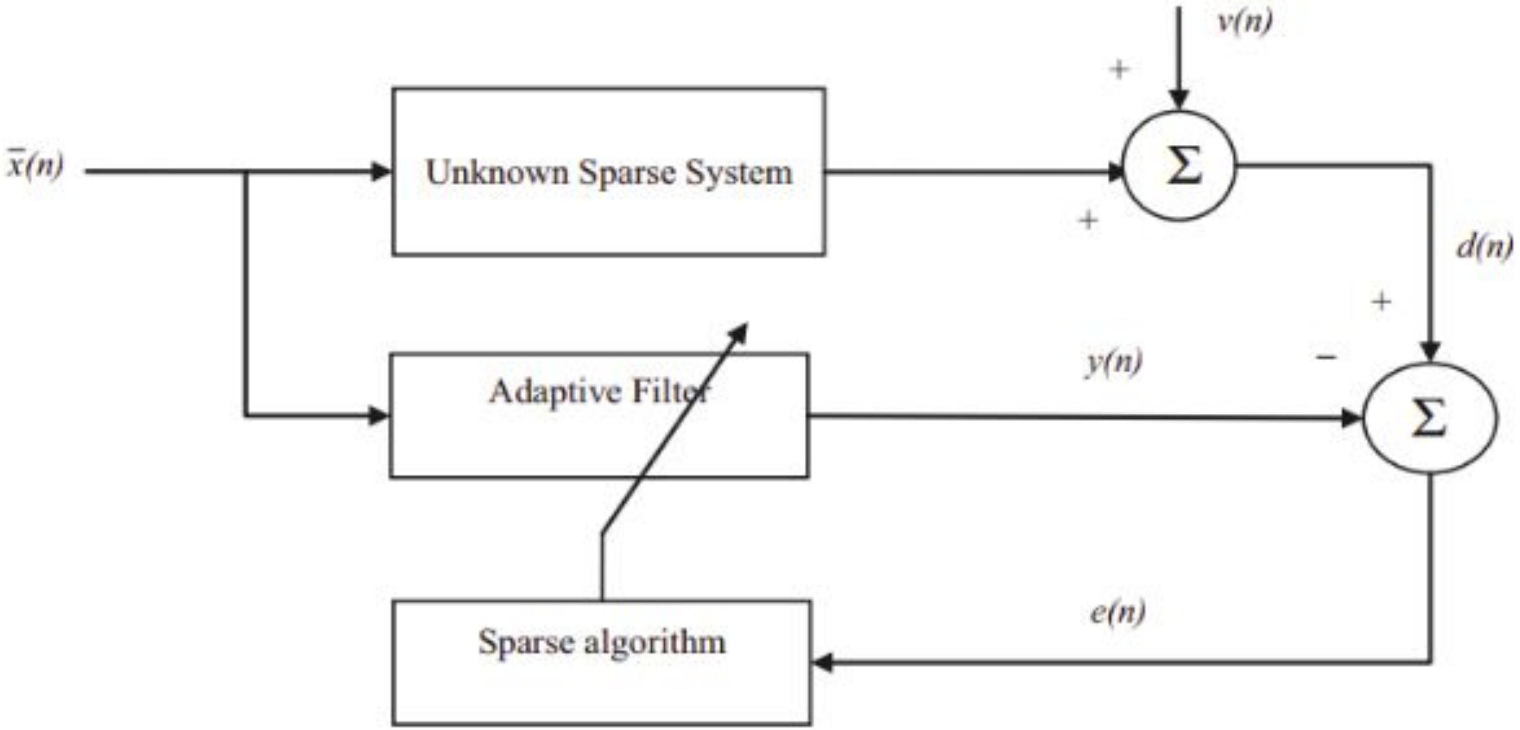


Fig. 1 Block diagram of sparse system identification

Exploiting sparsity :

Zero-Attracting NLMAT
Reweighted zero-attracting NLMAT
Reweighted l1-nprm NLMAT
Non-Uniform Norm Constraint NLMAT
Correntropy0Induced Metric CIM-NLMAT

I implemented NNC-NLMAT with following additions :

Proportionality Constraint
Set-Membership Constraint

NLMAT :

The objective function of LMAT algorithm is

$$
\begin{aligned}
J_{\mathrm{LMAT}}(n) &= |e(n)|^3 \\
&= |d(n) - y(n)|^3
\end{aligned}
$$

The weight update rule of LMAT is given as

$$\bar{W}(n+1) = \bar{W}(n) + \mu e^2(n)\operatorname{sgn}[e(n)]\bar{x}(n)$$

where the positive constant μ is the step-size parameter.

$\operatorname{sgn}(x)$ denotes the sign function of x which is defined as

$$
\begin{aligned}
\operatorname{sgn}(x) &= \frac{x}{|x|}, \quad x \neq 0 \\
&= 0, \quad x = 0
\end{aligned}
$$

The drawback of the LMAT algorithm is that its convergence performance is highly dependent on the power of the input signal.

To avoid the limitation of the LMAT algorithm, the NLMAT algorithm [43] is derived by considering the following minimization problem [34]:

$$\min_{\bar{W}(n+1)} \left\{ \frac{1}{3} \left| d(n) - \bar{W}^T(n+1)\bar{x}(n) \right|^3 + \frac{1}{2} \|\bar{x}(n)\|^2 \left\| \bar{W}(n+1) - \bar{W}(n) \right\|^2 \right\} \tag{5}$$

The weight update equation for the NLMAT algorithm is given by

$$\bar{W}(n+1) = \bar{W}(n) + \mu \frac{e^2(n)\mathrm{sgn}[e(n)]\bar{x}(n)}{\bar{x}^T(n)\bar{x}(n) + \delta} \tag{7}$$

where μ is a step-size parameter, and δ is a small positive constant to prevent division by zero when $\bar{x}^T(n)\bar{x}(n)$ vanishes.

Non-Uniform Norm Constraint :

In order to further improve the performance of sparse system identification, the non-uniform p-norm-like constraint is incorporated into NLMAT algorithm.

$$||\mathbf{w}(n)||_{p,L}^{p} = \sum_{i=1}^{L} |w_i(n)|_i^{p}, \quad 0 \leq p_i \leq 1.$$

the p-norm-like constraint will cause
an estimation error for the desired sparsity exploitation. To solve this problem, the non-uniform p-norm-like definition which uses a different value of p for each of the L entries in $\bar{W}$ (n) is provided,

$$w_i(n) > v(n) \text{ or } w_i(n) < v(n)$$

$$\begin{aligned} w_i(n+1) = & w_i(n) + \mu e(n) x(n-i) \\ & - \kappa f \operatorname{sgn}[w_i(n)], \quad \forall 0 \leq i < L \end{aligned}$$

$$f = \frac{\operatorname{sgn}[v(n) - |w_i(n)|] + 1}{2}, \quad \forall 0 \leq i < L$$

$$w_i(n+1) = w_i(n) + \mu e(n) x(n-i) - \frac{\kappa f \operatorname{sgn}[w_i(n)]}{1+\varepsilon|w_i(n)|}, \quad \forall 0 \leq i < L$$

Proportionality Constraint :

The updating equation of the PNLMS algorithm can be described as ;-

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \frac{\mu \mathbf{x}(n) \mathbf{Q}(n-1) e(n)}{\mathbf{x}^T(n) \mathbf{Q}(n-1) \mathbf{x}(n) + \varepsilon'},$$

where μ is an overall step size, and ε is a small regularization parameter. Q (n − 1) is the step
size
assignment matrix, which is diagonal and can be described as

$$\mathbf{Q}(n-1) = \mathrm{diag}\left\{ q_0(n-1), q_1(n-1), \ldots, q_{N-1}(n-1) \right\} .$$

The elements in Q (n − 1) are calculated by

$$q_j(n-1) = \frac{\alpha_j(n-1)}{\sum\limits_{i=0}^{N-1} \alpha_i(n-1)}, 0 \le j \le N-1,$$

Set-Membership Constraint :

The updating equation of the SM-PNLMS is

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \frac{\mu_{\text{SM}}\mathbf{x}(n)\mathbf{Q}(n-1)e(n)}{\mathbf{x}^T(n)\mathbf{Q}(n-1)\mathbf{x}(n) + \varepsilon_{\text{SM}}},$$

$$\mu_{\text{SM}} = \begin{cases} 1 - \frac{\gamma}{|e(n)|}, \text{if } |e(n)| > \gamma \\ 0, \text{ otherwise} \end{cases} .$$

The Algorithm :

Table 1 Pseudocodes

Initialization	$\bar{W}(0) = 0_{\mathrm{Lx1}}, \sigma_e(0) = 0, N_w = L$
Parameters	$\mu, \delta, \rho_{\mathrm{ZA}}, \rho_{\mathrm{RZA}}, \varepsilon_{\mathrm{RZA}}, \rho_{\mathrm{RL1}}, \delta_{\mathrm{RL1}}, \rho_{\mathrm{NNC}}, \varepsilon_{\mathrm{NNC}}, \rho_{\mathrm{CIM}}, \sigma$
Loop	For $n = 1, 2, 3 \ldots$ $y(n) = \bar{W}^T(n)\bar{x}(n)$ $e(n) = d(n) - y(n)$ The proposed algorithms can be written in a unifying form as $\bar{W}(n+1) = \bar{W}(n) + \mu f(e(n))\bar{x}(n) + \rho g(\bar{W}(n))$ where $f(e(n)) = \frac{\mathrm{sgn}[e(n)]}{\bar{x}^T(n)\bar{x}(n)+\delta}\min\left\{e^2(n), e_{\mathrm{up}}\right\}$ $e_{\mathrm{up}} = \frac{\sqrt{2\pi}\sigma_e(n)}{\mu}, \sigma_e(n) = \sqrt{\frac{O^T(n)T_w O(n)}{N_w - K}}$ and for ZA $-$ NLMAT : $g(\bar{W}(n) = -\mathrm{sgn}(\bar{W}(n))$ RZA $-$ NLMAT : $g(\bar{W}(n) = -\frac{\mathrm{sgn}(\bar{W}(n))}{1+\varepsilon_{\mathrm{RZA}}\lvert\bar{W}(n)\rvert}$ RL1 $-$ NLMAT : $g(\bar{W}(n) = -\frac{\mathrm{sgn}(\bar{W}(n))}{\delta_{\mathrm{RL1}}+\lvert\bar{W}(n-1)\rvert}$ NNC $-$ NLMAT : $g(\bar{W}(n) = -\frac{F\mathrm{sgn}(\bar{W}(n))}{1+\varepsilon_{\mathrm{NNC}}\lvert\bar{W}(n)\rvert}$ CIM $-$ NLMAT : $g(\bar{W}(n) = -\frac{1}{L\sigma^3\sqrt{2\pi}}\bar{W}(n)\exp\left(-\frac{(\bar{W}(n))^2}{2\sigma^2}\right)$

Computational Complexity :

Per Iteration :L is the length of unknown sparse system

Algo(-nlmat)	+	*	/	sqrt()	comp.	exp()
nnc	5L+3	6L+2	L+2	1	2L+2	-
Sm-nnc	5L+4	6L+4	L+3	1	2L+3	-
P-sm-nnc	7L+4	6L+ L*L+6	2L+3	1	2L+5	

Implementations :

All the files are almost the same with minor changes.

Nnc_nlmatXY.m
nnc_nlmatucXY.m

Where X is in [1,3] ;1 : vanilla nnc-nlmat, 2 : SM-nnc-nlmat,3 : P-SM-nnc-nlmat;
And Y is in [1,4];1 : Gaussian noise,2 : Uniform noise , 3 : Rayleigh noise , 4 : exponential noise
.
All files and images from results are available on this link :

https://github.com/cheapkai/gde

Simulations :

The below described model is from this simulation experiment : https://link.springer.com/article/10.1007%2Fs00034-019-01111-3

The desired response d(n) of the adaptive filter is calculated as d(n) h T x¯(n) + v(n), where superscript T indicates transpose of matrix or vector, h denotes the weight vector of the unknown system of length L, x¯(n) [x(n), x(n − 1), . . . x(n − L + 1)] T is the input vector of the system, and v(n) is the system background noise. The system noise consists of the impulsive noise along with different noise distributions (Gaussian, uniform, Rayleigh and exponential).

The unknown system, h, is of length L = 36, The adaptive filter is also assumed to be of the same length. The proposed algorithms are compared under different sparsity levels $S = 5$ and $S = 10$. The active coefficients,non-zero are uniformly distributed in the interval (− 1, 1),

. The Gaussian white noise with variance σ x 2 1 is considered as the input signal $x(n)$. The correlated signal $z^{-}(n)$ is obtained using a first-order autoregressive process, AR(1), with a pole 0.5 and is given by $z^{-}(n)\ \ 0.5z^{-}(n - 1) + x^{-}(n)$. The system background noise consists of impulsive noise combined with different noise distributions such as (1) white Gaussian noise with N (0, 1), (2) uniformly distributed noise within the range (-1, 1), (3) Rayleigh distribution with 1 and (4) an exponential distribution with 2. The impulsive noise is modeled by a Bernoulli–Gaussian (BG) process is given as ξ (n) a(n)I (n), where a(n) is a white Gaussian and signal with N 0, σ a 2 and I (n) is a Bernoulli process described by the probability

p{I (n) 1} Pr, p{I (n) 0} 1 − Pr, where Pr represents the probability of the impulsive noise occurrence.
The mean square deviation (MSD) :

as the performance metrics to measure the performance of the proposed algorithms which are expressed as :

$$\mathrm{MSD(dB)} = 10\log_{10}\left(\left\|\boldsymbol{h} - \bar{W}(n)\right\|_2^2\right)$$

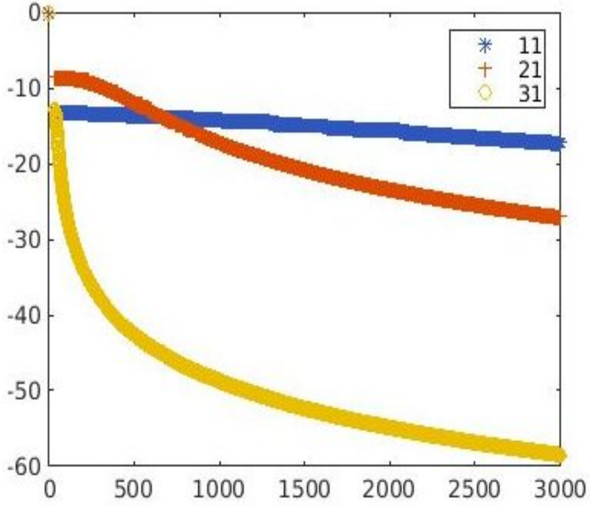
The simulation parameters for sparse
NLMAT algorithms are given as $\mu = 0.8$(initially-before SM), $\delta = 1\times10^{-3}$,
$\rho_{NNC} = 1 \times 10^{-3}$, $\varepsilon_{NNC} = 20$, $\sigma = 0.05$.

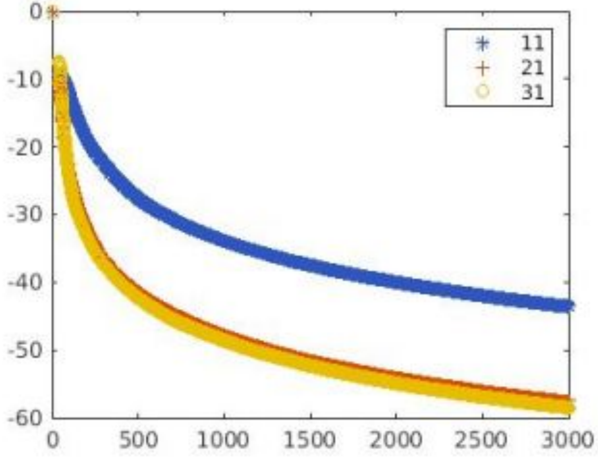
Results :

Simulated data :

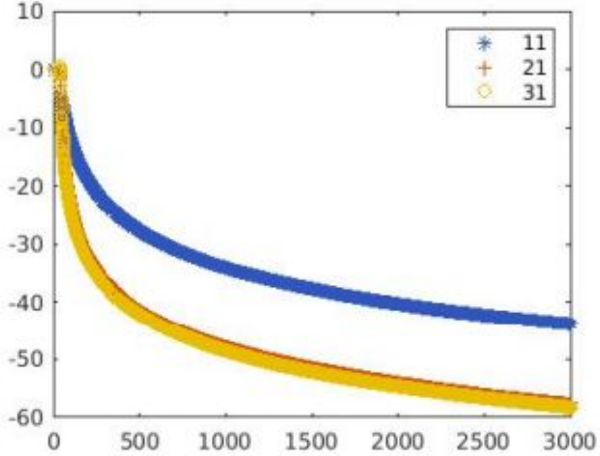
S = 10 - less sparsity

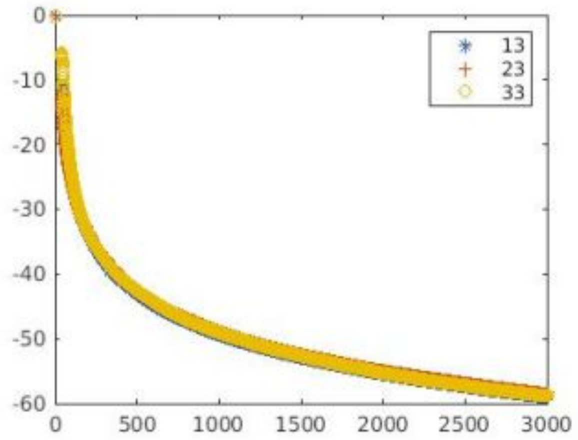
1 :





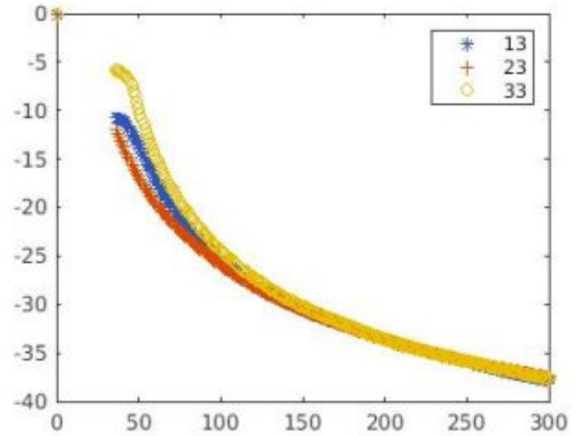
S = 2 , Very Sparse

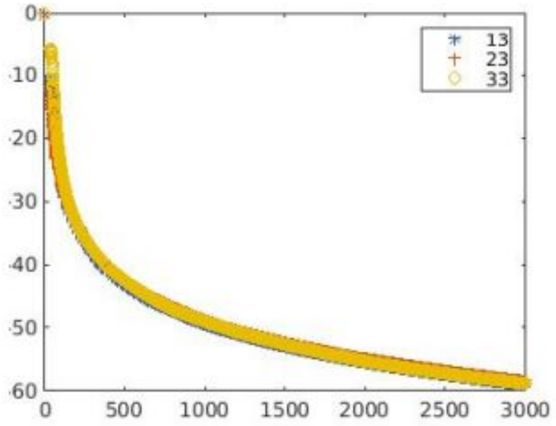




With fewer iterations :

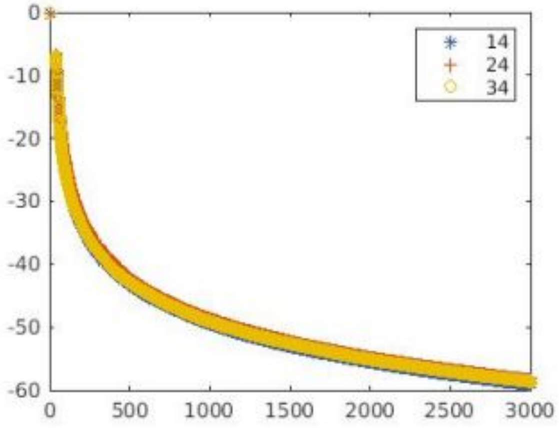
S = 4

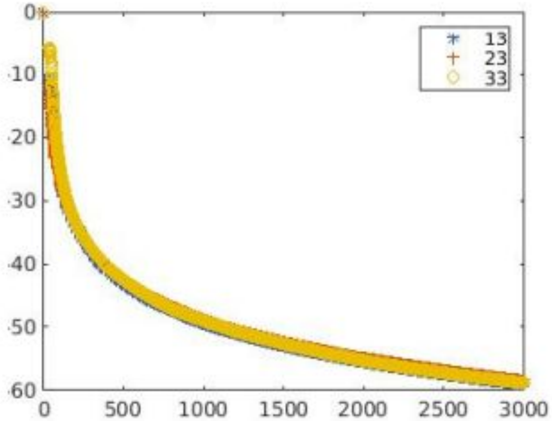




4 :

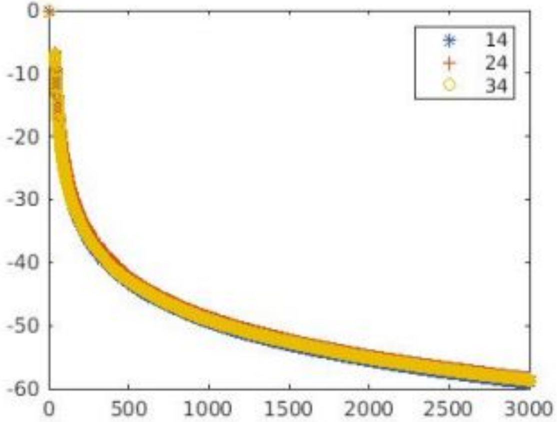
S = 10

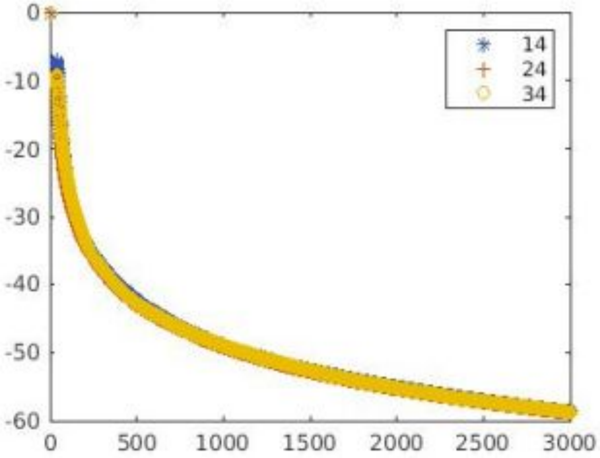




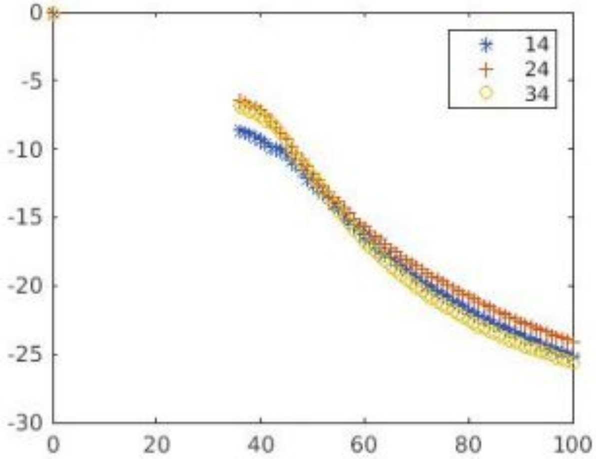
4 :

S = 10





S = 5 - till fewer iterations -



Real Data :
From MATLAB datasets -

System Description :

This case study concerns data collected from a laboratory scale "hairdryer". (Feedback's Process Trainer PT326; See also page 525 in Ljung, 1999). The process works as follows: Air is fanned through a tube and heated at the inlet. The air temperature is measured by a thermocouple at the outlet. The input is the

voltage over the heating device, which is just a mesh of resistor wires. The output is the outlet air temperature represented by the measured thermocouple voltage.

load dryer2.mat

Output :