

A few instructions

- Codes should be compatible with *Python3* and should run on Ubuntu.
- Code for each question should be placed in a separate stand-alone files.
- Codes should be well-commented.
- **Appropriate exceptions should be raised and handled.**

1. Write a program to visualize actual derivative ( $f'(x)$ ) and forward finite difference approximation ( $\delta_{0.01}^+(x)$ ) of the function  $\sin(x^2)$  in the interval  $[0, 1]$ . [10]
2. Write a program to visualize the absolute errors of approximation  $\delta_{0.01}^+(x)$ ,  $\delta_{0.01}^-(x)$  and  $\delta_{0.01}^c(x)$  of function  $\sin(x^2)$  in the interval  $[0, 1]$ . [10]
3. Write a program to visualize, as a function of  $h$ , the maximum absolute error of approximations  $\delta_h^+(x)$  and  $\delta_h^c(x)$  of function  $\sin(x^2)$  in the interval  $[0, 1]$ . In the same figure, also plot the theoretical maximum absolute error of approximations  $\delta_h^+(x)$  and  $\delta_h^c(x)$ . [20]
4. Write a program to visualize, as a function of  $M$  (number of intervals), area under the curve  $y(x) = 2x \cdot e^{x^2}$  in the interval  $[1, 3]$  computed using the trapezoidal formula. In the figure, also indicate the exact area. [20]
5. Write a program to visualize, as a function of  $u$ , area under the curve  $y(x) = 2x \cdot e^{x^2}$  in the interval  $[0, u]$  computed using various integration functions available in Python's `scipy.integrate` module. In the figure, also indicate the actual area under the curve. [20]
6. Enhance the class `Polynomial`, developed in the last coding assignment, as follows
  - Add a method `derivative` that will return the polynomial's derivative.

```
p = Polynomial([1, 2, 3])
pd = p.derivative()
print(pd)
```

Expected output:

```
Coefficients of the polynomial are:
2 6
```

- Add a method **area** that takes two arguments  $a$  and  $b$ , and returns the exact area under the polynomial in the interval  $[a, b]$

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```
p = Polynomial([1, 2, 3])  
print(p.area(1,2))
```

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Expected output:

Area in the interval [1, 2] is: 11

7. Write a program that uses the enhanced **Polynomial** class to approximate area under the curve  $y(x) = e^x \cdot \sin x$  in the interval  $[0, 1/2]$  within a guaranteed error of  $10^{-6}$ .

[10]

**NOTE: Your code should not use any numerical integration techniques, and should not compute the actual area under the curve.**