Априорное распр.:

$\mu \sim N(\mu_0, \sigma_0^2)$

μ_0, σ_0^2 — изв. конст.

① $X_1 \dots X_n \sim N(\mu, 1)$

$$p(D|\mu) = \prod_{i=1}^{n} \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{(x-\mu)^2}{2\sigma^2}} \qquad p(x|\mu) = \frac{1}{\sigma_0\sqrt{2\pi}} \cdot e^{\frac{(x-\mu_0)^2}{2\sigma_0^2}}$$

MLE: $\theta_0 = \arg\max_\theta p(D|\mu) \xrightarrow{\text{по байесу}} = \frac{p(X|\mu)p(\mu)}{p(X)}$ → не $\sim$ от μ

логарифм.

NLL: $$l(\theta) = -\sum_{i=1}^{n} \log p(x_i|\mu) - \log p(\mu) = -\sum_{i=1}^{n} \log \frac{1}{\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2}}$$

$$-\log \frac{1}{\sigma_0^2\sqrt{2\pi}} e^{-\frac{(\mu-\mu_0)^2}{2\sigma_0^2}}$$

↓ пр. по μ

$$\frac{dl}{d\mu} = \sum_{i=1}^{n} x_i - n\mu + \frac{\mu_0 - \mu}{\ln\sigma_0^2} = 0 \iff \sum_{i=1}^{n} x_i - \mu\left(n + \frac{1}{\ln\sigma_0^2}\right) + \frac{\mu_0}{\ln\sigma_0^2} = 0$$

$$\mu = \frac{\sum_{i=1}^{n} x_i + \frac{\mu_0}{\ln\sigma_0^2}}{n + \frac{1}{\ln\sigma_0^2}} = \frac{\sum_{i=1}^{n} x_i \cdot \ln\sigma_0^2 + \mu_0}{\cancel{\ln\sigma_0^2}} : \frac{n \cdot \ln\sigma_0^2 + 1}{\cancel{\ln\sigma_0^2}} =$$

$$= \frac{\sum_{i=1}^{n} x_i \cdot \ln\sigma_0^2 + \mu_0}{n\ln\sigma_0^2 + 1}$$