THROUGHPUT OPTIMIZATION

FOR WIRELESS DATA TRANSMISSION

THESIS

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Vita

Richard J. Lavery was born in Queens, NY on November 28, 1979. He attended elementary school at St. Pancras parish in Glendale, NY. Richard was then enrolled at Archbishop Molloy High School in Briarwood, NY from 1993 to 1997.

In September 1997, Richard entered the Brooklyn campus of Polytechnic University. He was enrolled in the University's BS/MS Honors program for a Bachelor of Science in Electrical Engineering and a Master of Science in Telecommunications Networks. Work experience gained while enrolled at the University included a position as a student researcher in the power lab as part of the Polytechnic University Summer Research Program in 1999. In the summer of 2000, Richard was admitted to the NASA Academy at Goddard Space Flight Center in Greenbelt, MD. There he worked on a prototype for an electrical circuit used to simulate time interval measurement circuits. Richard graduated with honors from Polytechnic University on June 3, 2001.

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for my family and friends…

for their love and support throughout my life

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AN ABSTRACT

THROUGHPUT OPTIMIZATION FOR WIRELESS DATA TRANSMISSION

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The throughput of a wireless data communications system depends on a number of variables. I've examined several of them including: packet size, transmission rate, the number of overhead bits in each packet, received signal power, received noise power spectral density, modulation technique, and channel conditions. Our main result is a mathematical technique for determining the optimum transmission rate and packet size as a function of the other variables. The key to maximizing the throughput rate is maintaining the signal-to-noise ratio at an optimum level determined by the nature of the modem and the channel. In an attempt to improve throughput performance, I have included an analysis using forward error correcting (FEC) block codes. The optimum amount of FEC coding was found to be dependent upon the packet length. As the packet length increases, the number of correctable errors to optimize the throughput also

increases. This optimum point roughly occurs when the amount of error correcting overhead bits is the same as the number of bits in the packet prior to encoding. The received power was also varied by changing the distance between the transmitter and receiver. Rate adaptation and adaptive FEC coding was shown as a way to extend the range of a wireless data transmission at the cost of decreased throughput. A comparison of the two methods showed adaptive FEC coding to yield the better results. BCH codes were used to provide a practical example of the work presented. The results were shown to agree with the previous conclusions made.

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Chapter 1

Introduction

Following the success of cellular telephone services in the 1990s, the technical community has turned its attention to data transmission. Throughput is a key measure of the quality of a wireless data link. It is defined as the number of information bits received without error per second and we would naturally like this quantity as to be high as possible. This thesis looks at the problem of optimizing throughput for a packet based wireless data transmission scheme from a general point of view. The purpose of this work is to show the very nature of throughput and how it can be maximized by observing its response to certain changing parameters.

There has been little previous work on the topic of optimizing throughput in general. Some things that have been investigated include choosing an optimal power level to maximize throughput [1]. Maximizing throughput in a direct sequence spread spectrum network by way of a link layer protocol termed the Transmission Parameter Selection Algorithm (TPSA) has also been discussed [2]. This provides real time distributed control of transmission parameters such as power level, data rate, and forward error correction rate. An analysis of throughput as a function of the data rate in a CDMA system has also been presented [3]. Most of the previous work found has taken a very specific look at throughput in different wireless voice systems such as TDMA, CDMA, GSM, etc. by taking into account many different system parameters in the analysis such as *Parameter Optimization of CDMA Data Systems* [4]. We have taken a more general look at throughput by considering its definition for a packet-based scheme and how it can be maximized based on the channel model being used. Unlike most of the work done on this topic, our research is focused on the transmission of data as opposed to that of voice. Most of the work done on data throughput analysis has been in wired networks (i.e. Ethernet, SONET, etc.). Even in this work, however, the analysis is mostly done with system specific parameters.

Many variables affect the throughput of a wireless data system including the packet size, the transmission rate, the number of overhead bits in each packet, the received signal power, the received noise power spectral density, the modulation technique, and the channel conditions. From these variables, we can calculate other important quantities such as the signal-to-noise ratio γ, the binary error rate $P_e(\gamma)$, and the packet success rate $f(\gamma)$. Throughput depends on all of these quantities. Chapter 2 begins the analysis by looking at throughput optimization as a function of the packet length with a fixed transmission rate followed by an analysis of throughput as a function of transmission rate with a fixed packet length. Using the optimization equations obtained, the throughput can be jointly optimized with respect to both the packet length and transmission rate, both written in terms of the SNR. Chapter 3 looks at the same problem, but this time with the inclusion of a forward error correcting (FEC) block code. Using the coding bounds (as opposed to actual block codes) we can see the effect of FEC coding on throughput and how to obtain the optimum amount of coding. Chapter 4 takes into account mobile receiving stations. Here we look at throughput as a function of increasing distance, and how changing the transmission rate, packet length, and FEC coding affects the results. Chapter 5 gives a practical example of the material presented in chapters 3 and 4 using the well know BCH codes for multiple error correction capability.

Chapter 2

Throughput Optimization Via the Packet Length and Transmission Rate For AWGN Channel and Rayleigh Fading Channel

In this chapter we have taken a general look at throughput by considering its definition for a packet-based scheme and how it is maximized based on the channel model being used. As an initial step in a theoretical study, we examine the influence of transmission rate and packet size in a noise-limited transmission environment. The transmitter, operating at R b/s, sends data in packets. Each packet contains L bits including a payload of K bits and a cyclic redundancy check error-detecting code with C bits. A forward error correction encoder produces the remaining L-K-C bits in each packet. The channel adds white noise with power spectral density N_0 watts/Hz and the signal arrives at the receiver at a power level of P watts. In this thesis we assume N_0 to be the sum of all noise and interference, which can be modeled as Gaussian white noise. The CRC decoder detects transmission errors and generates acknowledgments that cause packets with errors to be retransmitted. Table 2-1 displays a summary of the variables in our analysis and their notation.

Table 2-1 Variables in Analysis

Quantity	**Notation**	**Value**
Signal to Noise Ratio	γ	10
Received signal power	P (watts)	$5*10^{-9}$ W
Receiver noise power spectral density	N_0 (W/Hz)	10^{-15} W/Hz
Binary transmission rate	R bits/s	Varied
Packet size	L bits	Varied
Cyclic Redundancy Check	C bits	16 bits

2.1 Problem Statement

An important objective of data communications systems design and operation is to match the transmission rate to the quality of the channel. A good channel supports a high data rate, and conversely. For a given channel, there is a transmission rate that maximizes throughput. At low rates, transmitted data arrives without error with high probability and an increase in the data rate increases throughput. Above the optimum rate, the error probability is high and it is possible to increase throughput by decreasing the data rate, thereby increasing the probability of correct reception. Recognizing this fact, practical communications systems including facsimile, telephone modems, wireless local area networks, and cellular data systems incorporate rate adaptation to match the transmission rate to the quality of the channel. In some systems (facsimile and telephone modems), the adaptation is static, occurring at the beginning of a communication session only. In others, the adaptation is more dynamic with the rate rising and falling in response to changes in channel conditions.

2.2 Throughput Analysis

2.2.1 Assumptions and Definitions

Our analysis includes the following simplifying assumptions:

- The CRC decoder detects all errors in the output of the FEC decoder.
- Transmission of acknowledgments from the receiver to the transmitter is error-free and instantaneous.
- In the presence of errors, the system performs selective repeat ARQ retransmissions.

- The received signal power is P watts, either a constant or a random variable with a Rayleigh probability density function, representative of fading wireless channels. In this paper, we consider "fast fading" in which the values of P for the different bits in a packet are independent, identically distributed Rayleigh random variables.

System throughput (T) is the number of payload bits per second received correctly:

$$T = \frac{K}{L} Rf(g), \tag{2.1}$$

where (KR/L) b/s is the payload transmission rate and f(γ) is the packet success rate defined as the probability of receiving a packet correctly. This probability is a function of the signal-to-noise ratio

$$g = \frac{E_b}{N_o} = \frac{P}{N_o R}. \tag{2.2}$$

In which E_b=P/R joules is the received energy per bit. We will now look at maximizing the throughput in a Gaussian white noise channel with respect to the transmission rate and packet length.

2.2.2 Throughput vs. Transmission Rate: Fixed Packet Length

To find the transmission rate, R=R* b/s, that maximizes the throughput, we differentiate Equation (2.1) with respect to R to obtain:

$$\frac{dT}{dR} = (K/L)f(g) + (K/L)R\frac{df(g)}{dg}\frac{dg}{dR} = (K/L)\left(f(g) + R\frac{df(g)}{dg}(-P/N_0R^2)\right). \tag{2.3}$$

Next we set the derivative to zero:

$$f(g) - (P/N_0R)\frac{df(g)}{dg} = f(g) - g\frac{df(g)}{dg} = 0, \tag{2.4}$$

or

$$f(\gamma) = \gamma \frac{df(\gamma)}{d\gamma}. \tag{2.5}$$

We adopt the notation γ=γ* for a signal-to-noise ratio that satisfies Equation (2.5). The corresponding transmission rate is

$$R^* = \frac{P}{\gamma^* N_o}. \tag{2.6}$$

A sufficient condition for a locally maximum throughput at R=R* is:

$$\frac{d^2T}{dR^2}\Big|_{R=R^*} < 0. \tag{2.7}$$

The solution to Equation (2.5), γ*, is the key to maximizing the throughput of a packet data transmission. To operate with maximum throughput, the system should set the transmission rate to R* in Equation (2.6). γ* is a property of the function, f(γ), which is the relationship between packet success rate and signal to interference ratio. This function is a property of the transmission system including the modem, codecs, receiver structure and antennas. Each system has its own ideal signal-to-noise ratio, γ*. Depending on the channel quality, reflected in the ratio P/N_0, the optimum transmission rate is R* in Equation (2.6).

2.2.3 Throughput vs. Packet Length: Fixed Transmission Rate

Each packet, of length L bits, is a combination of a payload (K bits) and overhead (L-K bits). Because the packet success rate, f(γ), is a decreasing function of L, there is an optimum packet length, L*. When L<L*, excessive overhead in each packet limits the throughput. When L>L*, packet errors limit the throughput. When there is no forward error correction coding, which we shall assume for the entirety of this chapter, (K=L-C, where C bits is the length of the cyclic redundancy check), there is a simple derivation of L*. In this case,

$$f(\gamma) = \left(1 - P_e(\gamma)\right)^L, \tag{2.8}$$

where $P_e(\gamma)$ is the binary error rate of the modem. Therefore, in a system without FEC, the throughput as a function of L is

$$T = \frac{L - C}{L} R\left(1 - P_e(\gamma)\right)^L. \tag{2.9}$$

To maximize T with respect to L, we consider L to be a continuous variable and differentiate Equation (2.9) to obtain

$$\frac{dT}{dL} = R\frac{L - C}{L}\left(1 - P_e(\gamma)\right)^L \ln\left(1 - P_e(\gamma)\right) + R\frac{C}{L^2}\left(1 - P_e(\gamma)\right)^L. \tag{2.10}$$

Setting the derivative equal to zero produces a quadratic equation in L with the positive root:

$$L^* = \frac{1}{2}C + \frac{1}{2}\sqrt{C^2 - \frac{4C}{\ln\left(1 - P_e(\gamma)\right)}}. \tag{2.11}$$

As shown in Figure 2-1 (in which C=16), the optimum packet size is a decreasing function of $P_e(\gamma)$. As the binary error rate goes to zero, the packet error rate also approaches zero and the optimum packet size increases without bound. Because $P_e(\gamma)$ decreases with γ, L* increases monotonically with signal-to-noise ratio. Better channels support longer packets. Of course, in practice L is an integer and the optimum number of bits in a packet is either the greatest integer less than L* or the smallest integer greater than L*.

Equations (2.5) and (2.11) can be viewed as a pair of simultaneous equations in variables L and γ. Their simultaneous solution produces the jointly optimum packet size and signal-to-noise ratio of a particular transmission system. We will use the notation, L** and γ**, respectively for the jointly optimized variables.

2.2.4 Analysis For a Fast Fading Channel

When the received power is a Rayleigh random variable, the signal-to-noise ratio is a sample value on an exponential random variable, Γ, with probability density function:

$$\begin{aligned} f_\Gamma(g) &= \exp(-g/G)/G \quad \gamma \geq 0 \\ &= 0 \qquad \text{otherwise.} \end{aligned} \tag{2.12}$$

The expected value of the signal-to-noise ratio is E[Γ] = G, and the average packet success rate is $\overline{f}(G)$. To maximize the average throughput with respect to transmission rate, we employ the analysis in Equations (2.1) and (2.3) – (2.5) substituting $\overline{f}(G)$ for f(γ). The result is an optimum average signal-to-noise ratio G*, which is a solution to the equation corresponding to Equation (2.5):

$$\overline{f}(G) = G \frac{d\overline{f}(G)}{dG}. \tag{2.13}$$

The expected value of the probability of error is a function G:

$$\overline{P_e}(G) = \int_0^\infty f_\Gamma(g) P_e(g)\, dg\ . \tag{2.14}$$

The fast fading assumption implies that the average packet success rate is

$$\overline{f}(G) = \left(1 - \overline{P}_e(G)\right)^L. \tag{2.15}$$

Therefore, we find the optimum packet length in a fast fading channel by following Equation (2.9) – (2.11) with $\overline{P}_e(G)$ replacing $P_e(\gamma)$. The result is:

$$L^* = \frac{1}{2}C + \frac{1}{2}\sqrt{C^2 - \frac{4C}{\ln\left(1 - \overline{P}_e(G)\right)}}. \tag{2.16}$$

2.3 Numerical Examples

We confine our attention to transmission systems without forward error correction and examine the relationships between several variables. Table 2-1 lists the variables and the nominal value of each one. The nominal values are representative of wireless communications systems. In our numerical examples, we hold most of these variables at their nominal values and vary one or two others in order to examine their influence on another one. In our numerical analysis we consider a non-coherent frequency shift keying (FSK) transmission scheme in both a white Gaussian noise channel and a Rayleigh fading channel.

2.3.1 White Gaussian Noise Channel

For non-coherent FSK in a white Gaussian noise channel, the probability of a bit error is given by:

$$P_e(g) = \tfrac{1}{2}e^{-\frac{g}{2}}, \qquad (2.17)$$

and so from (2.11) above, we can get the length to maximize the throughput by plugging in (2.17) for $P_e(\gamma)$.

To see the effects of a varying packet length on throughput we choose a fixed transmission rate and graph the throughput (2.9) as a function of L. We illustrate how this graph changes with different values of R by showing three different plots on Figure 2-2. The solid line uses a transmission rate of 300 kbps (γ = 16.67) which, from (2.11), yields a length of $L^*(16.67)$ = 373 bits to maximize the throughput. The small dotted line uses a transmission rate of 400 kbps (γ = 12.5) which yields a length of $L^*(12.5)$ = 137 bits to maximum the throughput. The large dotted line uses a transmission rate of 600 kbps (γ = 8.33) which yields a length of $L^*(8.33) = 54$ bits to maximize the throughput. The relationship between the SNR, γ, and the transmission rate, R, is derived from (2.2).

Some important conclusions can be drawn from this information. We first notice that at high SNR values (low transmission rates) the packet length used to maximize the throughput must be large. When the transmission rate increases and the SNR drops, the packet length to maximize the throughput must also decrease. Another observation we make is how the throughput curve behaves for increasing values of L when different SNR values are used. From Figure 2-2 we can see that at high bit rates (low SNR) the choice of packet size is more critical (i.e. the peak is very localized). On the contrary, at low bit rates (high SNR) the packet length doesn't have much of an effect on the throughput. Also, it can be seen that the maximum throughput increases with decreasing SNR, up to a point. When the SNR gets too low, the maximum throughput begins to decrease. This suggests that the optimum SNR value to give the maximum throughput (γ^{**}) is between 8.33 and 16.67. This observation is confirmed when the throughput is optimized jointly with both the packet length and the SNR.

From (2.5) we cannot obtain an explicit solution for the rate (or SNR) that optimizes throughput directly as was done for the length, but the following result is obtained:

$$e^{-\frac{g^*}{2}} = \frac{4}{2 + Lg^*}. \qquad (2.18)$$

This solution results from substituting (2.8) for $f(\gamma)$ and (2.17) for $P_e(\gamma)$. For any value of L, there is a γ^* that maximizes the throughput. The rate to maximize the throughput can then be found by (2.6). The relationship between γ^* and L can be see in Figure 2-3. Notice that it is approximately linear in the log scale for the packet length.

To see the effects of varying the transmission rate we choose a fixed value of L and graph the throughput (2.9) as a function of R. To illustrate how this graph changes with different values of L we have shown three plots on Figure 2-4. The solid line uses a packet length of 50 bits. If we use this value of L in (2.18) we obtain as a solution γ^* = 9.58, which from (2.6) corresponds to a rate of R^* = 521.9 kbps to maximize the throughput. The small dotted line uses a packet length of 200 bits. If we use this value of L in (2.18) we obtain as a solution γ^* = 12.95, or a rate of R^* = 386.1 kbps to maximize the throughput. The large dotted line uses a packet length of 2000 bits. If we use this value of L in (2.18) we obtain as a solution γ^* = 18.24, or a rate of R* = 274.1 kbps to maximize the throughput.

We can see from Figure 2-4 that as the packet length increases the rate necessary to maximize the throughput decreases. Unlike Figure 2-2, however, the slope at which the throughput decays remains approximately constant for the different packet lengths. For rates less than the optimal rate, the throughput increases linearly with a slope of (L-C)/L. We can also make a general conclusion based on the shapes of the plots in Figures 2-2 and 2-4 by saying that the throughput is more sensitive to changes in the transmission rate than it is to changes in the packet length. Also, it can be seen that the maximum throughput achieved increases with increasing packet length, up to a point. If the packet length gets too large, then the maximum throughput begins to decrease. Based on the graphs in Figure 2-4, we can say that the optimum length to achieve the maximum throughput (L^{**}) is somewhere between 50 and 2000 bits. This observation is confirmed when we optimize the throughput with respect to both SNR and packet length.

To maximize the throughput with respect to both the packet length and the transmission rate, we can write the throughput solely as a function of the SNR by graphing (2.9) and substituting L^* (2.11) for the length, and R^* (2.6) for the rate. In Figures 2-1 through 2-4 we have assumed a constant value ($5*10^6$) for P/N_o. In reality, this value can change as a result of a number of different situations. P depends on the location of the mobile in relation to the base station and N_0 depends on the level of interference present at the mobile. To illustrate how the throughput is affected by these different values of P/N_o we put three different plots in Figure 2-5. The solid line uses a value of 10^6, the small dotted line uses a value of $5*10^6$, and the large dotted line uses a value of 10^7. A very important conclusion can be drawn from this. The actual value of P/N_o only determines the value of the maximum throughput. A high value of P/N_o indicates a high maximum throughput, and a low value indicates a low maximum throughput. The important thing to note from Figure 2-5 is that the value of the SNR to maximize the throughput is independent of the value of P/N_o. We can see that the SNR to maximize throughput for a Gaussian Channel is $\gamma^{**}=11.47$. This is indicated by the vertical line in Figure 2-5. We can now use this value in (2.11) to find that the packet length to achieve maximum throughput is $L^{**}(\gamma^{**})=108$ bits. This packet length is also independent of P/N_o. The rate to maximize the throughput, however, is dependent on P/N_o from (2.6).

2.3.2 Rayleigh Fading Channel

For a model that corresponds to mobile radio communications, we can perform the same analysis for a fast fading Rayleigh channel. For non-coherent FSK in a Rayleigh fading channel, the probability of a bit error is given by:

$$\overline{P}_e(G) = \frac{1}{2+G}. \tag{2.19}$$

We can see how a changing packet length affects the throughput by choosing a fixed transmission rate and graphing (2.9), with $P_e(G)$ replacing $P_e(\gamma)$, as a function of the packet length. To illustrate the effects of changing the transmission rate on the throughput graph, we have three plots on Figure 2-6. The solid line uses a transmission rate of 10 kbps corresponding to G = 500 from (2.6) which from (2.11) yields a packet length of $L^*(500) = 98$ bits to maximize the throughput. The small dotted line uses a transmission rate of 100 kbps corresponding to G = 50 which yields a packet length of $L^*(50) = 38$ bits to maximize the throughput. The large dotted line uses a transmission rate of 500 kbps corresponding to G = 10 which yields a packet length of $L^*(10) = 24$ bits to maximize the throughput. The same conclusions and observations can be made from Figure 2-6 as those made from Figure 2-2. The only real difference is the scale of the numbers used. Because a fading channel imposes more rigorous conditions on a transmission system, the achievable throughput will be lower than a Gaussian channel. Consequently, the system will have to operate at higher average SNR values and smaller average packet lengths.

From (2.13) the bit rate to maximize throughput is found to be:

$$R^* = \frac{P}{N_o}\left[\frac{L-3-\sqrt{L^2-6L+1}}{4}\right]. \tag{2.20}$$

This solution results from substituting (2.15) for $\overline{f}(G)$ and (2.19) for $\overline{P_e}(G)$. To see how throughput changes as a function of the transmission rate we graph the throughput as a function of R with L fixed. To illustrate the effects of changing the packet length we have three plots on Figure 2-7. The solid line uses a packet length of 20 bits. We can use this value in (2.20) to tell us that the transmission rate to maximize the throughput is R^* = 296.2 kbps (G^* = 16.88). The small dotted line uses a packet length of 40 bits. From (2.20), the transmission rate to maximize throughput is R^* = 135.3 kbps (G^* = 36.95). The large dotted line uses a packet length of 100 bits. From (2.20), the transmission rate to maximize throughput is R^* = 51.6 kbps (G^* = 96.98). The same conclusions and observations can be made from Figure 2-7 as those made from Figure 2-4. Again, the only real difference is the numbers used. The transmission rate and throughput values are much smaller and the G values are much larger. An interesting result that follows from (2.20) is:

$$G^* = \frac{4}{L-3-\sqrt{L^2-6L+1}} = \frac{1}{2}\left(L-3+\sqrt{L^2-6L+1}\right). \tag{2.21}$$

This allows us to determine the value of the SNR to achieve maximum throughput for a given packet length in a Rayleigh fading channel.

To maximize the throughput with respect to both the packet length and transmission rate we can write the throughput as a function of SNR by using equation (2.9) and substituting L^* (2.16) for the length, and R^* (2.6) for the rate. The result is in Figure 2-8. The same changes are made in P/N_o as were made in Figure 2-5 and the same conclusions can be drawn. The SNR value that maximizes throughput for a Rayleigh fading channel is G^{**} = 28.12 and is independent of the value of P/N_o. This can be seen by the vertical line in Figure 2-8. We can now use this value in (2.11) to find that the packet length to achieve maximum throughput is $L^{**}(G^{**})$ = 31 bits. This value is also independent of P/N_o. The rate to maximize throughput R^{**} is dependent on P/N_o from (2.6).

2.3.3 Different Modulation Schemes For Both Channel Models

In the context of this study, the main effect of the modem is on the optimum signal-to-noise ratio γ^*, the solution to Equation (2.5), with $f(\gamma)$ given by Equation (2.8). Table 2-2 presents data for four modems described in communications textbooks using a non-fading Gaussian channel: binary phase shift keying, differential phase shift keying, coherent frequency shift keying, and non-coherent frequency shift keying. The table contains the formula for $P_e(\gamma)$, the γ^* for L=80 bits per packet, L^* for γ=10 and the jointly optimized signal-to-noise ratio and packet length: γ^{**} and L^{**}. The maximum throughput for two different values of P/N_0 are also given.

Table 2-2 Non-fading Channel

	Binary PSK	Differential PSK	Coherent FSK	Non-coherent FSK
$P_e(g)$	$Q\left(\sqrt{2g}\right)$	$\frac{1}{2}\exp\left(-g\right)$	$Q\left(\sqrt{g}\right)$	$\frac{1}{2}\exp\left(-\frac{g}{2}\right)$
γ^* (L=80)	3.79	5.37	7.57	10.75
L^* bits (γ=10)	2041	848	151	77
γ^{**}	3.84	5.74	7.69	11.47
L^{**} bits	84	108	84	108
T^{**} ($P/N_o=5*10^6$)	833.4 kbps	623.7 kbps	416.7 kbps	311.8 kbps
T^{**} ($P/N_o=5*10^5$)	83.34 kbps	62.37 kbps	41.67 kbps	31.18 kbps

$$Q(x) = \frac{1}{\sqrt{2p}}\int_x^{\infty} \exp(-u^2/2)du$$

An interesting observation to note from Table 2-2 is the 2:1 relationship between Binary PSK and Coherent FSK as well as the 2:1 relationship between Differential PSK and

Non-coherent FSK. In comparing the respective PSK and FSK groups we can see that the PSK has an optimum SNR exactly half that of the FSK and a maximum throughput exactly twice that of the FSK. The respective PSK and FSK groups also have the same optimum packet lengths. Table 2-3 shows the same information for the four modems in a Rayleigh fading channel.

Table 2-3 Rayleigh Fading Channel

	Binary PSK	Differential PSK	Coherent FSK	Non-coherent FSK
$\overline{P}_e(G)$	$\frac{1}{2}\left\{1-\sqrt{\frac{G}{1+G}}\right\}$	$\frac{1}{2(1+G)}$	$\frac{1}{2}\left\{1-\sqrt{\frac{G}{2+G}}\right\}$	$\frac{1}{2+G}$
G^* (L=80)	18.74	38.49	37.48	76.97
L^* (G=10)	35	28	29	24
G^{**}	6.37	14.06	12.75	28.12
L^{**}	31	31	31	31
T^{**} ($P/N_o=5*10^6$)	125.3 kbps	60.4 kbps	62.6 kbps	30.2 kbps
T^{**} ($P/N_o=5*10^5$)	12.53 kbps	6.04 kbps	6.26 kbps	3.02 kbps

These tables show us that of the four different modulation schemes considered, Binary PSK gives the best results. In general, the phase shift keying is more desirable, however it is often more difficult to implement in the detection aspect at the receiver than the alternative frequency shift keying.

2.4 Conclusions

Maximizing throughput in a wireless channel is a very important aspect in the quality of a voice or data transmission. In this chapter, we have shown that factors such as the optimum packet length and optimum transmission rate are all functions of the signal to noise ratio. These equations can be used to find the optimum signal to noise ratio that the system should be operated at to achieve the maximum throughput. The key concept behind this research is that for each particular channel (AWGN or Rayleigh) and transmission scheme ($P_e(\gamma)$), there exists a specific value for the signal to noise ratio to maximize the throughput. Once the probability of error, $P_e(\gamma)$ is known, this optimal SNR value can be obtained.

This research has basically laid the framework for which future study can be built on. One of the most significant simplifying assumptions that was made stated that this analysis was done using no forward error correcting (FEC) codes. In the next chapter a similar analyses is done including FEC.

Figure 2-1
Optimum Packet Length as a Function of P_e

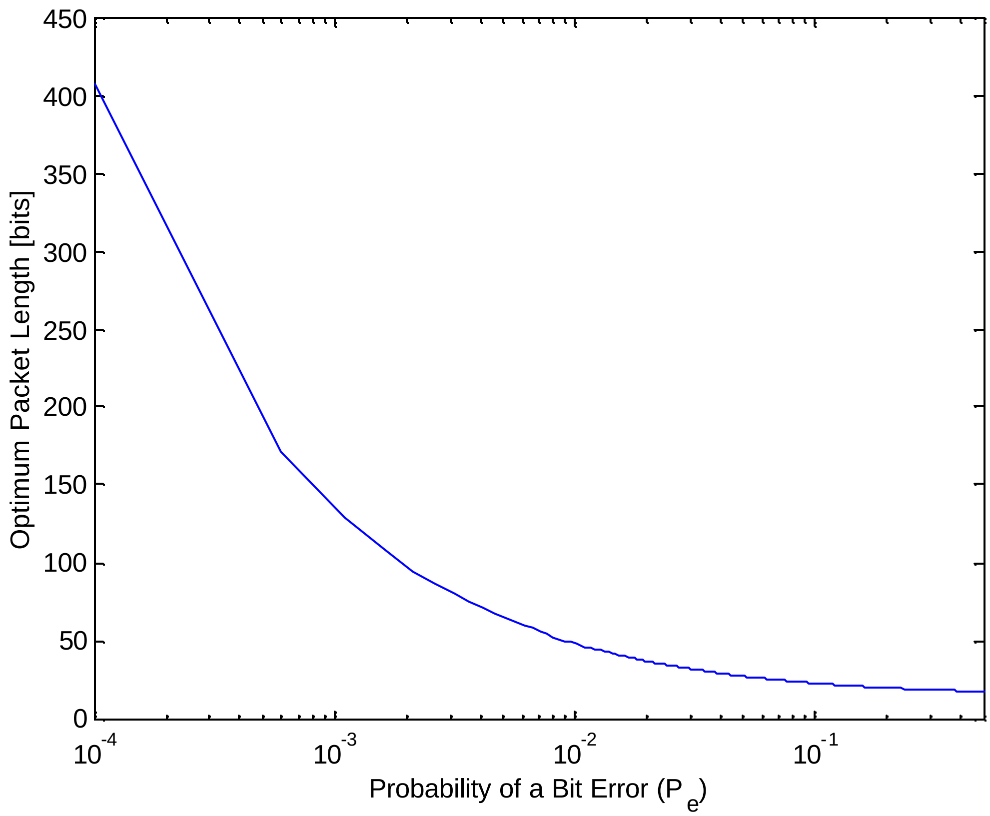


Figure 2-2
Throughput vs L for a Fixed Transmission Rate

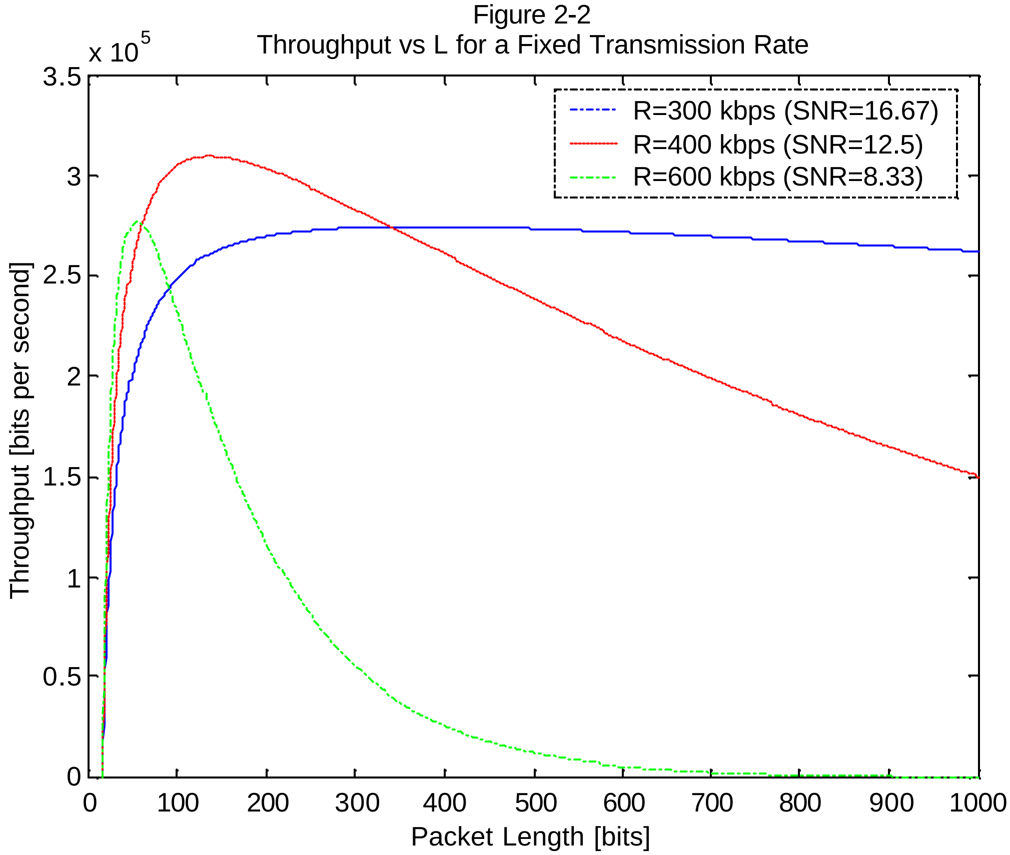


Figure 2-3

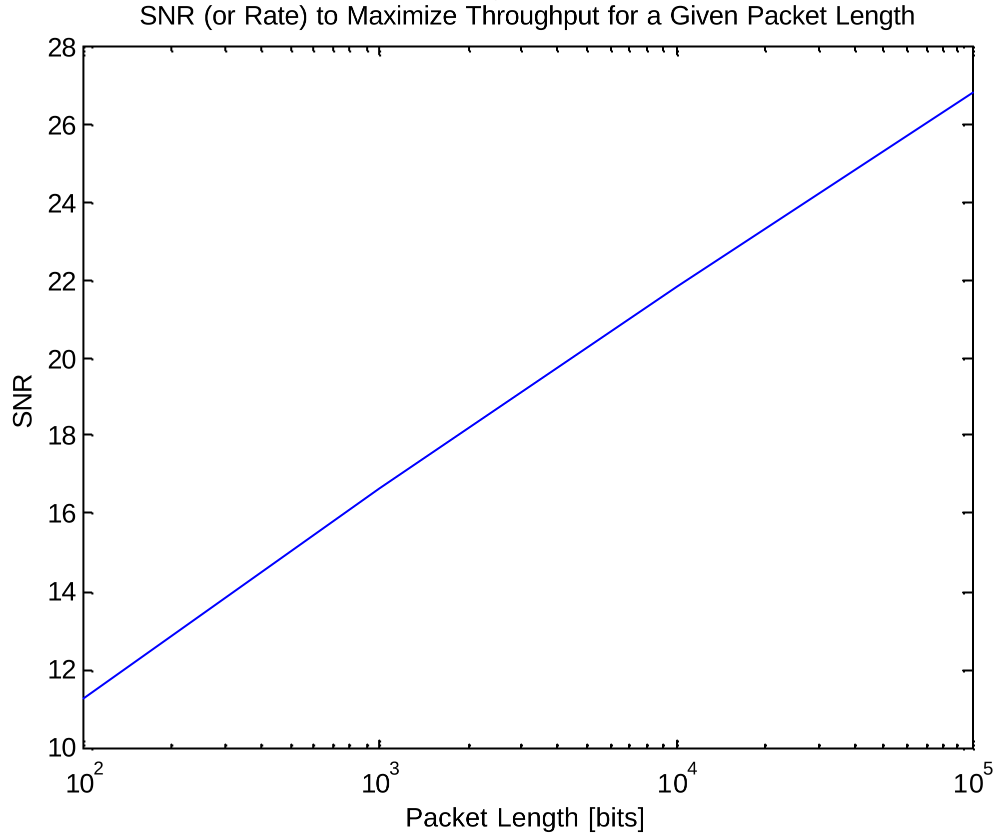


Figure 2-4

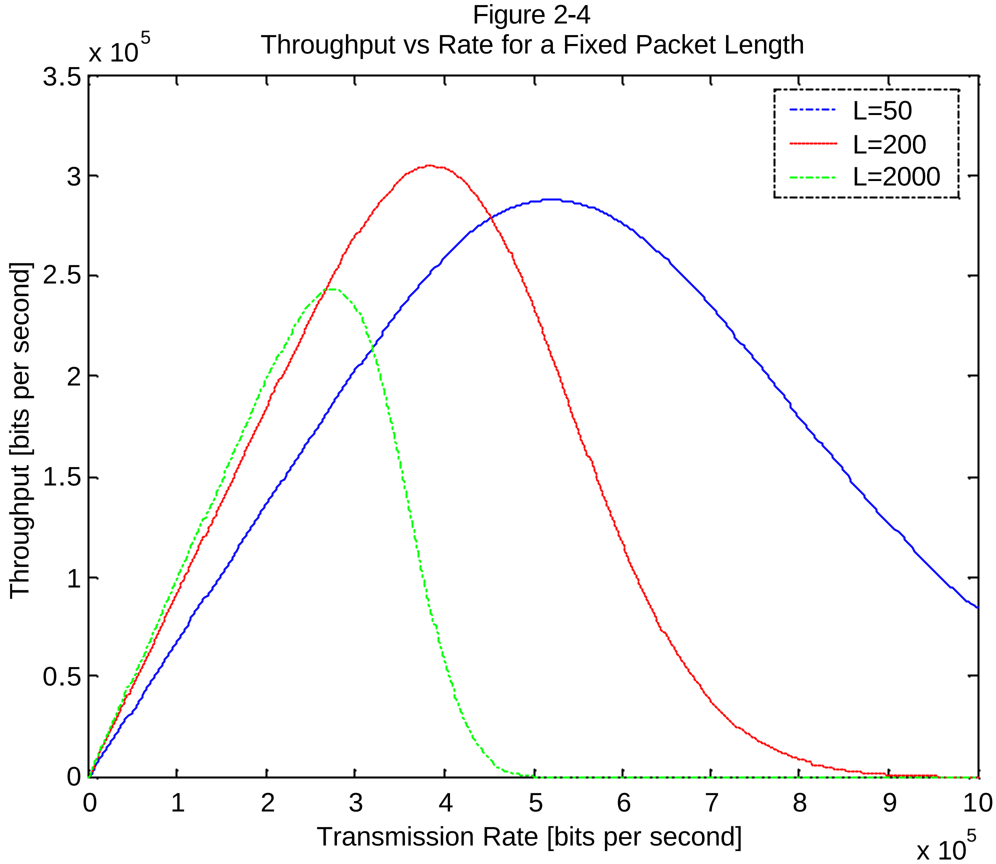


Figure 2-5

Throughput vs SNR Using Joint Optimization

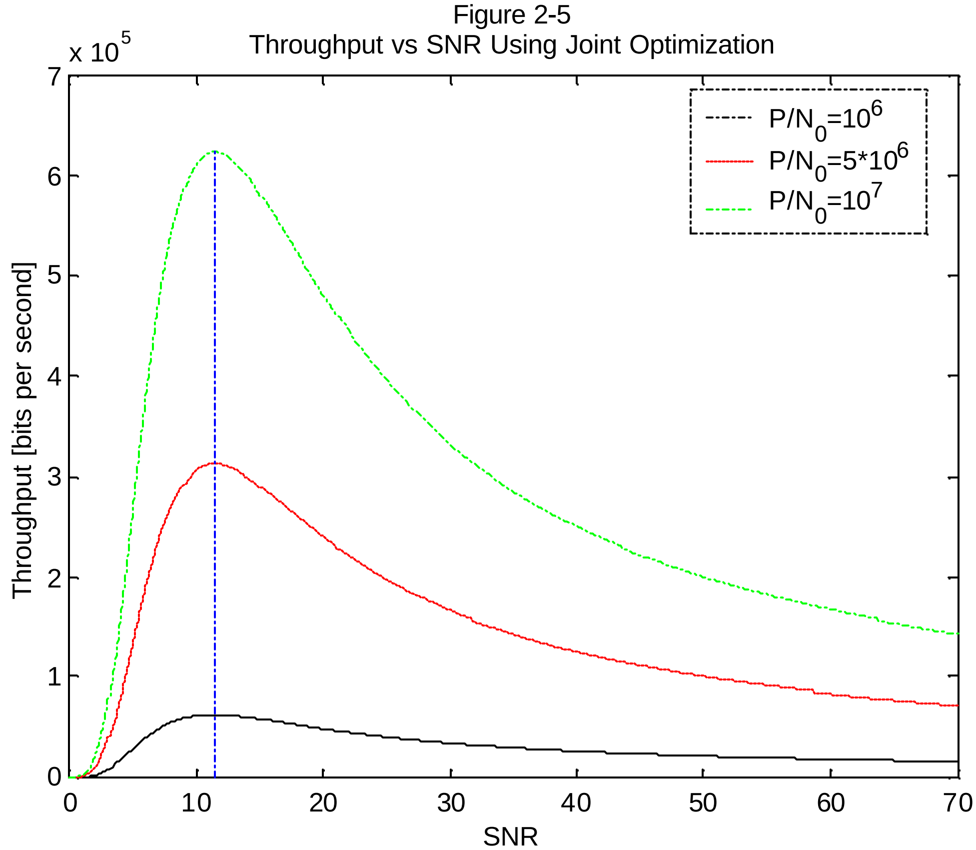


Figure 2-6

Throughput vs L for a Fixed Transmission Rate (Rayleigh Fading)

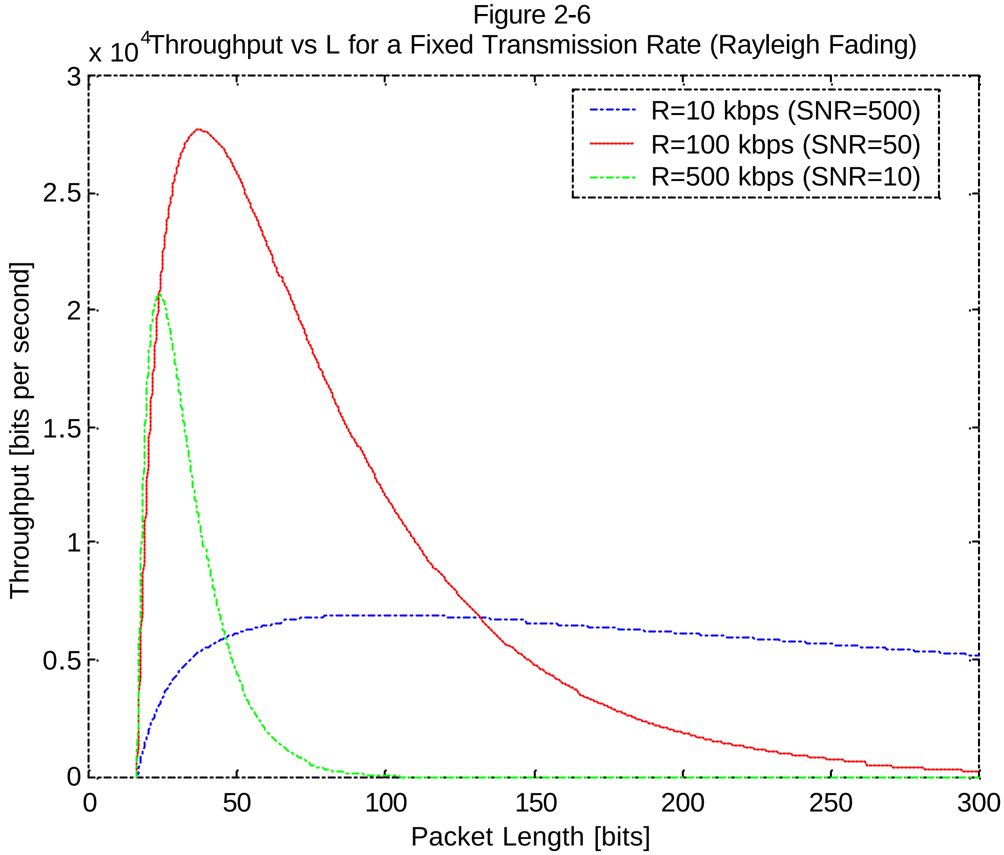


Figure 2-7

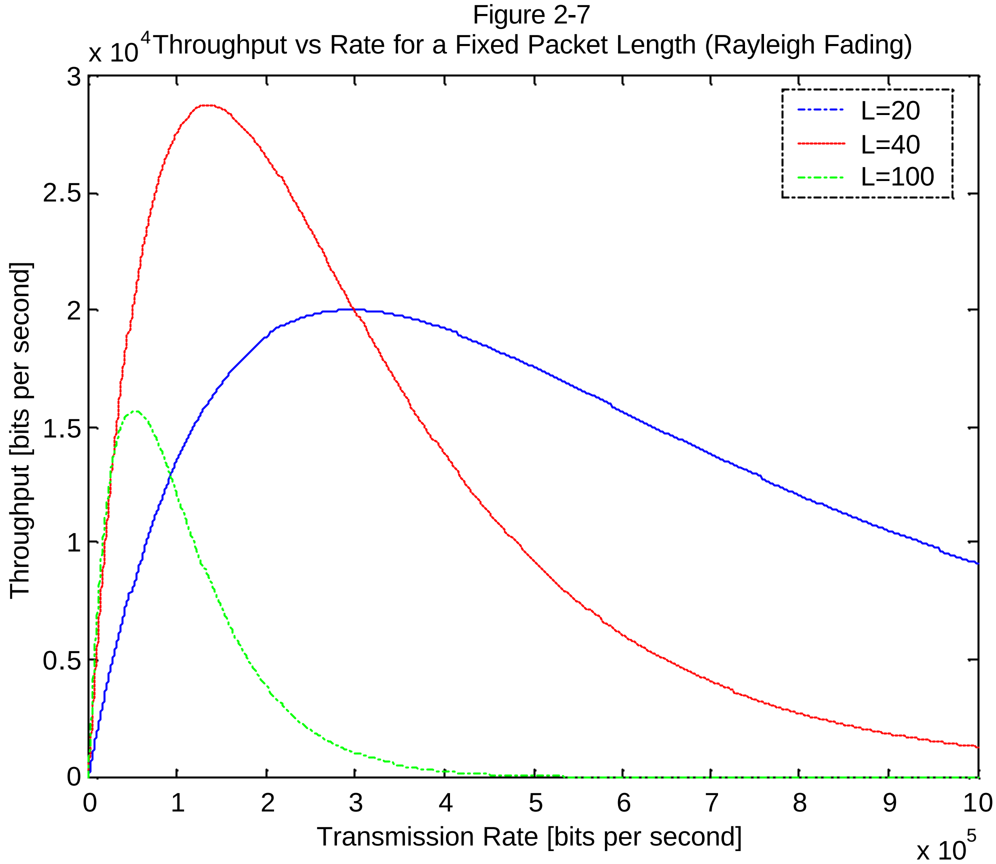
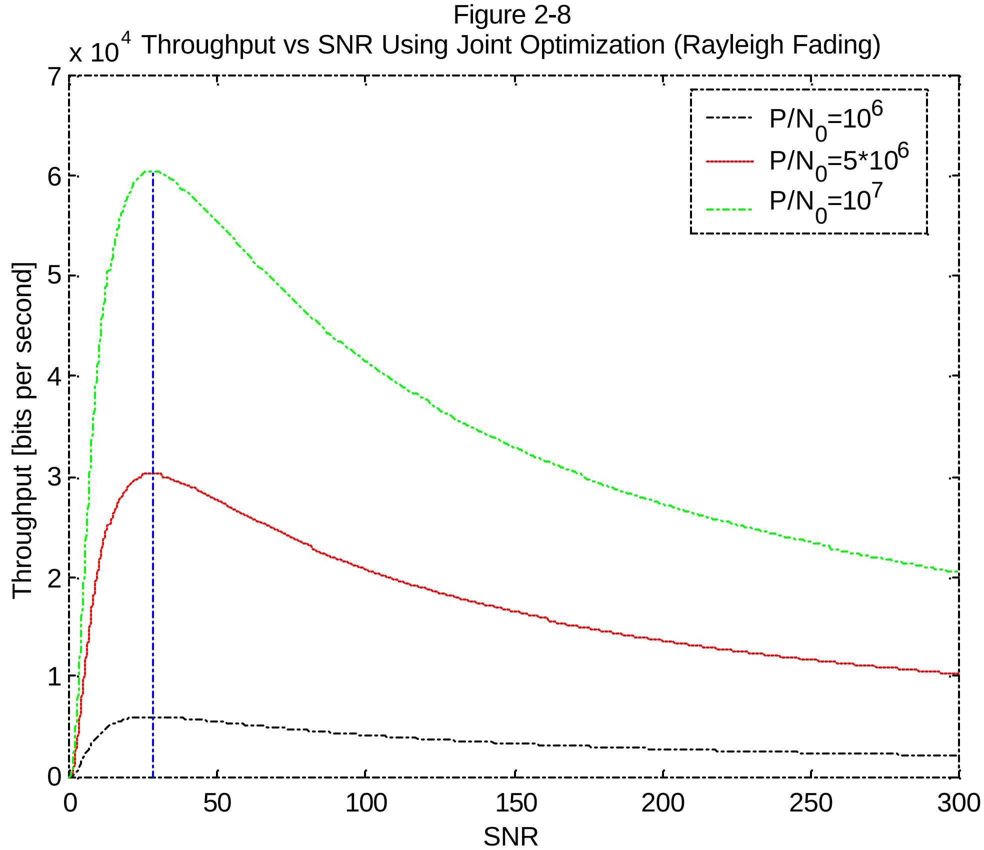


Figure 2-8



Chapter 3

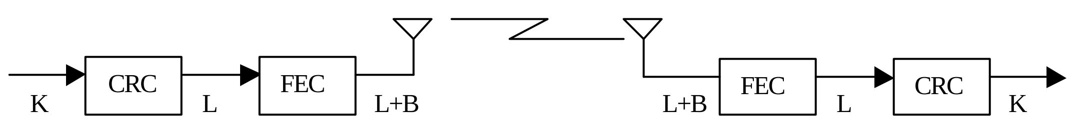
Throughput Analysis using Forward Error Correction

As a continuation of our previous work involved with throughput optimization, we now look at the effect of forward error correction on throughput. In the previous chapter, it was assumed that an automatic repeat request was used to retransmit any packets that contained errors detected by the cyclic redundancy check. As we would expect, the throughput should increase if we were able to keep any erroneous packets by simply correcting the errors instead of discarding the whole packet and re-transmitting everything.

3.1 Forward Error Correction Throughput Equations

Using the same notation as in the previous chapter, we will denote the number of information bits in the packet as K, and the number of CRC bits as C. L is defined as K+C. Now instead of transmitting those L bits with no error correction capability, we will now add B error correcting bits and transmit a total of L+B bits (see Diagram 3-1).

Diagram 3-1



Using a block code forward error correction scheme, the minimum number of bits B required to correct t errors is given by [5]:

$$B \geq \log_2\left[\sum_{n=0}^{t}\binom{L+B}{n}\right]. \tag{3.1}$$

Now that we can correct t errors, our packet success rate, f(γ) should be larger than its previous value with no error correction. Recall that f(γ) with t=0 is given by:

$$f(\gamma) = \left(1 - P_e(\gamma)\right)^L, \tag{3.2}$$

where $P_e(\gamma)$ is the probability of a bit error as a function of the SNR. Now, with error correction capability, the packet success rate for some arbitrary value of t is [6]:

$$f_t(\gamma) = \sum_{n=0}^{t}\binom{L+B}{n} P_e^{\,n}(\gamma)\left(1 - P_e(\gamma)\right)^{L+B-n}. \tag{3.3}$$

Our new equation for the throughput as a function of the signal to noise ratio is:

$$T(\gamma) = \left(\frac{L-C}{L+B}\right)\left(\frac{P/N_0}{\gamma}\right) f_t(\gamma). \tag{3.4}$$

3.2 Analysis

3.2.1 Optimum Error Correction

Looking at (3.4) we see that, with the exception of the constant B, this is the same as equation (2.1) which was analyzed in chapter 2. This means that mathematically, the situation is the same as was derived in the previous chapter. Namely, there is an optimum SNR for data throughput given the modulation scheme and channel model. The only addition made in this chapter is the number of errors that can be corrected, t. In chapter 2, t was always set to zero. Since f(γ) depends on t, we will obtain a different optimum SNR for each value of t.

To see the relationship between the minimum number of error correcting bits (B) and the number of correctable errors (t), we have graphed B as a function of t for L=216 bits in Figure 3-1. By the time t=49 the number of error correcting bits added is greater than the original number of bits in the packet (L). For this reason we can see that t cannot be made arbitrarily large. As t increases, there is a considerable increase in both the amount of overhead added and the FEC code complexity.

If we take equation (3.4) and plot it for different values of t using differential PSK ($P_e(\gamma)=0.5*\exp(-\gamma)$) we get a very interesting result. Figure 3-2 shows the throughput for t=0,20,60,100. In each of these plots, P/N_0 is fixed at $5*10^6$ and C at 16 bits. Notice that with the error correction, we get an increase by a factor of two in the maximum throughput. Also, the value of the SNR at which the optimum throughput is reached gets much lower as t increases. Perhaps the most important thing to note in this graph is that there is an optimum value for t at which higher values of t will not produce higher throughputs. From this graph the throughput appears to reach an optimum value somewhere around t=60. After a closer look, it was found to be t=55. The value for the maximum throughput at this optimum t is 1.29 Mbps.

3.2.2 Effect of Packet Length

The next thing we looked at was how the throughput responded to an increasing packet length, L. As L was increased in increments of 100 bits, the maximum throughput increased, quickly at first and then slower toward the end, to a value of 1.42 Mbps at L=757 bits (with t=100). Any further increase in L just resulted in a decrease in the maximum throughput. Another trend that was noticed when increasing L was that the optimum SNR slowly increased. These observations can be seen in Figures 3-3, 3-4, 3-5, 3-6, and 3-7. Although increasing L gives you the obvious advantage of a higher throughput, there are also a few drawbacks. As L gets larger, equation (3.1) tells us that the number of error correcting bits needed to correct t errors gets larger. If you notice the plots when L=216, there is a distinct maximum near t=60. When L=316, the maximum seems to occur around t=80, and for L=416 and higher, a maximum cannot even be reached if we limit ourselves to t=100. This would indicate that for these higher values of L, we would have to correct over 100 errors in order to reach the maximum throughput. A problem then arises with the implementation of this large forward error correction code. One important observation that was made concerning the optimum number of errors to correct was that this occurs when B is roughly equal to L. This means that we will get the best throughput results when we add on a number of overhead error correcting bits equal to the original size of the packet (L) without the error correction. This would indicate a rate ½ code where the code rate is given by

$$r = \frac{L}{L+B}\,. \tag{3.5}$$

Note the results: when L=216, maximum throughput was achieved when B=238 (t=55), when L=316, maximum throughput was achieved when B=352 (t=81).

3.2.3 Shannon's Throughput Limit

The addition of the forward error correction scheme described above has increased our throughput to more than twice its original value. Although this is a dramatic improvement, it is in no way the <u>absolute</u> best we can do. A well know theorem referred to as Shannon's Upper Bound tells us the maximum throughput (in bits per second) of a communication system, if we were to use the best coding scheme possible. It simply states:

$$T_{\max}(W) = W \log_2\left(1 + \frac{P}{WN_0}\right), \tag{3.6}$$

where W is the bandwidth of the channel in Hz, P is the total received signal power in watts and N_0 is the total noise power density in W/Hz [7]. In the coding scheme that I've described, the bandwidth is roughly equal to the rate at which the L+B coded bits are transmitted (R). If we replace the bandwidth (W) with the transmission rate (R) we get:

$$T_{\max}(R) = R \log_2\left(1 + \frac{P}{RN_0}\right). \tag{3.7}$$

If we use $\gamma = P/(R \cdot N_0)$, we can write T_{max} in terms of γ:

$$T_{\max}(\gamma) = \frac{P}{N_0 \gamma} \log_2(1+\gamma). \tag{3.8}$$

To get an idea of how well our throughput results size up to Shannon's limit, we have plotted them both in Figure 3-8. The best case FEC plot is for L=757 and t=100. Under these conditions, a maximum throughput of 1.42 Mbps is achieved at SNR=2.026. If we look at the value of Shannon's limit at an SNR of 2.026, from equation (3.8) we get a value of 3.942 Mbps (this is assuming $P/N_0 = 5*10^6$). Therefore, we were only able to achieve 36% of the absolute maximum. These results are not all that bad, considering the fact that we are only using a two level modulation scheme.

3.2.4 Effect of P/N_0

One more thing we can look at is how this FEC throughput is affected by different values of P/N_0. Recall from the previous chapter that P/N_0 is dependent upon the distance between the mobile and base station as well as the total interference present. Therefore, if our mobile were located at different distances, we would get different values for P/N_0. Figure 3-9 shows this situation for three different values. In this case, L was chosen to be 516 bits and t=60. The behavior is exactly the same as in the previous chapter without the FEC. Since P/N_0 is just a constant multiplying factor, it simply scales the throughput plot and does not change the optimum SNR value. We can see that if we increase P/N_0 by a factor of 5 then the maximum is increased by a factor of 5 and if we further increase it by a factor of 2, then the maximum is increased by a factor of 2.

3.3 Conclusions

The addition of FEC coding gives us a significant increase in throughput (about twice as much as without). Too much coding, however, can hurt us because the amount of overhead necessary will begin to overshadow the data. Therefore we must find the optimum amount of error correction to yield the best results. This optimum value depends on the packet length, L. As L gets larger, the optimum amount of correctable errors also increases. As a general rule of thumb, the optimum FEC occurs when the amount of overhead bits (B) is equal to the original packet length (L).

Figure 3-1
B vs t (L=216)

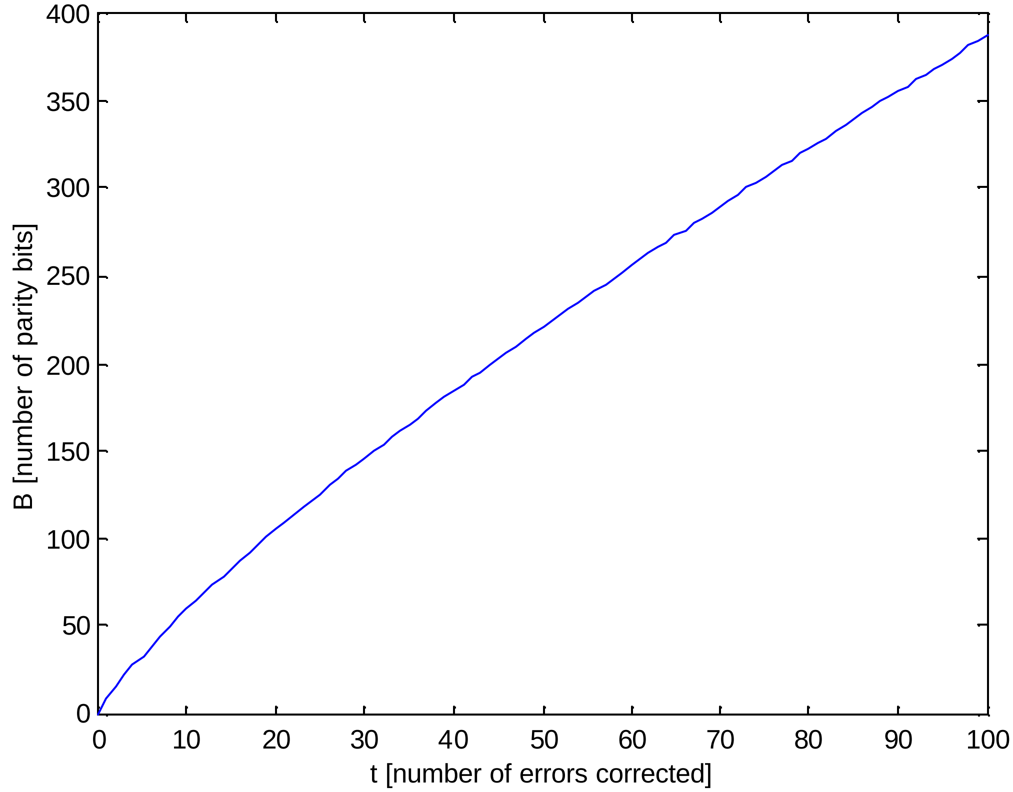


Figure 3-2
Throughput vs SNR (L=216 bits)

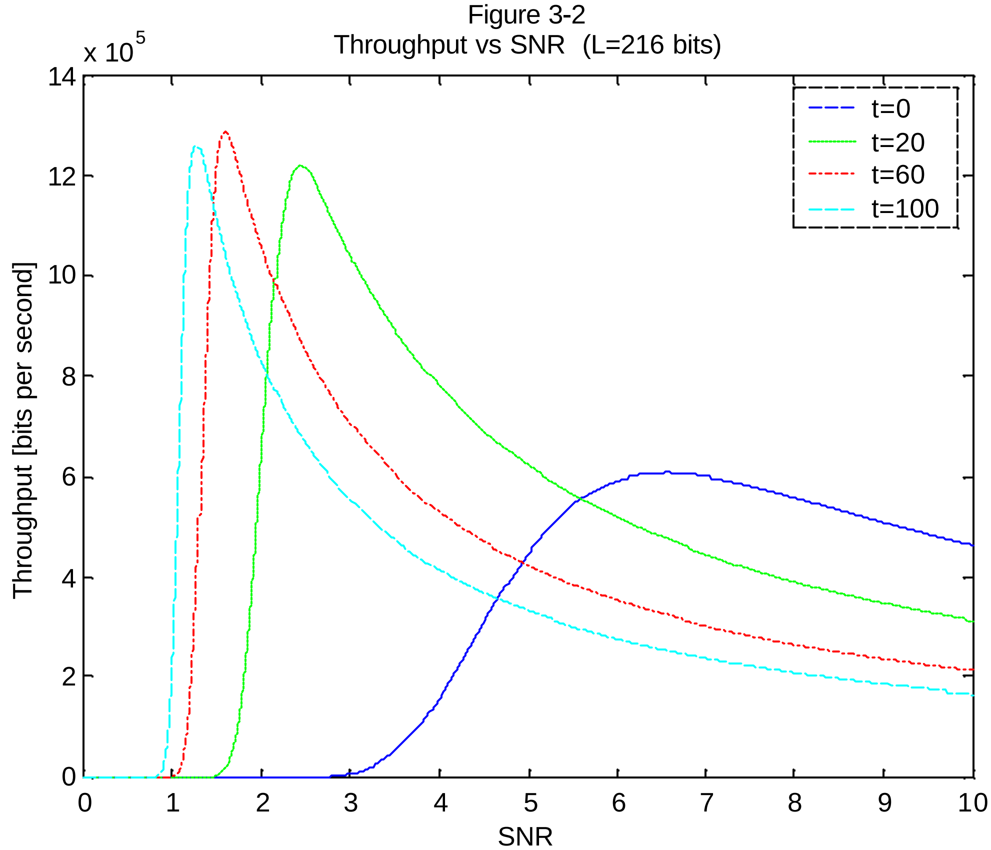


Figure 3-3
Throughput vs SNR (L=216 bits)

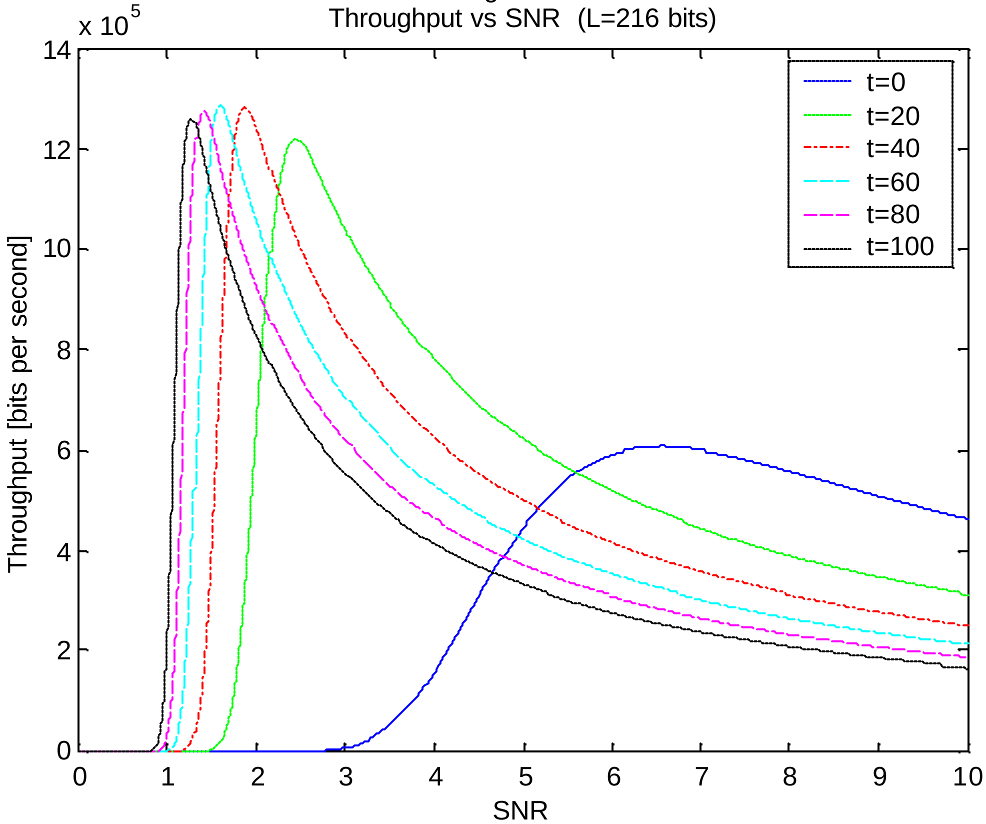
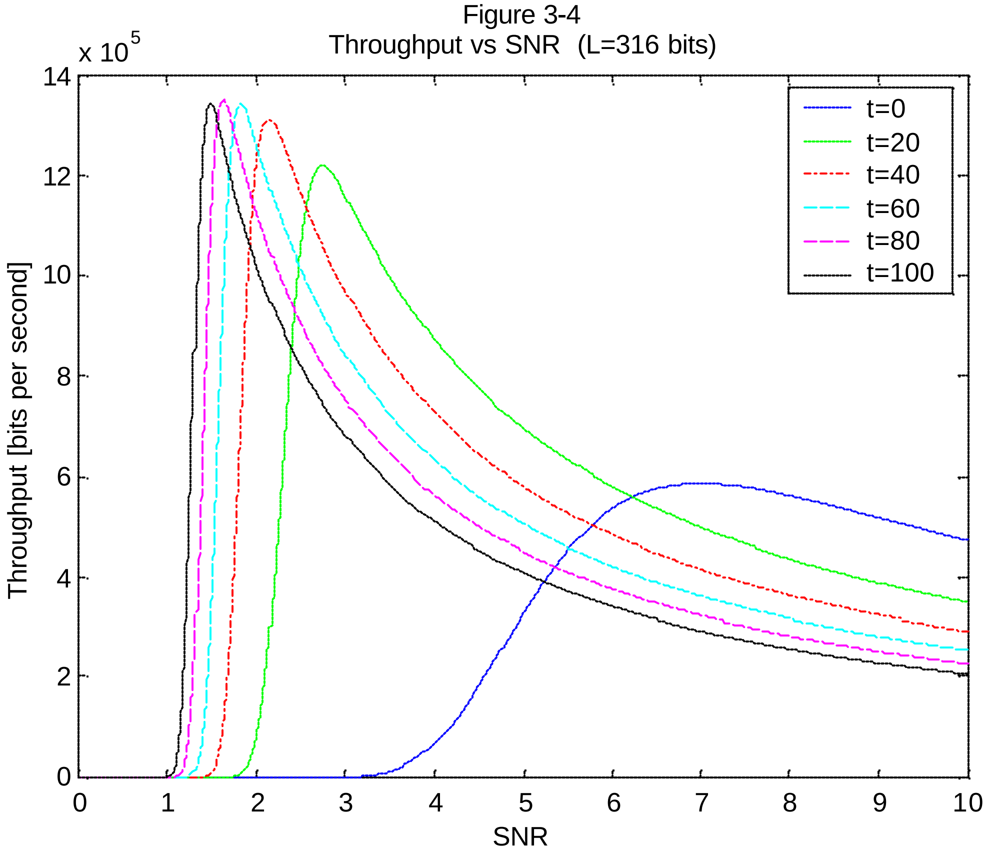


Figure 3-4
Throughput vs SNR (L=316 bits)



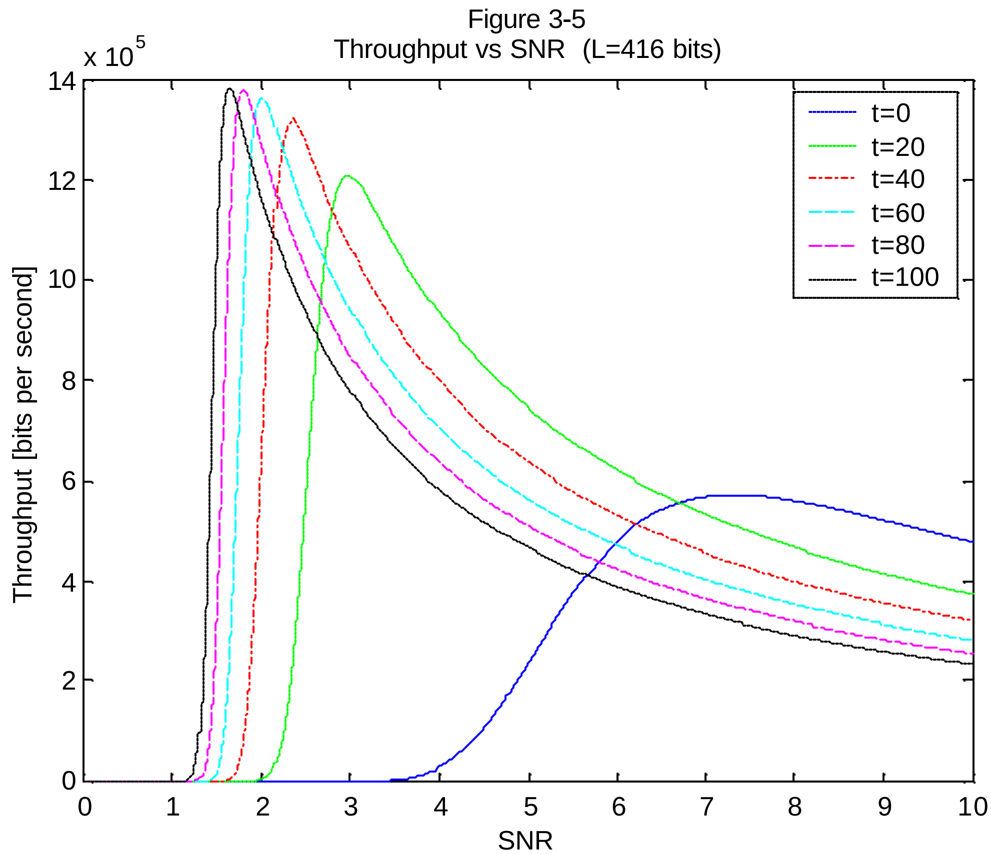


Figure 3-5
Throughput vs SNR (L=416 bits)
x 10^5
Throughput [bits per second]
SNR
0
1
2
3
4
5
6
7
8
9
10
0
2
4
6
8
10
12
14
t=0
t=20
t=40
t=60
t=80
t=100

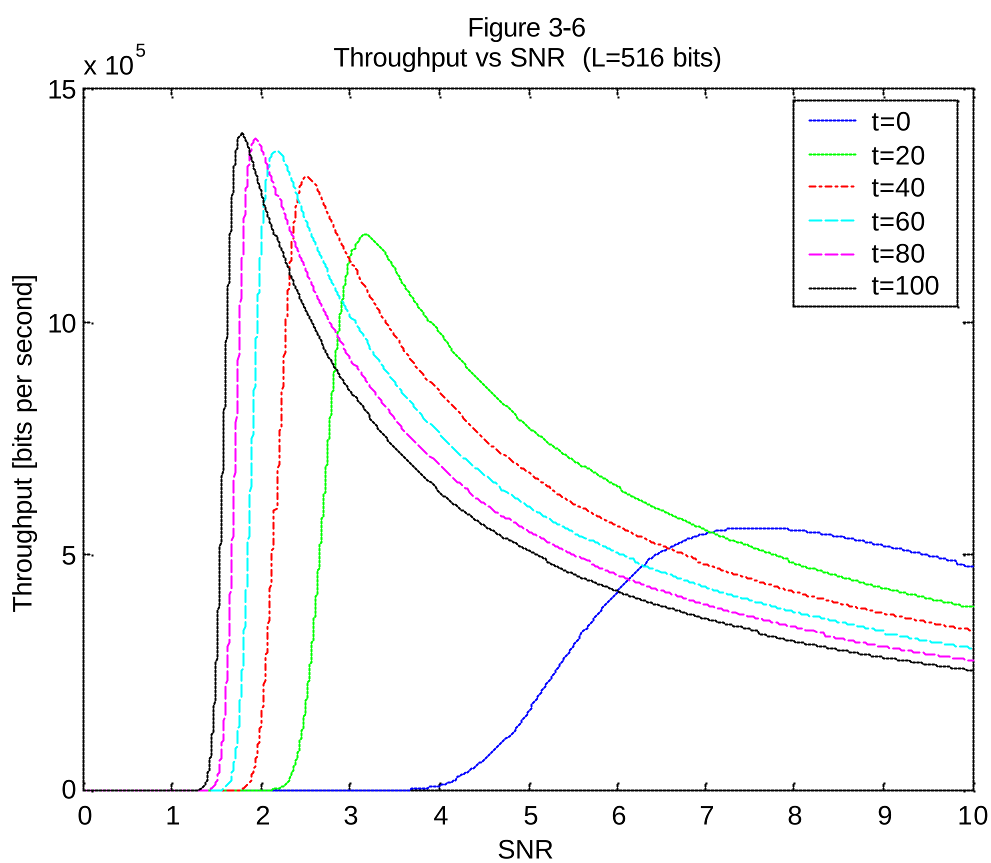


Figure 3-6
Throughput vs SNR (L=516 bits)
x 10^5
Throughput [bits per second]
SNR
0
1
2
3
4
5
6
7
8
9
10
0
5
10
15
t=0
t=20
t=40
t=60
t=80
t=100

Figure 3-7
Throughput vs SNR (t=100) ($P/N_0=5*10^6$)

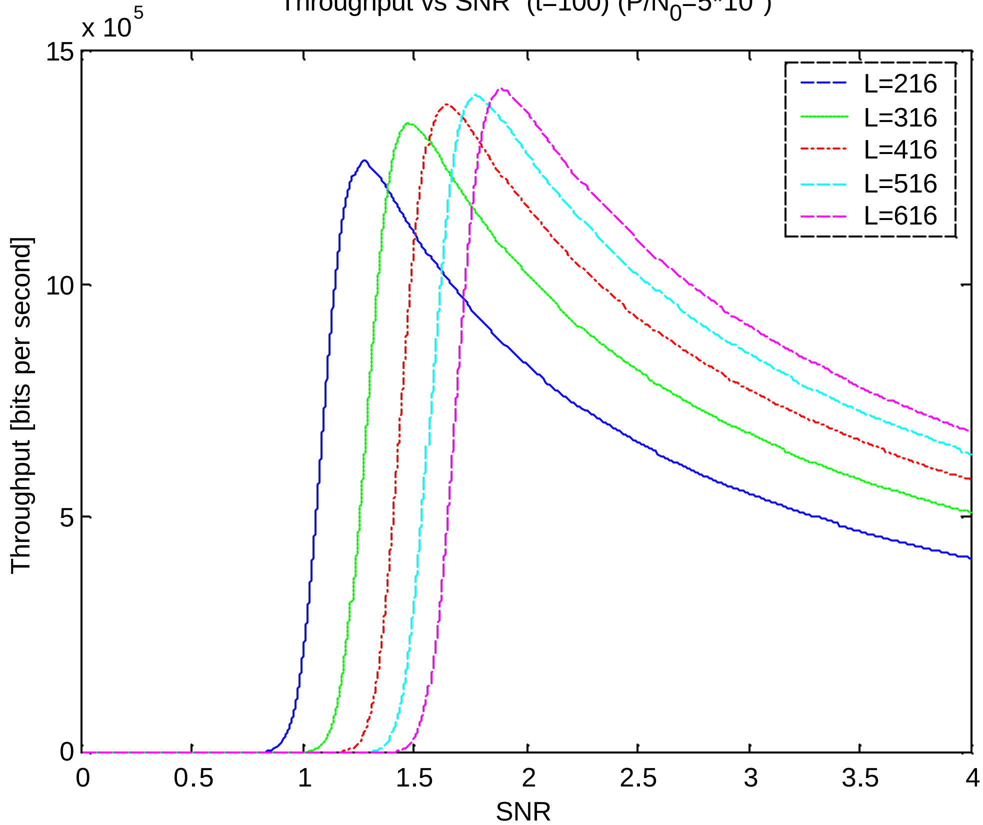


Figure 3-8
Shannons Throughput Limit (P/N_0 = $5*10^6$)

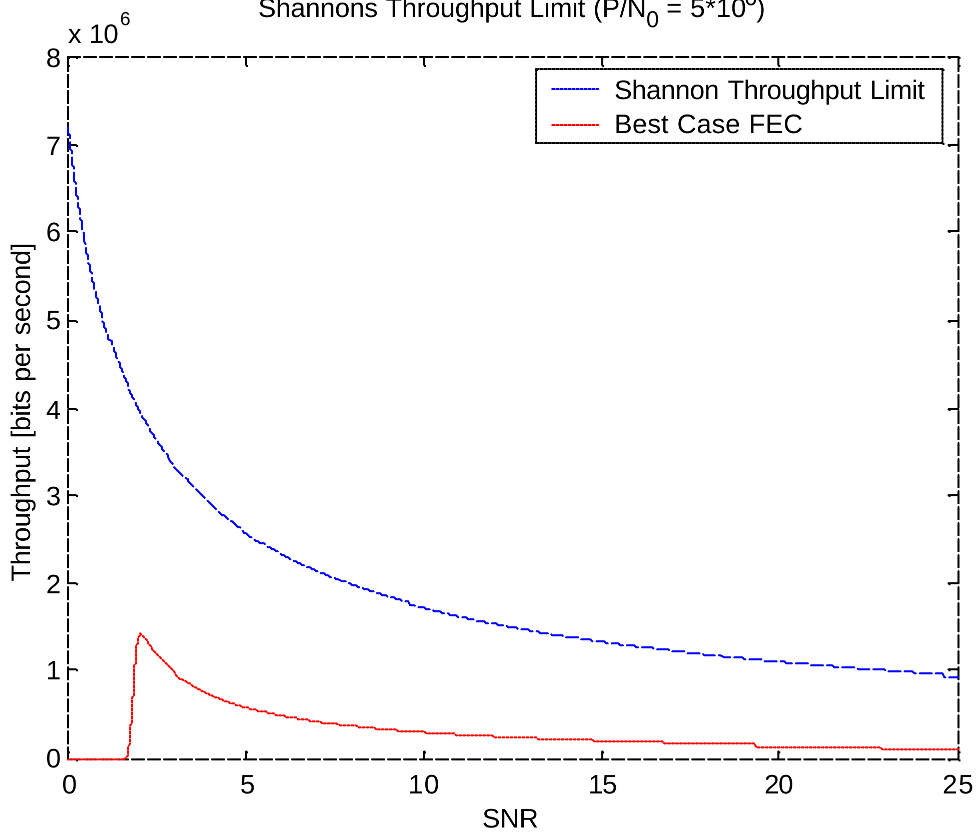
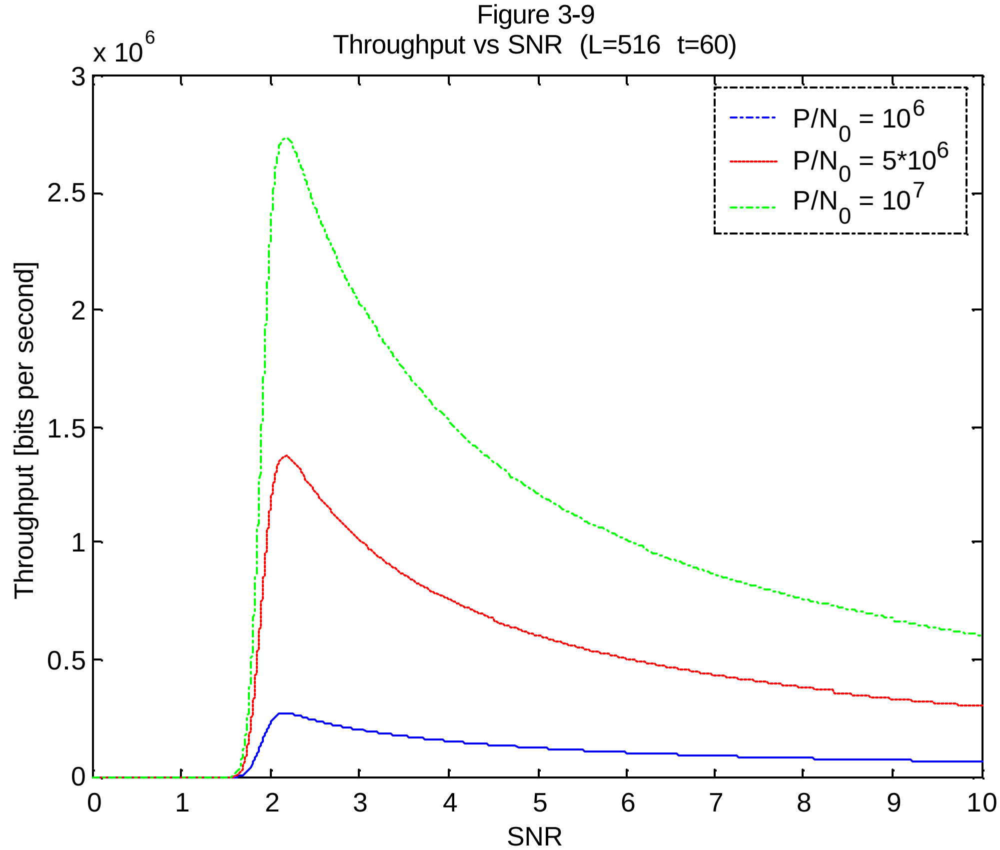


Figure 3-9
Throughput vs SNR (L=516 t=60)



Chapter 4

Throughput Analysis For a Varying Received Power

The analysis in earlier chapters assumes a constant ratio of the received power to the noise power spectral density, P/N_0. This could be considered a valid assumption if the mobile unit is stationary at any fixed location. Since we are considering factors influencing the mobile communications industry, it is crucial that we look at this problem from a mobile point of view. The preceding chapters have looked at throughput as a function of SNR, which is inversely proportional to the transmission rate. This implies that because throughput is an increasing function of SNR, it is a decreasing function of transmission rate. The purpose of the plots in chapters 2 and 3 was to indicate an optimal SNR (i.e. transmission rate) at which to operate the system. Now we want to see how the throughput behaves, using a fixed rate, as we move the terminal farther from the transmitter. The primary theme of this chapter is how we can change the transmission rate and forward error correction to get the best results for a particular received power.

4.1 New Equations For Mobile Receiver

Recall from Chapter 2 that we can express the SNR as:

$$g = \frac{P}{N_0 R}, \tag{4.1}$$

where P is the received power in watts, N_0 is the noise power spectral density in watts/Hz, and R is the transmission rate in bits per second. A simple model of the relationship between the received power and the distance is

$$P = A\frac{P_t}{d^a}\ , \quad \text{for d > 1 meter} \tag{4.2}$$

where P_t is the transmitter power in watts, d is the distance between the transmitter and receiver in meters, α is the range index, and A is a constant of proportionality. Substituting (4.2) for P in equation (4.1) will yield:

$$g(d) = A\frac{P_t}{N_0 R d^a}\ . \tag{4.3}$$

If the transmission rate is constant, γ varies with distance according to equation (4.3). P_t, α, A, and N_0 are all assumed to remain constant. To get an idea of how this behaves, we can plot throughput as a function of distance only.

$$T(d) = \frac{L-C}{L} R f\left(g(d)\right). \tag{4.4}$$

4.2 Numerical Results

In order to get a plot of (4.4) we must choose appropriate values for our constants. We have selected a transmit power P_t of 6 watts, a noise power spectral density of 10^{-15} W/Hz, and a proportionality constant A of 1. This means that the received power is 6 W when the terminal is 1 meter from the transmitter. For the packet success rate f(γ) I have chosen that of differential PSK in a Gaussian white noise channel for simplicity (that is $f(\gamma)=(1-P_e(\gamma))^L$ where $P_e(\gamma)=0.5*e^{-\gamma}$). This analysis can just as easily be done using a different modulation scheme by replacing $P_e(\gamma)$ with the appropriate function. We must then choose a packet length, CRC length, and transmission rate. This is an arbitrary choice, but I have chosen L=108 bits with C=16 bits (16 bit CRC) and R=871 kbps because these where the values found to give the optimum throughput in Chapter 2 (see Table 2-2). Please note that this rate was the optimum rate only when $P/N_0=5*10^6$. As the distance changes, the received P/N_0 will also change, yielding a different optimum rate. Nevertheless, we must choose a rate and hold it constant in order to do this analysis. Figure 4-1 shows the throughput as a function of distance for three different values of the range index, α. The range index is determined by a number of influencing factors in the surrounding environment. In free space $\alpha=2$, but for most places on earth α is in the range between 3 and 4.

4.2.1 Effect of Changing Packet Length

To observe the effects of changing certain parameters we will choose a fixed α of 3.5. Figure 4-2 shows the effect of an increasing packet length on the throughput. From this plot we can see that increasing the packet length gives us a higher throughput at relatively small distances (d<300 meters). The effect of the packet length on the critical distance at which the throughput falls off is minimal. We can see that the critical distance barely changes as the packet length increases, however the slope of the fall off gets increasingly steep. In the limit as L approaches infinity, the throughput approaches the transmission rate, R.

If you recall from chapter 2, we discussed the optimum packet length as a function of SNR (2.11). If we substitute (4.3) for γ in (2.11) we can graph the optimum packet length as a function of distance.

$$L^*(d) = \frac{1}{2}C + \frac{1}{2}\sqrt{C^2 - \frac{4C}{\ln(1-P_e(d))}}. \tag{4.5}$$

Equation (4.5) is plotted in Figure 4-3. When d<350 meters, the SNR is high enough to allow very large packets to be transmitted without errors. This is also evident in Figure 4-2. Notice that when d<350 meters, the throughput is limited by the values of L and R even though it has the potential to go higher, but when d>350 meters it begins to fall off and the optimum packet length then becomes important. From a mathematical point of view, this tells us that as we get closer to the transmitter, we can use enormous, almost limitless values of L. From a practical point of view, we may not be able to use such large values of L as it might cause other problems such as delay. This plot is simply telling us to use the largest feasible packet length when at close distances. As the distance increases, however, we must be careful in the selection of our optimum packet length so as to achieve the best possible throughput results.

4.2.2 Effect of Changing Transmission Rate

A similar but more dramatic effect can be seen from varying R. From Figure 4-4 we see that choosing a low transmission rate gives us a lower throughput for a longer distance. As we increase the transmission rate, the throughput falls off quicker. This plot nicely demonstrates how rate adaptation can work in a wireless network. When at close distances, a higher rate can be used and then when throughput begins to fall, a lower rate can be used to stabilize the throughput at the longer distances.

As was done in the previous section, we can also plot the optimum transmission rate as a function of distance. Solving (4.3) for R=R^* yields

$$R^*(d) = A \frac{P_t}{N_0 g^* d^a}, \qquad (4.6)$$

where γ^* is the jointly optimized SNR for DPSK (given in Table 2-2 as γ^{**}=5.74). Equation (4.6) is plotted in Figure 4-5. The same situation exists as was seen in Figure 4-

3 for $L^*(d)$. At close distances, the SNR is high enough to allow for a very high optimum transmission rate, but from a practical point of view we might not have a large enough bandwidth to transmit at such large rates. As the terminal moves farther from the transmitter, the optimum rate must decrease in order to obtain the best throughput results.

Another way to look at the figures in this chapter is to consider the throughput as a function of P/N_0. This could be helpful in trying to implement rate adaptation in a wireless system since it would be much easier for the receiver to measure the received power over the noise power spectral density than it would be to measure the distance between itself and the transmitter. Using the above equations, we can write this ratio as

$$\frac{P}{N_0} = A\frac{P_t}{N_0 d^a}. \tag{4.7}$$

Figure 4-6 uses the same throughput plots as Figure 4-4, only this time as a function of P/N_0. Naturally, since this is proportional to the received power, as the distance increases, this ratio will decrease.

4.2.3 Effect of Changing FEC

The effect of forward error correction on this situation is also very interesting. It has a similar effect as the rate adaptation. Using the equations from chapter 3 to find the number of error correcting bits B and the packet success rate $f(\gamma)$ we can get a feel for how this behaves. From Figure 4-7 we can see that as the number of correctable errors t, increases, the range of our transmission increases, however the throughput of our connection decreases. The reason for the decrease in throughput at higher values of t is because the increasing number of overhead bits (B) starts to overshadow the information bits. We can also see that the "payoff" resulting from an increasing t gets lower as t increases. From t=0 to t=20 we get an range increase of about 150 meters, but from t=20 to t=40 we get only about 40 meters and even less on the subsequent increases of t. This would indicate that it's not worth it to pay the large price in throughput reduction just for a few more meters of range.

To illustrate a more practical implementation of this system of changing FEC we can plot Figure 4-7 as a function of P/N_0 as was done in Figure 4-6. In this case we have only included t=0, 20, and 60 because it is more likely to be implemented then the six graphs plotted in Figure 4-7.

4.2.4 Comparison of Adaptive FEC and Rate Adaptation

After looking at Figures 4-4 and 4-7, one might ask which is the better option to use. "Will I get better results by lowering the transmission rate as I get farther away from the transmitter, or will I get better results by adding more FEC coding?" In order to do a fair comparison, we must establish similar operating conditions. To do this, please draw your attention to Figure 4-4 once more. Each of the four plots represents a different transmission rate that is kept constant. For each of these cases, the SNR is constantly decreasing as the distance from the transmitter increases. Now imagine an infinite number of plots all on the same graph with each plot differing from the next by a small change in the transmission rate. The edge of the envelope formed by these graphs would then represent the absolute best results that we could obtain by changing the rate. This envelope could be formed by plotting the throughput for the fixed optimum SNR (γ^*) and letting the rate change according to (4.6). Consequently, if we used a fixed SNR (in this case we will use the optimum SNR for DPSK of 5.74 – see Table 2-2), the packet success rate, $f(\gamma^*)$ would be a constant and we would have:

$$T(d) = \frac{L-C}{L} R^*(d) f(\gamma^*), \tag{4.8}$$

which is simply an exponentially decaying function of d. We will limit the rate to 871 kbps and use a packet length of 108 bits. Figure 4-9 shows four different rates along with the best case envelope limited to 871 kbps. Notice that it passes tangentially through each plot in such a way so that each is entirely contained beneath it.

Now that the concept of the continuously changing rate envelope has been established we can use it to make the comparison between changing the rate and changing the FEC. Figure 4-10 is similar to Figure 4-7 with one addition. It shows the effect of adding FEC capability as well as the envelope representing the best possible results of changing the transmission rate. As mentioned earlier, this basically tells us that the throughput plot of any attempt to lower the rate to increase the transmission range will always lie below this envelope. From this, we can clearly see that the better results are obtained when we keep the rate fixed and add more FEC as we increase our distance from the transmitter.

4.3 Conclusions

When looking at throughput as a function of distance there are basically two tradeoff factors involved: the throughput magnitude, and the range of distances that throughput magnitude can be sustained. Increasing the packet length has only a small effect on the range but does yield an increase in throughput. The parameters that have the greatest influence on these two factors are the transmission rate and FEC coding. Both are also easy to implement in an adaptive system. In both cases, decreasing the transmission rate or increasing the FEC have the similar effects of lowering the throughput magnitude and increasing the range. After a close comparison, it was found that adaptive FEC coding yields the best results.

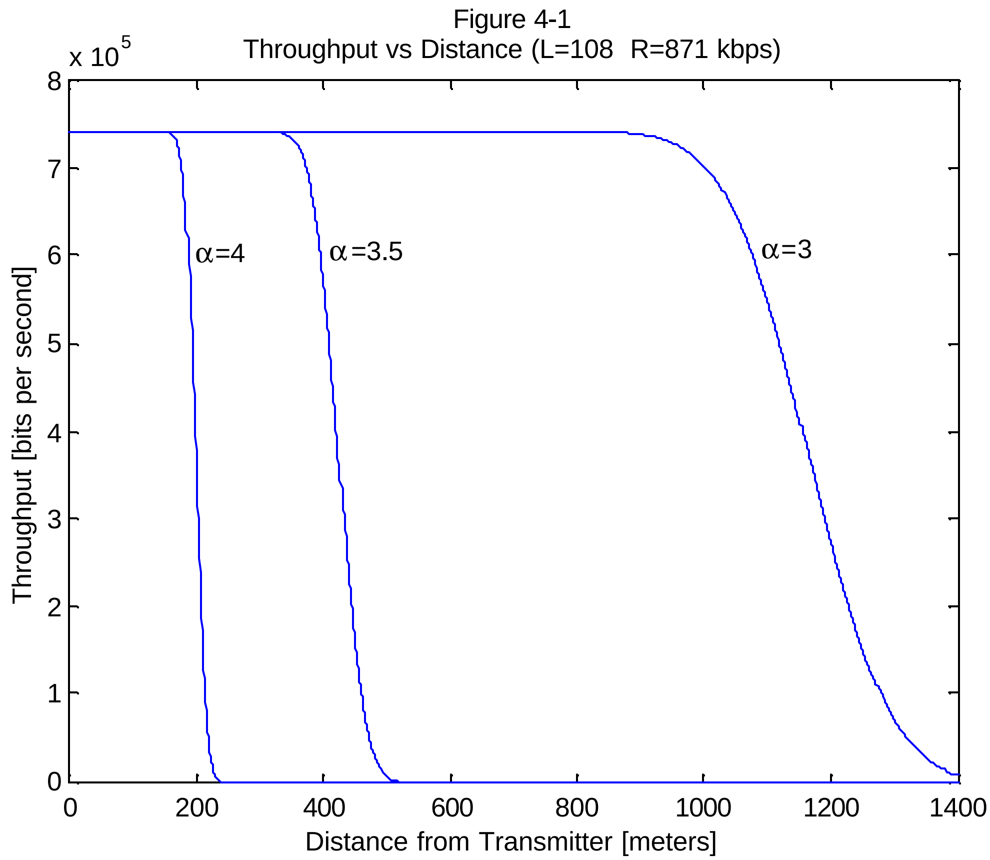


Figure 4-1
Throughput vs Distance (L=108 R=871 kbps)
x 10^5
Throughput [bits per second]
Distance from Transmitter [meters]
α=4
α=3.5
α=3
0
1
2
3
4
5
6
7
8
0
200
400
600
800
1000
1200
1400

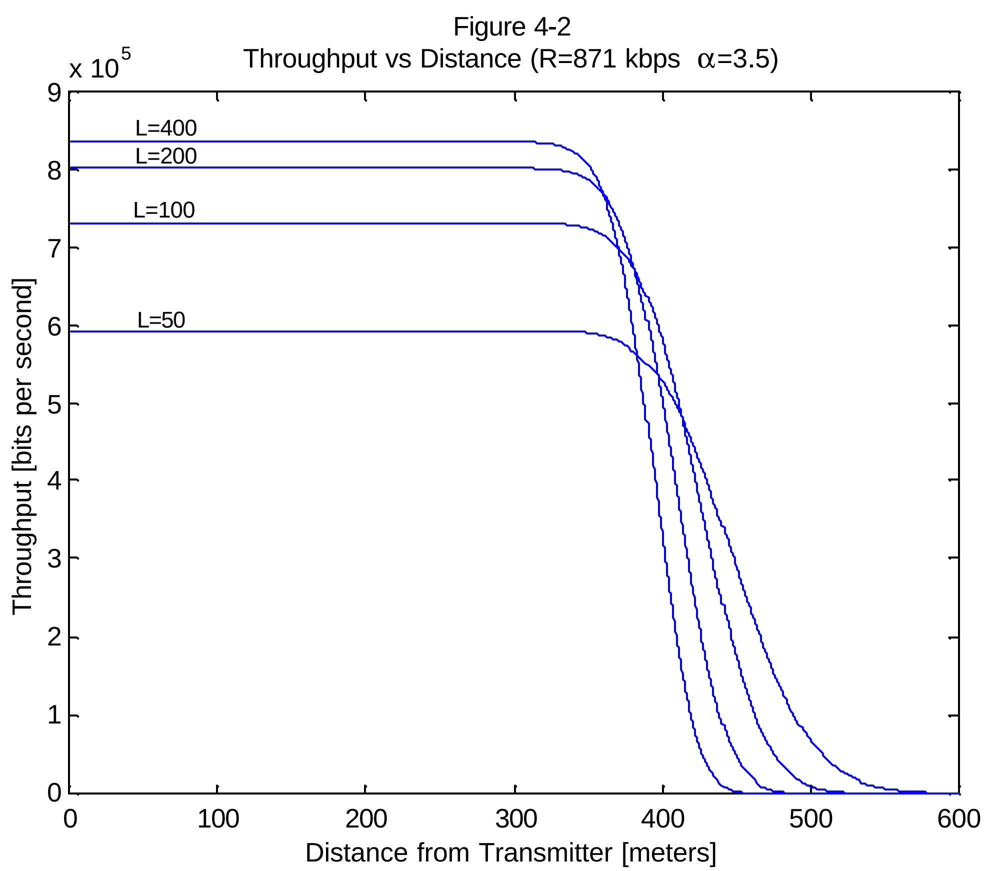


Figure 4-2
Throughput vs Distance (R=871 kbps α=3.5)
x 10^5
Throughput [bits per second]
Distance from Transmitter [meters]
L=400
L=200
L=100
L=50
0
1
2
3
4
5
6
7
8
9
0
100
200
300
400
500
600

Figure 4-3
Optimum Packet Length vs Distance (R=871 kbps)

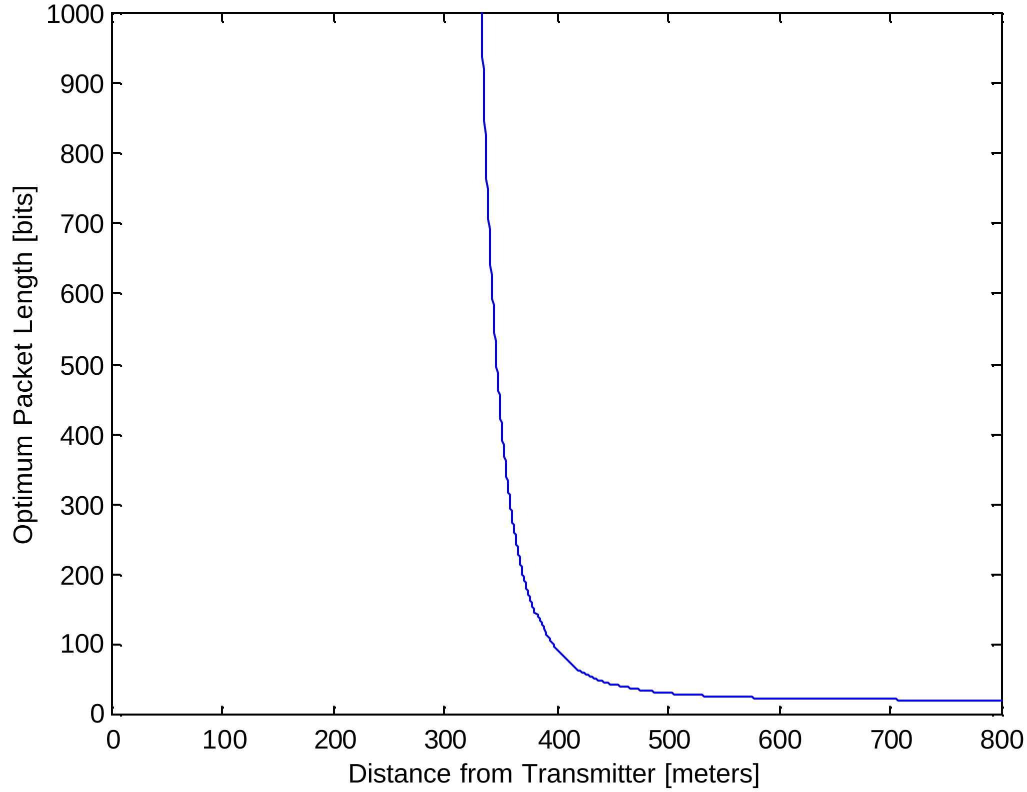


Figure 4-4
Throughput vs Distance (L=108 α=3.5)

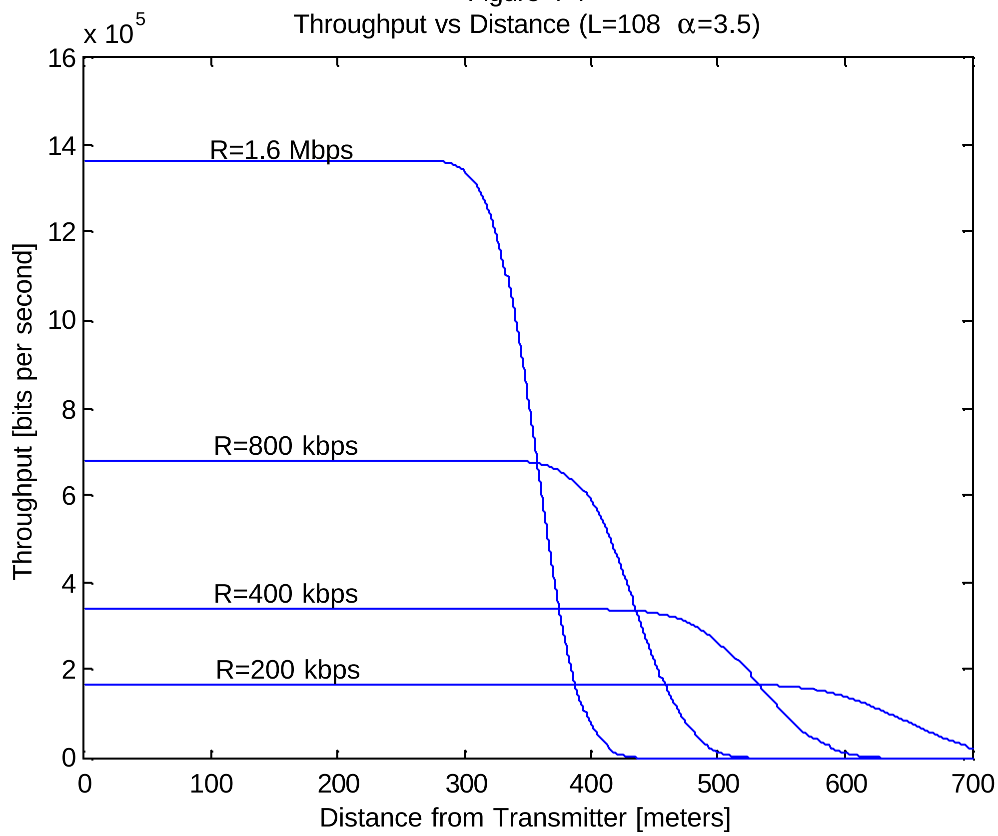


Figure 4-5
Optimum Transmission Rate vs Distance

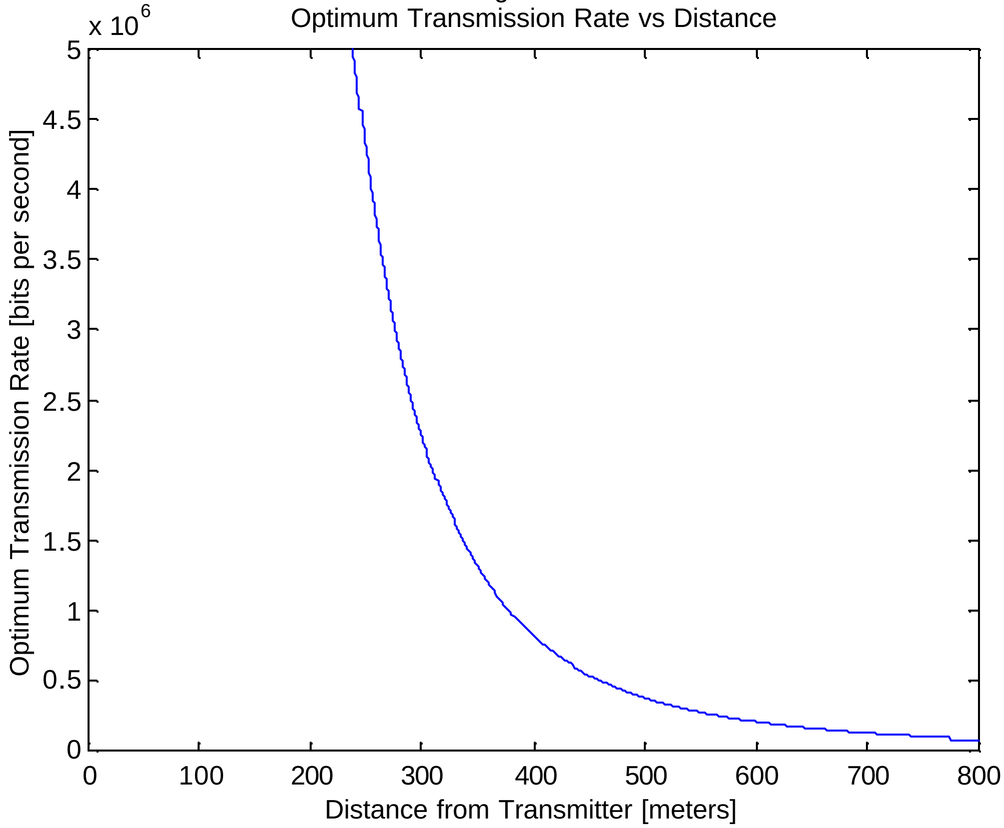


Figure 4-6
Throughput vs P/N_0 (L=108 α=3.5)

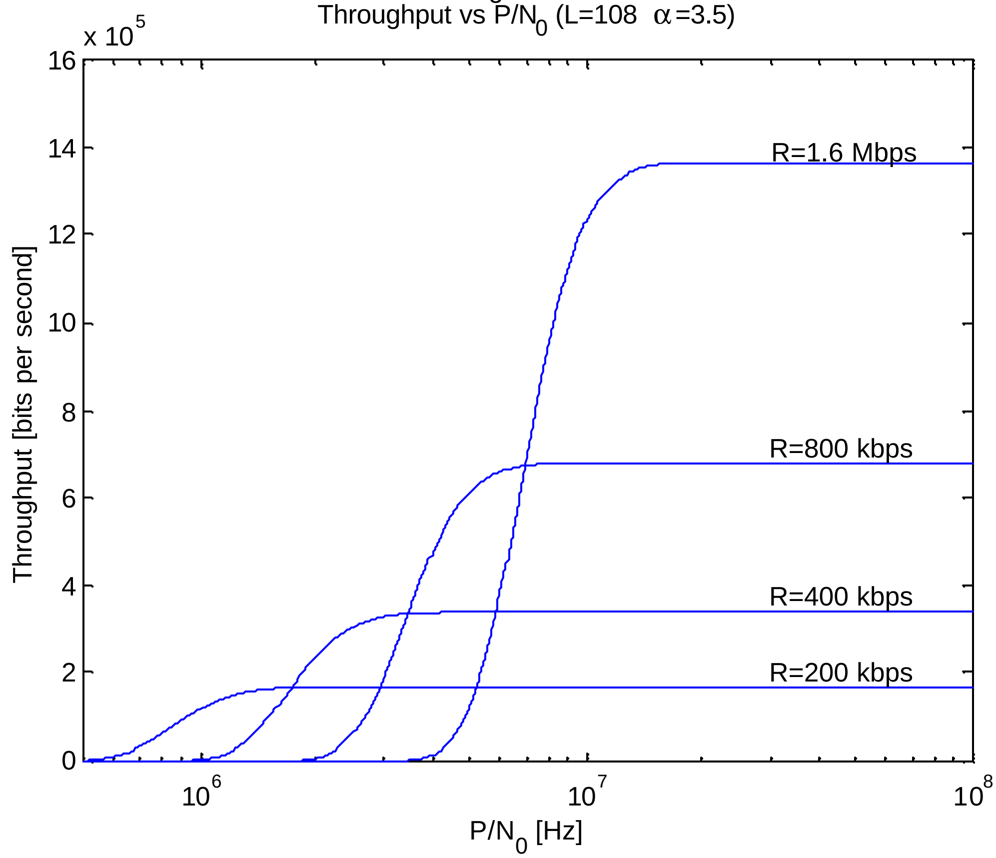


Figure 4-7

Throughput vs Distance (L=108 R=871 kbps α=3.5)

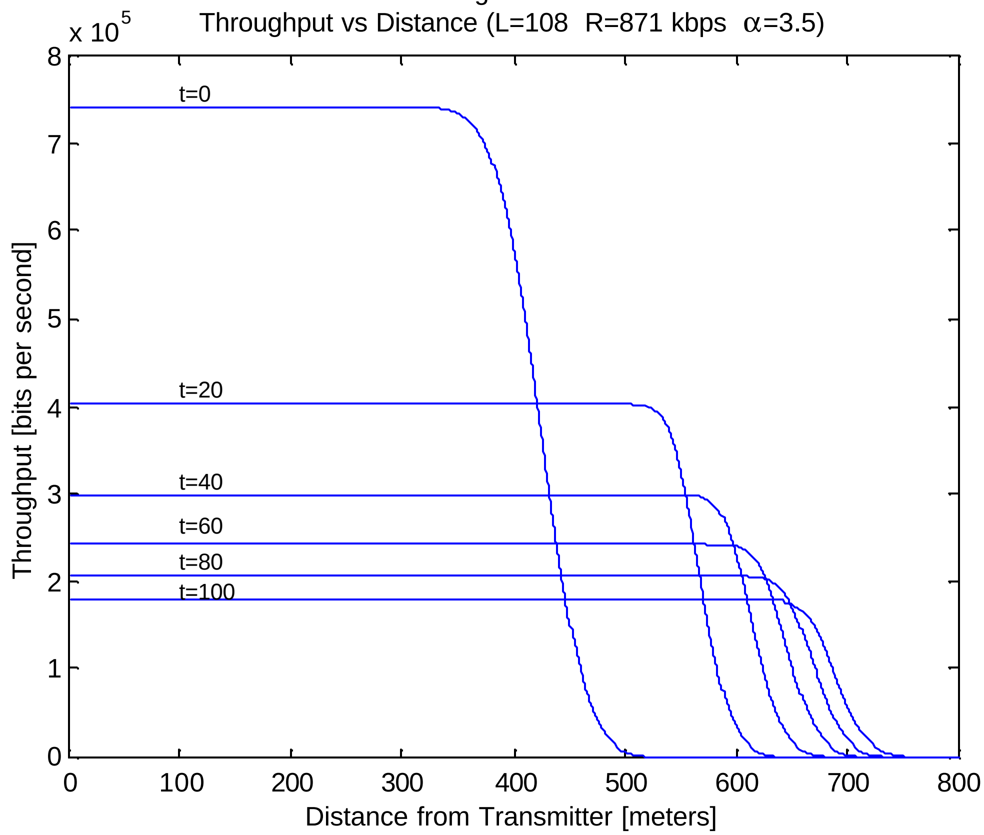


Figure 4-8

Throughput vs P/N_0 (L=108 R=871 kbps α=3.5)

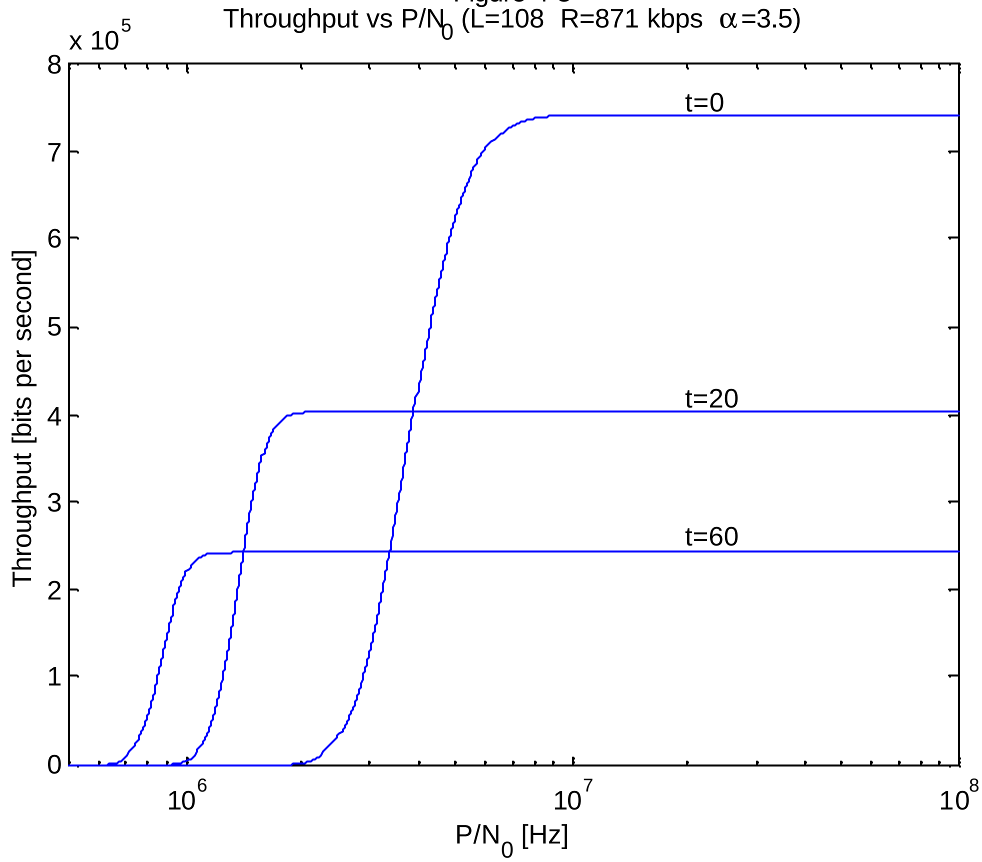


Figure 4-9

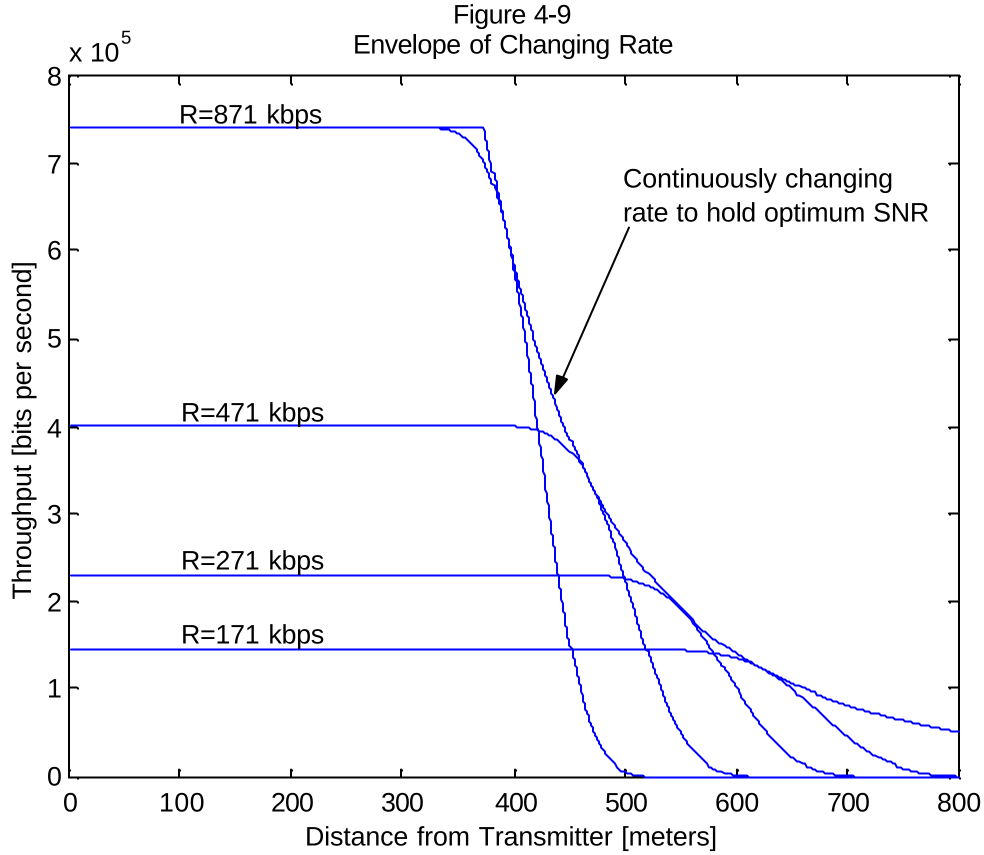
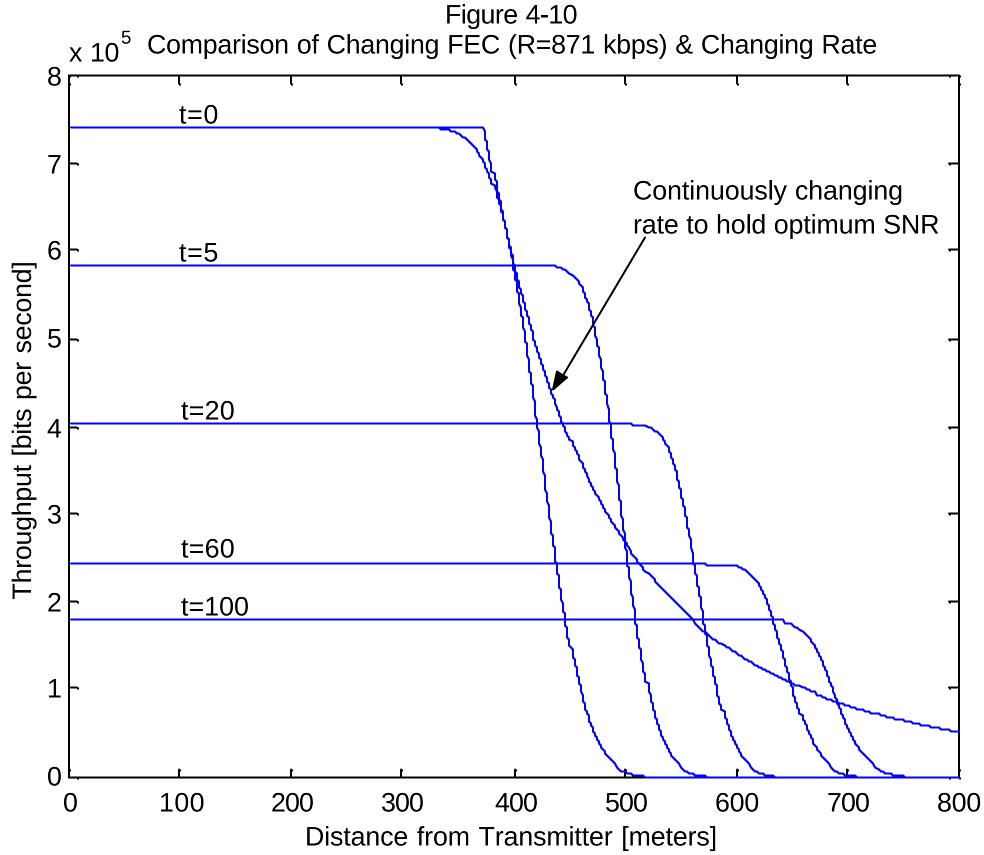


Figure 4-10



Chapter 5

Throughput Analysis Using BCH Codes

The forward error correction analysis presented in chapter 3 used block coding bounds to compute the number of error correcting bits needed to correct t errors in a packet of length L. These bounds represent the theoretical limits of block codes (i.e. there exists a code capable of correcting t errors in an L bit packet using as little as B additional bits). Although these results give us the best case, they are not necessarily very practical codes to implement. In this chapter we give a practical example of a block code that can be easily implemented for multiple error correction.

5.1 BCH Codes

BCH codes (named for their discoverers Bose – Chaudhuri – Hocquenghen) are among the most well known and widely implemented multiple error correcting block codes. The block length of the of code is denoted by L+B and must be of length:

$$L + B = 2^m - 1, \tag{5.1}$$

where m=3,4,5… In this chapter we will define L as L=K+C where K is the number of information bits and C is a 16 bit CRC. B represents the number of additional overhead bits needed to correct t errors. Appendix A gives a comprehensive summary of the code parameters for block lengths from m=3 to m=8.

5.2 Numerical Results

5.2.1 Throughput vs. SNR

Using these L, B pairs along with equations (3.3) and (3.4) we can plot these different throughput curves. Because we are using a 16 bit CRC, we are forced to only use block lengths of m=5, 6, 7 and 8 (L+B=31, 63, 127, 255). For m=3 and 4, L is always less than 16 leaving no bits for information. Figures 5-1 through 5-4 show the throughput plot using the BCH codes of block lengths 31, 63, 127, and 255. In each graph we have shown plots above and below the optimum throughput plot. Notice that if we start with a small block length of 31 bits (Figure 5-1), our best bet is to use no error correction at all. As soon as we try to correct any errors, the overhead introduced starts to overshadow the information being sent. At L+B=63 bits (Figure 5-2), the optimum error correction turns out to be t=2 correctable errors. For a block length of 127 bits (Figure 5-3), the optimum is at t=4 correctable errors and for L+B=255 bits (Figure 5-4), the best results occur at t=9 correctable errors. Another thing to note in this series of graphs is that the maximum throughput plot on each graph increases as the block length increases. We would hypothesize that this trend would not continue on indefinitely. Unfortunately, we were not able to see the point at which the increasing block length started to yield a decreasing optimum throughput. The reference used [8], only listed BCH code data for block lengths up to 255 bits. You'll notice that on each of these four figures, we've included the value of the packet success rate at the optimum SNR, $f(\gamma^*)$. These results are also summarized in Table 5-1. For each block length, there is an entry in bold indicating the optimum amount of error correction. One important observation from this data is that the optimum packet success rate increases with increasing block length.

Table 5-1 BCH Packet Success Rates

L+B	**t**	g^*	**f(g^*)**
31	**0**	**4.1698**	**0.7862**
	1	3.0030	0.8208
	2	2.3836	0.8297
63	0	5.0689	0.8200
	2	**3.2388**	**0.8735**
	4	2.4995	0.8835
	6	2.0360	0.8844
127	0	5.9241	0.8437
	1	4.6873	0.8833
	4	**3.2947**	**0.9116**
	14	1.8382	0.9198
255	0	6.7512	0.8614
	3	4.3876	0.9239
	9	**3.1169**	**0.9403**
	26	1.8761	0.9446

One should also note that the optimum $f(\gamma^*)$ is not necessarily the largest value. In all four cases, adding more error correction capability yields higher values for $f(\gamma^*)$, but not higher values of optimum throughput. This can be attributed to the large amount of overhead (B bits) that is implied with high values of t.

5.2.2 Throughput vs. Distance

Now that we have some practical BCH codes to work with, we will use this to obtain results for the distance dependence covered in chapter 4. Figures 5-5 through 5-8 show the throughput as a function of distance using the same block sizes as before. As in the previous chapter, these plots have a fixed N_0 of 10^{-15} W/Hz, a rate of 871 kbps, and a range index, α of 3.5. In Figure 5-5 with L+B=31 bits, we can see that putting in some error correcting capability does not seem to help us much. For each additional error that we want to correct, our throughput will decrease drastically and we will get a very small range increase. In Figure 5-6 the situation gets a little better with L+B=63 bits. The throughput values are higher and we get some range increase for increasing t. Figure 5-7 (L+B=127) is even better and Figure 5-8 (L+B=255) is still better.

5.2.3 Comparison of Coding Bounds and BCH codes

Recall in chapter 3 the use of FEC to obtain better throughput results. The number of overhead bits, B, was calculated using equation (3.1) using the packet length L and the number of correctable errors t. This value of B is a theoretical limit on the minimum number of additional bits needed to correct t errors. Any practical code will have a value of B greater than or, at best, equal to this limit. To see how the BCH codes size up to the best case derived in (3.1), we have plotted the two values as a function of t in Figure 5-9. The solid curve denotes the absolute minimum value of B as calculated from (3.1) and the dotted curve denotes the actual BCH value of B. Both graphs are plotted assuming L+B=255 bits. From this graph we can see that the two values differ by as little as zero at the ends at t=1 and t=127 and by as much as 67 bits at t=31.

For a comparison in terms of the throughput, we can plot the same curves in Figure 5-8 for L+B=255, C=16, and R=871 kbps, only this time, using the coding bounds in chapter 3 instead of the BCH codes used in this chapter. Figure 5-10 shows the throughput vs. distance for the coding bounds. In comparing Figure 5-8 and Figure 5-10 we can see how the ideal case differs from the practical case. An analysis of these graphs can lead one to conclude that the throughput magnitudes are greater using the coding bounds, which is what we would expect since there is less overhead (B) involved for each value of t. One thing that nearly remains unchanged is the distance at which the throughput begins to fall off.

5.3 Conclusions

When comparing the throughput results for the FEC coding bounds in Chapter 3, to the results for the BCH codes in this chapter we can see that the coding bounds yield the best results. This is what we would expect considering that the coding bounds represent the theoretical best-case forward error correction. The BCH codes, however, do perform quite well, attaining about 77% of the throughput using the coding bounds. When dealing with BCH codes, larger block lengths yield higher throughputs and better payoffs for the use of error correction, in general.

Figure 5-1

Throughput vs SNR (BCH codes L+B=31, C=16)

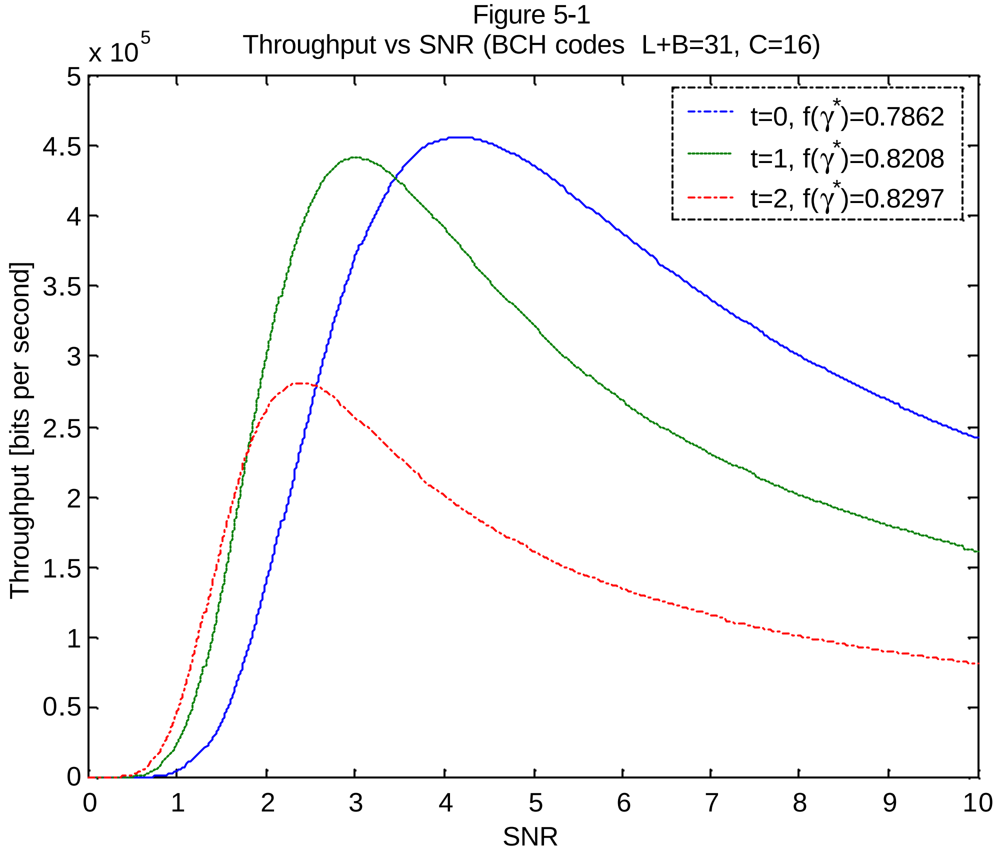


Figure 5-2

Throughput vs SNR (BCH codes L+B=63, C=16)

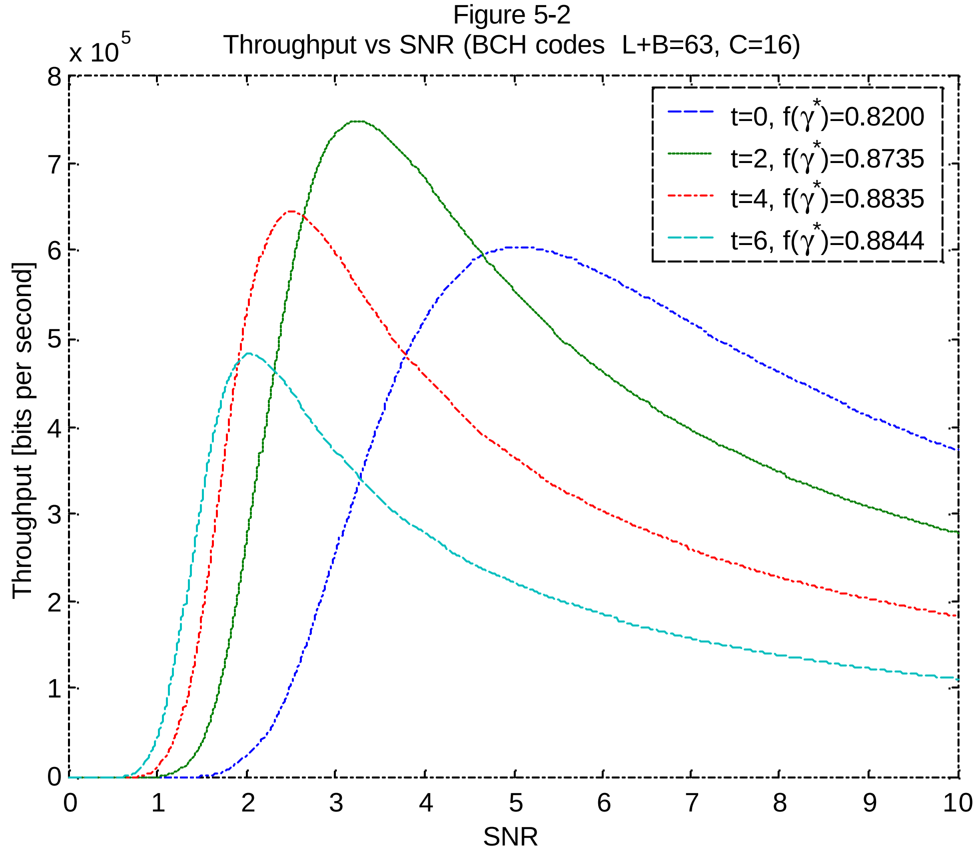


Figure 5-3

Throughput vs SNR (BCH codes L+B=127, C=16)

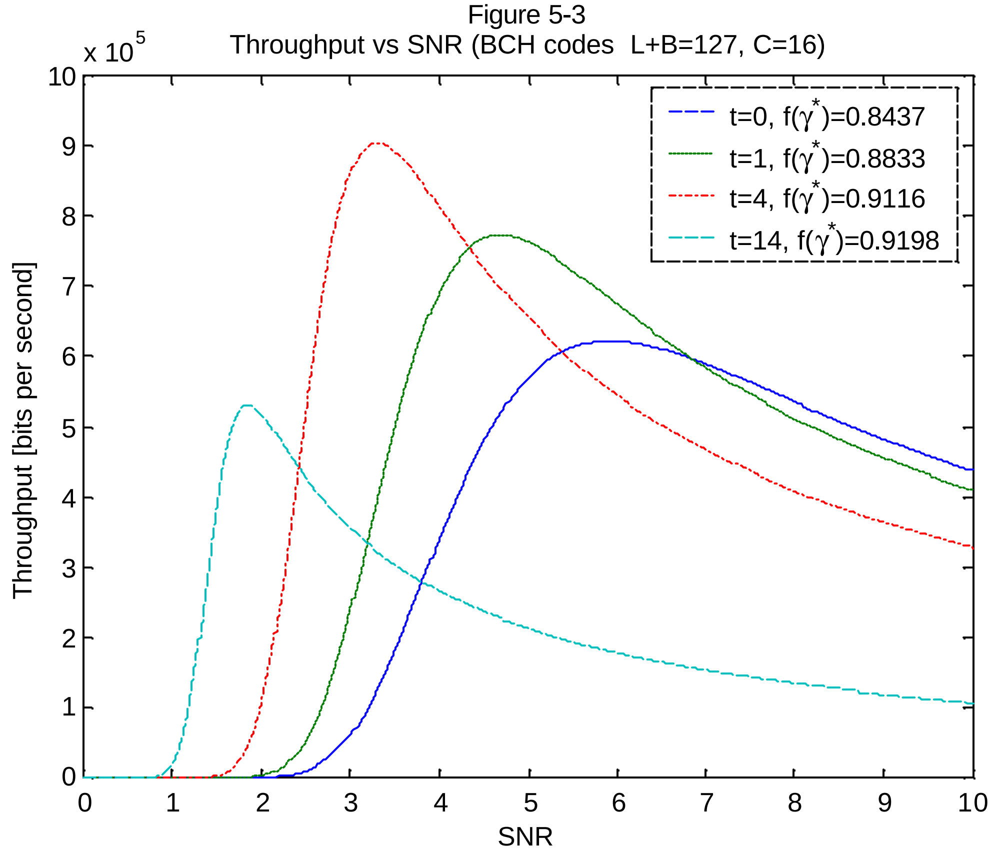


Figure 5-4

Throughput vs SNR (BCH codes L+B=255, C=16)

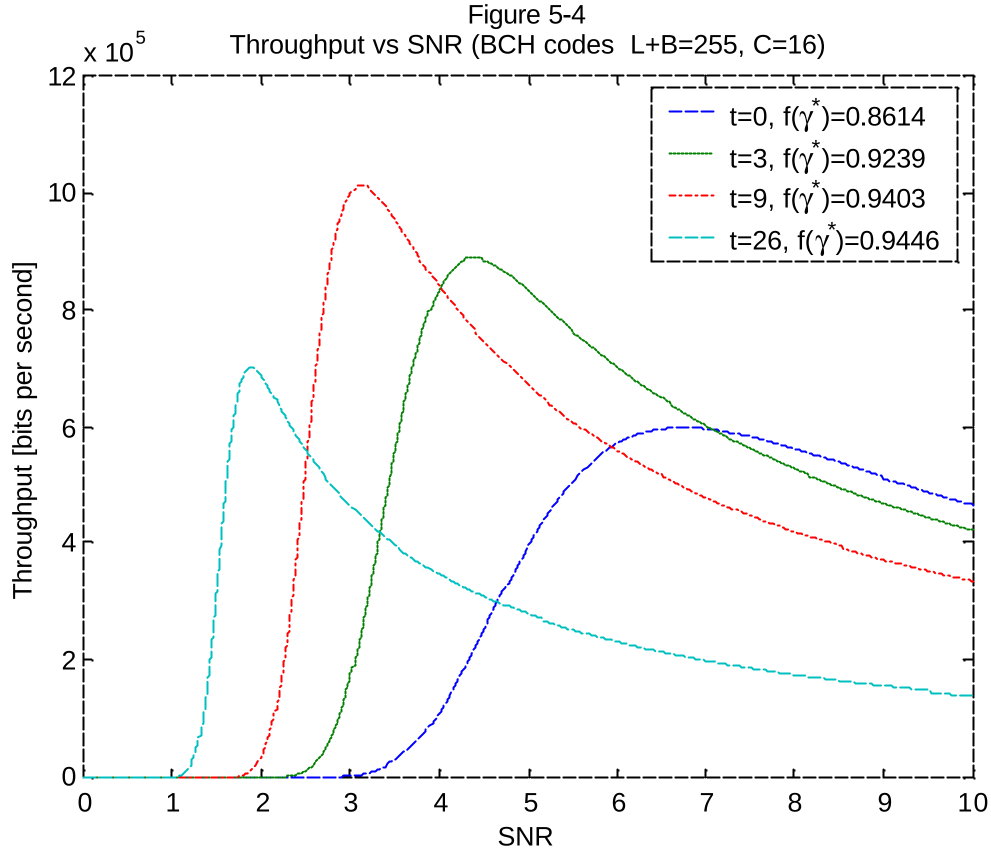


Figure 5-5

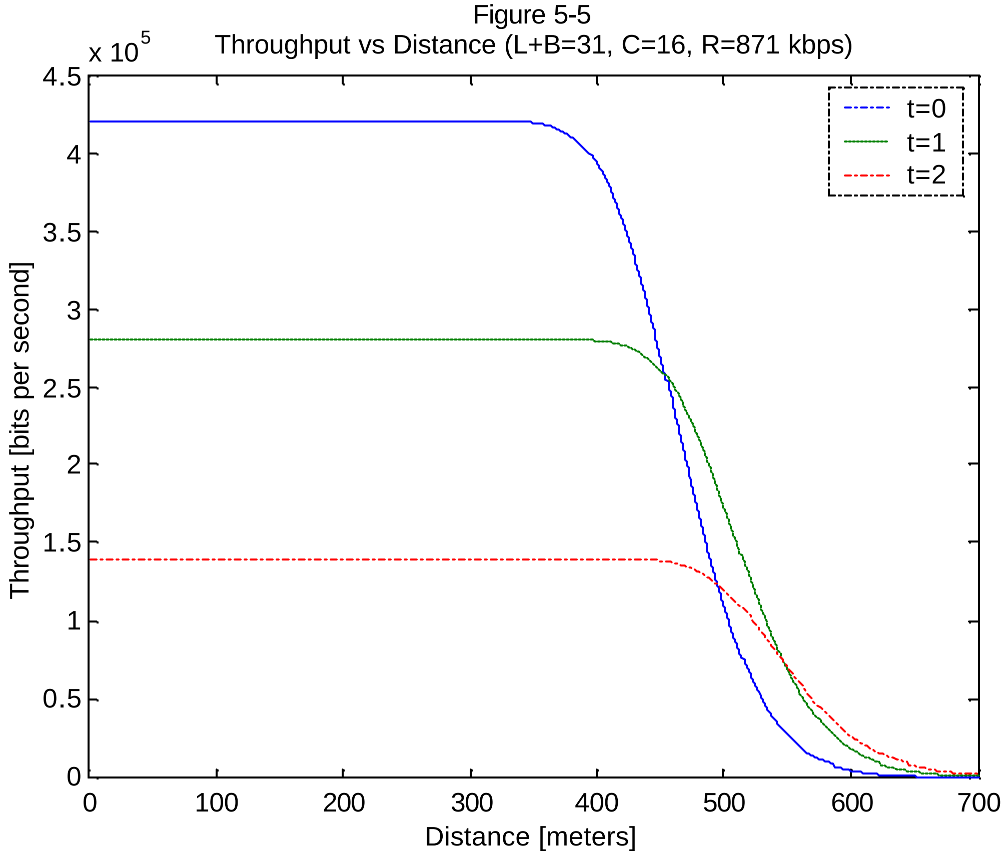


Figure 5-6

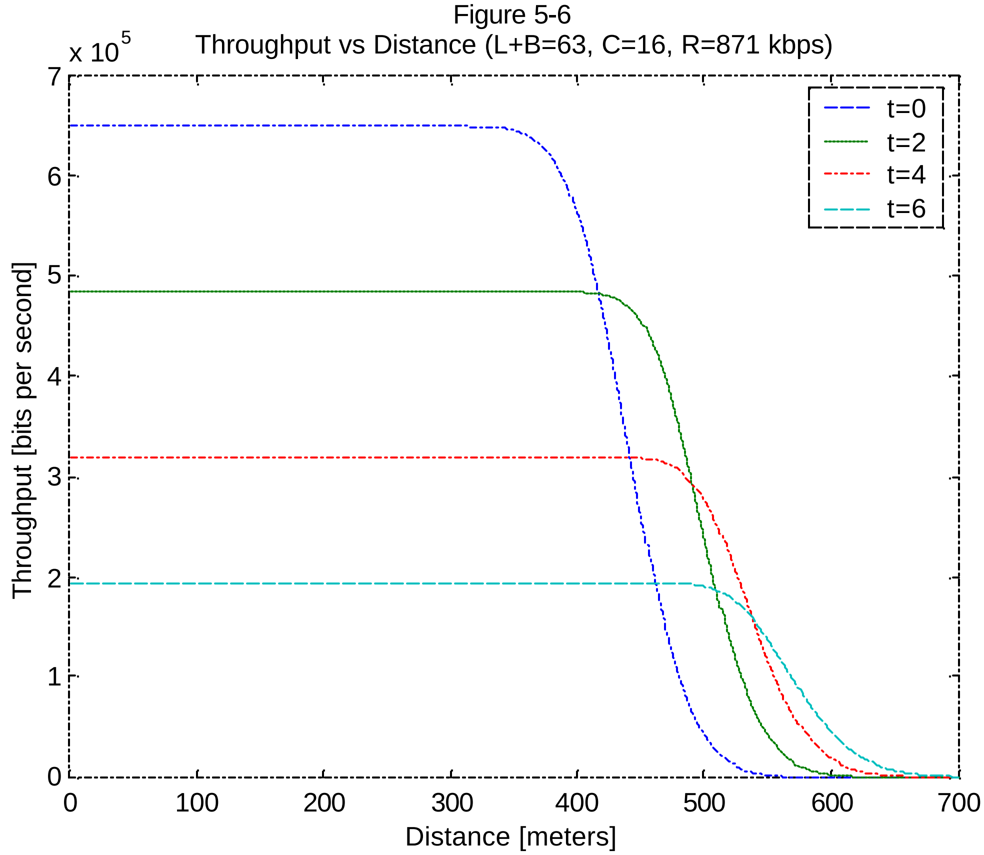


Figure 5-7

Throughput vs Distance (L+B=127, C=16, R=871)

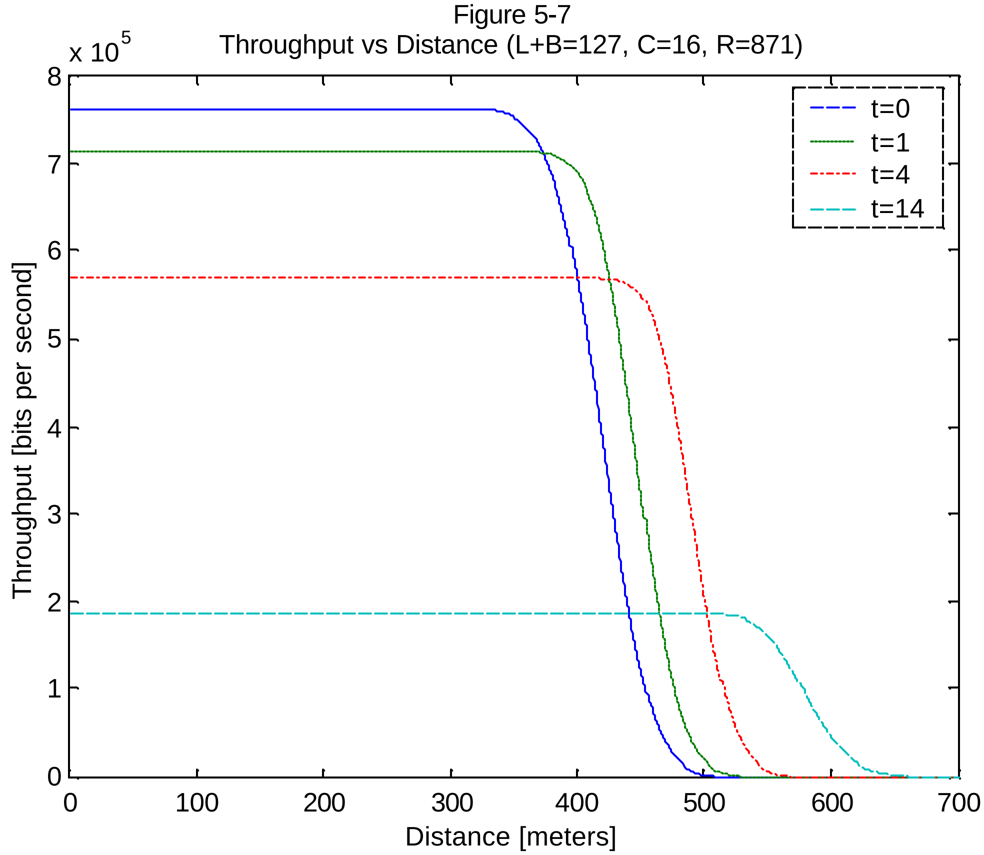


Figure 5-8

Throughput vs Distance (L+B=255, C=16, R=871 kbps)

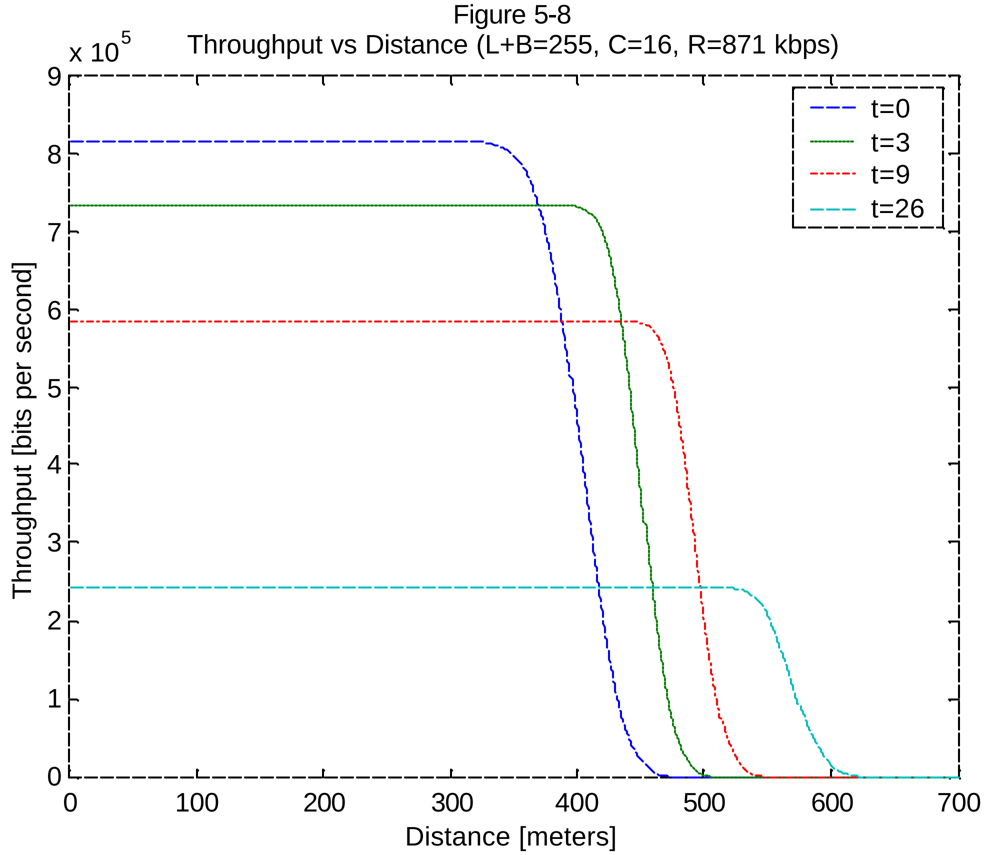


Figure 5-9
Bound on B vs BCH Value of B (L+B=255)

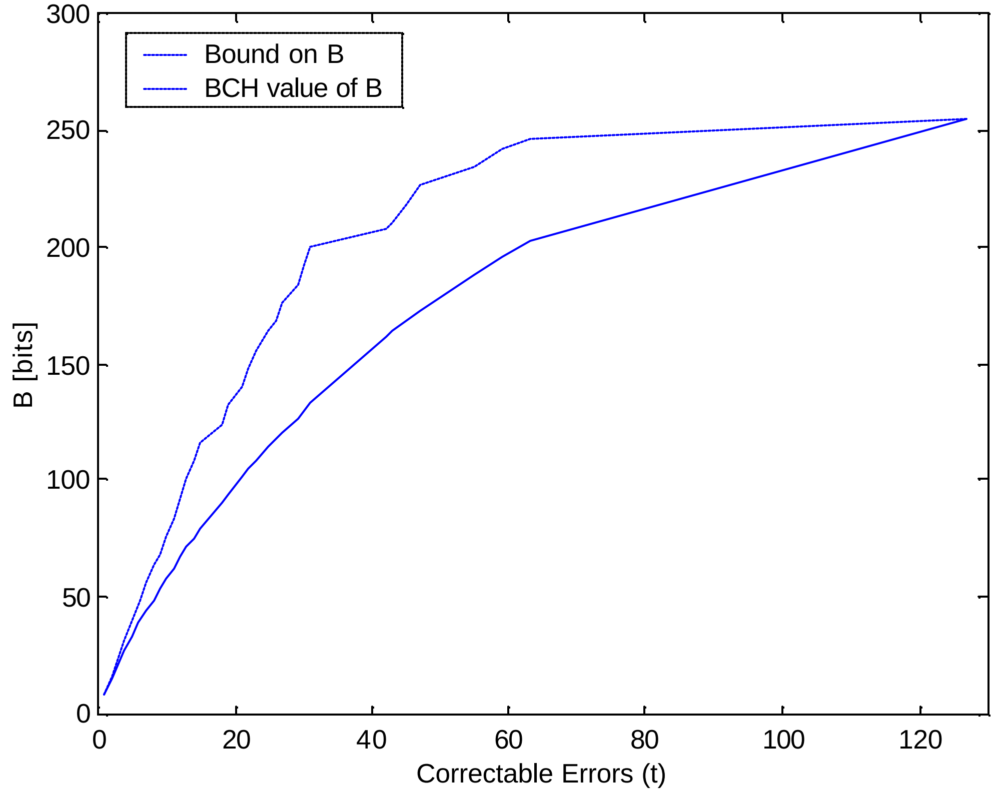
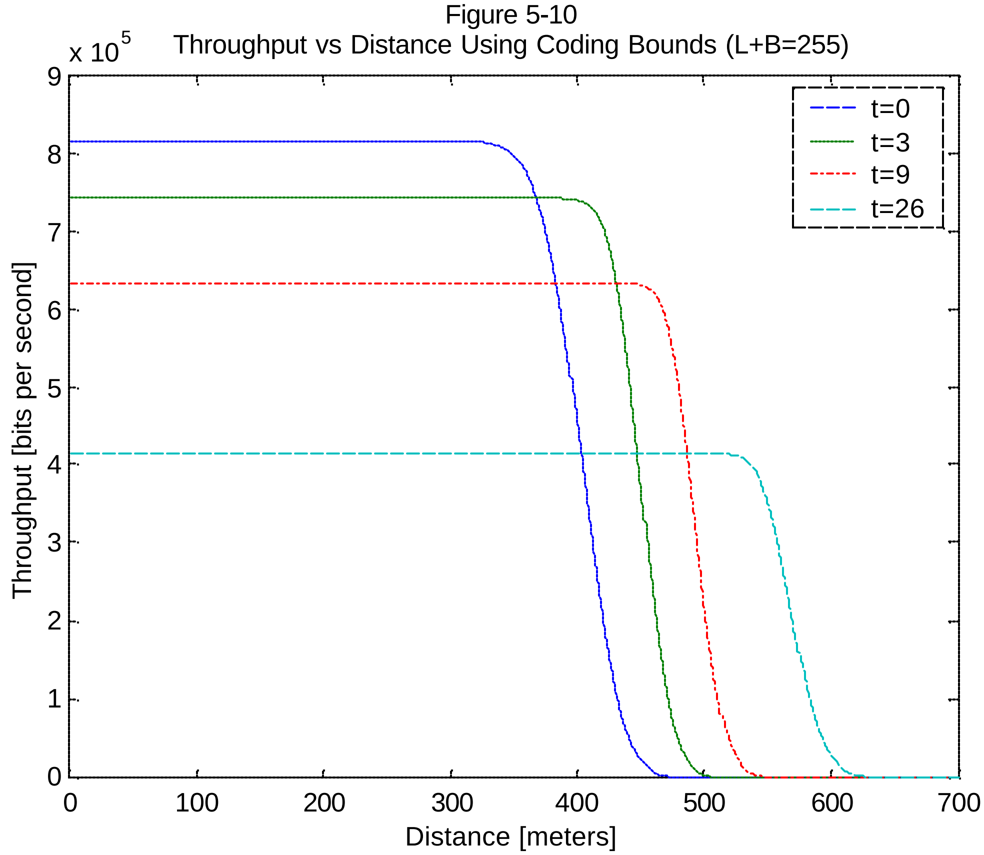


Figure 5-10
Throughput vs Distance Using Coding Bounds (L+B=255)



Chapter 6

Summary and Conclusions

When this work was initially started, the goal was to provide new insight into the understanding of data throughput in a wireless environment. This was not geared toward any system in particular. No system specific protocols and/or parameters were used in this work. Wireless systems engineers, in their design of wireless data systems, should use this research as a tool.

Some very important results were obtained. We have learned that data throughput depends on a wide variety of variables, some of which include the packet size, the transmission rate, the number of overhead bits in each packet, the received signal power, the received noise power spectral density, the amount of forward error correction used, the modulation technique, and the channel conditions. Given a modulation scheme, and the channel conditions, the optimum SNR at which to operate the system for maximum throughput can be derived. From this optimum SNR, the optimum packet length and (given the received signal power over the noise power spectral density) the optimum transmission rate can be determined. The addition of forward error correction codes has shown us a significant increase in throughput performance. Through a series of graphs, We have demonstrated how the optimum amount of FEC coding can be obtained and how that optimum amount depends on the packet length. Throughput as a function of distance was also investigated. From this we have shown how rate adaptation and the addition of FEC can be used to increase the range of a transmission at a lower throughput. We have also shown that between the two options, changing the FEC has a more beneficial result than changing the transmission rate. To show a practical example of a forward error correcting code, we demonstrated most of the findings of the previous chapters with the use of BCH block codes.

We have talked about many different variables and how changing certain parameters can yield better throughput performance. Implementing these concepts in real systems is not as easy as one might think. In order for a system to make parameter changes it must be able to make simple measurements in the system. For example, if rate adaptation is to be employed, the receiver cannot easily determine its distance from the transmitter, but it can easily measure the ratio of the received signal power to the noise power spectral density. As distance increases, this ratio decreases. When its value reaches a certain point, the receiver can tell the transmitter (on a control channel perhaps) to lower the transmission rate. In another example, consider a system that uses adaptive FEC coding. The receiver could use the packet success rate as a factor for determining how much FEC coding should be used. The receiver can easily calculate the packet success rate simply by finding the ratio of the number of packets received without error to the total number of packets considered when the sample space is large. At close distances, the signal arrives strong and few errors are made resulting in a high packet success rate. As the transmission distance increases, the packet success rate will decrease. Once it has reached a certain value the receiver can tell the transmitter to increase the FEC coding (i.e. increase the number of correctable errors, t).

6.1 Future Work

As was mentioned earlier, this work did not take any particular wireless system into account in its different analyses. One possible path for future work could be the application of these general results to a particular system such as the IEEE 802.11 standard.

Appendix A - BCH Code Data [8]

m	L+B	L	B	t
3	7	4	3	1
		1	6	3
4	15	11	4	1
		7	8	2
		5	10	3
		1	14	7
5	31	26	5	1
		21	10	2
		16	15	3
		11	20	5
		6	25	7
		1	30	15
6	63	57	6	1
		51	12	2
		45	18	3
		39	24	4
		36	27	5
		30	33	6
		24	39	7
		18	45	10
		16	47	11
		10	53	13
		7	56	15
		1	62	31
7	127	120	7	1
		113	14	2
		106	21	3
		99	28	4
		92	35	5
		85	42	6
		78	49	7
		71	56	9
		64	63	10
		57	70	11
		50	77	13
		43	84	14
		36	91	15
		29	98	21

m	L+B	L	B	t
7	127	22	105	23
		15	112	27
		8	119	31
		1	126	63
8	255	247	8	1
		239	16	2
		231	24	3
		223	32	4
		215	40	5
		207	48	6
		199	56	7
		191	64	8
		187	68	9
		179	76	10
		171	84	11
		163	92	12
		155	100	13
		147	108	14
		139	116	15
		131	124	18
		123	132	19
		115	140	21
		107	148	22
		99	156	23
		91	164	25
		87	168	26
		79	176	27
		71	184	29
		63	192	30
		55	200	31
		47	208	42
		45	210	43
		37	218	45
		29	226	47
		21	234	55
		13	242	59
		9	246	63
		1	254	127

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