A Quaternion-Based Orientation Estimation Algorithm Using an Inertial Measurement Unit

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A Quaternion-Based Orientation Estimation Algorithm Using an Inertial Measurement Unit

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Abstract **- This paper presents a real-time orientation estimation algorithm based on signals from a low-cost inertial measurement unit (IMU). The IMU consists of three MEMS accelerometers and three MEMS rate gyros. This approach is based on relationships between the quaternion representing the platform orientation, the measurement of gravity from the accelerometers, and the angular rate measurement from the gyros. Process and measurement models are developed based on these relations in order to implement them into an extended Kalman filter. The performances of each filter are evaluated in terms of the roll, pitch, and yaw angles. These are derived from the filter output since this orientation representation is more intuitive than the quaternion representation. Extensive testing of the filters with simulated and experimental data show that the filters perform very accurately in the roll and pitch angles, and even significantly corrects the yaw angle error drift.**

NOTATION

x_{bc}^{a}	Vector quantity x of frame c with respect to frame b represented in frame a
a	Acceleration vector
ω	Angular rate vector
f	Specific force vector
g	Gravity vector
C_b^a	Rotation matrix transforming from frame b to a
q_b^a	Quaternion transforming from frame b to a

I. INTRODUCTION

Real-time orientation tracking has a wide range of applications in virtual reality, autonomous vehicle navigation, and robotics, among many others. Recent research has focused on developing orientation trackers based on MEMS inertial sensors, attractive because of their low cost and small size. As well, the concept of a self-contained, or "source-less", tracking system is highly desirable.

This work presents a data fusion of MEMS sensors in a low-cost inertial measurement unit (IMU) to produce a better estimate of the IMU orientation than with gyros or accelerometers alone. The sensors in the IMU are three rate gyros and three accelerometers in triad configuration. With perfect gyro measurements, the estimate of the orientation could be determined quite accurately; however, using real sensors, the error in the estimate grows with time due to quantization, integration, and sensor errors.

There are several known facts about the IMU that can be exploited in a Kalman filter to produce an orientation estimation that is more accurate. For short durations of time in the order of minutes, the effects of the earth's rotation can be considered negligible. For localized tracking applications, i.e. within short distances in the order of metres, the gravitational vector can be considered to be constant and aligned in the vertical direction. By implementing a quaternion-based model of the system in a Kalman filter, accelerometers are used as a corrective measure by taking into account the gravitational force to curb the error of the orientation estimate.

Experimentation tests the effectiveness of the algorithm, showing that it effectively estimates the orientation in all three axes, given a relatively small amount of linear acceleration. An optical position tracking device measures the position of three LEDs attached to the IMU, providing an extremely accurate means of verification for the orientation tracker. The nonlinear Kalman filter used was the extended Kalman filter.

II. LITERATURE REVIEW

In developing a solution to the orientation problem, researchers have experimented with different models, estimation approaches, and sensor configurations. A literature review was conducted on four orientation tracking systems to find out what other researchers have come up with to estimate orientation using rate gyros as well as other redundant sensors.

Foxlin has developed a Kalman filter estimator for a head-mounted inertial orientation tracker [7] using rate gyros in triad formation, a fluid-filled inclinometer, and a magnetic compass. The differential equation for an Euler angle sequence was integrated into the process model:

$$\begin{bmatrix}\dot{\psi}\\ \dot{\theta}\\ \dot{\phi}\end{bmatrix}=\begin{bmatrix}1 & \tan(\theta)\sin(\psi) & \tan(\theta)\cos(\psi)\\ 0 & \cos(\psi) & -\sin(\psi)\\ 0 & \sec(\theta)\sin(\psi) & \sec(\theta)\cos(\psi)\end{bmatrix}\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} \quad (1)$$

The body-fixed sequence z, y, x was used, to represent yaw, pitch, and roll, respectively. This model was used to develop a nonlinear complementary Kalman filter, a Kalman filter designed to estimate the state errors, rather than the states themselves.

Azuma *et al* [8], developed an outdoor orientation tracker using a tri-axial rate gyro and an additional orientation sensor that contains a magnetic compass and a two-axis inclinometer, similar to the setup by Foxlin. The rate gyros used were Gyrochip II sensors from Systron-Donner. The compass/ inclinometer was the TCM2 from Precision Navigation. The six states of interest were the roll, pitch, and yaw angles that were measured by the TCM2 and the three angular rates that were measured by the gyros. Like Foxlin, a state space model was developed using the continuous Euler angle differential equation, except with a different Euler angle sequence. The resulting nonlinear Kalman filter combined the high-frequency performance of the rate gyros with the low-frequency stability of the TCM2 compass/inclinometer.

Vaganay *et al* [5] developed an orientation tracker for an all-terrain, outdoor, mobile robot. The sensor cluster that was used consisted of a low-cost tri-axial rate gyro and two low-cost accelerometers aligned to measure the roll and pitch of the strapdown platform. Given that the acceleration due to gravity is in the 'down' direction, the roll and pitch angles have a closed form solution that can be easily derived:

$$roll = -Sin^{-1}\left(g_x / \|g\|_2\right) \tag{2}$$

$$pitch = Sin^{-1}\left(g_y / \|g\|_2 \cos(roll)\right) \tag{3}$$

where $\|g\|_2$ is the magnitude of gravity

$$g = \begin{bmatrix} g_x & g_y & g_z \end{bmatrix}^T$$

The orientation estimate from the rate gyros is obtained using a quaternion differential equation. Due to integration and sensor errors, there is a drift in this orientation estimate. An extended Kalman filter was developed to track the drift in the roll and pitch angles, with the yaw angle left uncorrected. This filter produced reasonable estimations of roll and pitch; however, precise values of accuracy were not available due to a lack of a "truth" tracker to compare with the experimental results.

Marins *et al* [6] developed an extended Kalman filter for estimating the orientation using a MARG (magnetic, angular rate, gravity) sensor. A MARG sensor consists of a strapdown tri-axial rate gyro, tri-axial accelerometer, and tri-axial magnetometer. The orientation was defined by quaternions rather than Euler angles and rotation matrices. The process model for the system had a state vector with seven states, for the three angular rates and the four quaternion elements. As well, the quaternion state models include a normalization expression to ensure that the quaternion always has unity norm.

Two Kalman filter configurations were developed, both using the same process model, but different measurement models. The first approach can be considered the "brute force" approach. Each one of the measurements in the measurement vector, z, is an output signal from the MARG sensor, making z a nine-element vector. After rotation algorithms, these measurement equations become extremely complicated and nonlinear. The second approach uses an external algorithm (i.e. external to the Kalman filter) to calculate the quaternion. The measurement vector, z, in this case has seven equations – three angular rate measurements, and four quaternion elements. Therefore, the measurement vector equals the state vector, resulting in a linear state space model. This is considerably less complicated than the first approach; however, the Kalman filter is more or less reduced to a noise filter rather than a data fusion tool.

Euler angle formulations in orientation filters present several problems. Due to the formulation's reliance on trigonometry, the process model tends to be quite nonlinear. Euler angle formulations also have singularity issues. For the sequence, x,y,z, for Euler angles, ϕ, θ, ψ, respectively, the following singularities exist:

$$\lim_{\theta \to \pi/2^{(-)}} \left(\dot{\phi}\right) \to \infty \qquad \lim_{\theta \to -\pi/2^{(+)}} \left(\dot{\phi}\right) \to -\infty \tag{4}$$

$$\lim_{\theta \to \pi/2^{(-)}} \left(\dot{\psi}\right) \to \infty \qquad \lim_{\theta \to -\pi/2^{(+)}} \left(\dot{\psi}\right) \to \infty \tag{5}$$

If the tracker is expected to exceed the limit of, say, $-\pi/2 < \phi < \pi/2$, significant complications occur. These can be avoided using a discrete Euler angle operator, or by using algorithms that smoothes over the singularities. This unnecessarily complicates the filter and may cause normalization problems, so this should be avoided. Quaternions, on the other hand, can be put into a process model directly and avoids the long-standing singularity problem with Euler angles altogether.

Sensor redundancy is another important feature to improve estimation; at the same time, too many sensors can lead to a system that is too bulky. Marins *et al* [6] used nine sensors to obtain orientation; Vaganay *et al* [5] used five sensors, sacrificing the heading angle for the sake of fewer sensors. When selecting sensors for a cluster, a desire for accuracy must be tempered by the need for frugality.

III. System Modeling

A. Process Model

The orientation sensor that is modeled in this work is a standard strapdown IMU consisting of rate gyros and accelerometers in orthogonal triads. This orientation sensor is sparing in sensors in measuring three degree-of-freedom orientation compared to the systems in [5] and [7].

The following process model is developed and considered in the continuous domain, since the discrete case can be easily derived from this model. For modeling the rotational motion, quaternions are used rather than direction cosines and Euler angles because the quaternion representation does not have a problem with singularities; handles normalization better; and exchanges the bulky trigonometry of Euler angles with multiplication and addition, as demonstrated in Kuipers [4], an excellent primer on quaternion theory and usage.

There are two main frames used in this work. The GROUND frame, denoted by o, is considered to be the inertial frame and is attached to the ground, with the vertical axis aligned with the gravitational vector. The effects due to the rotation of the earth are considered negligible in this work. The BODY frame, denoted by b, is attached to the IMU. The orthogonal axes are aligned with the sensor axes.

The state vector consists of the quaternion associated with the orientation, q_o^b, the angular velocity, ω_{ob}^b, and the low-frequency gyro drift, δ :

$$x = \begin{bmatrix} q_o^b & \omega_{ob}^b & \delta \end{bmatrix}^T \tag{6}$$

The quaternion representation could be transformed into the familiar direction cosine matrix using the equation:

$$C_o^b = \begin{bmatrix} 2q_0^2 - 1 + 2q_1^2 & 2q_1q_2 + 2q_0q_3 & 2q_1q_3 - 2q_0q_2 \\ 2q_1q_2 - 2q_0q_3 & 2q_0^2 - 1 + 2q_2^2 & 2q_3q_2 + 2q_0q_1 \\ 2q_1q_3 + 2q_0q_2 & 2q_3q_2 - 2q_0q_1 & 2q_0^2 - 1 + 2q_3^2 \end{bmatrix} \tag{7}$$

where $q_o^b = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}^T$

The quaternion that is the transformation from frame o to frame b, q_o^b, is propagated according to the differential equation:

$$\dot{q}_o^b = \tfrac{1}{2}\left[\Omega_{ob}^b\right] q_o^b \tag{8}$$

where $\left[\Omega_{ob}^b\right] = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -\omega_z \\ \omega_x & 0 & \omega_z & -\omega_y \\ \omega_y & -\omega_z & 0 & \omega_x \\ \omega_z & \omega_y & -\omega_x & 0 \end{bmatrix}^T$

$$\omega_{ob}^b = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$$

The angular rate, ω_{ob}^b, is propagated according to an approximate power spectral density function that the three degree-of-freedom motion is contained in. In other words, the motion is expected to be within a certain frequency bandwidth and magnitude. This is modeled in state space format as a first order system with a time constant set according to the system's bandwidth and process noise set in relation to the limit of the magnitude of the three degree-of-freedom motion.

$$\dot{\omega}_{ob}^b = \begin{bmatrix} 1/\tau_\omega & & \\ & 1/\tau_\omega & \\ & & 1/\tau_\omega \end{bmatrix} \omega_{ob}^b + \mathbf{w}_\omega \tag{9}$$

The propagation of the rate gyro drift error, δ, is determined prior to operation by static testing. The error propagates as:

$$\dot{\delta} = \begin{bmatrix} 1/\tau_\delta & & \\ & 1/\tau_\delta & \\ & & 1/\tau_\delta \end{bmatrix} \delta + \mathbf{w}_\delta \tag{10}$$

B. Measurement Model

The measurements from the IMU are the rate gyro and accelerometer triads.

The measurement vector is:

$$z = \begin{bmatrix} f_{IMU} & \omega_{IMU} \end{bmatrix}^T \tag{11}$$

The angular rate is the direct measurement of the corresponding state in addition to the rate gyro drift error, δ, and stochastic error vector, ε.

$$\omega_{IMU} = \omega_{ob}^b + \delta + \varepsilon \tag{12}$$

The accelerometer measurements could be modeled as measuring only the gravitational force as well as a stochastic error vector ∇. Using the quaternion representation to rotate the constant gravitational force, the accelerometers measure:

$$\begin{aligned} f_{IMU} &\cong C_o^b \begin{bmatrix} 0 \\ 0 \\ -\|g\|_2 \end{bmatrix} + \nabla \\ &= \|g\|_2 \begin{bmatrix} -2q_1q_3 + 2q_0q_2 \\ -2q_0q_1 - 2q_2q_3 \\ -q_0^2 + q_1^2 + q_2^2 - q_3^2 \end{bmatrix} + \nabla \end{aligned} \tag{13}$$

where $\|g\|_2$ is the magnitude of gravity

The accelerometers certainly measure linear acceleration, so one aspect of the filter's performance is how well it rejects this disturbance.

IV. Experimentation

A proof-of-concept experiment was run to demonstrate the effectiveness of the filter. An IMU was constructed using three accelerometers (Silicon Designs 1210) and three rate gyros (ADXRS150, Analog Devices Inc.). The range of the accelerometers was ±5g and the measurement noise was rated at $32\,\mu g/\sqrt{Hz}$. The gyros had a range of 150*deg/s* and a noise rating of $0.05\,deg/s/\sqrt{Hz}$. Static testing showed that the rate gyros had practically zero drift over a one-hour test, which is good for the sensors but unfortunate for testing a Kalman filter that is supposed to estimate the rate gyro drift error. For filter evaluation purposes, the error, δ, was modeled as a slowly drifting random process and added onto the rate gyro signal.

For verification purposes, Optotrak®, an optical tracking system from Northern Digital Inc. (Waterloo, ON, Canada) supplied the "true" translational and rotational kinematics for the IMU. Three tracking LEDs were attached to the IMU so that the position and orientation could be determined (Fig. 1). From this information, the acceleration and angular rate could be calculated with a great degree of accuracy, so that they could be compared with the sensor signals from the IMU. The Optotrak cameras track the position of each LED with a positional resolution and accuracy rated at 0.002*mm*@2.5*m* and 0.05*mm*@2.5*m*, respectively, accurate enough to be considered trustworthy. The sensor axis misalignments and stochastic properties of the inertial sensors were calibrated according to the routine outlined in Kim and Golnaraghi [9].

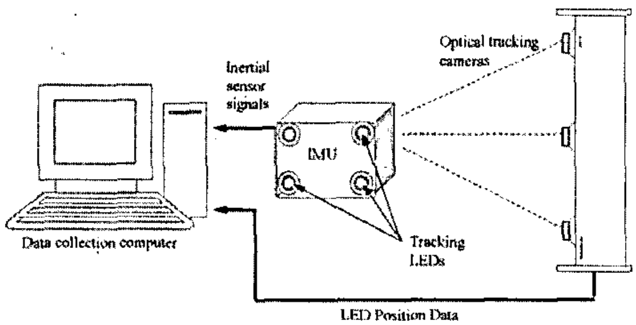

Fig. 1: Schematic of the experimental setup

In the experiment, the IMU was manually rotated in three degrees-of-freedom with some linear acceleration. The linear acceleration content for a typical test is shown in Fig. 2.

Fig. 3 shows the estimation of the four quaternion states compared to the actual orientation. The quaternion states were also decomposed into the more intuitive representation of roll, pitch, and yaw in Fig. 5. Intuitively, we would expect that the roll and the pitch errors would be corrected for due to the accelerometric measurements of the gravitational force. In Fig. 6, the roll and pitch errors are indeed stable for three separate experiments; however, the yaw angle is also stable, albeit with a slightly larger error variance. This is somewhat counter-intuitive, since the accelerometers do not measure heading angles.

The performance of the filter for estimating the gyro drift errors (Fig. 4) is also accurate, with the estimation closing in on the actual value within $10s$ and remaining stable after that. It is worthy to note that the drift in all three axes was tracked well, rather than just in the two axes that are ostensibly corrected by the accelerometers.

The yaw angle state, hidden in the quaternion representation, does have a weak observability from the accelerometer measurements. Heuristically, this sensor redundancy could be demonstrated by the propagation equation for the gravity vector as seen from the BODY frame:

$$\dot{g}^b = \begin{bmatrix} 0 & \omega_z & -\omega_y \\ -\omega_z & 0 & \omega_x \\ \omega_y & -\omega_x & 0 \end{bmatrix} g^b \tag{14}$$

where $\omega_{ob}^b = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$

Assuming that the accelerometers measure the gravitational force perfectly, then from equation (14), the orientation and the angular velocity could be determined in all three axes (for local tracking) with only the accelerometers.

The experimental results show that a Kalman filter could tolerate a modicum of linear acceleration (as shown in Fig. 2) and still correct for three orientation axes. For a more robust tracker, an additional heading sensor, such as a magnetometer, could be added to correct for the yaw angle state. Such a measurement could be modeled and added to equation (11), with the rest of the process and measurement equations the same as before.

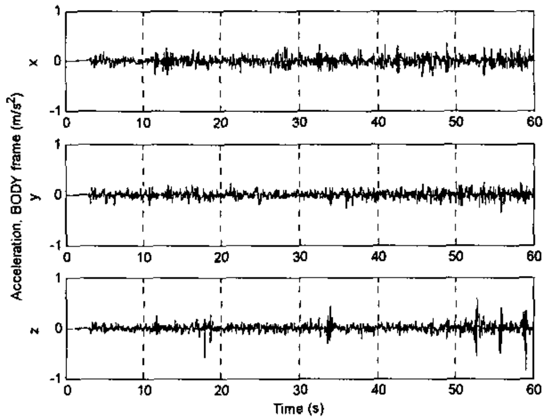

Fig. 2: Corruption of gravity measurements by translational acceleration

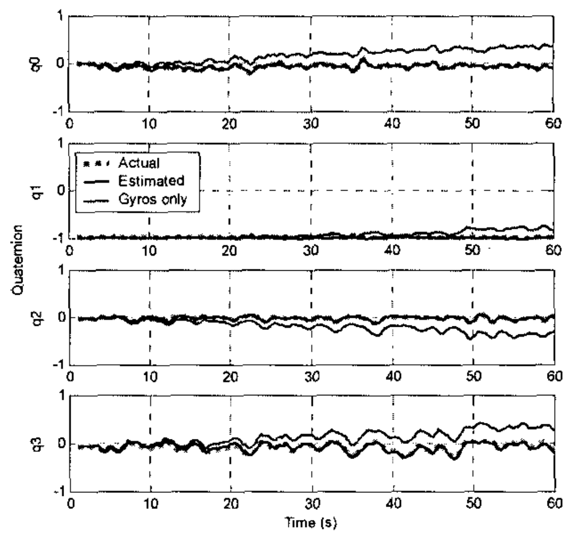

Fig. 3: Quaternion estimation

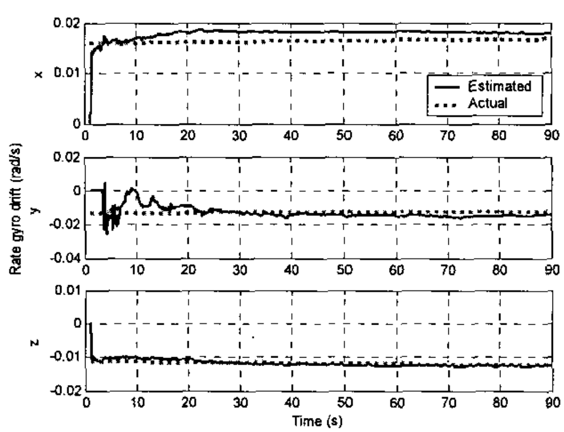

Fig. 4: Estimation of the gyro drift error

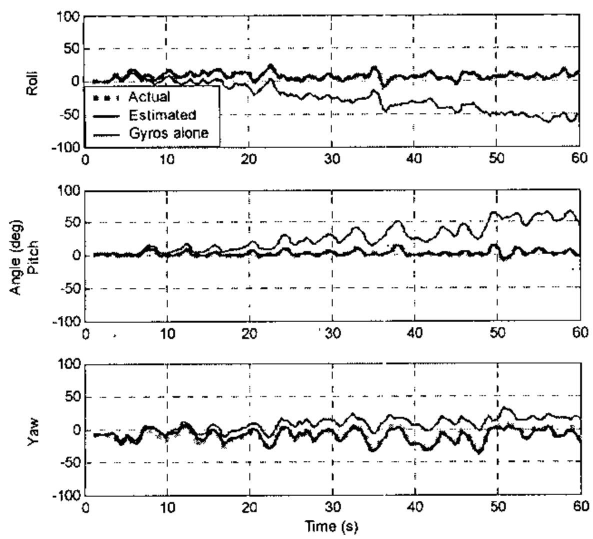

Fig. 5: Angular representation of the orientation estimation

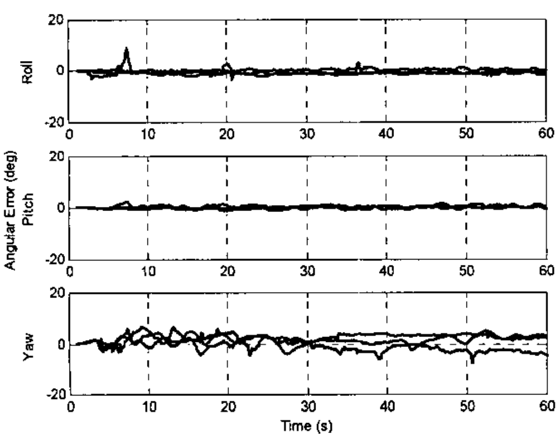

Fig. 6: Roll, pitch, and yaw estimation errors for three experiments

V. CONCLUSION

A Kalman filter is developed for orientation estimation using accelerometer and rate gyro measurements from a low-cost inertial measurement unit. A quaternion-based process model is used to avoid the problem of singularities in Euler angle representations. The accelerometers measure the gravitational force, providing redundancy in the orientation estimation. Experiments demonstrate that the filter tracks the orientation states quite accurately, even the yaw angle state. The Kalman filter also tracks the rate gyro drift error well in three axes. The results were verified with a highly accurate optical position location system configured to measure the translational and rotational kinematics of the IMU. A possible solution to increase the robustness of the tracker is add a heading sensor to the sensor suite for extra redundancy.

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