$$\operatorname{Cov}\left(\widehat{\beta_1^0}, \widehat{\beta_1^0} - \widehat{\beta_1}\right) = 0$$

$$\operatorname{cov}\left(\sum \alpha_i x_i, \sum \beta_j y_j\right) = \sum\sum \alpha_i \beta_j \cdots$$

$$\operatorname{cov}\left(\frac{\sum xy}{\sum x^2}, \frac{\sum xy}{\sum x^2} - \frac{\sum (x-\bar{x})y}{\sum (x-\bar{x})^2}\right) =$$

$$= \frac{1}{\left(\sum x^2\right)^2} \cdot \sum x^2 \operatorname{Var} y - \frac{1}{\sum x^2} \cdot \frac{1}{\sum (x-\bar{x})^2} \cdot \sum x(x-\bar{x}) \operatorname{Var} y =$$

$$= \sigma^2 \left[\frac{1}{\sum x^2} - \frac{1}{\sum x^2} \cdot \frac{1}{\sum (x-\bar{x})^2} \cdot \sum (x-\bar{x})^2\right] = 0 .$$

$$\operatorname{Var}\left(\hat{\beta}_1^0 - \hat{\beta}_1\right) = \operatorname{Var} \hat{\beta}_1^0 + \operatorname{Var} \hat{\beta}_1 - 2 \operatorname{cov}\left(\hat{\beta}_1^0, \hat{\beta}_1\right) =$$

$$= \frac{\sigma^2}{\sum x^2} + \frac{\sigma^2}{\sum (x-\bar{x})^2} - 2 \cdot \operatorname{cov}\left(\hat{\beta}_1 - \hat{\beta}_1^0, \hat{\beta}_1^0\right) - 2 \underbrace{\operatorname{cov}\left(\hat{\beta}_1^0, \hat{\beta}_1^0\right)} =$$

$$= \sigma^2 \left(\frac{1}{S_{xx}} - \frac{1}{S_{xx}^0}\right).$$