Run-Time of a TM

(standard, 1 tape ...)

= # of steps (on input w) to halt

transitions taken

$t_M : \Sigma^* \to \mathbb{N} \cup \{\infty\}$

not halting

runtime of $M(w)$

— consider machines that always halt starting now

Worst-case runtime

$$T_M(n) = \max_{|w|=n} t_M(w)$$

Complexity Classes

$$P = \bigcup_{k=1}^{\infty} \mathrm{TIME}(n^k)$$

— smallest invariant class capturing "decided by a fast alg." (by adding more languages)

polynomial time

p-time

alg. worst-case runtime $O(n^k)$, fixed k

polynomial-time algorithm

p-time alg.

$$\ldots \subseteq \Sigma^* \mid \exists M \quad T_M(n) = O(n^2),\ L(M) = B \}$$

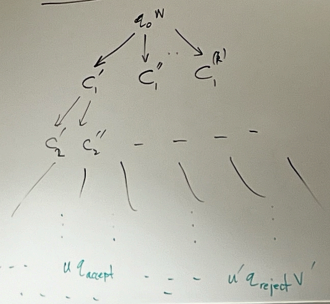
— not invariant

= # of steps (on input

transitions taken

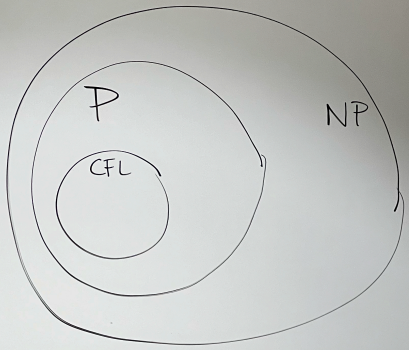
$t : \Sigma^* \to \mathbb{N} \cup \{\infty\}$

NTM



runtime

$t_M(w) =$ depth


P
NP
CFL

p-time

Classes

$TIME(n^2) = \{ B \subseteq \Sigma^* \mid \exists M \;\; T_M(n) = O(n^2) \;\; L(M) = B \}$

TIME

polynomial-time algorithm

p-time alg.

Verification

p-time

$B \subseteq \Sigma^*$

verifier for B :

$V =$ " on input $\langle w, c \rangle$

$\vdots$

$\cdots$ accept

$\cdots$ reject

//

$\overset{?}{\in} B$

certificate

e.g.

graph

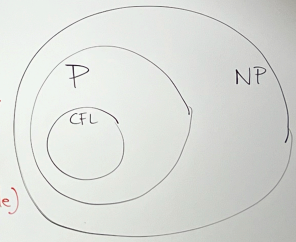
$w = \langle G \rangle$

$c = \langle v_1, v_2, \ldots v_n \rangle$

putative

HC

(Hamiltonian Cycle)



Worst-case runtime

$$T_M(n) = \max_{|w|=n} t_M(w)$$

$$P = \bigcup_{k=1}^{\infty} TIME(n^k)$$

polynomial time

p-time

smallest invariant class capturing "decided by a fast alg" (by adding more languages)

$\boxed{\text{alg. worst-case runtime } O(n^k), \text{ fixed } k}$

Verification

p-time

$B \subseteq \Sigma^*$

verifier for B :

$V =$ " on input $\langle w, c \rangle$

$\in B$?

certificate

⋮

accept

reject

if $w \in B$ (G has a HC)

then $\exists c$

so $V(\langle w, c \rangle)$ accepts

$\exists$ a HC $c = \langle v_1, \dots, v_n \rangle$

so V confirms c is a HC for real

" genuine certificate

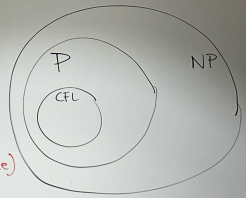
e.g.

$w = \langle G \rangle$ graph

$c = \langle v_1, v_2, \dots v_n \rangle$

permutative

HC (Hamiltonian Cycle)



w

pting config.