Equational Entropy: Quantifying the Informational Structure of Physical Laws
(Preliminary Draft – Work in Progress)

J. Dizon

April 22, 2025

Abstract

Note: This is a preliminary draft. The ideas presented are part of an ongoing conceptual and mathematical development process, and constructive feedback is welcome.

We propose a framework for quantifying the informational content of physical theories via a construct we call *Equational Entropy* ($\mathcal{E}$). This entropy-based metric evaluates the complexity of a physical equation through two components: conceptual novelty ($\mathcal{N}$), which captures the number of new physical principles and constants introduced, and structural complexity ($\mathcal{C}$), which reflects mathematical transformations, dimensionality, and the suppressive effects of symmetry. We demonstrate that $\mathcal{E}$ provides a consistent, interpretable way to compare physical laws and analyze theoretical unifications. The framework also suggests that symmetries act as compression mechanisms in the informational landscape of physics. Finally, we explore the potential of $\mathcal{E}$ as a predictive tool for evaluating new or AI-generated physical models. Future work will involve computational implementation and empirical testing against historical theory development.

1 Introduction

Physics is built on fundamental equations that describe the laws of nature. However, not all equations carry the same informational weight. Some equations introduce entirely new principles, while others are derived from existing ones. Additionally, mathematical complexity and symmetry influence how much information is encoded in an equation. We define **Equational Entropy** ($\mathcal{E}$) as a measure of the total information required to construct an equation.

2 Postulates of Equational Entropy

2.1 Postulate 1: Equational Entropy as a Measure of Information

For any physical equation, its **equational entropy** ($\mathcal{E}$) represents the total information required to construct it. This entropy consists of two contributions:

$$\mathcal{E} = \mathcal{N} + \mathcal{C} \tag{1}$$

where:

- $\mathcal{N}$: **Conceptual Novelty** – Measures new physical principles and constants introduced by the equation.
- $\mathcal{C}$: **Structural Complexity** – Measures mathematical complexity, transformations, and symmetry effects.

2.2 Postulate 2: Conceptual Novelty and the Number of Fundamental Principles

Conceptual entropy measures how much **new physics** an equation introduces. It is given by:

$$\mathcal{N} = \log(1 + P + C) \tag{2}$$

where:

- P: The number of **independent** physical principles introduced.
- C: The number of **new fundamental constants** required.

The logarithmic form prevents runaway entropy growth.

Example:

- Newton's Laws introduce basic principles of motion $\Rightarrow$ Low $\mathcal{N}$.
- General Relativity replaces Newtonian gravity with curved spacetime $\Rightarrow$ High $\mathcal{N}$.

2.3 Postulate 3: Structural Complexity and Transformations

Structural entropy accounts for the number of transformations, dimensionality, and symmetry in an equation:

$$\mathcal{C} = \log(1 + (T + D)e^{-S}) \tag{3}$$

where:

- T: Number of transformations required to derive the equation.
- D: Dimensionality of the equation (scalar, vector, tensor).
- S: Number of symmetries, which reduce entropy exponentially.

Example:

- Algebraic equations (low T, low D) $\Rightarrow$ Low $\mathcal{C}$.
- Einstein Field Equations (high T, high D, but high S) $\Rightarrow$ Moderate $\mathcal{C}$ due to symmetry suppression.

2.4 Postulate 4: Symmetry Reduces Equational Entropy

Symmetry constraints reduce the number of independent degrees of freedom, lowering entropy. Mathematically:

$$\frac{\partial \mathcal{C}}{\partial S} < 0 \tag{4}$$

or explicitly:

$$\mathcal{C} = \log(1 + (T + D)e^{-S}) \tag{5}$$

Example:

- Maxwell's Equations have gauge symmetry $\Rightarrow$ Lower entropy than expected.
- Einstein Field Equations are fully covariant $\Rightarrow$ Less entropy than classical field equations.

2.5 Postulate 5: Unification Does Not Follow Simple Superposition

When two theories T_1 and T_2 unify into a single framework T_U, entropy does not simply add:

$$\mathcal{E}U \neq \mathcal{E}T_1 + \mathcal{E}T_2 \tag{6}$$

Instead, unification introduces structural reorganization:

$$\mathcal{E}U = \alpha \sum \mathcal{E}_i - \beta \tag{7}$$

where:

- α: Accounts for emergent complexity or simplification.
- β: Accounts for redundant information removed.

Implications:

- Some unifications **increase** entropy (Quantum Field Theory).
- Others **reduce** entropy (Electroweak Unification).

3 Conclusion and Future Work

Equational Entropy provides a structured approach to quantify the information content of physical theories. It combines:

- **Conceptual Novelty** ($\mathcal{N}$) – How much new physics is introduced.
- **Structural Complexity** ($\mathcal{C}$) – The mathematical complexity of an equation.

Symmetry plays a crucial role in entropy suppression, and unification does not follow simple additive rules.

Future directions include:

- Empirical testing on historical and contemporary theories.
- Development of computational tools for EE calculation.
- Integrating EE with algorithmic information theory.
- Using EE as a heuristic in AI-driven theory generation.