## Make or Spy?

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#### Abstract

## 1 Setup

Upstream good P ("platform") and downstream good A ("application") are supplied by two monopoly suppliers. Product P can be consumed independently while product A can only be consumed in conjunction with P. Fixed costs of entry for A are  $C_A$  while those for P are normalized to zero and all marginal costs are zero. Consumers are indexed by i ( $i \in \mathbb{R}^+$ ) and have valuations  $V_P^i$  and  $V_A^i$  over the two items. These are defined as  $V_P^i = V_P - i - p$  and  $V_A^i = V_A - ki - (p + \rho)$  where p is the price of P and  $\rho$  the price for A. Each consumer consumes at most one unit of each good.

In this setting I study the following problem:

What are equilibrium revenues  $R_P$  and  $R_A$  when the two producers play a sequential pricing game where p is announced in stage 1 and conditional on entry  $\rho$  in stage 2?

### 2 Solution

The setup is completely described by three key parameters:  $\{V_P\}$  characterizing P and  $\{V_A, k\}$  characterizing A. With respect to  $V_P$ , I define four types of A by splitting the parameter space as follows:

	$V_P > V_A/k$	$V_P < V_A/k$
$V_A > V_P$	High Type	Good Type
$V_A < V_P$	Bad Type	Low Type

A solution for each of these four types is given below.

### 1. Bad Types

In equilibrium,  $R_A = 0$  and  $R_P = V_P^2/4$  with  $p = V_P/2$  and  $\rho = 0$  with no entry from A.

### 2. Good Types

In equilibrium,  $R_A = 0$  and  $R_P = \frac{V_P^2}{4}$  with  $p = \frac{V_P}{2}$  and  $\rho = 0$  with no entry from A.

In what follows I show that under certain conditions only sufficiently "specialized" ideas (sufficiently high or sufficiently low type) do not face hold-up. The nature of the problem is not symmetric between the high and low types and is more acute for low types. Because General ideas face hold-up at all times, a well-known result in the literature on bundling, I will focus on high/low types. (insert a section on how general applications relates to bundling, double marginalization etc)

Further, in the analysis both the platform and the developer have zero marginal costs. The fixed cost for the Platform is assumed to be zero for simplicity.

### 2.1 Timing

The two players, the platform and the developer play a pricing game that is described as follows:

- **Period 1**: Platform specifies access price p. In extensions of the model the platform will be able to specify a number of contracting mechanisms including charging a royalty based on application sales but in the baseline model the platform makes no direct revenue from developers (à la Microsoft Windows).
- **Period 2**: Developer makes entry decision by incurring fixed cost of entry  $C_E$  and sets price  $\rho$  for the application
- **Period 3**: Platform decides whether it wants to imitate the application's idea at cost  $C_I$ . If it imitates the application then (a) the only available product on the market is the platform with the application baked in<sup>1</sup>. The developer is driven out of business (has zero revenue).<sup>2</sup>
- **Period 4**: Consumers make purchase decisions and payoffs are realized.

### 3 Solution

Define  $i^*$  such that  $V_{i^*}^P = V_{i^*}^A$  i.e. the consumer who has no marginal (positive or negative) valuation for the application. Consumer  $i^*$  exists only for high/low type of applications and is unique. It is easy to see that  $i^* = \frac{(V_P - V_A)}{(1-k)}$ . Let  $V_P^{i^*} = V_A^{i^*} = V^{i^*}$ 

### 3.1 Period 2: Specifying the Developer's problem

In the section I derive the developer's demand  $D(p, \rho)$ .

Consumer i demands the application if she satisfies the following two constraints:

$$V_A^i > V_P^i \tag{1}$$

<sup>&</sup>lt;sup>1</sup>why no mixed bundling?

<sup>&</sup>lt;sup>2</sup>justification?

$$V_A^i > 0 (2)$$

I will call equation (??) the incentive constraint(IC) and equation (??) the participation constraint(PC).

Rewriting, IC becomes:

$$i(1-k) > p - (V_A - V_P)$$

and PC becomes

$$i < \frac{V_A - (p + \rho)}{k}$$

These two constraints bind in different ways for high and low types to generate different demand for each type as follows:

**Proposition 3.1** In Period 2 for high types, the best response to a price p set by the platform in Period 1 is given by:

$$\rho^* = \begin{cases} \frac{V_A - V_P}{2} & \text{if } p < \frac{V_P(2k-1) - V_A}{2(k-1)} \\ \frac{V_A - p}{2} & \text{if } p > \frac{2kV_P - V_A}{2k-1} \\ (k-1)p - (kV_P - V_A) & \text{otherwise} \end{cases}$$

#### **Proof** Proof goes here

The intuition for the result in ?? is as follows:

When the platform's price is sufficiently low the developer is able to target his entire captive audience (all i such that  $V_A^i > V_P^i$ ) and price like a monopolist over the consumer base. Note that the optimal price does not depend upon the exact value of p. Therefore effectively in this case every consumer who values the application sufficiently higher than the platform buys the application in equilibrium, while everyone else with  $V_P^i > 0$  buys just the platform.

When the platform's price is *sufficiently high* only the Participation Constraint binds. This means that buying just the platform is not valuable for

any consumer, and everyone who buys the platform must also purchase the app. Formally,  $V_P^i < 0 \,\forall i$ . In this case the platform is forgoing additional userbase (by pricing them out) in order to get a larger share of the application's revenue by increasing the platform's price.

**Proposition 3.2** In Period 1, against high types the platform plays the following strategy: Set equilibrium price in this period:

$$p^* = \begin{cases} \frac{V_A}{2} & \text{if } V_P^2 < V_A^2/2k \\ \frac{V_P}{2} & \text{if } V_P^2 > V_A^2/2k \end{cases}$$

#### **Proof** Proof goes here

The intuition for the result in ?? is straightforward. When a potential application is substantially more valuable (i.e. when  $V_A^2/2k > V_P^2$ ), the platform decides to forgo monopoly revenue from the platform itself to capturing additional revenue from those who have a high valuation from the application. In this case, everyone who buys the platform also buys the application.

Now we turn to analyzing the case where the potential application is of the "low" type.

**Proposition 3.3** When the potential application is of the low type, i.e. when  $V_A < V_P < V_A/k$  in equilibrium,

- 1. it is definitely the case that some consumers buy the platform without buying applications.
- 2. the application cannot make positive revenues when  $\frac{2V_A}{k+1} < V_P$
- 3. the platform always prices at  $p = \frac{V_P}{2}$

#### **Proof** Proof goes here

# 4 Data

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# 5 Conclusion

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