PART-B QB For CIE-2

① The joint PMF of 2 rvs X and Y is given by

$$P(x,y) = \begin{cases} kxy & x=1,2,3,\ y=1,2,3 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the value of the constant k.

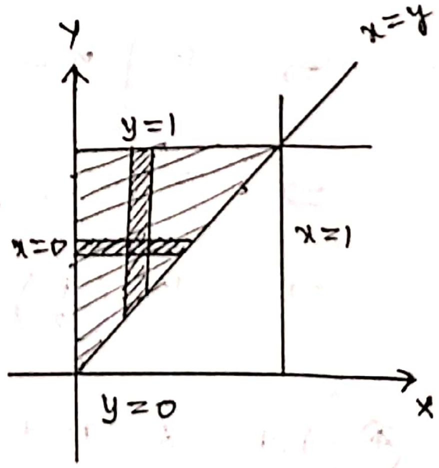
Ans:

X \ Y	1	2	3	$P_X(x)$
1	k	$2k$	$3k$	$6k$
2	$2k$	$4k$	$6k$	$12k$
3	$3k$	$6k$	$9k$	$18k$
$P_Y(y)$	$6k$	$12k$	$18k$	$36k$

$\sum\sum P(x_i, y_j) = 1 \Rightarrow 36k = 1 \quad \therefore \boxed{k = \frac{1}{36}}$

② The joint PDF of a rv (X,Y) is $f(x,y) = 4xy$, $0<x<y<1$. Find the marginal density function of X and Y.

Ans:-



$\therefore$ x varies from 0 to y & y varies from x to 1

Marginal density fn of X:

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_{x}^{1} 4xy\,dy$$

$$= 4x\left(\frac{y^2}{2}\right)_x^1 = 4x\left(\frac{1}{2} - \frac{x^2}{2}\right)$$

$$= 2(x - x^3), \quad 0<x<y$$

Marginal density fn of Y:

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{0}^{y} 4xy\,dx$$

$$= 4y\left(\frac{x^2}{2}\right)_0^y = 4y\left(\frac{y^2}{2}\right) = 2y^3$$

(3) Find the value of k if $f_{XY}(x, y) = \begin{cases} k(1-x)(1-y) & 0<x,y<1 \\ 0 & \text{elsewhere} \end{cases}$

is to be the joint density function.

→ Ans:-

We know that $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y)\, dx\, dy = 1$

$$\int_0^1\int_0^1 k(1-x)(1-y)\, dx\, dy = 1$$

$$\Rightarrow k\int_0^1\int_0^1 (1-y-x+xy)\, dx\, dy = 1$$

$$\Rightarrow k\int_0^1 \left[x - xy - \frac{x^2}{2} + \frac{x^2}{2}y\right]_0^1 dy = 1$$

$$\Rightarrow k\int_0^1 \left(1 - y - \frac{1}{2} + \frac{y}{2}\right) dy = 1$$

$$\Rightarrow k\int_0^1 \left(\frac{1}{2} - \frac{y}{2}\right) dy = 1$$

$$\Rightarrow \frac{k}{2}\left[y - \frac{y^2}{2}\right)_0^1 = 1$$

$$\Rightarrow \frac{k}{2}\left(1 - \frac{1}{2}\right) = 1$$

$$\Rightarrow \frac{k}{2}\left(\frac{1}{2}\right) = 1 \quad \Rightarrow \boxed{k=4}$$

(4) Let X and Y be 2 independent RV's with var(X) = 9 and var(Y) = 3. Find var(4X − 2Y + 6).

Ans:-

$$\text{Var}(4X-2Y+6) = 4^2 \cdot \text{var}(X) - 2^2\, \text{var}(Y)$$
$$= 16 \times 9 - 4 \times 3 = 132$$

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5) The joint PMF of 2 r.v's X and Y is given by

$$P(x,y) = \begin{cases} k(2x+y) & x=1,2 \; ; \; y=1,2 \\ 0 & \text{otherwise.} \end{cases}$$

Determine the value of the constant k.

Ans:-

$X \backslash Y$	1	2	$P_X(x)$
1	$3k$	$4k$	$7k$
2	$5k$	$6k$	$11k$
$P_Y(y)$	$8k$	$10k$	$18k$

We know that

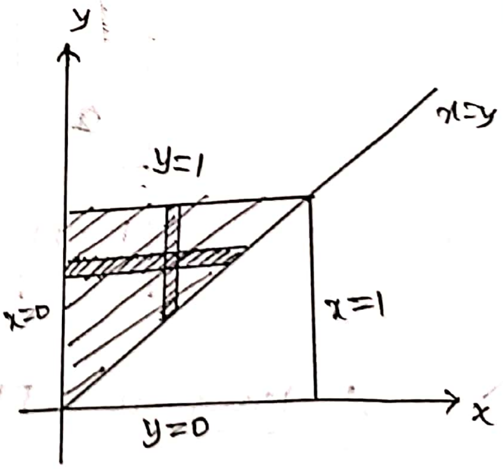
$$\sum \sum P(x_i, y_j) = 1$$

$$18k = 1$$

$$\boxed{k = \frac{1}{18}}$$

6) The joint PDF of a r.v (X,Y) is $f(x,y)=2$, $0 \le x \le y \le 1$. Find Marginal density function of X and Y.

Ans:-



Marginal density of X:

$$f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy = \int_x^1 2\,dy$$

$$= 2(y)_x^1 = 2(1-x), \quad 0<x<y$$

Marginal density of Y:

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_0^y 2\,dx$$

$$= 2(x)_0^y = 2y, \quad 0<y<1.$$

x varies from 0 to y

y varies from ~~x~~ to 1

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(7) Let X and Y be 2 RV's and a and b are constants. Prove that $\text{Cov}(aX, bY) = ab\,\text{Cov}(X,Y)$.

$\longrightarrow$

Ans.

$$\text{cov}(ax, by) = E(axby) - E(ax)E(by)$$

$$= E(ab\,xy) - aE(x)\,bE(y)$$

$$= ab\,E(xy) - ab\,E(x)E(y) = ab\,\text{cov}(x,y).$$

(8) The joint PMF of (X,Y) is $P(x,y) = k(2x+3y)$ for $x=0,1,2$; $y=1,2,3$. Find all Marginal distributions.

Ans:

X \ Y	1	2	3	$P_X(x)$
0	3k	6k	9k	18k
1	5k	8k	11k	24k
2	7k	10k	13k	30k
$P_Y(y)$	15k	24k	33k	72k

We know that

$$\sum\sum P(x_i, y_j) = 1$$

$$\Rightarrow 72k = 1$$

$$\Rightarrow \boxed{k = \frac{1}{72}}$$

$\therefore$ Marginal PMF of X:

X	0	1	2
$P_X(x)$	$\frac{18}{72}$	$\frac{24}{72}$	$\frac{30}{72}$

Marginal PMF of Y:

Y	1	2	3
$P_Y(y)$	$\frac{15}{72}$	$\frac{24}{72}$	$\frac{33}{72}$

(9) The joint PMF of (X,Y) is given by $P(x,y) = \frac{x+2y}{27}$, for $x = 0,1,2$ and $y = 0,1,2$. Find all Marginal distributions.

Ans:

X \ Y	0	1	2	$P_X(x)$
0	$\frac{0}{27}$	$\frac{2}{27}$	$\frac{4}{27}$	$\frac{6}{27}$
1	$\frac{1}{27}$	$\frac{3}{27}$	$\frac{5}{27}$	$\frac{9}{27}$
2	$\frac{2}{27}$	$\frac{4}{27}$	$\frac{6}{27}$	$\frac{12}{27}$
$P_Y(y)$	$\frac{3}{27}$	$\frac{9}{27}$	$\frac{15}{27}$	1

Marginal PMF of X

X	0	1	2
$P_X(x)$	$\frac{6}{27}$	$\frac{9}{27}$	$\frac{12}{27}$

Marginal PMF of Y:

Y	0	1	2
$P_Y(y)$	$\frac{3}{27}$	$\frac{9}{27}$	$\frac{15}{27}$

(10) The joint PDF of (X,Y) is given by

$$f(x,y) = \begin{cases} e^{-(x+y)} & x \geq 0,\ y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

. Are X and Y independent?

$\rightarrow$

Marginal PDF of Y: $f(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$

$$= \int_0^{\infty} e^{-(x+y)}\,dy = \int_0^{\infty} e^{-x}\cdot e^{-y}\,dy = e^{-x}\left(\frac{e^{-y}}{-1}\right)_0^{\infty}$$

$$= e^{-x}\left(0-(-1)\right) = e^{-x},\quad x \geq 0.$$

Marginal PDF of Y:

$$f(y) = \int_{-\infty}^{\infty} f(x,y)\,dx = \int_{0}^{\infty} e^{-(x+y)}\,dx = \int_{0}^{\infty} e^{-x} \cdot e^{-y}\,dx$$

$$= e^{-y}\left[\frac{e^{-x}}{-1}\right]_0^{\infty} = e^{-y}\left(0-(-1)\right) = e^{-y}$$

$$f(y) = e^{-y}, \quad y \geq 0$$

$$f(x) \cdot f(y) = e^{-x} \cdot e^{-y} = e^{-(x+y)} = f(x,y)$$

$\therefore$ X and Y are independent.

11) Check whether the random process $X(t) = \cos(t+\theta)$, where θ is a random variable with density function $f(\theta) = \frac{1}{\pi}, \; -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$ is a WSS process?

Ans:-

Given $X(t) = \cos(t+\theta)$, $f(\theta) = \frac{1}{\pi}$, $[-\pi/2, \frac{\pi}{2}]$

$$E[X(t)] = \int_{-\infty}^{\infty} X(t) f(\phi)\,d\phi = \int_{-\frac{\pi}{2}}^{\pi/2} \cos(t+\phi)\frac{1}{\pi}\,d\phi$$

$$= \frac{1}{\pi}\left[\sin(t+\phi)\right]_{-\pi/2}^{\pi/2} = \frac{1}{\pi}\left[\sin\left(t+\frac{\pi}{2}\right) - \sin\left(t-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{\pi}\left[\sin\left(\frac{\pi}{2}+t\right) + \sin\left(\frac{\pi}{2}-t\right)\right] = \frac{1}{\pi}\left[\cos t + \cos t\right]$$

$$= \frac{2}{\pi}\cos t \neq \text{constant.}$$

$\therefore$ X(t) is not a WSS process.

12) State the postulates of Poisson process?

The postulates of Poisson process $\{X(t)\}$ are follows:

i) $P[1 \text{ occurrence in } (t, t+\Delta t)] = \lambda \Delta t + O(\Delta t)$

ii) $P[0 \text{ occurrence in } (t, t+\Delta t)] = 1 - \lambda \Delta t + O(\Delta t)$

iii) $P[2 \text{ or more occurrence in } (t, t+\Delta t)] = O(\Delta t)$

iv) $X(t)$ is independent of the no. of occurrence of the event in ~~the~~ any interval before and after the interval $(0, t)$.

v) The probability that the event occurs a specified no. of times in (t_0, t_0+t) depends only on t, but not on t_0.

13) Define Markov Process.

A random process $X(t)$ is said to be Markov process if

$$P[X(t) \le x / X(t_1) = x_1, X(t_2) = x_2, \cdots X(t_n) = x_n] = P[X(t) \le x / X(t_n) = x_n]$$

14) Define Wide Sense and Strict sense stationary process

- If a random process is stationary to all order then it is said to be Strict sense stationary process.

* A random process is said to be Wide-sense stationary process if it satisfies:

(i) $E(X(t)) = \text{constant}$ (ii) $R(t_1, t_2) = E[X(t_1) X(t_2)] = R(t_1 - t_2)$.

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15) Define Discrete random sequence and give an example.

A random process $X\{s,t\}$ is called Discrete random sequence if both x and t are discrete.

<u>Example</u>: The outcome of n^{th} toss of a Fair die.

16) If tPM of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$. Find Limiting distribution of Markov chain.

→

Given $P = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ Let $\pi = (\pi_1, \pi_2)$ be Limiting distribution of the chain.

$\pi P = \pi \Rightarrow \left(\frac{\pi_2}{2} \quad \pi_1 + \frac{1}{2}\pi_2 \right) = (\pi_1 \quad \pi_2)$

$\Rightarrow \frac{\pi_2}{2} = \pi_1 \longrightarrow ①$ $\quad \pi_1 + \frac{1}{2}\pi_2 = \pi_2 \longrightarrow ②$

As $\pi_1 + \pi_2 = 1$

$① \Rightarrow \frac{\pi_2}{2} + \pi_2 = 1$

$3\pi_2 = 2$

$\boxed{\pi_2 = 2/3} \Rightarrow \boxed{\pi_1 = 1/3}$

$\therefore \pi = \left(\frac{1}{3} \quad \frac{2}{3} \right)$

17) How is random process classified?

1) continuous random process.
2) continuous random sequence.
3) Discrete random process.
4) Discrete random sequence.

18) Is Poisson process stationary? Justify.

As the statistical properties mean, auto-correlation are time dependent Poisson process is not stationary.

19) State Chapman-Kolmogorov theorem.

If P is the tpm of a homogeneous Markov chain, then the n^{th} step tpm $P^{(n)}$ is equal to P^n i.e. $\left[P_{ij}^{(n)}\right] = \left[P_{ij}\right]^n$.

20) What is a Markov chain? When can you say that a Markov chain is homogeneous.

If for all n

$$P[X_n = a_n / X_{n-1} = a_{n-1}; X_{n-2} = a_{n-2}; \dots X_0 = a_0\} = P\{X_n = a_n / X_{n-1} = a_{n-1}\}$$

is called a Markov chain.

If $P_{ij}(n-1, n) = P_{ij}(m-1, m)$ the Markov chain is called a Homogeneous Markov chain.