MATH1023/MATH1062 Calculas

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1 Week8

Function of one variable

- **Definition**: Recall that a function of one real variable

$$f : D \to \mathbb{R}, D \subseteq \mathbb{R}$$

  is a rule that assigns to each number $x \in D$ a number $f(x) \in \mathbb{R}$

- The **domain** of f is the set D of allowed inpus.
- The **natural domain** of f is the largest subset of R of allowed inputs.

Function of 2 variables

- **Definition:** A function of 2 real variables:

$$f : D \to \mathbb{R}, D \subseteq \mathbb{R}^2$$

  is a rule that assigns to each pair $(x, y) \in D$ a number $f(x, y) \in \mathbb{R}$

- The **domain** of f is the set D of allowed inpus
- The **natrual domian of** f is the largest subset of $\mathbb{R}^2$ of allowed inputs

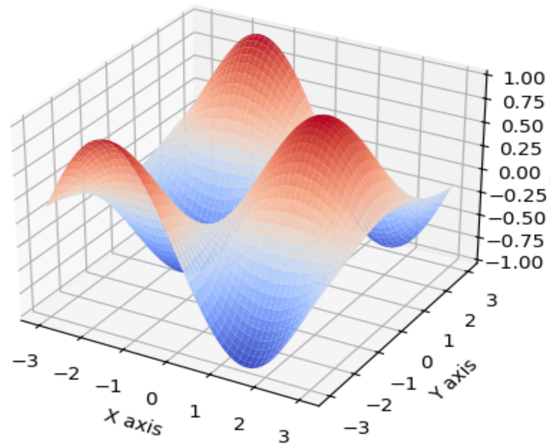
Graphs of functions

- The graph of a function of 2 variables:

$$f : D \to \mathbb{R}$$

  is the set of points

$$\{(x, y, f(x, y)) \in \mathbb{R}^3 | (x, y) \in D\}$$



- We can not get a full sphere as a function. It fails the vertical line test.

Level Curves

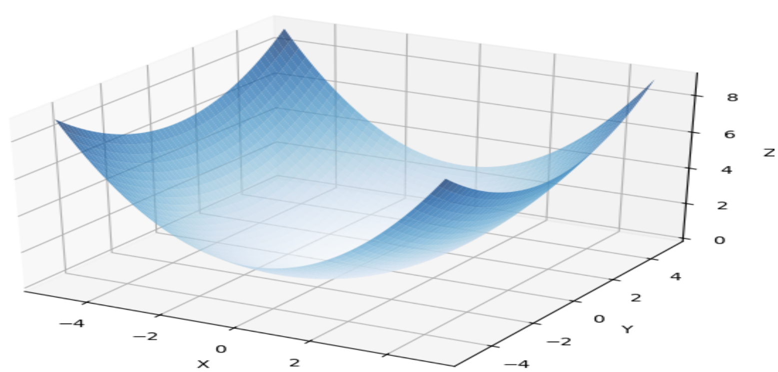
- **Definition:** A level cureve of a function $f(x, y)$ is a curve in $\mathbb{R}^2$ defined by

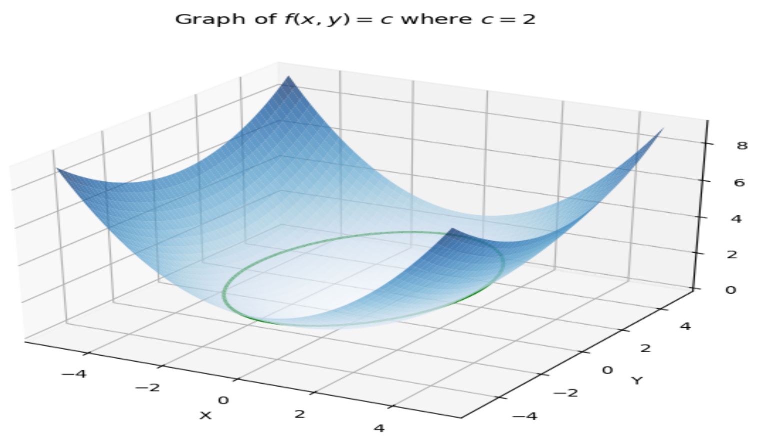
$$f(x, y) = c$$

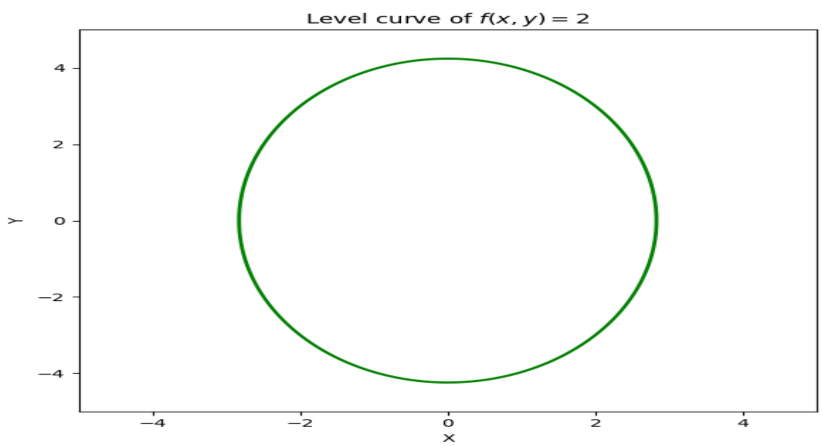
for a constant $c \in \mathbb{R}$

- The level curves $f(x, y) = c$ are the intersections of the surface $z = f(x, y)$ with the planes $z = c$

Graph of $f(x, y) = \frac{x^2}{4} + \frac{y^2}{9}$







Partial derivatives

- **Definition:** for a sufficiently smooth function of 2 variables

$$f : D \to \mathbb{R}, D \subseteq \mathbb{R}^2$$

The partial derivative of f with respect to x at $(x, y) = (a, b)$ is:

$$f_x(a,b) = \frac{\partial f}{\partial x}\bigg|_{(x,y)=(a,b)} = \lim_{h\to 0} \frac{f(a+h,b)-f(a,b)}{h}$$

and the partial derivative of f with respect to y at $(x, y) = (a, b)$ is

$$f_x(a,b) = \frac{\partial f}{\partial y}\bigg|_{(x,y)=(a,b)} = \lim_{h\to 0} \frac{f(a,b+h)-f(a,b)}{h}$$

- **Terminology**: If $f_x(a,b) = \left.\frac{\partial f}{\partial x}\right|_{(a,b)}$ exists for all $(a,b) \in D$, then we say that f is diffrentiable with respect to x on D and we write

$$f_x(x,y) = \frac{\partial f}{\partial x}(x,y)$$

for the derivative function of f w.r.t. x.

- Similarly, If $f_y(a,b) = \left.\frac{\partial f}{\partial y}\right|_{(a,b)}$ exists for all $(a,b) \in D$, then we say that f is diffrentiable with respect to y on D and we write

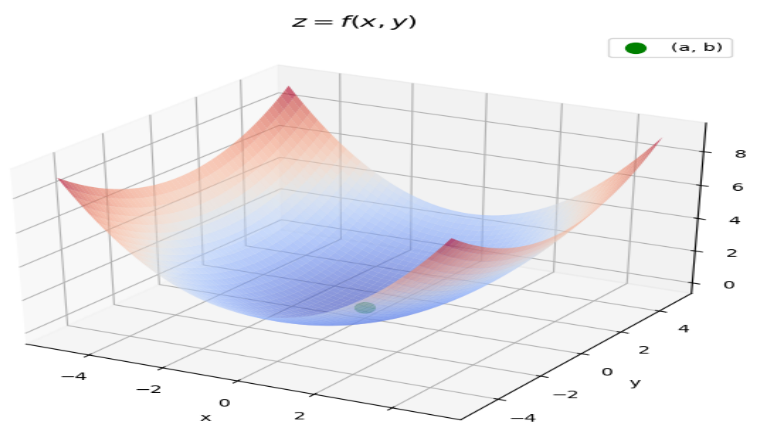
$$f_y(x,y) = \frac{\partial f}{\partial y}(x,y)$$

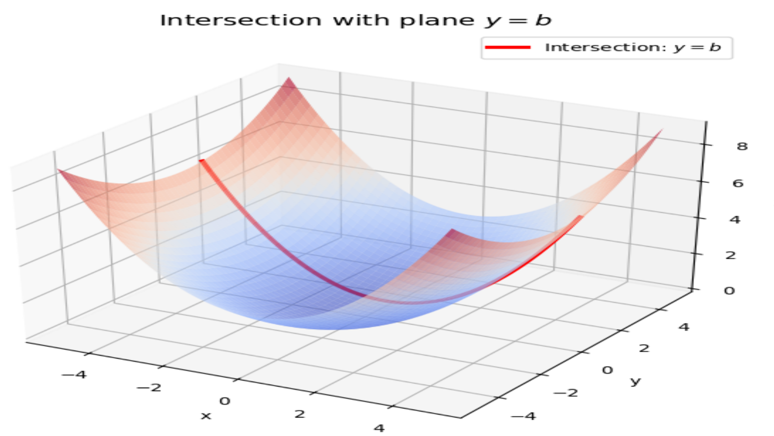
for the derivative function of f w.r.t. y.

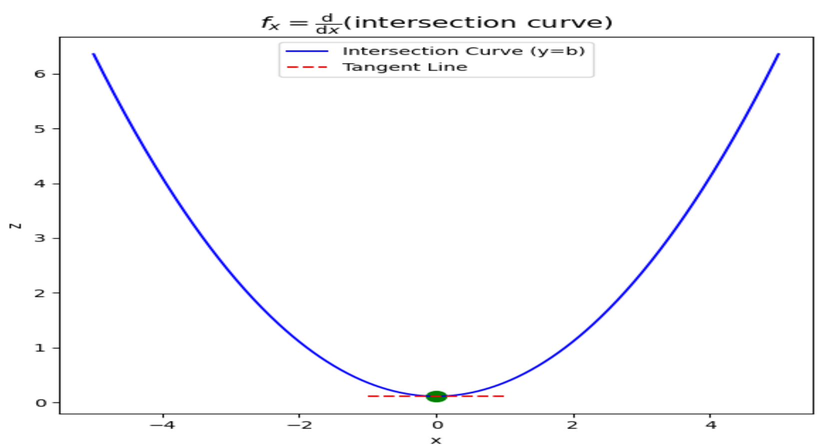
- What do partial derivatives measure? For a sufficiently smooth function:

$$f : D \to \mathbb{R}, D \subset \mathbb{R}$$

the partial derivatives $f_x = \frac{\partial f}{\partial x}$ measure the rate of change of f on the x direction.







- Here we have the intersection of the surface $z = f(x, y)$ and the plane $y = b$ is the function of one variable given by

$$g(x) = f(x, b)$$

  The gradient of the tangent to this curve at $x = a$ is given by

$$\begin{aligned} g'(a) &= \lim_{h \to 0} \tfrac{g(a+h)-g(a)}{h} \\ &= \lim_{h \to 0} \tfrac{f(a+h,b)-f(a,b)}{h} \\ &= f_x(a, b) \end{aligned}$$

- How do we calculate partial derivatives?
  - To calculate $f_x = \frac{\partial f}{\partial x}$
    1. Imagine y is a constant
    2. Differentiate as a function of one variable x.

- To calculate $f_y = \frac{\partial f}{\partial y}$
  1. Imagine x is a constant
  2. Differentiate as a function of one variable y.