MATH1023/MATH1062 Calculas

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1 Week1

1.1 Differential Equation

1. **Differential Equation(DE)**: A differential equation (DE) is a mathmatical equation that relates some function with its derivatives

2. **Order**: The order of a differential equation equals to a highest derivative occuring in it.
   - $\frac{dy}{dx} = -ky$ has order 1
   - $\frac{dy}{dx} = y^{18} + \frac{d^5y}{dx^2}y + x^2$ has order 5

3. **Standard Form**: The standard form of a first-order differential equation is

$$\frac{dy}{dx} = f(x, y)$$

4. **General Solution**: A general solution is a solution incoprating all constants of integration.

5. **Initial Condition**: An initial condition is a pair (x_0, y_0) such that $y(x_0) = y_0$

2 Week2

2.1 Direction Field

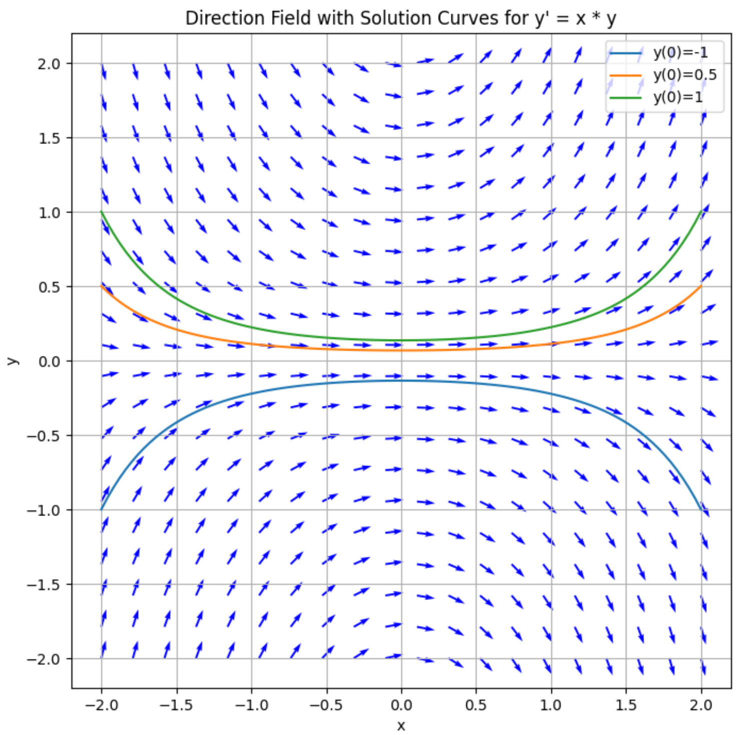
1. **Definition**: A direction field of a DE

$$y' = f(x, y)$$

   consists of a grid of short line segments with slope $f(a, b)$ drawn at points (a, b).So the line segment at (a, b) is tangent to any solution passing through (a, b)

2. **Example**:Draw some solution curves on the given direction field for the DE:

$$y' = xy$$



2.2 Separable equations

1. **Definition**: A first-order DE $y' = f(x, y)$ is called separable if there are functions $g(x)$ and $h(y)$ such that $f(x, y) = g(x)h(y)$, so a separable DE can be written

$$y' = g(x)h(y)$$

2. **Goal**: We want to find a method for solving separable DEs

3. **Method**: We can solve a separable DE:

$$\frac{dy}{dx} = g(x)h(y)$$

by separating variables.

Dividing both sides by $h(y)$ gives

$$\frac{1}{h(y)}\frac{dy}{dx} = g(x)$$

Intergrating both sides gives:

$$\int \frac{1}{h(y)} = \int g(x)dx$$

If we can find antiderivatives $H(y)$ for $\frac{1}{h(y)}$ and $G(x)$ for $g(x)$, then we have

$$H(y) = G(x) + C$$

3 Week3

3.1 Modelling Population Growth

1. **Constant Growth**: This occurs when the population x increases at a constant rate. The DE is

$$\frac{dx}{dt} = k$$

   where k is constant

2. **Exponential Growth**: The exponential growth model assumes the growth rate is proportional to the size of the population.

   The general form of a DE modelling exponential growth is

$$\frac{dx}{dt} = kx$$

   where k is constant

3. **Logistic Growth**: Exponential growth is not a realistic growth model for all values of t. A small animal population with unlimited resources of food and space may show exponential growth initially

   As the population gets larger there will be food shortages, overcrowding, and other factors that slow down the growth rate.

   The growth rate k should decrease as the population x increases.

   Since k is no longer constant, we write $k = g(x)$, so the DE becomes

$$\frac{dx}{dt} = g(x)x$$

A small population can growth exponentially, se we want $g(x) \approx k$ when $x \approx 0$. But as x increases $g(x)$ should decrease.

The simplest formula with this behaviour is

$$g(x) = k - ax$$

So the DE becomes

$$\frac{dx}{dt} = (k - ax)x$$

We introduce a new constant $b = \frac{k}{a}$ so

$$(k - ax)x = ax(\frac{k}{a} - x) = ax(b - x)$$

Let $\frac{b}{a} = b$, the logistic DE is then given by

$$\frac{dx}{dt} = ax(b - x)$$