

## BÀI TẬP LỚP 5

TƯƠN 1:

$$\begin{array}{r} \left( \begin{array}{cccc|c} 1 & 1 & 2 & -1 & -1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 5 & -2 & -1 & 1 \end{array} \right) \xrightarrow{\begin{matrix} d_2 - 2d_1 \\ d_3 - 5d_1 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 2 & -1 & -1 \\ 0 & -2 & 3 & 3 \\ 0 & -1 & 1 & 2 \end{array} \right) \end{array}$$

$$\begin{array}{r} \xrightarrow{\begin{matrix} d_1 + d_2 \\ d_2 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 0 & 1 & 1 & 1 \\ 0 & -2 & 3 & 3 & 1 \\ 0 & -1 & 1 & 2 & 1 \end{array} \right) \xrightarrow{\begin{matrix} d_1 + d_2 \\ d_3 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 0 & 0 & 1 & 1 \\ 0 & -2 & 3 & 3 & 1 \\ 0 & 0 & 1 & 2 & 1 \end{array} \right) \end{array}$$

$$\begin{array}{r} \xrightarrow{\begin{matrix} d_2 + d_3 \\ d_2 + d_3 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & -2 & -1 & 1 \\ 0 & -1 & 1 & 2 & 1 \end{array} \right) \xrightarrow{\begin{matrix} d_1 + 2d_2 \\ -d_2 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 2 & 1 \end{array} \right) \end{array}$$

$$\xrightarrow{\begin{matrix} d_3 - d_2 \\ d_3 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 0 & 0 & -2 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 3 & 1 \end{array} \right)$$

⇒ Hệ có nghiệm duy nhất  $x_1 = -2, x_2 = 1, x_3 = 1$

$$\begin{array}{r} \xrightarrow{\begin{matrix} d_2 \\ d_2 \end{matrix}} \left( \begin{array}{cccc|c} 1 & -2 & -1 & 1 & 1 \\ 2 & -1 & 1 & 6 & 1 \\ 3 & -5 & 0 & 2 & 1 \\ 4 & 0 & 5 & 9 & 1 \end{array} \right) \xrightarrow{\begin{matrix} d_2 + 2d_1 \\ d_3 - 3d_1 \\ d_4 - d_1 \end{matrix}} \left( \begin{array}{cccc|c} 1 & -2 & -1 & 1 & 1 \\ 0 & 1 & 3 & 4 & 1 \\ 0 & 1 & 1 & 4 & 1 \\ 0 & 0 & 6 & 8 & 1 \end{array} \right) \end{array}$$

$$\begin{array}{r} \xrightarrow{\begin{matrix} d_1 + 2d_2 \\ d_3 + 3d_2 \\ d_4 - 2d_2 \end{matrix}} \left( \begin{array}{cccc|c} 1 & 0 & 5 & 9 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \end{array}$$

⇒ Kết luận

Đkt  $x_3 = a$ , ta có hệ:

$$x_1 = -5a + 9$$

$$x_2 = -3a + 4$$

$$3. \left( \begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 3 & 5 & -1 & 6 & 12 \\ 2 & 4 & 1 & 2 & 12 \\ 2 & 0 & -2 & 11 & 2 \end{array} \right) \xrightarrow{d_2 \leftrightarrow d_4} \left( \begin{array}{cccc|c} 1 & 2 & 0 & 2 & 6 \\ 0 & -1 & -1 & 0 & -1 \\ 0 & 0 & 1 & -2 & 0 \\ 6 & -4 & -2 & 7 & -3 \end{array} \right)$$

$$\xrightarrow{d_1 + 2d_2} \left( \begin{array}{cccc|c} 1 & 0 & -2 & 2 & 6 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & -1 & 7 & -3 \end{array} \right) \xrightarrow{d_1 + d_3} \left( \begin{array}{cccc|c} 1 & 0 & 0 & -2 & 4 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & -15 & -9 \end{array} \right)$$

$$\xrightarrow{d_1 \leftrightarrow d_4} \left( \begin{array}{cccc|c} 1 & 0 & 0 & -15 & 4 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right) \xrightarrow{d_1 + 2d_4} \left( \begin{array}{cccc|c} 1 & 0 & 0 & 0 & 12 \\ 0 & 1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & -1 \end{array} \right)$$

$x_1 = 2, x_2 = 3, x_3 = 2, x_4 = -1 \Rightarrow$  Hệ có nghiệm nhất

$$4. \left( \begin{array}{ccc|c} 2 & -4 & -1 & 1 \\ 1 & -3 & 1 & 1 \\ 3 & -5 & -3 & 2 \end{array} \right) \xrightarrow{d_1 \leftrightarrow d_3} \left( \begin{array}{ccc|c} 3 & -5 & -3 & 2 \\ 1 & -3 & 1 & 1 \\ 2 & -4 & -1 & 1 \end{array} \right)$$

$$\xrightarrow{d_1 - 2d_2} \left( \begin{array}{ccc|c} 1 & -5 & 1 & 1 \\ 0 & 2 & -1 & -1 \\ 2 & -4 & -1 & 1 \end{array} \right) \xrightarrow{d_1 + d_3} \left( \begin{array}{ccc|c} 1 & -3 & 1 & 1 \\ 0 & 1 & -2 & -1 \\ 0 & 4 & -6 & -1 \end{array} \right)$$

$$\xrightarrow{d_1 + 9d_2} \left( \begin{array}{ccc|c} 1 & 0 & -11/2 & -5/2 \\ 0 & 1 & -3/2 & 1/2 \\ 0 & 0 & 0 & 1 \end{array} \right) \Rightarrow$$

ĐK:  $v_3 \neq 0$  nghĩa là

$$x_1 = \frac{-11}{2}a - \frac{5}{2}, x_2 = \frac{3}{2}a + \frac{1}{2}$$

$$5. \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 3 & -1 & 1 & 1 \\ -1 & 5 & -5 & 5 \end{array} \right) \xrightarrow{d_2 \rightarrow 3d_1} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & -7 & 4 & -8 \\ -1 & 5 & -5 & 5 \end{array} \right)$$

$$\xrightarrow{d_3 + d_1} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 2 \\ 0 & -2 & 5 & -8 \\ 0 & 0 & -12 & 0 \end{array} \right) \xrightarrow{\frac{1}{12}d_3} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 1 & 5/2 & -8/3 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

$$\xrightarrow{d_1 \rightarrow 2d_1} \left( \begin{array}{ccc|c} 2 & 0 & -4 & 6 \\ 0 & -2 & 5 & -8 \\ 0 & 0 & 1 & 0 \end{array} \right) \xrightarrow{\frac{1}{2}d_1} \left( \begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & -1 & 5/2 & -4 \\ 0 & 0 & 1 & 0 \end{array} \right)$$

$$\xrightarrow{d_3 + d_1} \left( \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & -2 & 2 & -8 \\ 0 & 2 & -2 & 0 \end{array} \right) \xrightarrow{\frac{1}{2}d_2} \left( \begin{array}{ccc|c} 1 & 2 & -2 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & 2 & -2 & 0 \end{array} \right)$$

$$\xrightarrow{d_3 - 2d_2} \left( \begin{array}{ccc|c} 1 & 2 & -1 & 3 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{d_2 - 2d_1} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 5/2 \\ 0 & 1 & -1 & -6 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$\therefore x_1 = 5/2, x_2 = -6, x_3 = 0$

$$x_1 = 5/2$$

$$6. \left( \begin{array}{ccc|c} 2 & -4 & 6 & 8 \\ 1 & -1 & 1 & -1 \\ 1 & -3 & 4 & 0 \end{array} \right) \xrightarrow{1/2d_1} \left( \begin{array}{ccc|c} 1 & -2 & 3 & 4 \\ 0 & -1 & -2 & -5 \\ 1 & -3 & 4 & 0 \end{array} \right)$$

$$\xrightarrow{\frac{1}{2}d_1 + 2d_2} \left( \begin{array}{ccc|c} 1 & 0 & -1 & -6 \\ 0 & 1 & -2 & -5 \\ 1 & -3 & 4 & 0 \end{array} \right) \xrightarrow{d_3 - 4d_1} \left( \begin{array}{ccc|c} 1 & 0 & 0 & 13 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 9 \end{array} \right)$$

$$x_1 = 3$$

$$x_2 = 1$$

$$x_3 = 5$$

$$\left( \begin{array}{cccccc|c} 2 & 4 & -2 & -4 & 2 & 1 & 1 \\ 6 & 3 & 0 & -5 & 1 & 1 & 1 \\ 8 & -4 & 20 & -44 & 1 & 1 & 1 \\ -8 & 4 & -4 & 12 & -5 & 1 & 1 \end{array} \right) \xrightarrow{d_1 + 6d_2} \left( \begin{array}{cccccc|c} 1 & 1 & -1 & -1 & 1 & 1 & 1 \\ 6 & 3 & 0 & -5 & 1 & 1 & 1 \\ 8 & -4 & 20 & -44 & 1 & 1 & 1 \\ -8 & 4 & -4 & 12 & -5 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{d_2 - 6d_1} \left( \begin{array}{cccccc|c} 1 & 1 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 6 & -8 & 5 & 1 & 1 \\ 0 & 0 & 36 & -48 & 9 & 1 & 1 \\ 0 & 0 & -12 & 16 & -3 & 1 & 1 \end{array} \right) \xrightarrow{d_3 + 6d_1} \left( \begin{array}{cccccc|c} 1 & 1 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 36 & -48 & 9 & 1 & 1 \\ 0 & 0 & -12 & 16 & -3 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{d_4 + 11d_1} \left( \begin{array}{cccccc|c} 1 & 1 & -1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \rightarrow \text{vô số nghiệm}$$

Đặt  $x_1 = a$ ,  $x_2 = b$ , ta có

$$x_1 = \frac{1}{2}a + \frac{5}{6}b + \frac{1}{2}$$

$$x_2 = \frac{1}{3}b + \frac{1}{6}$$

$$\left( \begin{array}{cccc|c} 1 & -2 & 3 & -3 & -1 \\ 2 & 2 & 0 & 0 & 6 \\ 0 & 1 & 4 & 1 & 1 \\ 1 & 0 & 1 & -1 & 2 \end{array} \right) \xrightarrow{d_2 + d_1} \left( \begin{array}{cccc|c} 1 & -2 & 3 & -3 & -1 \\ 0 & 0 & 0 & 0 & 6 \\ 0 & 3 & 4 & 1 & 1 \\ 0 & 2 & -2 & 2 & 2 \end{array} \right)$$

$$\xrightarrow{\frac{1}{6}d_2} \left( \begin{array}{cccc|c} 1 & -2 & 3 & -3 & -1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & -3 & 4 & 1 & 1 \\ 0 & 2 & -2 & 2 & 2 \end{array} \right) \xrightarrow{d_3 + 3d_1} \left( \begin{array}{cccc|c} 1 & -2 & 3 & -3 & -1 \\ 0 & 1 & -1 & 1 & 1 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right)$$

$$\xrightarrow{d_2 + d_1} \left( \begin{array}{cccc|c} 1 & -2 & 3 & -3 & -1 \\ 0 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{d_1 - 3d_2} \left( \begin{array}{cccc|c} 1 & 0 & 0 & -5 & 0 \\ 0 & 1 & 0 & 5 & 0 \\ 0 & 0 & 1 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \rightarrow \text{Ph. có nghịch}\text{chung nhau}$$

điều kiện

$$\lambda_1 = -5$$

$$\lambda_2 = 5$$

$$\lambda_3 = 4$$

$$9. \left( \begin{array}{cccc} 2 & -1 & 3 & -3 \\ -1 & 5 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 3 & -1 & 2 & -4 \end{array} \right) \xrightarrow{d_1 + d_2} \left( \begin{array}{cccc} 1 & -1 & 1 & -1 \\ -1 & 5 & 2 & 4 \\ 0 & -4 & 2 & 2 \\ 3 & -1 & 2 & -4 \end{array} \right)$$

$$\xrightarrow{d_2 - 5d_1} \left( \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 0 & -6 & 3 & 3 \\ 0 & -4 & 2 & 2 \\ 0 & 2 & -1 & -10 \end{array} \right) \xrightarrow{-\frac{1}{6}d_2} \left( \begin{array}{cccc} 1 & -1 & 1 & -1 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & -4 & 2 & 2 \\ 0 & 2 & -1 & -10 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} d_3 + d_4 \\ d_3 + 4d_2 \end{array}} \left( \begin{array}{cccc} 1 & 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{-1} \text{Ke} \text{ v} \text{v} \text{ s} \text{t} \text{t} \text{ n} \text{g} \text{ h} \text{v} \text{v} \text{m}$$

$$\text{Đ} \hat{\text{a}} \text{t } \lambda_1 = c \Rightarrow \lambda_1 = -\frac{1}{2}a - \frac{3}{2}$$

$$\lambda_2 = \frac{1}{2}a - \frac{1}{2}$$

$$10. \left( \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 2 & -1 & 4 & -2 & 0 \end{array} \right) \xrightarrow{d_2 - 2d_1} \left( \begin{array}{cccc|c} 1 & -1 & 1 & -3 & 0 \\ 0 & 1 & 2 & 4 & 0 \end{array} \right) \xrightarrow{d_1 + d_2} \left( \begin{array}{cccc|c} 1 & 0 & 3 & 1 & 0 \\ 0 & 1 & 2 & 4 & 0 \end{array} \right)$$

$\text{Đ} \hat{\text{a}} \text{t } \lambda_2 = c, \text{ v} \text{v} \text{v} \text{v} \text{v} \text{v}$

$$\Rightarrow \lambda_1 = -3a - b$$

$$\lambda_2 = -2a - 4b$$

$$11. \left( \begin{array}{cccc|c} 2 & -3 & 4 & -1 & 0 \\ 6 & 1 & -8 & 9 & 0 \\ 2 & 6 & 1 & -1 & 0 \end{array} \right) \xrightarrow{\begin{array}{l} d_2 - 3d_1 \\ d_3 - 6d_1 \end{array}} \left( \begin{array}{cccc|c} 1 & -7 & 2 & -1/2 & 0 \\ 0 & 1 & -8 & 8 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right)$$

$$\xrightarrow{\begin{array}{l} d_2 - 8d_3 \\ d_3 - 1d_1 \end{array}} \left( \begin{array}{cccc|c} 1 & -7 & 2 & -1/2 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right) \xrightarrow{\begin{array}{l} 1/7d_2 \\ d_3 - 10d_1 \end{array}} \left( \begin{array}{cccc|c} 1 & -3/2 & 1 & -1/2 & 0 \\ 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 \end{array} \right)$$

$$\begin{array}{c} d_1 - 9d_2 \\ \hline 0, 1, -2, -\frac{6}{5}, 0 \\ 0, 0, 15, -50, 0 \end{array} \xrightarrow{d_1 + 15d_2} \begin{array}{c} 1, -3, 1, 0 \\ 0, 1, -2, -\frac{6}{5}, 0 \\ 0, 0, 1, -\frac{1}{5}, 0 \end{array}$$

$$\begin{array}{c} d_1 + 2d_3 \\ \hline 0, 1, -2, -\frac{1}{2}, 0 \\ 0, 1, 0, -\frac{6}{15}, 0 \\ 0, 0, 1, -\frac{1}{15}, 0 \end{array} \xrightarrow{d_1 - 2d_3} \begin{array}{c} 1, -3, 0, 0 \\ 0, 1, 0, -\frac{6}{15}, 0 \\ 0, 0, 1, -\frac{1}{15}, 0 \end{array}$$

$$\begin{array}{c} d_1 + \frac{3}{5}d_2 \\ \hline 0, 1, 0, -\frac{6}{15}, 0 \\ 0, 0, 1, -\frac{1}{5}, 0 \end{array} \Rightarrow \text{Hệ số số nguyên}$$

$$d_1/2a = a \Rightarrow \lambda_1 = -\frac{25}{50}a$$

$$\lambda_2 = \frac{15}{25}a$$

$$\begin{array}{c} 12, 1, 6, 4, 0 \\ 2, 4, -1, 0 \\ -1, 2, 5, 0 \end{array} \xrightarrow{d_1 - 2d_2} \begin{array}{c} 1, 0, 4, 0 \\ 0, -8, -9, 0 \\ 0, 0, 9, 0 \end{array} \xrightarrow{d_3 + d_1} \begin{array}{c} 1, 0, 4, 0 \\ 0, -8, -9, 0 \\ 0, 0, 9, 0 \end{array}$$

$$\begin{array}{c} d_1 - 6d_2 \\ \hline 1, 0, 4, 0 \\ 0, 1, \frac{9}{8}, 0 \\ 0, 8, \frac{9}{8}, 0 \end{array} \xrightarrow{d_2 - 9d_3} \begin{array}{c} 1, 0, 4, 0 \\ 0, 1, \frac{9}{8}, 0 \\ 0, 0, 0, 0 \end{array} \xrightarrow{d_1 - 6d_2} \begin{array}{c} 1, 0, 4, 0 \\ 0, 1, \frac{9}{8}, 0 \\ 0, 0, 0, 0 \end{array} \Rightarrow \text{Hệ số số nguyên}$$

$$\lambda_1 = a \Rightarrow \lambda_1 = \frac{9}{8}a, \lambda_2 = -\frac{1}{8}a$$

$$\begin{array}{c} 1, 2, -1, -1 \\ 2, 1, 1, 1 \\ 3, 5, -2, -1 \end{array} \xrightarrow{d_1 - 2d_2} \begin{array}{c} 1, 2, -1, -1 \\ 0, -1, \frac{3}{2}, -\frac{3}{2} \\ 3, 0, -\frac{7}{2}, \frac{1}{2} \end{array} \xrightarrow{d_1 - 3d_3} \begin{array}{c} 1, 2, -1, -1 \\ 0, 1, -\frac{3}{2}, \frac{3}{2} \\ 0, 0, 1, -\frac{1}{2} \end{array}$$

$$\xrightarrow{d_1 + d_2} \begin{array}{c} 1, 2, -1, -1 \\ 0, 1, -\frac{3}{2}, \frac{3}{2} \\ 0, 0, 1, -\frac{1}{2} \end{array} \xrightarrow{d_1 - 2d_3} \begin{array}{c} 1, 2, -1, -1 \\ 0, 1, 0, -\frac{1}{2} \\ 0, 0, 1, -\frac{1}{2} \end{array} \xrightarrow{d_1 + d_2} \begin{array}{c} 1, 2, -1, -1 \\ 0, 1, 0, -\frac{1}{2} \\ 0, 0, 1, -\frac{1}{2} \end{array}$$

$$\lambda_1 = a, \lambda_2 = -1, \lambda_3 = -1 \Rightarrow \text{Hệ số số nguyên duy nhất}$$

SUVÄN 2:

$$A \cdot AB = \begin{pmatrix} 1 & 0 & 2 \\ 1 & 1 & 0 \\ 1 & -1 & -2 \end{pmatrix} \begin{pmatrix} 7 & 2 & -2 \\ 0 & 1 & 4 \\ 0 & 1 & 4 \end{pmatrix}$$

 $A(3B + C)$ 

$$3B + C = \begin{pmatrix} 4 & 0 & 9 \\ 2 & 4 & -7 \\ 2 & 4 & -7 \end{pmatrix} \Rightarrow A(3B + C) = \begin{pmatrix} 1 & 2 & 0 \\ 3 & 1 & 1 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 4 & -2 & 9 \\ 2 & 4 & -7 \\ 2 & 4 & -7 \end{pmatrix}$$

$$= \begin{pmatrix} 15 & 6 & -5 \\ 9 & -2 & 5 \\ 9 & -2 & 5 \end{pmatrix}$$

ABC (khi thực hiện phép viết bù có ma trận  $2 \times 3$  giữ c)

$$B^T A = \begin{pmatrix} 1 & 3 \\ 0 & 1 \\ 2 & -2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 3 & 1 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 10 & 5 \\ 3 & 1 \\ -4 & 2 \end{pmatrix}$$

$$BC^{-1} \begin{pmatrix} 1 & 0 & 2 \\ 3 & 1 & -2 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -2 \\ 2 & 1 \\ 3 & -1 \end{pmatrix} = \begin{pmatrix} 2 & -4 \\ -5 & -3 \\ 3 & -1 \end{pmatrix}$$

c)  $\begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 3 \\ 2 & 3 & 4 \end{pmatrix}$  ( $|A| = 2 \neq 0$ ) ma trận không nghịch

$$\rightarrow \left( \begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 2 & 3 & 0 & 1 & 0 \\ 2 & 3 & 4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} d_2 - 2d_1 \\ d_3 - 2d_1 \end{array}} \left( \begin{array}{ccc|ccc} 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array} \right)$$

$$\xrightarrow{d_1 - 1d_3} \left( \begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 2 & 2 & -5 & 1 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} d_1 - d_2 \\ d_3 - d_2 \end{array}} \left( \begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 5 & 1 & 1 \end{array} \right)$$

$$\xrightarrow{d_1 - d_3} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 5 & 1 & 1 \end{array} \right)$$

$$b) \begin{pmatrix} 1 & -1 & 2 \\ 1 & 1 & -2 \\ 1 & 1 & 4 \end{pmatrix} \xrightarrow{\text{det } A = 12 \neq 0 \Rightarrow \text{matrix has inverse}} \quad$$

$$\begin{array}{c} \left( \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 1 & 1 & -2 & 0 & 1 & 0 \\ 1 & 1 & 4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} d_1-d_1 \\ d_2-d_1 \\ d_3-d_1 \end{array}} \left( \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 2 & -4 & -1 & 1 & 0 \\ 0 & 2 & 2 & -1 & 0 & 1 \end{array} \right) \\ \xrightarrow{\begin{array}{l} d_3-d_2 \\ d_2 \cdot \frac{1}{2} \end{array}} \left( \begin{array}{ccc|ccc} 1 & -1 & 2 & 1 & 0 & 0 \\ 0 & 1 & -2 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 4 & -1 & 0 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} d_3 \cdot \frac{1}{4} \\ d_1-d_3 \\ d_2+d_3 \end{array}} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{array} \right) \end{array}$$

$$\xrightarrow{-d_3} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & 0 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & \frac{1}{4} & \frac{1}{4} \end{array} \right)$$

$$c) \begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix} \xrightarrow{\text{det } A = -1 \neq 0 \Rightarrow \text{matrix has inverse}}$$

$$\begin{array}{c} \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 5 & 3 & 0 & 1 & 0 \\ 1 & 0 & 8 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} d_1-2d_2 \\ d_3-d_1 \end{array}} \left( \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1 & 1 & 0 \\ 0 & -2 & 5 & -1 & 0 & 1 \end{array} \right) \\ \xrightarrow{\begin{array}{l} d_3+2d_2 \\ d_1-2d_2 \end{array}} \left( \begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -1 & 0 \\ 0 & 1 & -1 & -2 & 1 & 0 \\ 0 & 0 & 1 & -5 & 2 & 1 \end{array} \right) \xrightarrow{\begin{array}{l} d_2-d_3 \\ d_1-9d_3 \end{array}} \left( \begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \end{array}$$

$$\begin{array}{c} \left( \begin{array}{ccc|ccc} 1 & 0 & 9 & 5 & -2 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \\ \xrightarrow{d_2+d_3} \left( \begin{array}{ccc|ccc} 1 & 0 & 0 & -40 & 16 & 9 \\ 0 & 1 & 1 & 12 & -5 & -3 \\ 0 & 0 & 1 & 5 & -7 & -1 \end{array} \right) \end{array}$$

$$\begin{pmatrix} 1 & -1 & 3 & -4 \\ 2 & 1 & 1 & 0 \\ -1 & 2 & -9 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\text{c1} \leftrightarrow \text{c4}, \text{c2} \leftrightarrow \text{c3}} \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

có định thức  $|A|=0=1$  mà  $\det A \neq 0$   
chứng minh

$$3. \quad \mathbf{x} = (1, 2, 3)$$

$$\mathbf{y} = (y_1, y_2, y_3)$$

$$\mathbf{z} = (4, 1, 1)$$

$$\mathbf{w} = (2, 4, 6)$$

$$-3\mathbf{y} = (-3y_1, -3y_2, -3y_3)$$

$$\mathbf{x}(2\mathbf{y} - \mathbf{z}) = (-11 + 1y_1, -6 + 1y_2, 2y_3)$$