

Pricing-Based Decentralized Spectrum Access Control in Cognitive Radio Networks

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Abstract—This paper investigates pricing-based spectrum access control in cognitive radio networks, where primary users (PUs) sell the temporarily unused spectrum and secondary users (SUs) compete via *random access* for such spectrum opportunities. Compared to existing market-based approaches with centralized scheduling, pricing-based spectrum management with random access provides a platform for SUs contending for spectrum access and is amenable to decentralized implementation due to its low complexity. We focus on two market models, one with a monopoly PU market and the other with a multiple-PU market. For the monopoly PU market model, we devise decentralized pricing-based spectrum access mechanisms that enable SUs to contend for channel usage. Specifically, we first consider SUs contending via slotted Aloha. Since the revenue maximization problem therein is nonconvex, we characterize the corresponding Pareto-optimal region and obtain a Pareto-optimal solution that maximizes the SUs' throughput subject to their budget constraints. To mitigate the spectrum underutilization due to the "price of contention," we revisit the problem where SUs contend via CSMA, which results in more efficient spectrum utilization and higher revenue. We then study the tradeoff between the PU's utility and its revenue when the PU's salable spectrum is controllable. Next, for the multiple-PU market model, we cast the competition among PUs as a three-stage Stackelberg game, where each SU selects a PU's channel to maximize its throughput. We explore the existence and the uniqueness of Nash equilibrium, in terms of access prices and the spectrum offered to SUs, and develop an iterative algorithm for strategy adaptation to achieve the Nash equilibrium. Our findings reveal that there exists a unique Nash equilibrium when the number of PUs is less than a threshold determined by the budgets and elasticity of SUs.

Index Terms—Cognitive radio, nonconvex optimization, Pareto optimality, pricing, random access, spectrum access control.

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I. INTRODUCTION

COGNITIVE radio is expected to capture temporal and spatial "spectrum holes" in the spectrum white space and to enable spectrum sharing for secondary users (SUs). One grand challenge is how SUs can discover spectrum holes and access them efficiently, without causing interference to the primary users (PUs), especially when the demand for available spectrum nearly outstrips the supply. Market-based mechanisms have been explored as a promising approach for spectrum access, where PUs can dynamically trade unused spectrum to SUs [2]–[17]. In particular, auction-based spectrum access mechanisms have been extensively studied, including incentive compatibility [3]–[6], spectrum reuse [3], [4], [7], [8], auctioneer's revenue maximization [4], social welfare maximization [8], and power allocation for the SUs with interference protection for the PU [9] (and the references therein). These works focus on on-demand auctions where each SU requests spectrum based on its traffic demand, and it is worth noting that the overhead can be significant in the auction procedure (e.g., market setup time, bidding time, and pricing clearing time).

Compared to auction-based spectrum access, pricing-based spectrum access incurs lower overhead (see [10]–[16] and the references therein). Notably, [10] studied pricing policies for a PU to sell unused spectrum to multiple SUs. Recent works [11], [12] considered competition among multiple PUs that sell spectrum, whereas [13] focused on competition among multiple SUs to access the PU's channels. Reference [14] considered spectrum trading across multiple PUs and multiple SUs. Reference [15] studied the investment and pricing decisions of a network operator under spectrum supply uncertainty. One common assumption used in these studies is that *orthogonal* multiple access is used among SUs, either in time or frequency domain, where a central controller is needed to handle SUs' admission control, to calculate the prices, and to charge the SUs. However, the computational complexity for dynamic spectrum access and the need of centralized controllers can often be overwhelming or even prohibitive. To address these problems, a recent work [16] proposed a two-tier market model based on the decentralized bargain theory, where the spectrum is traded from a PU to multiple SUs on a larger timescale, and then redistributed among SUs on a smaller timescale. Due to the decentralized nature, coordination among SUs remains a challenge when SUs of different networks coexist [17], simply because contention between SUs is unavoidable.

As a less-studied alternative in cognitive radio networks, random access can serve as a platform for the contention among SUs (e.g., in [18] and [19]) and can be employed for decentralized spectrum access to mitigate the overwhelming complexity.

With this insight, we focus on pricing-based spectrum control with random access. In particular, we study the behaviors of PUs and SUs in two spectrum-trading market models based on random access: one with a monopoly PU market and the other with a multiple-PU market.

We first consider the monopoly PU market model where the PU's unused spectrum is fixed. We study pricing-based dynamic spectrum access based on slotted Aloha, aiming to characterize the optimal pricing strategy maximizing the PU's revenue. Due to the nonconvexity of the optimization problem, the global optimum is often unattainable. Instead, we first characterize the Pareto-optimal region associated with the throughput vector of SUs, based on the observation that the global optimum has to be Pareto-optimal. Roughly speaking, for any Pareto-optimal solution, the throughput of any individual SU cannot be improved without deteriorating some other SU's throughput. Then, by maximizing the SUs' throughput subject to the budget constraints, we provide a Pareto-optimal solution that is near-optimal. Furthermore, the structural properties of this Pareto-optimal solution indicate that the access probabilities can be computed by the SUs locally. With this insight, we develop a decentralized pricing-based spectrum access control algorithm accordingly. To mitigate the spectrum underutilization due to the "price of contention," we next turn to dynamic spectrum access using CSMA and quantify the improvements in spectrum utilization and PU's revenue. We also consider the case when PU's salable spectrum is controllable, i.e., the PU can flexibly allocate the spectrum to its ongoing transmissions so as to balance its own utility and revenue.

Next, for the multiple-PU market model, we treat the competition among PUs as a three-stage Stackelberg game, where each PU seeks to maximize its net utility and each SU selects a PU's channel to maximize its own throughput. We explore optimal strategies to adapt the prices and the offered spectrum for each PU and show that the Nash equilibria of the game exist. We further prove that the Nash equilibrium is unique when the number of PUs is less than a threshold, whose value is determined by the budgets and elasticity of SUs. Intuitively, this threshold criterion can be used by PUs to decide whether to join in the competition or not, i.e., when the number of PUs grows larger than the threshold, the competition among PUs is too strong, indicating that it is unprofitable for a PU to sell spectrum to SUs. An iterative algorithm is devised to compute the Nash equilibrium accordingly.

The rest of this paper is organized as follows. In Section II, we study the monopoly PU market and present the Pareto-optimal pricing strategy for the PU's revenue maximization problem. We also characterize the tradeoff between the PU's utility and its revenue. We study in Section III the competition among PUs in the multiple-PU market, which is cast as a three-stage Stackelberg game, and analyze the Nash equilibria of the game. Finally, we conclude the paper in Section IV.

II. MONOPOLY PU MARKET

A. System Model

We first consider a monopoly PU market with a set of SUs, denoted by \mathcal{M} . The PU sells the available spectrum

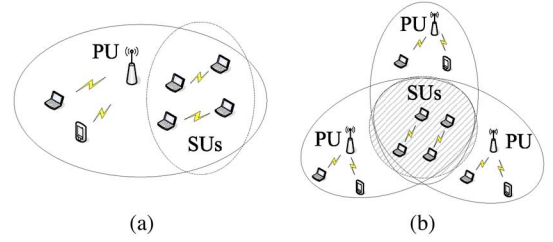


Fig. 1. System model: (a) monopoly PU market; (b) multiple-PU market.

TABLE I
SUMMARY OF THE KEY NOTATIONS AND DEFINITIONS

Notation	Definition
\mathcal{M}	set of SUs
\mathcal{N}	set of PUs
c	available spectrum opportunity
p_i (in vector \mathbf{p})	usage price of SU i
g_i (in vector \mathbf{g})	flat price of SU i
s_i (in vector \mathbf{s})	successful channel access probability of SU i
z_i (in vector \mathbf{z})	channel access probability of SU i
$U_i(\cdot)$	utility function of SU i
$d_i(\cdot)$	demand function of SU i
κ	spectrum utilization percentage
$R(\cdot)$	net utility function of PU
$ \mathcal{N} $	cardinality of the set \mathcal{N}

opportunity c in each period,¹ in terms of time-slots in a slotted wireless system based on random access, to SUs who are willing to buy the spectrum opportunities [Fig. 1(a)]. When one SU decides to buy the spectrum opportunity, it sends a request message together with the budget information (to be elaborated in the sequel) to the PU. Then, at the beginning of each period, the PU broadcasts to the SUs the salable spectrum opportunities and the prices to access them. Observe that message passing involved in this scheme is infrequent and minimum (instead of sending the control message at each slot to manage the spectrum access).²

We study two cases: 1) the spectrum opportunity c is fixed, and the PU desires to find the optimal prices (i.e., usage price and flat price) to maximize its revenue; 2) the spectrum opportunity c is a control parameter that the PU can use to balance its own utility and revenue. In both cases, each SU i seeks to set its demand that maximizes its payoff, given the spectrum opportunity c , the usage price p_i , and the flat price g_i . For ease of reference, the key notation in this paper is listed in Table I.³

B. Case With Fixed Spectrum Opportunity

We first study the case where the spectrum opportunity c is fixed. We begin with the channel access model for SUs, assuming a linear pricing strategy, i.e., the PU charges each SU

¹The period refers to a time frame where the PU sells its unused part, denoted by c . Please refer to Fig. 2 for an illustration of a period in the slotted wireless system.

²In cognitive radio networks, the control channel often does not exist [20]. In this study, assuming that no common control channel exists, we design algorithms that have minimal computational complexity and message passing overhead. The salable spectrum opportunity c is a fraction of a period. When selling the spectrum at the beginning of each period, the PU does not know which slots will be idle, as this depends on the PU's traffic, and SUs would have to detect the opportunities through spectrum sensing.

³We use bold symbols (e.g., \mathbf{p}) to denote vectors, and calligraphic symbols (e.g., \mathcal{M}) to denote sets.

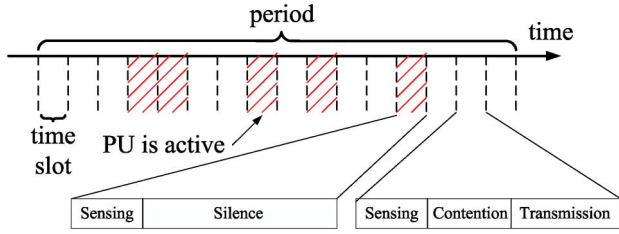


Fig. 2. Random access model for SUs.

a flat price and a usage price proportional to its *successful* transmissions. We will show that the Pareto-optimal usage price is the same for all SUs and is uniquely determined by the total demand of SUs and the available spectrum opportunity c .

1) *Slotted Aloha Model for SUs' Channel Access*: Each SU first carries out spectrum sensing to detect the PU's activity.⁴ When the sensing result reveals that the PU is idle, SUs will contend for channel access by random access; otherwise, SUs will remain silent as illustrated in Fig. 2. As in the standard slotted Aloha model [21], we assume that SUs under consideration are within the contention ranges of the others, and all transmissions are slot-synchronized. We assume that SUs always have packets to transmit, and traffic demands of SUs are elastic. Denote by z_i the transmission probability of the i th SU. The probability that the i th SU's packet is successfully received is $s_i = z_i \prod_{j \neq i} (1 - z_j)$. The expected number of successfully transmitted packets of the i th SU in one period can be written as $\hat{s}_i = cs_i$, where $\hat{s}_i \in [0, c]$. Accordingly, the i th SU receives a utility in one period equal to $U_i(\hat{s}_i)$, where $U_i(\cdot)$ denotes the utility function of the i th SU. The optimal demand \hat{s}_i^* is the solution to the following optimization problem:

$$\begin{aligned} & \text{maximize} && U_i(\hat{s}_i) - (p_i \hat{s}_i + g_i) \\ & \text{subject to} && 0 \leq \hat{s}_i \leq c \\ & \text{variables} && \{\hat{s}_i\}. \end{aligned} \quad (1)$$

As is standard, we define the demand function that captures the successful transmissions \hat{s}_i given the price p_i as

$$d_i(p_i) = \begin{cases} U_i'^{-1}(p_i), & \text{if } g_i \leq U_i(\hat{s}_i) - p_i \hat{s}_i \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

Assuming α -fair utility functions, the utility and the demand functions of each SU can be written, respectively, as

$$U_i(\hat{s}_i) = \begin{cases} \frac{\sigma_i}{1-\alpha} (\hat{s}_i)^{1-\alpha}, & 0 \leq \alpha < 1 \\ \sigma_i \log(\hat{s}_i), & \alpha = 1 \end{cases} \quad (3)$$

$$d_i(p_i) = \begin{cases} \sigma_i^{\frac{1}{1-\alpha}} p_i^{-\frac{1}{1-\alpha}}, & \text{if } g_i \leq U_i(\hat{s}_i) - p_i \hat{s}_i \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where σ_i , the multiplicative constant in the α -fair utility function, denotes the utility level⁵ of the i th SU, which reflects the *budget*⁶ of the i th SU (see [22] and the references therein).

⁴In this study, we assume that sensing errors are negligible.

⁵The PU collects the budget information of SUs from the SUs' request message at the beginning of each period, and this allows the PU to infer σ_i .

⁶Given the prices, the SUs with larger budget, i.e., larger σ_i , would buy more spectrum from the PU based on (4).

Note that we mainly consider the case of $0 \leq \alpha \leq 1$ because $1/\alpha = -(p_i d_i'(p_i)/d_i(p_i))$ is the elasticity of the demand seen by the PU, and it has to be strictly larger than 1 so that the monopoly price is finite [23]. Clearly, the α -fairness boils down to the weighted proportional fairness when $\alpha = 1$ and to selecting the SU with the highest budget when $\alpha = 0$.

2) *PU's Pricing Strategy*: We have the following revenue maximization problem for the monopoly PU market:

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{M}} (g_i + p_i \hat{s}_i) \\ & \text{subject to} && \hat{s}_i \leq cz_i \prod_{k \neq i} (1 - z_k) \quad \forall i \in \mathcal{M} \\ & && \hat{s}_i = d_i(p_i) \quad \forall i \in \mathcal{M} \\ & && U_i(\hat{s}_i) - g_i - p_i \hat{s}_i \geq 0 \quad \forall i \in \mathcal{M} \\ & \text{variables} && \{\mathbf{g}, \mathbf{p}, \hat{\mathbf{s}}, \mathbf{z}\}. \end{aligned} \quad (5)$$

The constraint $U_i(\hat{s}_i) - g_i - p_i \hat{s}_i \geq 0$ ensures that SUs have nonnegative utility under the prices g_i and p_i ; otherwise, SUs may not transmit. Let $\{g_i^*, p_i^*, \hat{s}_i^*, z_i^*\}$ denote the optimal solution to (5). It is clear that $U_i(\hat{s}_i^*) - g_i^* - p_i^* \hat{s}_i^* = 0, \forall i \in \mathcal{M}$; otherwise, the PU can always increase its revenue by increasing g_i^* to make the SU's net utility equal to zero.

Lemma 2.1: The optimal prices for (5) are given by

$$g_i = U_i(\hat{s}_i) - p_i \hat{s}_i \quad \forall i \in \mathcal{M}. \quad (6)$$

Based on Lemma 2.1, (5) can be rewritten as

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{M}} U_i(d_i(p_i)) \\ & \text{subject to} && d_i(p_i) \leq cz_i \prod_{k \neq i} (1 - z_k) \quad \forall i \in \mathcal{M} \\ & \text{variables} && \{\mathbf{p}, \mathbf{z}\}. \end{aligned} \quad (7)$$

Since the utility function $U_i(\cdot)$ is increasing, the optimal solution to (7) is achieved at the point when $d_i(p_i) = cz_i \prod_{k \neq i} (1 - z_k), \forall i \in \mathcal{M}$. Also, the objective function of (7) can be written as $U_i(d_i(p_i)) = (1/(1-\alpha)) p_i d_i(p_i)$. Since $1/(1-\alpha)$ is a constant, maximizing $(1/(1-\alpha)) \sum_{i \in \mathcal{M}} p_i d_i(p_i)$ is equivalent to maximizing $\sum_{i \in \mathcal{M}} p_i d_i(p_i)$, i.e., solving (5) is equivalent to solving the following problem without considering flat prices

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{M}} p_i d_i(p_i) \\ & \text{subject to} && d_i(p_i) = cz_i \prod_{k \neq i} (1 - z_k) \quad \forall i \in \mathcal{M} \\ & \text{variables} && \{\mathbf{p}, \mathbf{z}\}. \end{aligned} \quad (8)$$

In general, (8) is nonconvex, and therefore it is difficult to find the global optimum. Observing that the global optimum of (8) is Pareto-optimal, we shall confine our attention to the Pareto-optimal region, i.e., the set consisting of Pareto-optimal solutions to (8).

Definition 2.1: A feasible allocation $\hat{\mathbf{s}}$ is Pareto-optimal if there is no other feasible allocation $\hat{\mathbf{s}}'$ such that $\hat{s}_i' \geq \hat{s}_i$ for all $i \in \mathcal{M}$ and $\hat{s}_j' > \hat{s}_j$ for some $j \in \mathcal{M}$.

Lemma 2.2: The Pareto-optimal region corresponding to (8) has the following properties.

1) The global optimum is in the Pareto-optimal region.

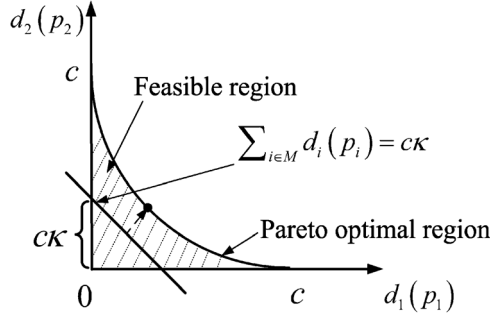


Fig. 3. Sketch of the Pareto-optimal region: the case with two SUs.

- 2) The solution to (8) is Pareto-optimal if and only if $\sum_{i \in \mathcal{M}} z_i = 1$.

The proof of Lemma 2.2 is given in Appendix A. By Lemma 2.2, for any Pareto-optimal allocation $d_i(p_i), \forall i \in \mathcal{M}$, we have $\sum_{i \in \mathcal{M}} z_i = 1$. Let $\mathcal{A} = \{z | \sum_{i \in \mathcal{M}} z_i = 1, z_i \geq 0, \forall i \in \mathcal{M}\}$ be the Pareto-optimal region. Therefore, it suffices to search for the points in \mathcal{A} that can maximize (8). In light of Lemma 2.2, instead of tackling the original problem given by (8), hereafter we focus on obtaining a Pareto-optimal solution to (8) that maximizes the sum of SUs' throughput given the SUs' budget constraints by confining the search space to the hyperplane $\sum_{i \in \mathcal{M}} d_i(p_i) = c\kappa$, where $\kappa \in [0, 1]$ denotes the *spectrum utilization percentage* under the allocation $d_i(p_i), \forall i \in \mathcal{M}$.

We now consider this “constrained” version of (8) for finding the maximum feasible spectrum utilization κ^* , i.e., the tangent point of the hyperplane and \mathcal{A} , as illustrated in Fig. 3

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{M}} p_i d_i(p_i) \\ & \text{subject to} && \sum_{i \in \mathcal{M}} d_i(p_i) = c\kappa \\ & && d_i(p_i) = cz_i \prod_{k \neq i} (1 - z_k) \quad \forall i \in \mathcal{M} \\ & \text{variables} && \{p, z, \kappa\}. \end{aligned} \quad (9)$$

We note that the solution to (9) is in general suboptimal for (8). However, by exploring the connections between the pricing strategy and the spectrum utilization, we derive a closed-form solution to (9) that is also a near-optimal solution to (8).

Proposition 2.1: For $\alpha \in (0, 1)$, the optimal solution to (9) is given by

$$\begin{aligned} p_i^* &= \left(\frac{G}{c\kappa^*} \right)^\alpha \quad \forall i \in \mathcal{M} \\ g_i^* &= U_i(d_i(p_i^*)) - p_i^* d_i(p_i^*) \quad \forall i \in \mathcal{M} \\ \kappa^* &= \sum_{i \in \mathcal{M}} z_i^* \prod_{j \neq i} (1 - z_j^*) \\ z_i^* &= \frac{w_i}{w_i + e^{-u}} \quad \forall i \in \mathcal{M} \end{aligned} \quad (10)$$

where $w_i = \sigma_i(1/\alpha)G^{-1}$, $G = \sum_{i \in \mathcal{M}} \sigma_i^{1/\alpha}$, and u is the unique solution of

$$\sum_{i \in \mathcal{M}} \frac{w_i}{w_i + e^{-u}} = 1.$$

Proof Outline: In what follows, we sketch an outline for proving the above proposition (see Appendixes B and C for the proof details). First, the following result establishes the relationship between the optimal pricing strategy of (9) and κ when $\alpha \in (0, 1)$.

Lemma 2.3: Given κ , the optimal pricing strategy of (9) for $\alpha \in (0, 1)$ is given by

$$p_i^* = \left(\frac{G}{c\kappa} \right)^\alpha \quad \forall i \in \mathcal{M}. \quad (11)$$

Next, a key step is to find κ^* . By Lemma 2.3, we have $d_i(p_i^*) = \sigma_i^{1/\alpha} G^{-1} c\kappa, \forall i \in \mathcal{M}$. Utilizing those constraints, we can find κ^* by solving the following problem:

$$\begin{aligned} & \text{maximize} && \kappa \\ & \text{subject to} && w_i \kappa \leq z_i \prod_{j \neq i} (1 - z_j) \quad \forall i \in \mathcal{M} \\ & \text{variables} && \{\kappa, z\}. \end{aligned} \quad (12)$$

Still, (12) is nonconvex, but by first taking logarithms of both the objective function and the constraints and then letting $\kappa' = \log(\kappa)$, we can transform (12) into the following convex problem:

$$\begin{aligned} & \text{maximize} && \kappa' \\ & \text{subject to} && \forall i \in \mathcal{M} \\ & && \log(w_i) + \kappa' \leq \log(z_i) + \sum_{j \neq i} \log(1 - z_j) \\ & \text{variables} && \{\kappa', z\}. \end{aligned} \quad (13)$$

Thus, the optimal solution to (12) can be summarized by the following lemma.

Lemma 2.4: The optimal solution to (12) is given by

$$\begin{aligned} \kappa^* &= \sum_{i \in \mathcal{M}} z_i^* \prod_{j \neq i} (1 - z_j^*) \\ z_i^* &= \frac{w_i}{w_i + e^{-u}} \quad \forall i \in \mathcal{M} \end{aligned} \quad (14)$$

where u is the unique solution of

$$\sum_{i \in \mathcal{M}} \frac{w_i}{w_i + e^{-u}} = 1. \quad (15)$$

Furthermore, when the number of SUs in the network is large, i.e., $|\mathcal{M}| \rightarrow \infty$, we can approximate κ^* by e^{-1} .

Based on Lemmas 2.3 and 2.4, the optimal solution to (9) is given by Proposition 2.1, which is a Pareto-optimal solution to (5). \square

Remarks: The “constrained” revenue maximization problem (9) is nonconvex. However, (9) exhibits a *hidden convexity* property, i.e., by utilizing Lemma 2.3, (9) can be transformed into a convex problem (13). Intuitively speaking, this property ensures that the line, $\sum_{i \in \mathcal{M}} d_i(p_i) = c\kappa$, as illustrated in Fig. 3, can touch the Pareto-optimal region with one unique point.

So far, we have focused on the case with $\alpha \in (0, 1)$. Next, we consider the special cases when $\alpha = 0$ and 1. Interestingly, we will see that the Pareto-optimal solutions are also global-optimal in those special cases.

Corollary 2.1: When $\alpha = 0$, the Pareto-optimal solution to (5) is also the global optimal solution, which is given by

$$\begin{aligned} p_i^* &= \max_{i \in \mathcal{M}} \sigma_i \\ z_i^* &= \begin{cases} 1, & i = k \\ 0, & i \neq k \end{cases} \\ g_i^* &= 0 \end{aligned} \quad (16)$$

for all i , where $k = \arg \max_{j \in \mathcal{M}} \sigma_j$.

Corollary 2.1 implies that when $\alpha = 0$, the PU selects the SU with the highest budget and only allows that SU to access the channel with probability 1 to maximize its revenue.

Remarks: The Pareto-optimal solution in (10) converges to the globally optimal solution as α goes to zero, i.e.,

$$p_i^* = \left(\frac{G}{c\kappa^*} \right)^\alpha = \left(\frac{1}{c\kappa^*} \right)^\alpha \left(\sum_{j \in \mathcal{M}} \sigma_j^{\frac{1}{\alpha}} \right)^\alpha \rightarrow \max_{j \in \mathcal{M}} \sigma_j.$$

When $\alpha = 1$, (7) can be transformed into a convex problem by taking logarithms of the constraints, and the global optimal access probabilities of SUs in this case are

$$z_i^* = \frac{\sigma_i}{\sum_{k \in \mathcal{M}} \sigma_k} \quad \forall i \in \mathcal{M}$$

i.e., the random access probability is proportional to SU's utility level, where the revenue is dominated by the flat rate. This is similar to the observation made in [22]. As α approaches 1, the revenue computed by (10) also converges to the global optimal solution, since the Pareto-optimal flat rates in (10) converge to the global optimal ones.

3) *Decentralized Implementation:* Based on the above study, we next develop decentralized implementation of the pricing-based spectrum access control. Based on the structural properties of the Pareto-optimal solution given in Proposition 2.1, we develop a decentralized pricing-based spectrum access control algorithm (Algorithm 1). In particular, the PU only needs to compute and broadcast the common parameters p^* , G , and u , based on which each SU can compute its access probability locally. It is clear that Algorithm 1 significantly reduces the complexity and the amount of the message passing, which would otherwise require a centralized coordination for the PU.

Algorithm 1: Decentralized Pricing-based Spectrum Access Control

Initialization:

- 1) The PU collects the budget information of SUs, i.e., $\{\sigma_i\}$.
- 2) The PU computes p^* , g^* , G , and u by (10) and broadcasts p^* , G , and u to SUs.
- 3) Each SU computes z_i^* by (10) based on G and u and infers g^* from its own utility and p^* by (10).

Repeat at the beginning of each period:

If New SUs join the system or SUs leave the system then

The PU updates the budget information of SUs. Then run Steps 2 to 3.

Endif

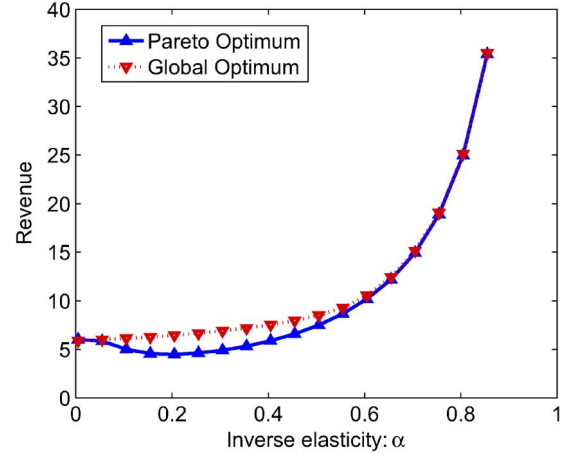


Fig. 4. Pareto optimum versus global optimum.

C. Numerical Example: Pareto Optimum vs. Global Optimum

To reduce the computational complexity in solving the global optimum of (5), we first solve (9). To examine the efficiency of this Pareto-optimal solution, we exhaustively search for the global optimum of (5) to compare to the Pareto optimum, in a small network with three SUs so as to efficiently generate the true global optimum as the benchmark. In this example, c is set to 5, and each SU's σ_i is generated uniformly in the interval $[0, 4]$ and fixed for different α for the sake of comparison. As shown in Fig. 4, the Pareto-optimal solution is close to the global optimal solution. Furthermore, the gap between the objective value evaluated at the Pareto-optimal solution and the global optimal value diminishes as α approaches 1. In addition, the gap goes to zero as α goes to zero, corroborating Corollary 2.1.

D. CSMA Model for SUs' Channel Access

Needless to say, the contention among SUs leads to spectrum underutilization. For the slotted Aloha model, the spectrum utilization κ approaches $1/e$, when the network size grows large, indicating that the unused spectrum is $1 - (1/e)$.

Definition 2.2: We define the unused spectrum, $1 - \kappa$, as the “price of contention.”

Obviously, the “price of contention” using slotted Aloha is high, compared to orthogonal access that requires centralized control. It is well known that spectrum utilization can be enhanced by using CSMA. Thus motivated, next we consider a CSMA-based random access for the SUs' channel access.

When CSMA is employed, a SU can successfully access the channel after an idle period if no other SUs attempt to access the channel at the same time. Let β denote the idle time of the channel. For a given \mathbf{z} , the network service throughput for a large network can be approximated by [21]

$$C(T(\mathbf{z})) = \frac{T(\mathbf{z})e^{-T(\mathbf{z})}}{\beta + 1 - e^{-T(\mathbf{z})}} \quad (17)$$

and the successful channel access probability of the i th SU can be approximated by

$$s_i(\mathbf{z}) = \frac{z_i e^{-T(\mathbf{z})}}{\beta + 1 - e^{-T(\mathbf{z})}} \quad \forall i \in \mathcal{M} \quad (18)$$

where $T(\mathbf{z}) = \sum_{i \in \mathcal{M}} z_i$ denotes the rate at which the SUs attempt to access the channel at the end of an idle period.

Since $C(T(\mathbf{z}))$ is maximized when $T(\mathbf{z}) = \sqrt{2\beta}$, the PU can always adjust the prices to make the SUs' channel access rates satisfy $\sum_{i \in \mathcal{M}} z_i = \sqrt{2\beta}$. In this case, $c/(1 + \sqrt{2\beta})$ can be utilized by the SUs per period, i.e., the spectrum utilization under CSMA is $1/(1 + \sqrt{2\beta})$. Similar to the approach for solving (5), by confining the search space to $\sum_{i \in \mathcal{M}} z_i = \sqrt{2\beta}$, the "constrained" revenue maximization problem under CSMA is given by⁷

$$\begin{aligned} & \text{maximize} && \sum_{i \in \mathcal{M}} (g_i + p_i \hat{s}_i) \\ & \text{subject to} && \hat{s}_i \leq \frac{cz_i e^{-\sqrt{2\beta}}}{\beta + 1 - e^{-\sqrt{2\beta}}} \quad \forall i \in \mathcal{M} \\ & && \hat{s}_i = d_i(p_i) \quad \forall i \in \mathcal{M} \\ & && \sum_{i \in \mathcal{M}} z_i = \sqrt{2\beta} \\ & && U_i(\hat{s}_i) - g_i - p_i \hat{s}_i \geq 0 \quad \forall i \in \mathcal{M} \\ & \text{variables} && \{\mathbf{g}, \mathbf{p}, \hat{\mathbf{s}}, \mathbf{z}\}. \end{aligned} \quad (19)$$

The above problem can be solved by following the same approach to (5), and the optimal solution to (19) is given by the following proposition.

Proposition 2.2: For $\alpha \in (0, 1)$, the optimal solution to (19) is given by

$$\begin{aligned} p_i^* &= \left(\frac{(1 + \sqrt{2\beta})G}{c} \right)^\alpha \quad \forall i \in \mathcal{M} \\ g_i^* &= U_i(d_i(p_i^*)) - p_i^* d_i(p_i^*) \quad \forall i \in \mathcal{M} \\ z_i^* &= \frac{\sigma_i^{\frac{1}{\alpha}} \sqrt{2\beta}}{G} \quad \forall i \in \mathcal{M}. \end{aligned} \quad (20)$$

Remarks: Note that (17) and (18) offer good approximations for large networks, in which case we can compare the performance of slotted Aloha and CSMA, e.g., revenue and spectrum utilization. As expected, the spectrum utilization under CSMA is higher, resulting in higher revenue from the SUs. We caution that when the network size is small, such comparisons may not be accurate since the system capacity under CSMA is unknown. Under slotted Aloha, the results hold for an arbitrary number of SUs. In the remainder of the paper, we focus on the system under slotted Aloha only.

E. Case With Controllable Spectrum Opportunity

When the spectrum opportunity c is a control parameter, there exists a tradeoff between the PU's utility and its revenue. For ease of exposition, we use the logarithmic utility function to quantify the PU's satisfaction

$$V(c) = a \log \left(1 - \frac{c}{C} \right) \quad (21)$$

where C denotes the total length of a period, and the utility level a is a positive constant depending on the application

⁷The solution to (19) is a Pareto-optimal solution to the original revenue maximization problem under CSMA without the constraint $\sum_{i \in \mathcal{M}} z_i = \sqrt{2\beta}$.

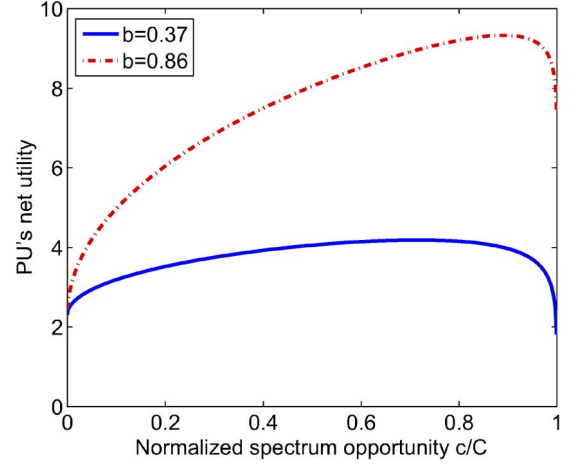


Fig. 5. PU's profit under different b .

type [14]. Given the PU's utility function, the net utility (or profit) of the PU for $\alpha \in (0, 1)$ can be expressed as

$$\begin{aligned} R(c, \mathbf{p}) &= a \log \left(1 - \frac{c}{C} \right) + \sum_{i \in \mathcal{M}} (g_i + p_i d_i(p_i)) \\ &= a \log \left(1 - \frac{c}{C} \right) + \frac{1}{1 - \alpha} \sum_{i \in \mathcal{M}} p_i d_i(p_i). \end{aligned} \quad (22)$$

Based on Proposition 2.1, the Pareto-optimal price is a function of the spectrum opportunity c . Then, the optimal c^* can be found by solving the following problem:

$$\begin{aligned} & \text{maximize} && a \log \left(1 - \frac{c}{C} \right) + bc^{1-\alpha} \\ & \text{subject to} && c \in \{0, 1, \dots, C\} \\ & \text{variable} && \{c\} \end{aligned} \quad (23)$$

where $b = (1/(1 - \alpha))\kappa^{*1-\alpha}G^\alpha$ is a positive constant.

Note that c is constrained to be an integer in the slotted system noted above. However, the objective function of (23) has the unimodal property, and therefore (23) can be solved efficiently (e.g., by the Fibonacci search algorithm [24]).

Lemma 2.5: The objective function of (23) is unimodal for $c \in \{0, 1, \dots, C\}$.

The unimodal property of the objective function of (23) directly follows from that of its continuous version, whose optimal solution c^* can be determined by the first order condition and the boundary conditions, which is the solution to

$$-\frac{a}{C - c^*} + \frac{b}{1 - \alpha} c^{*- \alpha} = 0. \quad (24)$$

When the length of each period, C , is reasonably large, c^* can be approximated by the solution to (24). In what follows, we adopt this continuous approximation of c .

As an illustration, we plot two possible curves of (22) for different b in Fig. 5. In this example, we set $a = 20$, $C = 100$, $\alpha = 0.5$, and $|\mathcal{M}| = 20$. Each SU's σ_i is generated uniformly in the interval $[\sigma_{\min}, \sigma_{\max}]$. For the two realizations of σ_i , $\forall i \in \mathcal{M}$, the optimal tradeoff decision, corresponding to the highest point of each curve within $(0, C)$, increases with b . Intuitively speaking, the PU would allocate more spectrum opportunity to those SUs who would pay more.

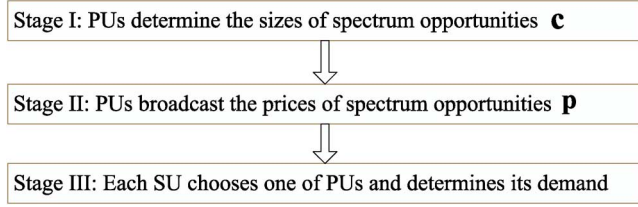


Fig. 6. Three-stage Stackelberg game.

III. MULTIPLE-PU MARKET

A. System Model

When there are multiple PUs in a cognitive radio network, they compete with each other in terms of prices and spectrum opportunities in order to maximize their net utilities. It can be seen from Lemma 2.1 that PU's flat prices depend on usage prices and that each SU wishes to choose the PU with the lowest usage price for its transmission.⁸ Thus, we focus on usage prices in what follows, and the corresponding flat prices can be obtained accordingly.

We assume that both PUs and SUs are selfish and yet rational. As the PUs are spectrum providers, they have the right to decide the prices and spectrum opportunities, so as to maximize their net utilities. Based on PU's decisions, each SU then chooses a PU's channel to maximize its transmission rate. Observe that it is a typical leader–follower game that can be analyzed by using the Stackelberg game framework. Specifically, we cast the competition among the PUs as a three-stage Stackelberg game, as summarized in Fig. 6, where the PUs and the SUs adapt their decisions dynamically to reach an equilibrium point. The PUs first simultaneously determine in Stage I their available spectrum opportunities, and then in Stage II simultaneously announce the prices to the SUs. Finally, each SU accesses only one PU's channel to maximize its throughput in Stage III. Here, we consider a set \mathcal{N} of PUs. We assume that all SUs are within the intersection of those PU's coverage areas shown in Fig. 1(b).

In the sequel, we focus on the game for $\alpha \in (0, 1)$ and use the index $i \in \mathcal{M}$ for SUs and the index $j \in \mathcal{N}$ for PUs.

B. Backward Induction for the Three-Stage Game

We analyze the subgame perfect equilibrium of the game by using the backward induction method [23], which is a popular technique for determining the subgame perfect equilibrium. First, we start with Stage III and analyze SU's behaviors, under given PU's spectrum opportunities and prices. Then, we turn our focus to Stage II and analyze how PUs determine prices given spectrum opportunities and the possible reactions of SUs in Stage III. Finally, we study how PUs determine spectrum opportunities given the possible reactions in Stages II and III.

1) *Channel Selection in Stage III*: In this stage, each SU determines which PU's channel to access based on the set of prices \mathbf{p} . The admission of SUs also depends on the available spectrum opportunities \mathbf{c} in Stage I. Since (4) decreases with

⁸Based on Lemma 2.1, each SU ends up with zero profit in any PU's channel. However, the SU can maximize its transmission rate by choosing the PU with the lowest usage price. Furthermore, the access probability of each SU is determined by (10).

price p_j , the i th SU would choose the j th PU's channel if $p_j = \min_{k \in \mathcal{N}} p_k$.

Given the set of prices \mathbf{p} , the total demand of SUs in the j th PU's channel can be written as

$$D_j(p_j, \mathbf{p}_{-j}) = \sum_{i \in \mathcal{M}_j} d_i(p_j) = \sum_{i \in \mathcal{M}_j} \sigma_i^{\frac{1}{\alpha}} p_j^{-\frac{1}{\alpha}}$$

where \mathcal{M}_j denotes the set of SUs choosing the j th PU, and \mathbf{p}_{-j} denotes the set of prices of PUs other than the j th PU. Both \mathcal{M}_j and D_j depend on prices \mathbf{p} and are independent of \mathbf{c} . Therefore, the demand function can be written as

$$D_j(p_j, \mathbf{p}_{-j}) = \begin{cases} \frac{G}{|\mathcal{J}|} p_j^{-\frac{1}{\alpha}}, & j \in \mathcal{J} \\ 0, & j \notin \mathcal{J} \end{cases}$$

where G is defined in Proposition 2.1, and $\mathcal{J} = \{j | p_j = \min_{k \in \mathcal{N}} p_k, j \in \mathcal{N}\}$ denotes the set of PUs with the smallest price in \mathcal{N} . In this paper, we assume that the SUs randomly pick one PU in \mathcal{J} with equal probability.

Given the size of available spectrum opportunities c_j , the j th PU always adjusts its price to make the demand of SUs equal to the supply so as to maximize its revenue (based on Proposition 2.1). It follows that at the Nash equilibrium point

$$D_j(p_j, \mathbf{p}_{-j}) = c_j \kappa_j^* \quad \forall j \in \mathcal{N} \quad (25)$$

where κ_j^* denotes the maximum feasible spectrum utilization of the j th PU's channel and is given by Lemma 2.4. Since κ_j^* depends on the budgets of SUs, it is difficult for each PU to decide whether or not to admit new SUs based on the current demand. Note that, in the multiple-PU market, the available spectrum is much larger than the single PU case, which can accommodate a large number of SUs. Since κ_j^* approaches e^{-1} when the number of SUs is reasonably large based on Lemma 2.4, we will approximate using this asymptote. Accordingly, (25) can be rewritten as

$$D_j(p_j, \mathbf{p}_{-j}) = \frac{c_j}{e} \quad \forall j \in \mathcal{N}. \quad (26)$$

2) *Pricing Competition in Stage II*: In this stage, the PUs determine their pricing strategies while considering the demands of SUs in Stage III, given the available spectrum opportunities \mathbf{c} in Stage I. The profit of the j th PU can be expressed as

$$R_j(c_j, p_j, \mathbf{c}_{-j}, \mathbf{p}_{-j}) = a_j \log \left(1 - \frac{c_j}{C} \right) + \frac{1}{1 - \alpha} p_j D_j(p_j, \mathbf{p}_{-j}).$$

Since c_j is fixed at this stage, the j th PU is only interested in maximizing the revenue $p_j D_j(p_j, \mathbf{p}_{-j})$. Obviously, if the j th PU has no available spectrum to sell, i.e., $c_j = 0$, it would not compete with other PUs by price reduction to attract the SUs. For convenience, define $\mathcal{N}' = \{j | c_j > 0, \forall j \in \mathcal{N}\}$ as the set of PUs with positive spectrum opportunity.

Game at Stage II: The competition among PUs in this stage can be modeled as the following game:

- *Players*: the PUs in the set \mathcal{N}' ;
- *Strategy*: each PU can choose a price p_j from the feasible set $\mathcal{P} = [p_{\min}, \infty)$;
- *Objective function*: $p_j D_j(p_j, \mathbf{p}_{-j}), j \in \mathcal{N}'$;

where p_{\min} denotes the minimum price that each PU can choose and is determined by (10) at $c = C$.

Proposition 3.1: A necessary and sufficient condition for PUs to achieve a Nash equilibrium price is $\sum_{j \in \mathcal{N}'} c_j \leq C$. Moreover, when $\sum_{j \in \mathcal{N}'} c_j \leq C$, there exists a unique Nash equilibrium price, and the Nash equilibrium price is given by $p_j^* = p^*, \forall j \in \mathcal{N}'$, where

$$p^* = \left(\frac{eG}{\sum_{j \in \mathcal{N}'} c_j} \right)^\alpha. \quad (27)$$

The proof of Proposition 3.1 is given in Appendix D. Proposition 3.1 shows that no PU would announce a price higher than its competitors to avoid losing most or all of its demand to its competitors, and the optimal strategy is to make the same decision as its competitors. Since $c_j = 0, \forall j \notin \mathcal{N}'$, the equilibrium price (27) can be rewritten as

$$p^* = \left(\frac{eG}{\sum_{j \in \mathcal{N}} c_j} \right)^\alpha \quad (28)$$

where $\sum_{j \in \mathcal{N}} c_j \leq C$.

3) *Spectrum Opportunity Allocation in Stage I:* In this stage, the PUs need to decide the optimal spectrum opportunities to maximize their profits. Based on Proposition 3.1, the j th PU's profit can be written as

$$\begin{aligned} R_j(c_j, \mathbf{c}_{-j}) &= a_j \log \left(1 - \frac{c_j}{C} \right) + \frac{c_j}{e(1-\alpha)} \left(\frac{eG}{\sum_{k \in \mathcal{N}} c_k} \right)^\alpha \\ &= \frac{e^{\alpha-1} G^\alpha}{1-\alpha} \left(\hat{a}_j \log \left(1 - \frac{c_j}{C} \right) + \frac{c_j}{\left(\sum_{k \in \mathcal{N}} c_k \right)^\alpha} \right) \end{aligned}$$

where $\hat{a}_j = (1-\alpha)e^{1-\alpha}G^{-\alpha}a_j$. For convenience, define $\hat{R}_j(c_j, \mathbf{c}_{-j}) = \hat{a}_j \log(1 - (c_j/C)) + (c_j/(\sum_{k \in \mathcal{N}} c_k)^\alpha)$. Since $e^{\alpha-1}G^\alpha/(1-\alpha)$ is a constant, maximizing $R_j(c_j, \mathbf{c}_{-j})$ is equivalent to maximizing $\hat{R}_j(c_j, \mathbf{c}_{-j})$.

Game at Stage I: The competition among the PUs in this stage can be modeled as the following game:

- *Players:* the PUs in the set \mathcal{N} ;
- *Strategy:* the PUs will choose \mathbf{c} from the feasible set $\mathcal{C} = \{\mathbf{c} | \sum_{j \in \mathcal{N}} c_j \leq C, c_j \in [0, C], \forall j \in \mathcal{N}\}$;
- *Objective function:* $\hat{R}_j(c_j, \mathbf{c}_{-j}), \forall j \in \mathcal{N}$.

We first examine the existence of the Nash equilibrium of the game at this stage. Based on [25], the existence of the Nash equilibrium can be obtained by showing the concavity of $\hat{R}_j(c_j, \mathbf{c}_{-j})$ in terms of c_j .

Proposition 3.2: There exists a Nash equilibrium in the spectrum opportunity allocation game, which satisfies the following set of equations:

$$\hat{R}_j(c_j^*, \mathbf{c}_{-j}^*) = \hat{a}_j \log \left(1 - \frac{c_j^*}{C} \right) + \frac{c_j^*}{\left(\sum_{k \in \mathcal{N}} c_k^* \right)^\alpha} \quad \forall j \in \mathcal{N}. \quad (29)$$

In general, the Nash equilibrium that satisfies (29) is not necessarily unique, as illustrated by the following example. Suppose a market with two heterogeneous PUs, with $\hat{a}_1 = 2.5$ and $\hat{a}_2 = 3.8$. Let $C = 20$ and $\alpha = 0.3$. Then, $\mathbf{c}^* = (12.45, 7.55)$ and $\mathbf{c}^* = (11.36, 8.64)$ are two possible Nash equilibria that satisfy (29).

In what follows, we provide a necessary and sufficient condition for the uniqueness of Nash equilibrium in the market with homogeneous PUs (i.e., $\hat{a}_j = \hat{a}, \forall j \in \mathcal{N}$).

To find the Nash equilibrium of the game at this stage, we first examine the strategy of the j th PU given other PU's decisions. By checking the first order condition $\partial \hat{R}_j(c_j, \mathbf{c}_{-j}) / \partial c_j = 0$ and the boundary conditions, we can obtain the best response strategy of the j th PU. As expected, the best response strategy for the j th PU depends on \hat{a} and its competitors' decision $\mathbf{1}^T \mathbf{c}_{-j}$. Let c^L and c^H be the thresholds for PU's decision making associated with $0 < \hat{a} \leq C^{1-\alpha}$ and $\hat{a} > C^{1-\alpha}$, respectively. They are given explicitly by $c^L = (1/2\alpha)(\alpha - 1 + \sqrt{(\alpha - 1)^2 + 4\alpha\hat{a}C^{\alpha-1}})$ and $c^H = (\hat{a}/C)^{-(1/\alpha)}$ (derivation in Appendix E). We now establish the response strategy for the PUs.

Proposition 3.3: The best response strategy for the j th PU in the above game is outlined as follows.

- 1) The case with $0 < \hat{a} \leq C^{1-\alpha}$:

If $\mathbf{1}^T \mathbf{c}_{-j} \in [0, Cc^L]$, then c_j^* is the solution to

$$(c_j^* + \mathbf{1}^T \mathbf{c}_{-j})^{-\alpha} - \alpha c_j^* (c_j^* + \mathbf{1}^T \mathbf{c}_{-j})^{-\alpha-1} - \frac{\hat{a}}{C - c_j^*} = 0. \quad (30)$$

If $\mathbf{1}^T \mathbf{c}_{-j} \in [Cc^L, C]$, then $c_j^* = C - \mathbf{1}^T \mathbf{c}_{-j}$.

- 2) The case with $\hat{a} > C^{1-\alpha}$:

If $\mathbf{1}^T \mathbf{c}_{-j} \in [0, c^H]$, then c_j^* is the solution to

$$(c_j^* + \mathbf{1}^T \mathbf{c}_{-j})^{-\alpha} - \alpha c_j^* (c_j^* + \mathbf{1}^T \mathbf{c}_{-j})^{-\alpha-1} - \frac{\hat{a}}{C - c_j^*} = 0. \quad (31)$$

If $\mathbf{1}^T \mathbf{c}_{-j} \in [c^H, C]$, then $c_j^* = 0$.

The proof of Proposition 3.3 is given in Appendix E. As expected, the Nash equilibrium of the spectrum opportunity allocation game depends on \hat{a} , α , and the number of PUs. We have the following necessary and sufficient condition for the uniqueness of Nash equilibrium.

Proposition 3.4: The Nash equilibrium of the spectrum opportunity allocation game is unique if and only if $c^L > (|\mathcal{N}| - 1)/|\mathcal{N}|$, and at the spectrum opportunity equilibrium, $c_j^* = c^*, \forall j \in \mathcal{N}$, where c^* is the solution to

$$(|\mathcal{N}|c^*)^{-\alpha} \left(1 - \frac{\alpha}{|\mathcal{N}|} \right) = \frac{\hat{a}}{C - c^*}. \quad (32)$$

The proof is given in Appendix F. Note that there exists a *threshold* for the number of PUs, denoted by N_{PU} , in the case with $0 < c^L < 1$, where N_{PU} is given by $\lceil 1/(1 - c^L) \rceil$.⁹ Accordingly, Proposition 3.4 can be treated as a criterion for the PUs to decide whether to join in the competition or not because each PU can calculate the pricing equilibrium when it gathers the necessary information based on Proposition 3.4. In the case with $0 < c^L < 1$, it needs to check whether the condition $N_{PU} > |\mathcal{N}|$ holds. This is because if $N_{PU} \leq |\mathcal{N}|$, the pricing equilibrium will be p_{\min} , which indicates that it is unprofitable to sell spectrum to SUs due to the strong competition.

C. Algorithm for Computing Nash Equilibria

To achieve the Nash equilibrium of the dynamic game, we present an iterative algorithm for each PU. Based on Proposition 3.1, if $\sum_{j \in \mathcal{N}} c_j > C$, the spectrum allocation is inefficient, i.e., there always exists some PU whose supply is larger than the demand. Thus, each PU first updates its spectrum opportunity based on the demand to fully utilize its spectrum.

⁹ $\lceil x \rceil$ denotes the largest integer not greater than x .

After the necessary condition $\sum_{j \in \mathcal{N}} c_j \leq C$ is satisfied, each PU can update its spectrum opportunity in the Stage I based on Proposition 3.3. We assume that the “total budget” G of SUs is available to each PU. The proposed algorithm for computing the market equilibrium is summarized in Algorithm 2. Based on Propositions 3.1, 3.3, and 3.4, Algorithm 2 provably converges to the equilibrium of the game, as also verified by simulation.

Algorithm 2: Computing the Nash equilibrium of the multiple-PU market

Initialization: Each PU collects the budget information of SUs, i.e., $\{\sigma_i\}$.

At the beginning of each period

- 1) **If** $\sum_{j \in \mathcal{N}} c_j > C$ **then**
 Each PU sets $c_j = eD_j(p_j, \mathbf{p}_{-j})$, and broadcasts c_j .
 Else
 Each PU sets c_j based on Proposition 3.3, and broadcasts c_j .
 Endif
 - 2) Each PU sets $p_j = \max(p_{\min}, (eG / \sum_{j \in \mathcal{N}} c_j)^\alpha)$, and broadcasts p_j .
 - 3) Each SU randomly chooses a PU's channel from the set of PUs with the lowest price in \mathcal{N} with equal probability.
 - 4) Each PU admits new SUs when $c_j/e > D_j(p_j, \mathbf{p}_{-j})$.
-

Remarks: Algorithm 2 is applicable to the scenarios where the PUs can vary their spectrum opportunities. When the spectrum opportunities are fixed, the three-stage game reduces to a two-stage game without the stage of spectrum opportunity allocation. In this case, the equilibrium of the game is given by Proposition 3.1.

D. Numerical Examples: Equilibria for Competitive PUs

In this section, we examine the Nash equilibrium of the three-stage game in the market with homogeneous PUs. First, we illustrate the existence and the uniqueness of the Nash equilibrium for two PUs, in the case with $c^L > 0.5$. Then, we consider a more general system model with four PUs and examine the convergence performance of Algorithm 2. In the end, we demonstrate how the equilibrium price evolves under different elasticities of SUs and different numbers of PUs. In each experiment, C is equal to 20, and each SU's budget σ_i is generated uniformly in the interval $[0, 4]$ and is fixed for different α for the sake of comparison.

The existence and the uniqueness of the Nash equilibrium, corresponding to the competitive spectrum opportunity of two PUs, is illustrated in Fig. 7. Based on Proposition 3.4, when $c^L > 0.5$, there exists a unique Nash equilibrium, as verified in Fig. 7. In particular, we change the inverse elasticity (i.e., α) of SUs from 0.3 to 0.4 in order to show how the spectrum opportunity equilibrium evolves. We observe that the spectrum equilibrium lies on the line with slope one, due to the symmetry of the best response functions, and increases with α since the SUs become more insensitive to prices, which motivates the PUs to offer more spectrum to the SUs.

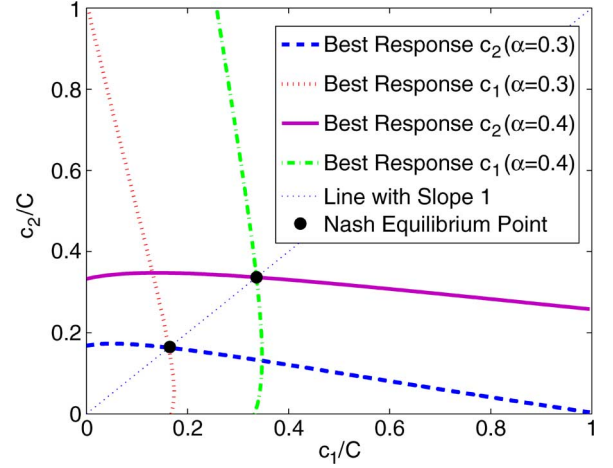


Fig. 7. Illustrating the existence and uniqueness of spectrum opportunity equilibrium.

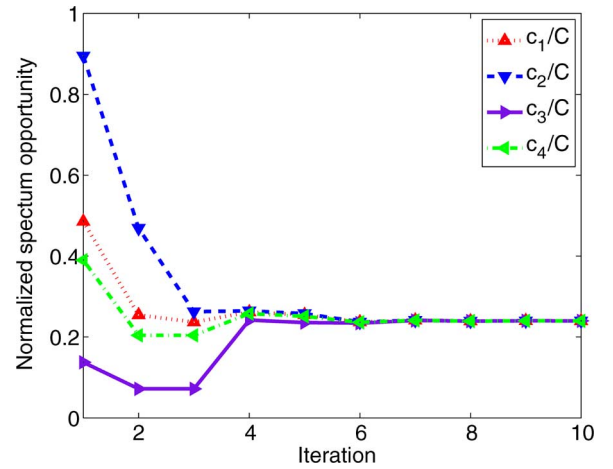


Fig. 8. Convergence of spectrum opportunity equilibrium.

Next, we examine the convergence performance of Algorithm 2 in the case of four PUs. Here, we choose $\alpha = 0.3$ and $a = 30$ such that $0 < \hat{a} \leq C^{1-\alpha}$ and $c^L > 0.75$. As illustrated in Fig. 8, the sum of initial normalized spectrum opportunities¹⁰ is greater than 1, i.e., $\sum_{j \in \mathcal{N}} c_j > C$, in which case there is no equilibrium point based on Proposition 3.1. Each PU then updates its offered spectrum opportunity based on its current demand (this process corresponds to the iterations from 1 to 3 in Fig. 8). Once the total supply is within the feasible region, each PU adjusts its supply based on Proposition 3.3. According to Proposition 3.4, there exists a unique Nash equilibrium, which is further verified in Fig. 8.

Fig. 9 depicts how the equilibrium price evolves under different elasticities of SUs and different numbers of PUs. Specifically, we choose $a = 30$ and $|\mathcal{M}| = 200$. As expected, the equilibrium price increases with α . For each α , the equilibrium price decreases with the number of PUs due to more competition among the PUs. In other words, the SUs can benefit from the competition among the PUs. Moreover, the equilibrium price approaches p_{\min} as the number of PUs increases.

¹⁰The spectrum opportunity of each PU c_j is normalized by C .

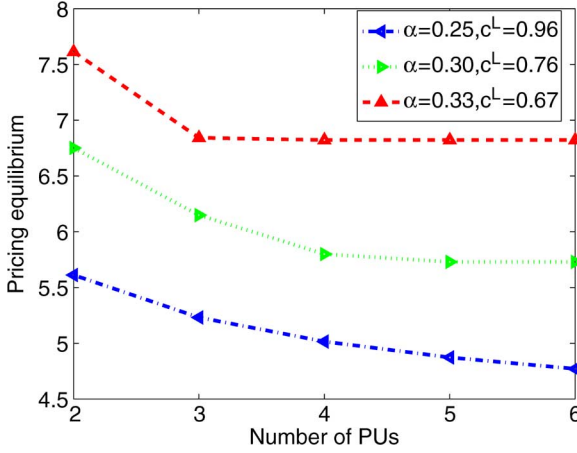


Fig. 9. Price equilibrium corresponding to different elasticities of SUs and different numbers of PUs.

IV. CONCLUSION

This paper studied pricing-based decentralized spectrum access in cognitive radio networks, where SUs compete via random access for available spectrum opportunities. We developed two models: one with the monopoly PU market and the other with the multiple-PU market. For the monopoly PU market model, we applied the revenue maximization approach to characterize the appropriate choice of flat and usage prices, and derived a Pareto-optimal solution, which was shown to be near-optimal. More importantly, this Pareto-optimal solution exhibits a decentralized structure, i.e., the Pareto-optimal pricing strategy and access probabilities can be computed by the PU and the SUs locally. We also analyzed a PU profit maximization problem by examining the tradeoff between the PU's utility and its revenue.

We then studied the multiple-PU market model by casting the competition among PUs as a three-stage Stackelberg game, in terms of access prices and the offered spectrum opportunities. We showed the existence of the Nash equilibrium and derived a necessary and sufficient condition for the uniqueness of Nash equilibrium for the case with homogeneous PUs. Intuitively, this condition can be used by PUs to decide whether to join in the competition or not, i.e., when the number of PUs grows larger than a certain threshold, the competition among PUs is too strong, indicating that it is unprofitable for a PU to sell spectrum to SUs. Then, we developed an iterative algorithm for strategy adaption to achieve the Nash equilibrium.

It remains open to characterize the condition for the uniqueness of Nash equilibrium for the case with heterogeneous PUs. Another interesting direction is to investigate transient behaviors corresponding to dynamic spectrum access in the presence of spectrum hole dynamics.

APPENDIX A PROOF OF LEMMA 2.2

The proof of the first property is contained in that for the second one, which can be derived from the Lagrangian of (8) by

utilizing the Karush–Kuhn–Tucker (KKT) conditions. Specifically, the Lagrangian of (8) is given by

$$L(\mathbf{z}, \mathbf{p}, \boldsymbol{\lambda}) = \sum_{i \in \mathcal{M}} (p_i - \lambda_i) d_i(p_i) + c \sum_{i \in \mathcal{M}} \lambda_i z_i \prod_{j \neq i} (1 - z_j).$$

By the KKT conditions, for optimal \mathbf{z}^* and \mathbf{p}^* , the system must satisfy the following equations:

$$\frac{\partial L}{\partial z_i} = c \lambda_i \prod_{j \neq i} (1 - z_j) - c \sum_{k \neq i} \lambda_k z_k \prod_{j \neq i, j \neq k} (1 - z_j) = 0 \quad \forall i \in \mathcal{M}.$$

Based on [26, Theorem 1], the solution to the above equations is

$$z_i = \frac{\sum_{k \neq i} \lambda_k - (|\mathcal{M}| - 2) \lambda_i}{\sum_{k \in \mathcal{M}} \lambda_k} \quad \forall i \in \mathcal{M}. \quad (33)$$

It follows that

$$\sum_{i \in \mathcal{M}} z_i = 1.$$

To prove the converse for the second property, note that the set of z_i given in (33) is a stationary point for the function $L(\cdot)$. It is straightforward to see that the set of z_i given in (33) cannot achieve a minimum point of the function $L(\cdot)$. Hence, the set of z_i given in (33) must maximize $L(\cdot)$.

APPENDIX B PROOF OF LEMMA 2.3

The Lagrangian of (9) is given by

$$L(\mathbf{p}, \lambda) = \sum_{i \in \mathcal{M}} p_i d_i(p_i) - \lambda \left(\sum_{i \in \mathcal{M}} d_i(p_i) - c\kappa \right).$$

By the KKT conditions, the optimal \mathbf{p}^* of the system must satisfy the following equations:

$$\frac{\partial L}{\partial p_i^*} = d_i(p_i^*) + p_i^* d_i'(p_i^*) - \lambda d_i'(p_i^*) = 0 \quad \forall i \in \mathcal{M}.$$

The above equations can be written as

$$1 - \frac{\lambda}{p_i^*} = -\frac{d_i(p_i^*)}{p_i^* d_i'(p_i^*)} = \alpha \quad \forall i \in \mathcal{M}$$

where $1/\alpha = -(p_i \hat{s}_i'(p_i)/\hat{s}_i(p_i))$ is the elasticity of SUs. Therefore, the optimal prices $p_i^* = \lambda/(1 - \alpha), \forall i \in \mathcal{M}$ are the same. Further based on the constraint of (9), $\sum_{i \in \mathcal{M}} d_i(p_i) = c\kappa$, the optimal price can be derived as (11) by substituting the demand function $d_i(p_i)$ for (4).

APPENDIX C PROOF OF LEMMA 2.4

Since (13) is strictly convex, we can solve it by first considering its dual problem. The Lagrangian of (13) is given by

$$L(\kappa', \mathbf{z}) = \kappa' + \sum_{i \in \mathcal{M}} \lambda_i \left(\log z_i + \sum_{j \neq i} \log(1 - z_j) - \log w_i - \kappa' \right).$$

By the KKT conditions, we have $\sum_{i \in \mathcal{M}} \lambda_i = 1$ and $z_i = \lambda_i$. Thus, the solution of the dual problem satisfies the Pareto-optimal condition, i.e., $\sum_{i \in \mathcal{M}} z_i = 1$, given by Lemma 2.2.

The dual problem can be written as

$$\begin{aligned} & \text{minimize} && \sum_{i \in \mathcal{M}} \lambda_i \left(\log \lambda_i + \sum_{j \neq i} \log(1 - \lambda_j) - \log w_i \right) \\ & \text{subject to} && \sum_{i \in \mathcal{M}} \lambda_i = 1 \\ & && 0 \leq \lambda_i \leq 1, i \in \mathcal{M} \\ & \text{variables} && \{\lambda\}. \end{aligned} \quad (34)$$

By manipulating the summations and utilizing the constraint, the above problem can be written as

$$\begin{aligned} & \text{minimize} && \sum_{i \in \mathcal{M}} \left(\lambda_i \log \frac{\lambda_i}{w_i} + (1 - \lambda_i) \log(1 - \lambda_i) \right) \\ & \text{subject to} && \sum_{i \in \mathcal{M}} \lambda_i = 1 \\ & && 0 \leq \lambda_i \leq 1, i \in \mathcal{M} \\ & \text{variables} && \{\lambda\}. \end{aligned} \quad (35)$$

Since (35) is strictly convex, therefore the optimal solution can be derived by the KKT conditions, which imply that $\lambda_i = w_i / (w_i + e^{-u})$, where u is given by (15).

When the number of SUs is large, let $w_{\min} = \sigma_{\min}^{1/\alpha} / G$, where $\sigma_{\min} = \min_{i \in \mathcal{M}} \sigma_i$. By (15), we have

$$\begin{aligned} 1 &= \sum_{i \in \mathcal{M}} \frac{1}{1 + w_i^{-1} e^{-u}} \\ &\leq |\mathcal{M}| \frac{1}{1 + w_{\min}^{-1} e^{-u}}. \end{aligned}$$

Then, we have $e^{-u} \geq w_{\min} (|\mathcal{M}| - 1)$. For any z_i , we have

$$\begin{aligned} z_i &= \frac{1}{1 + w_i^{-1} e^{-u}} \leq \frac{1}{1 + w_i^{-1} w_{\min} (|\mathcal{M}| - 1)} \\ &= \frac{1}{1 + \left(\frac{\sigma_{\min}}{\sigma_i} \right)^{\frac{1}{\alpha}} (|\mathcal{M}| - 1)} \\ &\rightarrow 0, \text{ as } |\mathcal{M}| \rightarrow \infty. \end{aligned}$$

Thus, $\max_{i \in \mathcal{M}} z_i \rightarrow 0$, as $|\mathcal{M}| \rightarrow \infty$. Since $\sum_{i \in \mathcal{M}} z_i = 1$, we have $\prod_{i \neq j} (1 - z_j) \rightarrow (1/e)$. Therefore, Lemma 2.4 is proved.

APPENDIX D PROOF OF PROPOSITION 3.1

We first show that $\sum_{j \in \mathcal{N}} c_j \leq C$ is the sufficient condition for the existence of the Nash equilibrium price, and that under that condition, the equilibrium price is uniquely determined by (27).

First, we consider the case where there are only two PUs. We show that there does not exist a pricing equilibrium such that $p_1^* \neq p_2^*$. Suppose that PU 1 and PU 2 have positive spectrum opportunities c_1 and c_2 . Without loss of generality, we assume that $c_1 \geq c_2$. The optimal prices for PU 1 and PU 2 are shown in Fig. 10, where the curve $Gp^{1-(1/\alpha)}$ represents the optimal revenue that each PU can earn when there is no competition.

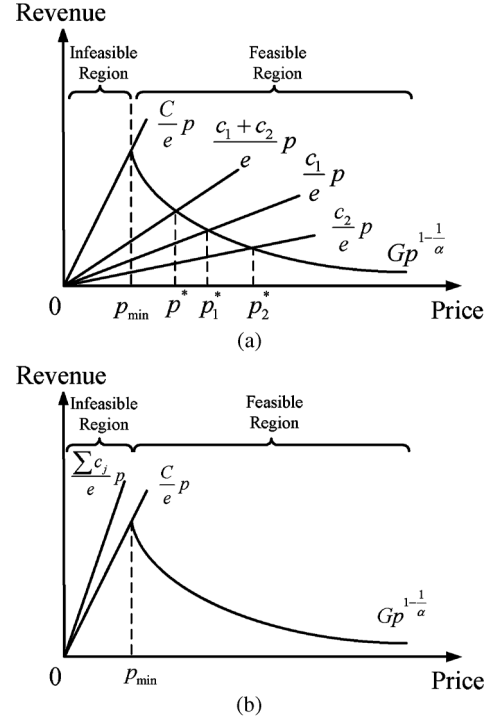


Fig. 10. PU's Revenue at different prices and spectrum opportunities. (a) $\sum_{j \in \mathcal{N}} c_j \leq C$. (b) $\sum_{j \in \mathcal{N}} c_j > C$.

Since it is only when the demand equals to the supply that each PU can achieve its Pareto-optimal revenue, the optimal price for each PU is at the intersection point of the curve and the line shown in Fig. 10. Obviously, $p_1^* \leq p_2^*$ when $c_1 \geq c_2$. From the analysis of Stage I, we know that SUs will choose PU 1, which makes PU 2 have no revenue. In this case, PU 2 will decrease its price at least less than or equal to PU 1 to get some revenue. Also, the price reduction will not end until both PUs announce the same price.

Next, we show that the equilibrium price is at the point p^* as shown in Fig. 10. Suppose that the equilibrium price p'^* is not at the point p^* . In this case, the total demand is not equal to the total supply. Thus, at least one PU's demand is not equal to its supply. For the case $p'^* > p^*$, the total demand is less than the total supply, which means at least one PU's demand is less than its supply. Without loss of generality, we assume that the PU 1's demand is less than its supply. Based on Lemma 2.2, PU 1 will decrease its price to make its demand equal to its supply, which will also make PU 2 decrease its price to achieve pricing equilibrium. Thus, when $p'^* > p^*$, both PUs will decrease their prices. For the other case $p'^* < p^*$, the total demand is larger than the total supply, which means at least one PU's demand is larger than its supply. Without loss of generality, we assume that the PU 1's demand is larger than its supply. Based on (26), PU 1 will increase its price to make its demand equal to its supply. In this case, the price of PU 1 will be larger than the price of PU 2, which makes all SUs choose PU 2. Therefore, the demand of PU 2 will be larger than its supply, which forces PU 2 to increase its price to achieve more revenue. Thus, when $p'^* < p^*$, both PUs will increase their prices. Hence, the equilibrium price is at p^* , which can be determined by (27). Since $p^* \in [p_{\min}, \infty)$, the total supply $\sum_{j \in \mathcal{N}} c_j$ is less than or equal to C .

Thus far, we have discussed the case for two PUs. The above results can be easily generalized to the case with more than two PUs by following similar steps. Therefore, the equilibrium price is uniquely determined by (27), when $\sum_{j \in \mathcal{N}} c_j \leq C$.

To show that $\sum_{j \in \mathcal{N}} c_j \leq C$ is the necessary condition for the existence of the Nash equilibrium price, it suffices to show that no equilibrium exists when $\sum_{j \in \mathcal{N}} c_j > C$. By the definition of *Game at Stage II*, each PU can choose a price p_j from the feasible set $\mathcal{P} = [p_{\min}, \infty)$, where p_{\min} is determined by (10) at $c = C$. When $\sum_{j \in \mathcal{N}} c_j > C$, the equilibrium price would be smaller than p_{\min} , which means that the total supply is greater than the total demand. In other words, some PU's spectrum opportunities are unused. Thus, those PUs can always decrease the supplied spectrum to improve their own utilities and make the demand equal to their supply in the end. Intuitively speaking, since the revenue curve has no intersection point with the line with slope $\sum_{j \in \mathcal{N}} c_j > C$ as shown in Fig. 10, this means that in this region there is no equilibrium point. Therefore, the necessary condition for PUs to achieve the Nash equilibrium is $\sum_{j \in \mathcal{N}} c_j \leq C$.

APPENDIX E PROOF OF PROPOSITION 3.3

Due to the concavity of $\hat{R}_j(c_j, \mathbf{c}_{-j})$, we can obtain the best response function by checking the first-order condition and boundary conditions. The first-order condition is

$$\frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} = (c_j + \mathbf{1}^T \mathbf{c}_{-j})^{-\alpha} - \alpha c_j (c_j + \mathbf{1}^T \mathbf{c}_{-j})^{-\alpha-1} - \frac{\hat{a}}{C - c_j}$$

and the boundary conditions can be written as

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=0} = (\mathbf{1}^T \mathbf{c}_{-j})^{-\alpha} - \frac{\hat{a}}{C} \quad (36)$$

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=C-\mathbf{1}^T \mathbf{c}_{-j}} = C^{-\alpha} \left(1 - \alpha + \frac{\alpha(\mathbf{1}^T \mathbf{c}_{-j})}{C} - \frac{\hat{a}C^\alpha}{\mathbf{1}^T \mathbf{c}_{-j}} \right) \quad (37)$$

both of which depend on its competitors spectrum opportunities and the parameter \hat{a} . For different \mathbf{c}_{-j} and \hat{a} , the j th PU's best response strategy can be written as follows.

1) Case $0 < \hat{a} \leq C^{1-\alpha}$:

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=0} \geq 0$$

i.e., the j th PU would sell spectrum opportunity to the SUs. Then, the optimal spectrum opportunity c_j^* depends on the boundary condition (37). From Fig. 11, we know that there exists a decision threshold $c^L = (1/2\alpha)(\alpha - 1 + \sqrt{(\alpha - 1)^2 + 4\alpha\hat{a}C^{\alpha-1}})$ for PU j , which is the solution to

$$1 - \alpha + \alpha c^L = \frac{\hat{a}C^{\alpha-1}}{c^L}.$$

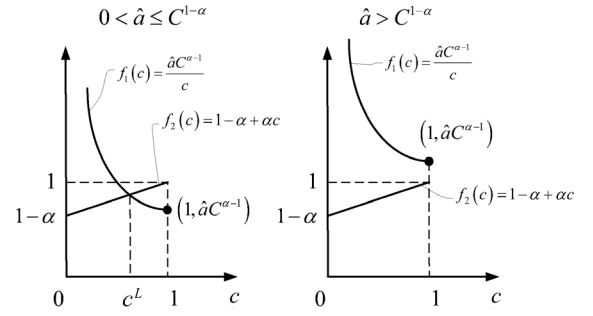


Fig. 11. Decision threshold c^L .

Based on its competitors' spectrum opportunities, the j th PU's decision is outlined as follows.

a) $\mathbf{1}^T \mathbf{c}_{-j} \in [0, Cc^L]$:

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=C-\mathbf{1}^T \mathbf{c}_{-j}} \leq 0.$$

The best response strategy of the j th PU is determined by its first-order condition (30).

b) $\mathbf{1}^T \mathbf{c}_{-j} \in [Cc^L, C]$:

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=C-\mathbf{1}^T \mathbf{c}_{-j}} \geq 0.$$

The best response strategy of the j th PU is to sell as much spectrum opportunity as possible, i.e., $c_j^* = C - \mathbf{1}^T \mathbf{c}_{-j}$.

2) Case $\hat{a} > C^{1-\alpha}$:

From Fig. 11, we know

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=C-\mathbf{1}^T \mathbf{c}_{-j}} \leq 0.$$

The optimal spectrum opportunity c_j depends on the boundary condition (36), from which we know there exists a decision threshold $c^H = (\hat{a}/C)^{-(1/\alpha)}$. Based on c^H and its competitors' spectrum opportunities, the decisions of the j th PU are as follows.

a) $\mathbf{1}^T \mathbf{c}_{-j} \in [0, c^H]$:

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=0} \geq 0.$$

The best response strategy of the j th PU is determined by its first-order condition (31).

b) $\mathbf{1}^T \mathbf{c}_{-j} \in [c^H, C]$:

$$\left. \frac{\partial \hat{R}_j(c_j, \mathbf{c}_{-j})}{\partial c_j} \right|_{c_j=0} \leq 0.$$

The best response strategy of the j th PU is not to sell any spectrum opportunity, i.e., $c_j^* = 0$.

APPENDIX F

PROOF OF PROPOSITION 3.4

Due to the concavity of $\hat{R}_j(c_j, \mathbf{c}_{-j})$ in c_j , the existence of equilibrium can be readily shown based on [25]. In what follows, we will derive the necessary and sufficient condition for the uniqueness of Nash equilibrium, based on the best response strategy.

1) Case $0 < \hat{a} \leq C^{1-\alpha}$:

Define $\mathcal{N}_1 = \{j | c_j^* \text{ is the solution to (30), } \forall j \in \mathcal{N}\}$ and $\mathcal{N}_2 = \{j | c_j^* = C - \mathbf{1}^T \mathbf{c}_{-j}, \forall j \in \mathcal{N}\}$ as the sets of PUs choosing decision c_j^* as the solution to (30) and $c_j^* = C - \mathbf{1}^T \mathbf{c}_{-j}$, respectively, where $|\mathcal{N}_1| + |\mathcal{N}_2| = |\mathcal{N}|$. Let $c_{j_1}^*, \forall j_1 \in \mathcal{N}_1$ and $c_{j_2}^*, \forall j_2 \in \mathcal{N}_2$ denote the spectrum opportunity equilibrium for PUs in the set \mathcal{N}_1 and \mathcal{N}_2 , respectively. Assume that both sets \mathcal{N}_1 and \mathcal{N}_2 are nonempty. Based on Proposition 3.3, we know $\sum_{j \in \mathcal{N}} c_j^* = C$, and $c_{j_1}^*$ is the solution of

$$\begin{aligned} C^{-\alpha} - \alpha c_{j_1}^* C^{-\alpha-1} - \frac{\hat{a}}{C - c_{j_1}^*} &= 0 \\ \Leftrightarrow 1 - \alpha + \alpha \frac{C - c_{j_1}^*}{C} &= \frac{\hat{a} C^\alpha}{C - c_{j_1}^*} \\ \Leftrightarrow 1 - \alpha + \alpha c^L &= \frac{\hat{a} C^{\alpha-1}}{c^L}. \end{aligned}$$

Thus, $c_{j_1}^* = C(1 - c^L), \forall j_1 \in \mathcal{N}_1$. Since $C - c_{j_2}^* \geq Cc^L, \forall j_2 \in \mathcal{N}_2$, we can get $\sum_{j_2 \in \mathcal{N}_2} c_{j_2}^* \leq |\mathcal{N}_2|C(1 - c^L)$. Utilizing the condition $\sum_{j_2 \in \mathcal{N}_2} c_{j_2}^* + |\mathcal{N}_1|C(1 - c^L) = C$ (i.e., $\sum_{j \in \mathcal{N}} c_j^* = C$), c^L needs to satisfy

$$c^L \leq \frac{|\mathcal{N}| - 1}{|\mathcal{N}|}.$$

Therefore, we can summarize the spectrum opportunity equilibria as follows.

a) Case $c^L \leq ((|\mathcal{N}| - 1)/|\mathcal{N}|)$:

In this case, there exist infinitely many spectrum opportunity equilibria that satisfy

$$\begin{aligned} \sum_{j_2 \in \mathcal{N}_2} c_{j_2}^* &= C - |\mathcal{N}_1|C(1 - c^L) \\ c_{j_1}^* &= C(1 - c^L), \forall j_1 \in \mathcal{N}_1 \\ C - c_{j_2}^* &\geq Cc^L, \forall j_2 \in \mathcal{N}_2. \end{aligned}$$

b) Case $c^L > ((|\mathcal{N}| - 1)/|\mathcal{N}|)$:

In this case, we have either $\mathcal{N}_1 = \mathcal{N}, \mathcal{N}_2 = \emptyset$ or $\mathcal{N}_1 = \emptyset, \mathcal{N}_2 = \mathcal{N}$. For the case $\mathcal{N}_1 = \emptyset, \mathcal{N}_2 = \mathcal{N}$, we have $\sum_{j \in \mathcal{N}} c_j^* = C$ and $C - c_j^* \geq Cc^L, \forall j \in \mathcal{N}$, by which we have $\sum_{j \in \mathcal{N}} (C - c_j^*) \geq |\mathcal{N}|Cc^L$, i.e., $|\mathcal{N}| - 1 \geq |\mathcal{N}|c^L$, which yields $1 \geq (|\mathcal{N}|/(|\mathcal{N}| - 1))c^L > 1$ due to $c^L > ((|\mathcal{N}| - 1)/|\mathcal{N}|)$. Obviously, this contradicts the fact $1 = 1$. Hence, the only possible case is $\mathcal{N}_1 = \mathcal{N}, \mathcal{N}_2 = \emptyset$. Due to the homogeneity of the best response function, there exists a unique spectrum opportunity equilibrium [27], i.e., $c_j^* = c^*, \forall j \in \mathcal{N}$, where c^* is the solution to

$$(|\mathcal{N}|c^*)^{-\alpha} - \alpha c^* (|\mathcal{N}|c^*)^{-\alpha-1} = \frac{\hat{a}}{C - c^*}.$$

2) Case $\hat{a} > C^{1-\alpha}$:

Define $\mathcal{N}_1 = \{j | c_j^* \text{ is the solution to (31), } \forall j \in \mathcal{N}\}$ and $\mathcal{N}_2 = \{j | c_j^* = 0, \forall j \in \mathcal{N}\}$ as the sets of PUs choosing decision c_j^* as the solution to (31) and $c_j^* = 0$, respectively, where $|\mathcal{N}_1| + |\mathcal{N}_2| = |\mathcal{N}|$. Let $c_{j_1}^*, \forall j_1 \in \mathcal{N}_1$ and $c_{j_2}^*, \forall j_2 \in \mathcal{N}_2$ denote the spectrum opportunity equilibrium for PUs in the set \mathcal{N}_1 and \mathcal{N}_2 , respectively. Based on Proposition 3.3, we know that $c_{j_2}^* = 0$, and we can use (31) to calculate $c_{j_1}^*$. Then, $c_{j_1}^* = c^*, \forall j_1 \in \mathcal{N}_1$, where c^* is the solution to

$$(|\mathcal{N}_1|c^*)^{-\alpha} - \alpha c^* (|\mathcal{N}_1|c^*)^{-\alpha-1} = \frac{\hat{a}}{C - c^*}.$$

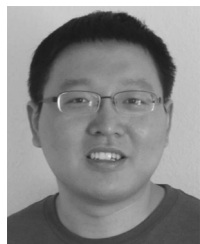
After some algebra, we have $|\mathcal{N}_1|c^* < (\hat{a}/C)^{-(1/\alpha)}$. Since for each PU in \mathcal{N}_2 , $|\mathcal{N}_1|c^* < (\hat{a}/C)^{-(1/\alpha)}$, i.e., $\mathbf{1}^T \mathbf{c}_{-j} \in [0, c^H]$, the best response for PUs in \mathcal{N}_2 is the decision (31), which shows that all PUs will choose the decision (31). Due to the homogeneity of best response function, there exists a unique spectrum opportunity equilibrium [27], which can be determined by (32).

When $\hat{a} > C^{1-\alpha}$, we have $c^L > 1$ based on the proof of Proposition 3.3. Therefore, $c^L > ((|\mathcal{N}| - 1)/|\mathcal{N}|)$. In summary, when $c^L > ((|\mathcal{N}| - 1)/|\mathcal{N}|)$, there exists a unique Nash equilibrium. The other direction follows directly based on the above discussion of spectrum opportunity equilibria.

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