Max-Min Weighted SINR in Coordinated Multicell MIMO Downlink

Desmond W. H. Cai*, Tony Q. S. Quek†, Chee Wei Tan‡, Steven H. Low*
*California Institute of Technology, 1200 E. California Blvd., Pasadena, CA 91125
†Institute for Infocomm Research, A*STAR, 1 Fusionopolis Way, #21-01 Connexis, Singapore 138632
‡City University of Hong Kong, Tat Chee Avenue, Kowloon, Hong Kong
Email: wccai@caltech.edu, qsquek@i2r.a-star.edu.sg, cheewtan@cityu.edu.hk, slow@caltech.edu

***Abstract*—This paper studies the optimization of a multicell multiple-input-single-output (MISO) downlink system in which each base station serves multiple users, and each user is served by only one base station. First, we consider the problem of maximizing the minimum weighted signal-to-interference-plus-noise ratio (SINR) of all users subject to a single weighted-sum power constraint, where the weights can represent relative power costs of serving different users in each cell. We apply concave Perron-Frobenius theory to propose a joint power control and linear beamforming algorithm which converges geometrically fast to the optimal solution. As a by-product, we resolve an open problem of convergence of a previously proposed algorithm by Wiesel, Eldar, and Shamai in 2006. Next, we study the max-min weighted SINR problem subject to multiple weighted-sum power constraints and we show that it can be decoupled into its associated single-constrained subproblems.**

I. Introduction

This paper considers a wireless network with arbitrary topology, in which each node is equipped with multiple antennas. In conventional wireless systems, there is limited coordination between base stations from different cells. When a base station communicates with its receiver, intercell interference is typically treated as background noise. Unfortunately, this approach often degrades the performance of users on the cell edge who suffer from both high signal attenuation and severe intercell interference. Clearly, significant performance gains can be realized if base stations from different cells coordinate their processing to reduce or cancel intercell interference.

This paper considers the goal of maximizing the minimum weighted signal-to-interference-plus-noise ratio (SINR) subject to multiple weighted-sum power constraints in a multiple-input-single-output (MISO) downlink system which employs linear transmit beamforming. Multiple power constraints are especially relevant to coordinated multicell processing because each base station could be subject to its own transmit power constraint. The weights can represent relative power costs of serving different users. While an important challenge in coordinated multicell processing is to achieve the best performance gains with minimal backhaul burden [1], in this paper, we focus specifically on obtaining optimal solutions to the problem.

Initial work on the max-min weighted SINR problem was in the context of power control subject to a single total power constraint. Specifically, the earliest strategy proposed to solve this problem was the extended coupling matrix approach [2]. Subsequently, this approach was extended to the MISO and multiple-input-multiple-output (MIMO) problems in [3]–[6]. The extended coupling matrix approach is computationally intensive as it requires a centralized power update involving an eigenvector computation.

The work in this paper is motivated by two recent approaches toward obtaining more efficient algorithms for the max-min weighted SINR problem. The first related approach is in [7], where the authors proposed a fixed-point algorithm for the MISO max-min SINR problem. Their algorithm was motivated from the fixed-point algorithm for the closely-related total power minimization problem. Although no convergence proof was given, rapid convergence was observed. The second related approach is in [8] and [9], where the authors showed a connection between concave Perron-Frobenius theory and the max-min weighted SINR power control problem subject to a single total power constraint [8] and individual power constraints [9]. By exploiting this connection, the authors proposed an algorithm which resembled the distributed power control (DPC) algorithm for the total power minimization problem [10], [11] and avoided the centralized computation necessary in the extended coupling matrix approach. Our work extends the application of concave Perron-Frobenius theory to the MISO max-min weighted SINR problem and, as a by-product, resolves the convergence of the proposed algorithm in [7].

In the context of multiple power constraints, one strategy was proposed for the MISO sum-rate maximization problem subject to transmit covariance constraints [12], and subsequently for arbitrary linear transmit power constraints [13]. It involved forming a relaxed single-constrained problem by taking a parameterized sum of the original constraints. It turns out that the solution to the relaxed problem is a tight upper-bound to the solution of the original multiple-constrained problem. Hence, the multiple-constrained problem can be solved using an inner-outer (subgradient) iterative algorithm

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[12], [13]. We derive a similar approach for the max-min weighted SINR problem subject to multiple weighted-sum power constraints.

This paper is organized as follows. We begin in Section II by introducing the problem formulation. Next, in Section III, we apply concave Perron-Frobenius theory to the MISO max-min weighted SINR problem subject to a single weighted-sum power constraint. Specifically, we derive a fixed-point equation for the problem, and we apply concave Perron-Frobenius theory to propose an algorithm for solving the fixed-point equation. As a by-product, we resolve the open problem of convergence of the previously proposed algorithm in [7]. In Section IV, we apply our result for the single-constrained problem to show that the multiple-constrained problem can be decoupled into its associated single-constrained subproblems. In the process, we show that the multiple-constrained problem can also be solved using an inner-outer (subgradient) iterative algorithm. In Section V, we provide numerical examples to illustrate the convergence of the inner-outer (subgradient) iterative algorithm. Finally, Section VI concludes the paper.

The following notations are used in this paper:

- Boldface upper-case letters denote matrices, boldface lowercase letters denote column vectors, italics denote scalars.
- For $\mathbf{u}, \mathbf{v} \in \mathbb{R}^L$, $\mathbf{u} \leq \mathbf{v}$ denotes $u_i \leq v_i$ for $1 \leq i \leq L$, $\mathbf{u} < \mathbf{v}$ denotes $u_i < v_i$ for $1 \leq i \leq L$, and $\mathbf{u} \lneqq \mathbf{v}$ denotes $\mathbf{u} \leq \mathbf{v}$ and $\mathbf{u} \neq \mathbf{v}$.
- $\rho(\mathbf{F})$ denotes the Perron-Frobenius eigenvalue of a nonnegative matrix $\mathbf{F}$.
- $\mathbf{x}(\mathbf{F})$ denotes the Perron right eigenvector of $\mathbf{F}$ associated with $\rho(\mathbf{F})$.
- $\text{diag}(\mathbf{v})$ denotes the diagonal matrix formed by the components of the vector $\mathbf{v}$.
- $(\cdot)^\mathsf{T}$ and $(\cdot)^\dagger$ denote transpose and complex conjugate transpose respectively.
- $\mathbf{e}_j$ denotes the unit vector with a 1 at the jth entry and 0's elsewhere.
- $\mathbb{R}_+$ denotes the set of nonnegative real numbers, $\mathbb{R}_{++}$ denotes the set of positive real numbers, and $\mathbb{C}$ denotes the set of complex numbers.
- $\|\cdot\|$ denotes the ℓ^2 norm.

II. System Model

We consider a MIMO network where L independent data streams are transmitted over a common frequency band. The transmitter and receiver of the lth stream, denoted by t_l and r_l respectively, are equipped with N_{t_l} antennas and N_{r_l} antennas respectively. The downlink channel can be modeled as a Gaussian broadcast channel given by

$$\mathbf{y}_l = \sum_{i=1}^{L} \mathbf{H}_{li}\mathbf{x}_i + \mathbf{z}_l, \quad l = 1, \dots, L,$$

where $\mathbf{y}_l \in \mathbb{C}^{N_{r_l}}$ is the received signal at r_l, $\mathbf{H}_{li} \in \mathbb{C}^{N_{r_l} \times N_{t_i}}$ is the channel vector between t_i and r_l, $\mathbf{x}_i \in \mathbb{C}^{N_{t_i}}$ is the transmitted signal vector of the ith stream, and $\mathbf{z}_l \sim \mathcal{CN}(\mathbf{0}, n_l\mathbf{I})$ is the circularly symmetric Gaussian noise vector with covariance $n_l\mathbf{I}$ at r_l, where $n_l \in \mathbb{R}_{++}$. The transmitted signal vector of the ith stream can be written as $\mathbf{x}_i = d_i\sqrt{p_i}\mathbf{u}_i$, where $\mathbf{u}_i \in \mathbb{C}^{N_{t_i}}$ is the normalized transmit beamformer, and d_i and $p_i \in \mathbb{R}_{++}$ are the information signal and transmit power respectively for that stream. The lth stream is decoded using a normalized receive beamformer $\mathbf{v}_l \in \mathbb{C}^{N_{r_l}}$. Define the power vector $\mathbf{p} = [p_1, \dots, p_L]^\mathsf{T}$, the tuples of beamforming matrices $\mathbb{U} = (\mathbf{u}_1, \dots, \mathbf{u}_L)$ and $\mathbb{V} = (\mathbf{v}_1, \dots, \mathbf{v}_L)$, and the noise vector $\mathbf{n} = [n_1, \dots, n_L]^\mathsf{T}$. Define the matrix $\mathbf{G} \in \mathbb{R}_{++}^{L \times L}$ where the entry $G_{li} = |\mathbf{v}_l^\dagger \mathbf{H}_{li}\mathbf{u}_i|^2$ is the effective link gain between t_i and r_l. We assume the receive beamformers are chosen such that all the effective links are coupled so $G_{li} > 0$ for all l and i. The SINR of the lth stream can be expressed as

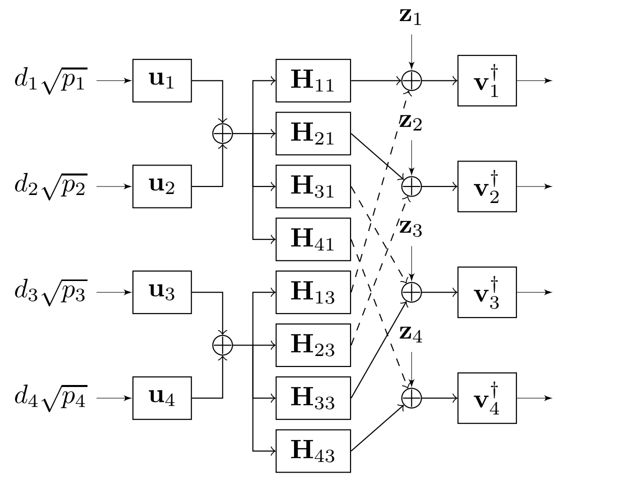

Fig. 1. Block diagram of a two-cell MIMO downlink system. Each cell serves two users. Dotted lines denote the inter-cell interfering paths.

$$\mathsf{SINR}_l^{\mathsf{DL}}(\mathbf{p}, \mathbb{U}) := \frac{p_l G_{ll}}{\sum_{i \neq l} p_i G_{li} + n_l}.$$

Define the priority vector $\boldsymbol{\beta} = [\beta_1, \dots, \beta_L]^\mathsf{T}$ where $\beta_l \in \mathbb{R}_{++}$ is the priority assigned by the network to the lth stream. For ease of notation, we also define the (cross channel interference) matrix $\mathbf{F} \in \mathbb{R}_{++}^{L \times L}$ and the normalized priority vector $\hat{\boldsymbol{\beta}} \in \mathbb{R}_{++}^L$ with the following entries:

$$F_{li} := \begin{cases} 0, & \text{if} \quad l = i, \\ G_{li}, & \text{if} \quad l \neq i, \end{cases}$$

$$\hat{\boldsymbol{\beta}} := \left(\frac{\beta_1}{G_{11}}, \dots, \frac{\beta_L}{G_{LL}} \right)^\mathsf{T}.$$

With these definitions, the weighted SINR of the lth stream can be expressed as

$$\frac{\mathsf{SINR}_l^{\mathsf{DL}}(\mathbf{p}, \mathbb{U})}{\beta_l} = \frac{p_l}{(\text{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p} + \mathbf{n}))_l}.$$

We are interested in the J-constrained max-min weighted

SINR problem in the downlink given by

$$\mathcal{U}^{\mathsf{DL}} := \begin{cases} \max\limits_{\mathbf{p},\mathbb{U}} & \min\limits_{l} \frac{p_l}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p}+\mathbf{n}))_l} \\ \text{subject to} & \mathbf{w}_j^{\mathsf{T}}\mathbf{p} \le \bar{P}_j, \quad j=1,\dots,J, \\ & \mathbf{p} > \mathbf{0}, \\ & \|\mathbf{u}_l\| = 1, \quad l = 1,\dots,L, \end{cases} \tag{1}$$

where $\bar{P}_j \in \mathbb{R}_{++}$ is the given jth power constraint and $\mathbf{w}_j = [w_{j1},\dots,w_{jL}]^{\mathsf{T}}$ is the weight vector such that $w_{jl} \in \mathbb{R}_{++}$ is the weight associated with p_l in the jth power constraint. The restriction that the weights be positive is for technical purposes and is not a stringent limitation on the model. In cases where a certain stream must be excluded from a power constraint, the weight on that stream in the power constraint can simply be chosen to be extremely small. Since $G_{li} > 0$ for all l and i, we have that at the optimal solution to $\mathcal{U}^{\mathsf{DL}}$, all the weighted SINR's are equal. Moreover, at least one power constraint must be active.

We also consider the simpler single-constrained downlink subproblem given by

$$\mathcal{U}_j^{\mathsf{DL}} := \begin{cases} \max\limits_{\mathbf{p},\mathbb{U}} & \min\limits_{l} \frac{p_l}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p}+\mathbf{n}))_l} \\ \text{subject to} & \mathbf{w}_j^{\mathsf{T}}\mathbf{p} \le \bar{P}_j, \quad \mathbf{p} > \mathbf{0}, \\ & \|\mathbf{u}_l\| = 1, \quad l = 1,\dots,L. \end{cases}$$

For reasons similar to the case of $\mathcal{U}^{\mathsf{DL}}$, we have that at the optimal solution to $\mathcal{U}_j^{\mathsf{DL}}$, all the weighted SINR's are equal and the single power constraint must be tight.

III. Single Power Constrained Analysis

In this section, we exploit the concave Perron-Frobenius theory to propose an algorithm for solving $\mathcal{U}_j^{\mathsf{DL}}$. We begin by considering the subproblem obtained by fixing the transmit beamformers $\mathbb{U}$ in $\mathcal{U}_j^{\mathsf{DL}}$:

$$\mathcal{S}_j^{\mathsf{DL}}(\mathbb{U}) := \begin{cases} \max\limits_{\mathbf{p}} & \min\limits_{l} \frac{p_l}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p}+\mathbf{n}))_l} \\ \text{subject to} & \mathbf{w}_j^{\mathsf{T}}\mathbf{p} \le \bar{P}_j, \quad \mathbf{p} > \mathbf{0}. \end{cases}$$

For notational brevity, we will often omit the dependence on $\mathbb{U}$. By introducing an auxiliary variable τ into $\mathcal{S}_j^{\mathsf{DL}}$ and making a change of variables $\tilde{\tau} = \log \tau$ and $\tilde{p}_l = \log p_l$, we arrive at the following equivalent convex problem:

$$\begin{array}{ll} \min\limits_{\tilde{\tau},\tilde{\mathbf{p}}} & -\tilde{\tau} \\ \text{subject to} & \log\left(\frac{e^{\tilde{\tau}}(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}e^{\tilde{\mathbf{p}}}+\mathbf{n}))_l}{(e^{\tilde{\mathbf{p}}})_l}\right) \le 0, \quad l = 1,\dots,L, \\ & \log\left(\frac{1}{\bar{P}_j}\mathbf{w}_j^{\mathsf{T}}e^{\tilde{\mathbf{p}}}\right) \le 0. \end{array} \tag{2}$$

The Lagrangian associated with (2) is

$$\mathcal{L}(\tilde{\tau},\tilde{\mathbf{p}},\boldsymbol{\lambda},\mu) = -\tilde{\tau} + \sum_l \lambda_l \log\left(\frac{e^{\tilde{\tau}}(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}e^{\tilde{\mathbf{p}}}+\mathbf{n}))_l}{(e^{\tilde{\mathbf{p}}})_l}\right) + \mu \log\left(\frac{1}{\bar{P}_j}\mathbf{w}_j^{\mathsf{T}}e^{\tilde{\mathbf{p}}}\right),$$

where λ_l and μ are the Lagrange dual variables.

It is easy to check that the convex problem given by (2) satisfies Slater's condition. Hence, the Karush-Kuhn Tucker (KKT) conditions of (2), which are given on the next page, are necessary and sufficient conditions for optimality of (2). Recall that, at the optimal solution to (2), all the inequality constraints must be active. Hence, we can replace the inequality constraints in the primal feasibility conditions with equality, and drop the complementary slackness conditions. Making a change of variables back into τ and $\mathbf{p}$, we arrive at the following necessary and sufficient conditions for the optimal solution of $\mathcal{S}_j^{\mathsf{DL}}$:

$$\tau_* = \frac{p_{*l}}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p}_* + \mathbf{n}))_l}, \quad l = 1,\dots,L, \tag{3}$$

$$\mathbf{w}_j^{\mathsf{T}}\mathbf{p}_* = \bar{P}_j, \tag{4}$$

$$\tau_* = \frac{q_{*l}}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}^{\mathsf{T}}\mathbf{q}_* + \mathbf{w}_j))_l}, \quad l = 1,\dots,L, \tag{5}$$

$$\mathbf{1}^{\mathsf{T}}\boldsymbol{\lambda}_* = 1, \tag{6}$$

$$\mathbf{q}_* = \frac{\tau_*\bar{P}_j}{\mu_*}\cdot\left(\frac{\lambda_{*1}\hat{\beta}_1}{p_{*1}},\dots,\frac{\lambda_{*L}\hat{\beta}_L}{p_{*L}}\right)^{\mathsf{T}}, \tag{7}$$

$$\lambda_{*l} > 0, \quad l = 1,\dots,L, \tag{8}$$

$$\mu_* > 0, \tag{9}$$

where $\tau_*, \mathbf{p}_*$ and $\boldsymbol{\lambda}_*, \mu_*$ are the optimal primal and dual variables respectively. Here, (5) follows from $\partial\mathcal{L}/\partial\tilde{p}_l$, and (8)-(9) follows from the fact that λ_{*l} and μ_* must be strictly positive in order for $\tilde{p}_{*l}$ and $\mathcal{L}(\tilde{\tau}_*,\tilde{\mathbf{p}}_*,\boldsymbol{\lambda}_*,\mu_*)$ to be bounded from below. Notice that $\mathbf{q}_*$ is strictly positive and finite.

From (5), we conclude that $\mathbf{q}_*$ is the optimal *dual uplink power* and $\mathbf{w}_j$ is the noise vector in the *dual uplink network*[1] [11], [14], [15]. To obtain the equivalent power constraint in the dual uplink network, we first rewrite (5) in vector form as

$$\mathbf{F}^{\mathsf{T}}\mathbf{q}_* + \mathbf{w}_j = \frac{1}{\tau_*}\mathrm{diag}(\hat{\boldsymbol{\beta}})^{-1}\mathbf{q}_*.$$

Then, taking the inner product with $\mathbf{p}_*$ and substituting for $\mathbf{F}\mathbf{p}_*$ and $\mathbf{w}_j^{\mathsf{T}}\mathbf{p}_*$ from (3) and (4), we obtain

$$\mathbf{n}^{\mathsf{T}}\mathbf{q}_* = \bar{P}_j. \tag{10}$$

Notice that (3)-(9) are also necessary conditions for the optimal solution of $\mathcal{U}_j^{\mathsf{DL}}$. To obtain necessary conditions for the optimal transmit beamformers $\mathbb{U}_*$, first apply the constraint $\|\mathbf{u}_{*l}\| = 1$ and rewrite (5) as

$$q_{*l}\mathbf{u}_{*l}^{\dagger}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l\mathbf{v}_l^{\dagger}\mathbf{H}_{ll}\mathbf{u}_{*l} = \beta_l\tau_*\mathbf{u}_{*l}^{\dagger}\left(\sum_{i\neq l} q_{*i}\mathbf{H}_{il}^{\dagger}\mathbf{v}_i\mathbf{v}_i^{\dagger}\mathbf{H}_{il} + w_{jl}\mathbf{I}\right)\mathbf{u}_{*l}, \tag{11}$$

for $l = 1,\dots,L$. Let the mapping $\mathbf{M}_l : \mathbb{R}_+^L \to \mathbb{C}^{L\times L}$ be defined by:

$$\mathbf{M}_l(\mathbf{q}_*) := \sum_{i\neq l} q_{*i}\mathbf{H}_{il}^{\dagger}\mathbf{v}_i\mathbf{v}_i^{\dagger}\mathbf{H}_{il} + w_{jl}\mathbf{I},$$

[1] The dual uplink network is a network in which the directions of the original links are reversed, and the original transmit and receive beamformers become receive and transmit beamformers respectively. For a given set of downlink powers, the associated dual uplink powers are the transmit powers in the uplink network such that the SINR of each uplink stream is equal to the SINR of the original downlink stream.

Karush-Kuhn Tucker (KKT) conditions of (2):

Primal Feasibility:

$$\log\left(\frac{e^{\tilde{\tau}}(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}e^{\tilde{\mathbf{p}}}+\mathbf{n}))_l}{(e^{\tilde{\mathbf{p}}})_l}\right)\leq 0,\quad l=1,\ldots,L,$$

$$\log\left(\frac{1}{\bar{P}_j}\mathbf{w}_j^{\mathsf{T}}e^{\tilde{\mathbf{p}}}\right)\leq 0,$$

Dual Feasibility:

$$\lambda_l\geq 0,\quad l=1,\ldots,L,$$

$$\mu\geq 0,$$

Complementary Slackness:

$$\lambda_l\log\left(\frac{e^{\tilde{\tau}}(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}e^{\tilde{\mathbf{p}}}+\mathbf{n}))_l}{(e^{\tilde{\mathbf{p}}})_l}\right)=0,\quad l=1,\ldots,L,$$

$$\mu\log\left(\frac{1}{\bar{P}_j}\mathbf{w}_j^{\mathsf{T}}e^{\tilde{\mathbf{p}}}\right)=0,$$

Stationarity:

$$\frac{\partial\mathcal{L}}{\partial\tilde{p}_l}=\sum_{i\neq l}\lambda_i\frac{(e^{\tilde{\mathbf{p}}})_i}{e^{\tilde{\tau}}(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}e^{\tilde{\mathbf{p}}}+\mathbf{n}))_i}\cdot\frac{e^{\tilde{\tau}}\hat{\beta}_iF_{il}(e^{\tilde{\mathbf{p}}})_l}{(e^{\tilde{\mathbf{p}}})_i}-\lambda_l+\mu\frac{w_{jl}(e^{\tilde{\mathbf{p}}})_l}{\mathbf{w}_j^{\mathsf{T}}e^{\tilde{\mathbf{p}}}}=0,\quad l=1,\ldots,L,$$

$$\frac{\partial\mathcal{L}}{\partial\tilde{\tau}}=-1+\sum_i\lambda_i=0.$$

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for $l=1,\ldots,L$. Since $w_{jl}>0$, we have that $\mathbf{M}_l(\mathbf{q}_*)$ is positive definite for any $\mathbf{q}_*$ and hence it is invertible. By applying the fact that $\mathbf{u}_{*l}$'s are normalized minimum-variance distortionless response (MVDR) beamformers in the dual uplink network [16]:

$$\mathbf{u}_{*l}=\frac{\mathbf{M}_l(\mathbf{q}_*)^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l}{\|\mathbf{M}_l(\mathbf{q}_*)^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l\|},\quad l=1,\ldots,L,$$

we eliminate $\mathbf{u}_{*l}$ from (11) and obtain the following fixed-point equation:

$$\frac{1}{\tau_*}\mathbf{q}_*=f(\mathbf{q}_*),\tag{12}$$

where the mapping $f:\mathbb{R}_+^L\to\mathbb{R}_+^L$ is defined by:

$$f_l(\mathbf{q}_*):=\frac{\beta_l}{\mathbf{v}_l^{\dagger}\mathbf{H}_{ll}\mathbf{M}_l(\mathbf{q}_*)^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l},$$

for $l=1,\ldots,L$. We now state our key result for the single-constrained problem.

Theorem 1: Define the norm $\|\cdot\|_\xi$ on $\mathbb{R}^L$ as follows: $\|\mathbf{q}\|_\xi:=(1/\bar{P}_j)\sum_l n_l|q_l|$. The conditional eigenvalue problem given by $(1/\tau)\mathbf{q}=f(\mathbf{q})$ and $\|\mathbf{q}\|_\xi=1$ has a unique solution $\mathbf{q}=\mathbf{q}_*$, $\tau=\tau_*$, and $\mathbf{q}_*>\mathbf{0}$, $\tau_*>0$. Moreover, the normalized fixed-point iteration $\tilde{f}(\mathbf{q}^{(n+1)})=(1/\|f(\mathbf{q}^{(n)})\|_\xi)f(\mathbf{q}^{(n)})$ converges to $\mathbf{q}_*$ geometrically fast.

Proof: First, we show that f is a concave mapping. It was shown in [7] that for a positive semidefinite matrix $\mathbf{A}$ and a vector $\mathbf{c}$ in the range of $\mathbf{A}$, we have

$$\frac{1}{\mathbf{c}^{\dagger}\mathbf{A}^{-1}\mathbf{c}}=\min_{\mathbf{z}:\mathbf{c}^{\dagger}\mathbf{z}=1}\mathbf{z}^{\dagger}\mathbf{A}\mathbf{z},$$

where the matrix inverse denotes the Moore Penrose pseudoinverse. Recall that $\mathbf{M}_l(\mathbf{q})$ is invertible so it has full rank. Hence, $\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l$ is in the range of $\mathbf{M}_l(\mathbf{q})$. Choosing $\mathbf{A}=\mathbf{M}_l(\mathbf{q})$ and $\mathbf{c}=\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l$, we have that

$$f_l(\mathbf{q})=\min_{\mathbf{z}:\mathbf{v}_l^{\dagger}\mathbf{H}_{ll}\mathbf{z}=1}\beta_l\mathbf{z}^{\dagger}\mathbf{M}_l(\mathbf{q})\mathbf{z}.$$

Since $\mathbf{M}_l(\mathbf{q})$ is an affine mapping of $\mathbf{q}$, and the point-wise minimum of affine mappings is concave, we have that $f_l(\mathbf{q})$ is a concave mapping of $\mathbf{q}$. Since $\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l\neq\mathbf{0}$ (because $G_{li}>0$ by assumption) and $w_{jl}>0$, we have that $f_l(\mathbf{q})>0$ for all $\mathbf{q}\gneqq\mathbf{0}$. The rest of the proof follows from applying Theorem 1 in [17]. That the fixed-point iteration converges geometrically fast follows from the remark after Theorem 1 in [17]. □

Theorem 1 implies that the optimal dual uplink power $\mathbf{q}_*$ in $\mathcal{U}_j^{\mathsf{DL}}$ is unique. With the optimal dual uplink power, $\mathbb{U}_*$ can be computed using the normalized MVDR beamformers, and $\mathbf{p}_*$ can be computed using the max-min weighted SINR power control algorithm in [15]. In fact, since the iteration for $\mathbf{q}$ does not depend on that for $\mathbf{p}$ and $\mathbb{U}$ (cf. steps 1 and 2 of Algorithm 1 below), the latter two quantities can be updated in parallel with $\mathbf{q}$. The following algorithm converges to the optimal solution of $\mathcal{U}_j^{\mathsf{DL}}$.

Algorithm 1: Single-constrained max-min weighted SINR

- Initialize: arbitrary $\mathbf{q}^{(0)}>\mathbf{0}$, $\mathbf{p}^{(0)}>\mathbf{0}$, $\{\mathbb{U}^{(0)}:\|\mathbf{u}_l^{(0)}\|=1\ \forall\ l\}$.

1) Update dual auxiliary variables:

$$q_l^{(n+1)}=\frac{1}{\mathbf{v}_l^{\dagger}\mathbf{H}_{ll}\mathbf{M}_l(\mathbf{q}^{(n)})^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l},$$

for $l=1,\ldots,L$.

2) Normalize dual auxiliary variables to update the dual uplink power $\mathbf{q}^{(n+1)}$:

$$\mathbf{q}^{(n+1)}\leftarrow\left(\frac{\bar{P}_j}{\mathbf{n}^{\mathsf{T}}\mathbf{q}^{(n+1)}}\right)\cdot\mathbf{q}^{(n+1)}.$$

3) Update auxiliary variables:
$$p_l^{(n+1)} = \frac{\beta_l}{\mathsf{SINR}_l^{\mathsf{DL}}(\mathbf{p}^{(n)}, \mathbb{U}^{(n)})},$$
for $l = 1, \dots, L$.
4) Normalize auxiliary variables to update the downlink power $\mathbf{p}^{(n+1)}$:
$$\mathbf{p}^{(n+1)} \leftarrow \left(\frac{\bar{P}_j}{\mathbf{w}_j^\mathsf{T}\mathbf{p}^{(n+1)}}\right) \cdot \mathbf{p}^{(n+1)}.$$
5) Compute MVDR transmit beamformers $\mathbb{U}^{(n+1)}$:
$$\mathbf{u}_l^{(n+1)} = \left(\sum_{i \neq l} q_i^{(n+1)} \mathbf{H}_{il}^\dagger \mathbf{v}_i \mathbf{v}_i^\dagger \mathbf{H}_{il} + w_{jl}\mathbf{I}\right)^{-1} \mathbf{H}_{ll}^\dagger \mathbf{v}_l,$$
$$\mathbf{u}_l^{(n+1)} \leftarrow \frac{1}{\|\mathbf{u}_l^{(n+1)}\|} \cdot \mathbf{u}_l^{(n+1)},$$
for $l = 1, \dots, L$.

Remark 1: Observe that Algorithm 1 resembles a previously proposed algorithm for the max-min weighted SINR problem [8]. However, there are two differences between Algorithm 1 and the algorithm in [8]. First, the algorithm in [8] was derived by applying concave Perron-Frobenius theory to the downlink power control problem and then exploiting uplink-downlink duality to update the transmit beamformers. On the other hand, Algorithm 1 was derived by applying concave Perron-Frobenius theory to the dual uplink fixed-point equation and then exploiting uplink-downlink duality to update the downlink power and transmit beamformers (the relationship between both algorithms is further investigated in [18]). Second, the algorithm in [8] only treated the specific case of a total power constraint while Algorithm 1 handles a general weighted-sum power constraint.

Remark 2: In [7], the authors studied $\mathcal{U}_j^{\mathsf{DL}}$ for the special case when $\boldsymbol{\beta} = \mathbf{1}$ and proposed a different fixed-point iteration for the problem. Although no convergence proof was provided, rapid convergence was observed. It is easy to modify the proof of Theorem 1 to prove the geometric convergence of the proposed fixed-point iteration in [7].

IV. Multiple Power Constrained Analysis

In this section, we study the multiple-constrained max-min weighted SINR problem $\mathcal{U}^{\mathsf{DL}}$. Specifically, we use Theorem 1 to show that $\mathcal{U}^{\mathsf{DL}}$ can be solved by solving J associated single-constrained subproblems. In the process, we also show that $\mathcal{U}^{\mathsf{DL}}$ can be divided into an inner and outer optimization problem, which can be solved by iteratively implementing Algorithm 1 and a subgradient projection algorithm.

We begin our analysis by considering the multiple-constrained subproblem obtained by fixing the beamformers $\mathbb{U}$ in $\mathcal{U}^{\mathsf{DL}}$:
$$\mathcal{S}^{\mathsf{DL}}(\mathbb{U}) := \begin{cases} \max\limits_{\mathbf{p}} & \min\limits_{l} \frac{p_l}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p}+\mathbf{n}))_l} \\ \text{subject to} & \mathbf{w}_j^\mathsf{T}\mathbf{p} \le \bar{P}_j, \quad j = 1, \dots, J, \\ & \mathbf{p} > \mathbf{0}. \end{cases}$$

For notational brevity, we will often omit the dependence on $\mathbb{U}$. In [15], it was shown that the concave Perron-Frobenius theory [19] can be used to obtain closed-form expressions for the optimal value and solution to $\mathcal{S}^{\mathsf{DL}}$. For completeness, we state here the following result from [15].

Theorem 2 ([15]): Define $\mathbf{B}_j := \mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F} + (1/\bar{P}_j)\mathbf{n}\mathbf{w}_j^\mathsf{T})$ and $k := \arg\min_j (1/\rho(\mathbf{B}_j))$. The optimal value and solution of $\mathcal{S}^{\mathsf{DL}}$ are given by
$$\tau_* = 1/\rho(\mathbf{B}_k),$$
$$\mathbf{p}_* = (\bar{P}_k/\mathbf{w}_k^\mathsf{T}\mathbf{x}(\mathbf{B}_k))\mathbf{x}(\mathbf{B}_k).$$

From Theorem 2, it is easily seen that the optimal value of $\mathcal{S}^{\mathsf{DL}}$ is also the optimal value of a power control problem subject to a single constraint given by $\mathbf{w}_k^\mathsf{T}\mathbf{p} \le \bar{P}_k$ (where k was defined in Theorem 2). The constraint $\mathbf{w}_k^\mathsf{T}\mathbf{p} \le \bar{P}_k$ has the intuitive interpretation as the "limiting" constraint of the network, in the sense that the optimal SINR cannot increase unless this constraint is relaxed. Now, consider the partial Lagrange dual of $\mathcal{S}^{\mathsf{DL}}$:
$$\min_{\boldsymbol{\theta} \gneqq \mathbf{0}} \quad h(\boldsymbol{\theta}, \mathbb{U}), \tag{13}$$
where
$$h(\boldsymbol{\theta}, \mathbb{U}) := \begin{cases} \max\limits_{\mathbf{p}} & \min\limits_{l} \frac{\mathsf{SINR}_l^{\mathsf{DL}}(\mathbf{p}, \mathbb{U})}{\beta_l} \\ \text{subject to} & \sum_j \theta_j \mathbf{w}_j^\mathsf{T}\mathbf{p} \le \sum_j \theta_j \bar{P}_j, \\ & \mathbf{p} > \mathbf{0}. \end{cases}$$

By definition, (13) is an upper-bound to the optimal value of $\mathcal{S}^{\mathsf{DL}}$. From Theorem 2, we conclude that this upper-bound is tight since it can be achieved by choosing $\boldsymbol{\theta} = \mathbf{e}_k$ for some $k \in \{1, \dots, J\}$. Hence, (13) is equivalent to $\mathcal{S}^{\mathsf{DL}}$. By including the outer maximization over $\mathbb{U}$, we obtain the following reformulation of $\mathcal{U}^{\mathsf{DL}}$:
$$\mathcal{U}^{\mathsf{DL}} = \left\{ \max_{\|\mathbf{u}_l\|=1 \forall l} \quad \min_{\boldsymbol{\theta} \gneqq \mathbf{0}} \quad h(\boldsymbol{\theta}, \mathbb{U}). \right. \tag{14}$$

Let $\mathbf{p}_*$ and $\mathbb{U}_*$ be the optimal solution of $\mathcal{U}^{\mathsf{DL}}$, and let $\boldsymbol{\theta}_*$ be the optimal $\boldsymbol{\theta}$ in (14). We will show that $h(\boldsymbol{\theta}, \mathbb{U})$ satisfies the strong max-min property (Chap. 5, [20]):
$$\min_{\boldsymbol{\theta} \gneqq \mathbf{0}} h(\boldsymbol{\theta}, \mathbb{U}_*) = h(\boldsymbol{\theta}_*, \mathbb{U}_*) = \max_{\|\mathbf{u}_l\|=1 \forall l} h(\boldsymbol{\theta}_*, \mathbb{U}). \tag{15}$$

Since the left equality follows from the definition of $\boldsymbol{\theta}_*$, it suffices to show the right equality.

There are two key ideas behind the proof that the right equality in (15) holds. The first is that the optimal $\mathbb{U}$ for both the left problem and the right problem are normalized MVDR beamformers in their respective dual uplink network. The second is that the optimal dual uplink power of the left problem is equal to that of the right problem. First, consider $h(\boldsymbol{\theta}_*, \mathbb{U}_*)$. By definition, $h(\boldsymbol{\theta}_*, \mathbb{U}_*)$ is a single-constrained max-min weighted SINR problem with optimal solution $\mathbf{p}_*$. By specializing (3)-(9), we obtain the following necessary and

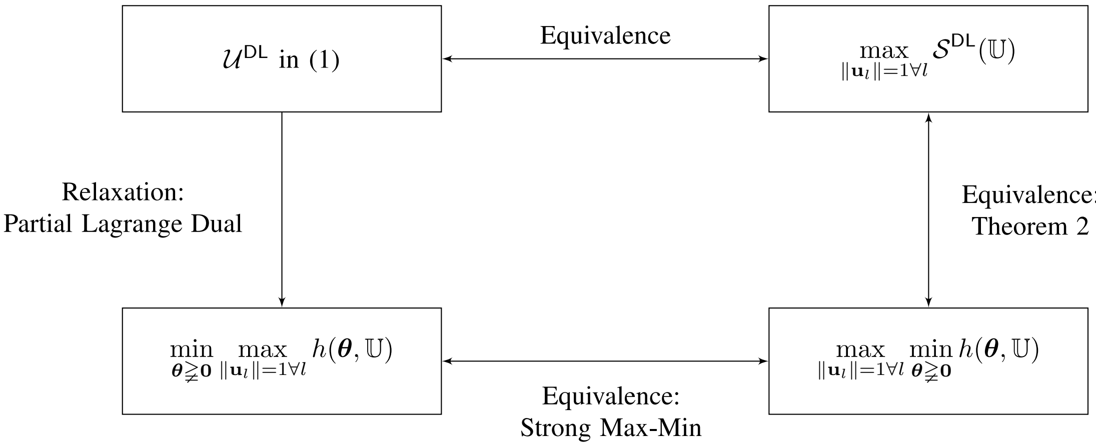

Fig. 2. The steps establishing the equivalence between $\mathcal{U}^{\mathsf{DL}}$ and its partial Lagrange dual. The equivalence via Theorem 2 is based on the zero duality gap between the power control problem $\mathcal{S}^{\mathsf{DL}}(\mathbb{U})$ and its partial Lagrange dual.

sufficient conditions for $\mathbf{p}_*$:

$$\tau_* = \frac{p_{*l}}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}\mathbf{p}_* + \mathbf{n}))_l}, \quad l = 1, \ldots, L, \tag{16}$$

$$\sum_j \theta_{*j} \mathbf{w}_j^{\mathsf{T}} \mathbf{p}_* = \sum_j \theta_{*j} \bar{P}_j, \tag{17}$$

$$\tau_* = \frac{q_{*l}}{(\mathrm{diag}(\hat{\boldsymbol{\beta}})(\mathbf{F}^{\mathsf{T}}\mathbf{q}_* + \sum_j \theta_{*j}\mathbf{w}_j))_l}, \quad l = 1, \ldots, L, \tag{18}$$

$$\mathbf{1}^{\mathsf{T}} \boldsymbol{\lambda}_* = 1, \tag{19}$$

$$\mathbf{q}_* = \frac{\tau_* \sum_j \theta_{*j} \bar{P}_j}{\mu_*} \cdot \left(\frac{\lambda_{*1}\hat{\beta}_1}{p_{*1}}, \ldots, \frac{\lambda_{*L}\hat{\beta}_L}{p_{*L}} \right)^{\mathsf{T}}, \tag{20}$$

$$\lambda_{*l} > 0, \quad l = 1, \ldots, L, \tag{21}$$

$$\mu_* > 0, \tag{22}$$

where $\boldsymbol{\lambda}_*$ and μ_* are the optimal dual variables, and $\tau_* = h(\boldsymbol{\theta}_*, \mathbb{U}_*)$ is the optimal value. From (18), we conclude that $\mathbf{q}_*$ is the optimal dual uplink power and $\sum_j \theta_{*j}\mathbf{w}_j$ is the noise vector in the dual uplink network. By the definition of $\mathbb{U}_*$ as the optimal transmit beamformers of $\mathcal{U}^{\mathsf{DL}}$, we have that $\mathbb{U}_*$ are normalized MVDR beamformers in the dual uplink network:

$$\mathbf{u}_{*l} = \frac{\mathbf{N}_l(\mathbf{q}_*)^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l}{\|\mathbf{N}_l(\mathbf{q}_*)^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l\|}, \quad l = 1, \ldots, L,$$

where the mapping $\mathbf{N}_l : \mathbb{R}_+^L \to \mathbb{C}^{L\times L}$ is defined by:

$$\mathbf{N}_l(\mathbf{q}_*) := \sum_{i\neq l} q_{*i}\mathbf{H}_{il}^{\dagger}\mathbf{v}_i\mathbf{v}_i^{\dagger}\mathbf{H}_{il} + \sum_j \theta_{*j} w_{jl}\mathbf{I}.$$

Since $w_{jl} > 0$ for all j and $\boldsymbol{\theta}_* \gneqq \mathbf{0}$, we have that $\mathbf{N}_l(\mathbf{q}_*)$ is positive definite and always invertible. It follows that $\mathbf{q}_*$ satisfies the following fixed-point equation (cf. (12)):

$$\frac{1}{\tau_*}\mathbf{q}_* = f'(\mathbf{q}_*), \tag{23}$$

where the mapping $f' : \mathbb{R}_+^L \to \mathbb{R}_+^L$ is defined by:

$$f_l'(\mathbf{q}_*) := \frac{\beta_l}{\mathbf{v}_l^{\dagger}\mathbf{H}_{ll}\mathbf{N}_l(\mathbf{q}_*)^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l},$$

for $l = 1, \ldots, L$.

Next, observe that $\max_{\{\|\mathbf{u}_l\|=1\forall l\}} h(\boldsymbol{\theta}_*, \mathbb{U})$ can be rewritten as the following single-constrained max-min weighted SINR problem:

$$\begin{array}{ll} \underset{\mathbf{p},\mathbb{U}}{\max} & \underset{l}{\min} \frac{\mathsf{SINR}_l^{\mathsf{DL}}(\mathbf{p},\mathbb{U})}{\beta_l} \\ \text{subject to} & \sum_j \theta_{*j}\mathbf{w}_j^{\mathsf{T}}\mathbf{p} \leq \sum_j \theta_{*j}\bar{P}_j, \quad \mathbf{p} > \mathbf{0}, \\ & \|\mathbf{u}_l\| = 1, \quad l = 1, \ldots, L. \end{array} \tag{24}$$

Let τ_*' be the optimal value of (24), let $\mathbf{p}_*'$ and $\mathbb{U}_*'$ be the optimal solution, and let $\mathbf{q}_*'$ be the optimal dual uplink power. It is straightforward to see that (16)-(22), with $\tau_* = \tau_*'$, $\mathbf{p}_* = \mathbf{p}_*'$, $\mathbb{U}_* = \mathbb{U}_*'$, and $\mathbf{q}_* = \mathbf{q}_*'$, give necessary conditions for τ_*', $\mathbf{p}_*'$, $\mathbb{U}_*'$, and $\mathbf{q}_*'$. By the definition of $\mathbb{U}_*'$ as the optimal transmit beamformers of (24), we have that $\mathbb{U}_*'$ are normalized MVDR beamformers in the dual uplink network:

$$\mathbf{u}_{*l}' = \frac{\mathbf{N}_l(\mathbf{q}_*')^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l}{\|\mathbf{N}_l(\mathbf{q}_*')^{-1}\mathbf{H}_{ll}^{\dagger}\mathbf{v}_l\|}, \quad l = 1, \ldots, L.$$

Via familiar algebraic manipulations, we conclude that $\mathbf{q}_*'$ satisfies the fixed-point equation in (23) with $\tau_* = \tau_*'$ and $\mathbf{q}_* = \mathbf{q}_*'$. By Theorem 1, it follows that $\tau_* = \tau_*'$. This shows the right equality in (15).

Due to the strong max-min property, it follows that we can interchange the max-min operators in (14) and arrive at the following reformulation of $\mathcal{U}^{\mathsf{DL}}$:

$$\mathcal{U}^{\mathsf{DL}} = \left\{ \quad \underset{\boldsymbol{\theta}\gneqq\mathbf{0}}{\min} \quad g(\boldsymbol{\theta}), \right. \tag{25}$$

where

$$g(\boldsymbol{\theta}) := \left\{ \begin{array}{ll} \underset{\mathbf{p},\mathbb{U}}{\max} & \underset{l}{\min} \frac{\mathsf{SINR}_l^{\mathsf{DL}}(\mathbf{p},\mathbb{U})}{\beta_l} \\ \text{subject to} & \sum_j \theta_j\mathbf{w}_j^{\mathsf{T}}\mathbf{p} \leq \sum_j \theta_j\bar{P}_j, \\ & \mathbf{p} > \mathbf{0}, \\ & \|\mathbf{u}_l\| = 1, \quad l = 1, \ldots, L. \end{array} \right.$$

It is easy to see that (25) is the partial Lagrange dual of $\mathcal{U}^{\mathsf{DL}}$. The complete sequence of analysis is illustrated in Fig. 2.

Since $g(\boldsymbol{\theta})$ is convex in $\boldsymbol{\theta}$, (25) can be solved via a subgradient projection method [12], [13]. For any fixed $\boldsymbol{\theta}$, Algorithm 1 can be used to compute $g(\boldsymbol{\theta})$. The subgradient of $g(\boldsymbol{\theta})$ can be easily computed to be $\left[\bar{P}_1 - \mathbf{w}_1^{\mathsf{T}}\mathbf{p}_*(\boldsymbol{\theta}), \ldots, \bar{P}_J - \mathbf{w}_J^{\mathsf{T}}\mathbf{p}_*(\boldsymbol{\theta})\right]^{\mathsf{T}}$ where $\mathbf{p}_*(\boldsymbol{\theta})$ is the optimal downlink power of $g(\boldsymbol{\theta})$.

Algorithm 2: Multiple-constrained max-min weighted SINR

- Define: $\mathcal{C} = \left\{\boldsymbol{\theta} \in \mathbb{R}_+^J : \boldsymbol{\theta} \gneqq \mathbf{0}\right\}$.
- Parameters: threshold $\epsilon > 0$ for Algorithm 1, step size $\alpha_n > 0$ for subgradient update.
- Initialize: arbitrary $\boldsymbol{\theta}^{(0)} \in \mathcal{C}$, $\mathbf{q}^{(0)} > \mathbf{0}$, $\mathbf{p}^{(0)} > \mathbf{0}$, $\{\mathbb{U}^{(0)} : \|\mathbf{u}_l^{(0)}\| = 1\}$.

1) Fix $\boldsymbol{\theta}^{(n)}$ and impose the power constraint:

$$\sum_j \theta_j^{(n)} \mathbf{w}_j^{\mathsf{T}} \mathbf{p}^{(n)} \leq \sum_j \theta_j^{(n)} \bar{P}_j.$$

Use Algorithm 1 to find the optimal $\mathbf{p}^{(n)}$, $\mathbf{q}^{(n)}$, and $\mathbb{U}^{(n)}$ subject to the above power constraint until

$$\|\mathbf{q}^{(n+1)} - \mathbf{q}^{(n)}\| < \epsilon.$$

2) Update $\boldsymbol{\theta}^{(n+1)}$ using the subgradient projection method:

$$\boldsymbol{\theta}^{(n+1)} = \mathcal{P}_{\mathcal{C}}\left\{\boldsymbol{\theta}^{(n)} - \alpha_n \hat{\mathbf{g}}(\mathbf{p}^{(n)})\right\},$$

where $\hat{\mathbf{g}}(\mathbf{p}^{(n)}) = [\bar{P}_1 - \mathbf{w}_1^{\mathsf{T}}\mathbf{p}^{(n)}, \ldots, \bar{P}_J - \mathbf{w}_J^{\mathsf{T}}\mathbf{p}^{(n)}]^{\mathsf{T}}$ and $\mathcal{P}_{\mathcal{C}}$ is the projection operator onto $\mathcal{C}$.

Remark 3: In [12], the authors showed that the max-min weighted SINR problem subject to multiple transmit covariance constraints can be reformulated into a form with an outer minimization over some parametrization variables and an inner maximization over the primal variables. (25) is different from the result in [12] in that (25) applies to the case of multiple weighted-sum power constraints. Moreover, the derivation in [12] was based on the KKT conditions of the non-convex problem. In contrast, our derivation is based on the uniqueness-of-solution to the single-constrained problem and provides additional insight on the strong max-min property of the subproblem $h(\boldsymbol{\theta}, \mathbb{U})$.

Our derivation of (25) has the advantage that it allows us to decouple $\mathcal{U}^{\mathsf{DL}}$. Recall from Theorem 2 that the optimal value of the multiple-constrained power control problem $\mathcal{S}^{\mathsf{DL}}$ is also the optimal value of an associated single-constrained power control problem. The following theorem gives a similar result for $\mathcal{U}^{\mathsf{DL}}$.

Theorem 3: The optimal value of the multiple-constrained MISO downlink problem $\mathcal{U}^{\mathsf{DL}}$ can be obtained from at least one of the J single-constrained MISO downlink subproblems. Specifically,

$$\mathcal{U}^{\mathsf{DL}} = \min_j \mathcal{U}_j^{\mathsf{DL}}.$$

Proof: From the discussion following Theorem 2, it follows that there exist some $k \in \{1, \ldots, J\}$ such that $g(\mathbf{e}_k)$ achieves the optimal value of (25). □

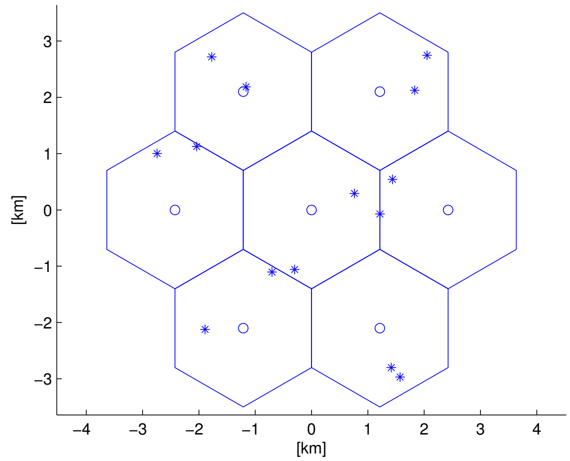

Fig. 3. Layout of 7-cell network used for convergence analysis. Each base station serves two users and is subject to a total power constraint of 10W.

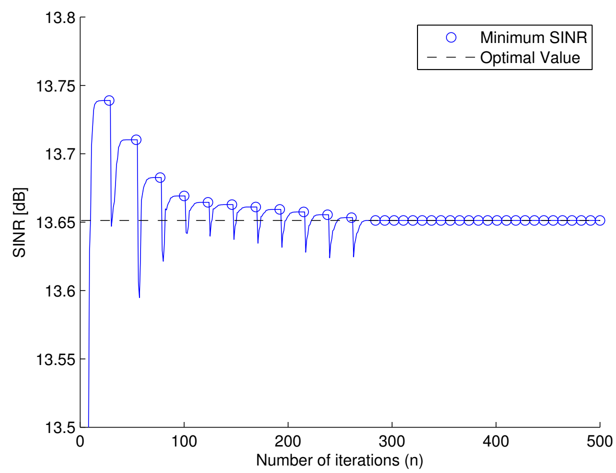

Fig. 4. Convergence of minimum SINR in Algorithm 2. The optimal value is computed using Theorem 3.

Theorem 3 implies that to solve $\mathcal{U}^{\mathsf{DL}}$, one could use Algorithm 1 to solve J independent single-constrained MISO downlink problems and choose the solution with the minimum objective. In the following section, we provide numerical examples to illustrate that this approach is typically less computationally intensive than the inner-outer (subgradient) iterative approach employed in Algorithm 2

V. Numerical Examples

In this section, we demonstrate the convergence of Algorithm 2 in a 7-cell network with 2 randomly located users per cell, as illustrated in Fig. 3. The cell radius is 1.4km. Each base station has a total power constraint of 10W and each user is served one independent data stream from its base station. It will be assumed for simplicity that every data stream has the same priority, i.e., $\boldsymbol{\beta} = \mathbf{1}$. The base stations are equipped with $N_{t_l} = 4$ antennas and each user is equipped with $N_{r_l} = 2$ antennas. The users are assumed to carry out Maximum Ratio Combining (MRC) beamforming with respect to its channel. The noise power spectral density is set to -162dBm/Hz. Each user communicates with the base station over independent MIMO rayleigh fading channels, with a path loss of $L = 128.1 + 37.6 \log_{10}(d)$dB, where d is the distance in kilometers.

Fig. 4 shows the minimum SINR of Algorithm 2 as a function of the iteration index n. In each monotone increasing segment, the network is optimized subject to a fixed single power constraint determined by $\boldsymbol{\theta}$ (i.e. step 1 of Algorithm 2). Each dip in the minimum SINR corresponds to a subgradient update of $\boldsymbol{\theta}$ (i.e. step 2 of Algorithm 2). As a result of the subgradient update, each increasing segment converges to a value closer to the optimal value, which is computed using Theorem 3. Although $\boldsymbol{\theta}^{(0)}$ could have been arbitrarily initialized, in this study, we chose $\boldsymbol{\theta}^{(0)} = \mathbf{1}$.

Observe that Algorithm 2 converges after roughly 11 subgradient updates, that is, after 11 passes of Algorithm 1. In contrast, using Theorem 3 to compute the optimal solution would require only 7 passes of Algorithm 1 (one for each cell). Although this example is not a representative study, multiple experiments conducted on various other network configurations demonstrate that Theorem 3 is typically more computationally efficient than Algorithm 2.

VI. Conclusion

We studied the max-min weighted SINR problem in the downlink subject to multiple weighted-sum power constraints. First, we studied the single-constrained problem and applied concave Perron-Frobenius theory to derive an algorithm which converges geometrically fast to the optimal solution. As a by-product, we resolve an open problem of convergence of a previously proposed algorithm in [7]. Next, we exploited the uniqueness-of-solution to the single-constrained problem to show that there is zero duality gap between the multiple-constrained problem and its partial Lagrange dual. This allowed us to extend the result that the multiple-constrained power control problem can be decoupled and conclude that the multiple-constrained MISO problem can also be decoupled.

There are many further issues that can be explored with regards to the work in this paper. As mentioned, Algorithm 2 is fairly computationally intensive. Hence, one issue is to propose efficient algorithms for the multiple-constrained MISO problem by exploiting the fact that the problem can be decoupled into single-constrained subproblems. Moreover, having all base stations fully coordinate their power updates could lead to high latencies. Hence, another interesting issue is the modification of Algorithm 2 into a suboptimal algorithm and a characterization of the resultant loss in performance.

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