# Wireless Network Optimization

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Convex Optimization and its Applications to Computer Science

# Outline

- What network utility are important to users in wireless networks characterized by mutual interference and power constraints?

- How to design distributed algorithms that converge fast to global optimal solution of non-convex wireless optimization problems?

- To tackle non-convexity, we can leverage the three great pillars of the theory of inequalities:[1] Positivity, Monotonicity, Convexity

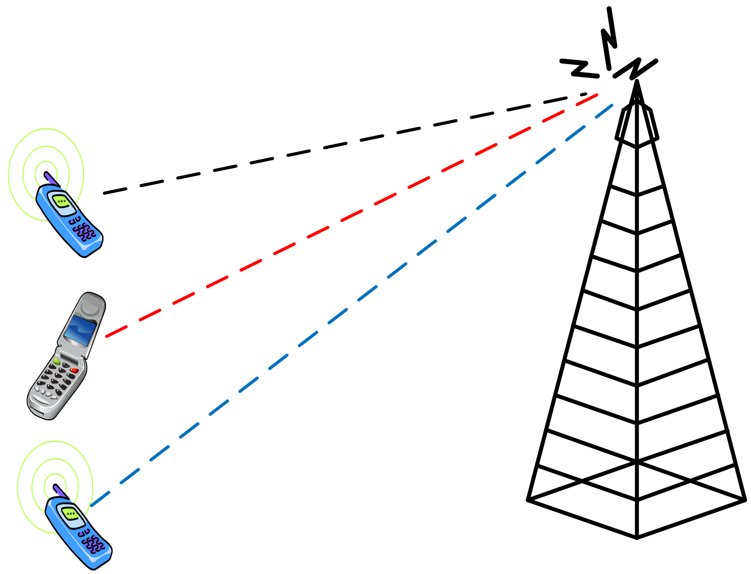
- A unifying approach to wireless network optimization based on Nonlinear Perron-Frobenius Theory

[1] The Cauchy–Schwarz Master Class: An Introduction to the Art of Mathematical Inequalities, J. Michael Steele, Cambridge University Press, 2004

# Wireless Networks

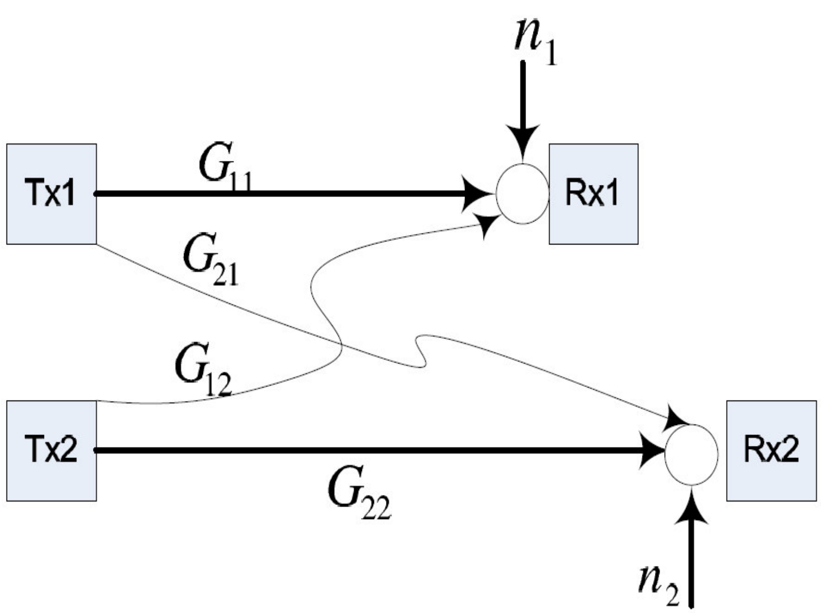
- Wireless user maximizes the network utility using power control in wireless networks such as next-generation mobile cellular networks, IoT/fog computing, satellite networks, deep space communications

- Distributed algorithm design for wireless network optimization



# System Model

- Interference channel with single-user decoding: Treat interference as Additive White Gaussian Noise
- Maximize network utility while controlling interference using power control



# Performance Metrics and Specifications

- Signal-to-Interference-Noise Ratio:

$$\mathsf{SINR}_l(\mathbf{p}) = \frac{G_{ll}p_l}{\sum_{j \neq l} G_{lj}p_j + n_l}$$

  with $G_{lj}$ the channel gains from transmitter $j$ to receiver $l$ and $n_l$ the additive white Gaussian noise (AWGN) power at receiver $l$

- Wireless resources are valuable – impose power constraints such as $\mathbf{1}^\top \mathbf{p} \leq \bar{\mathbf{P}}$ or $p_l \leq {p_l}^{max}$

- Communication qualities are important – impose Signal-to-Interference-Noise Ratio threshold constraints: $\mathsf{SINR}_l(\mathbf{p}) \geq \gamma_l, \ \ \forall \ l$

# Interference Parameters

- Let $\mathbf{F}$ be a nonnegative matrix with entries:

$$F_{ij} = \begin{cases} 0, & \text{if } i = j \\ \frac{G_{ij}}{G_{ii}}, & \text{if } i \neq j \end{cases}$$

and

$$\mathbf{v} = \left( \frac{n_1}{G_{11}}, \frac{n_2}{G_{22}}, \ldots, \frac{n_L}{G_{LL}} \right)^\top .$$

- Assume $\mathbf{F}$ is an irreducible nonnegative matrix – a user experiences interference from at least another user

- Reformulate the SINR constraints $\mathsf{SINR}_l(\mathbf{p}) \geq \gamma_l, \quad \forall\ l$ as linear constraints. Is this system of linear inequalities always feasible for any given $\mathbf{F}$, $\mathbf{v}$ and $\gamma_l$?

# Total Power Minimization

- Minimizing the total power of all users

$$\begin{array}{rl} \text{minimize} & \sum_l p_l \\ \text{subject to} & \mathsf{SINR}_l(\mathbf{p}) := \frac{G_{ll}p_l}{\sum_{j\neq l} G_{lj}p_j + n_l} \geq \gamma_l \;\; \forall\, l. \end{array}$$

- Observe that all SINR inequality-constraints tight at optimality
  Can easily reformulate as a Linear Program (LP) in vector form:

$$\begin{array}{rl} \text{minimize} & \mathbf{1}^\top \mathbf{p} \\ \text{subject to} & (\mathbf{I} - \mathsf{diag}(\boldsymbol{\gamma})\mathbf{F})\mathbf{p} \geq \mathsf{diag}(\boldsymbol{\gamma})\mathbf{v}. \end{array}$$

- Can also reformulate as a Geometric Program (GP)[2]

[2] S. Boyd, S.-J. Kim, L. Vandenberghe, and A. Hassibi, A Tutorial on Geometric Programming, Optimization and Engineering, 8(1):67-127, 2007.

# Total Power Minimization

- The optimal solution $\mathbf{p}^\star$ is:

$$\mathbf{p}^\star = (\mathbf{I} - \text{diag}(\boldsymbol{\gamma})\mathbf{F})^{-1}\text{diag}(\boldsymbol{\gamma})\mathbf{v}.$$

- Iterative algorithm: $\mathbf{p}(k+1) = \text{diag}(\boldsymbol{\gamma})\mathbf{F}\mathbf{p}(k) + \text{diag}(\boldsymbol{\gamma})\mathbf{v}$

- Rewrite iterative algorithm as Distributed Power Control (DPC) algorithm (allowing decentralized computation):[3]

$$p_l(k+1) = \frac{\gamma_l}{\text{SINR}_l(\mathbf{p}(k))} p_l(k) \quad \forall\, l.$$

- Geometric convergence to $\mathbf{p}^\star$ if and only if $\rho(\text{diag}(\boldsymbol{\gamma})\mathbf{F}) < 1$

[3] The DPC algorithm is used in CDMA wireless cellular networks such as Qualcomm IS-95 systems.

# Dualities

- GP duality: Application to energy-robustness tradeoff

$$\begin{array}{rl} \text{minimize} & \sum_l e^{\tilde{p}_l} \\ \text{subject to} & \log(\gamma_l/\mathsf{SINR}_l(\tilde{\mathbf{p}})) \leq 0 \quad \forall\, l, \\ \text{variables:} & \tilde{p}_l \quad \forall\, l. \end{array} \tag{1}$$

- LP Duality: Application to Beamforming uplink-downlink duality

- Perron-Frobenius Duality (later on max-min weighted SINR): Application to distributed fast algorithm

# Max-min SINR Problem Special Case

Consider the special case without background noise (i.e., $\mathbf{v} = \mathbf{0}$) and to maximize the worst-case SINR among all users:[4]

$$\max_{p_l \geq 0\ \forall\, l} \quad \min_{l=1,\ldots,L} \frac{G_{ll} p_l}{\sum_{j \neq l} G_{lj} p_j}.$$

- Wielandt's characterization of spectral radius:

$$\rho(\mathbf{F}) = \max_{\mathbf{p} \geq \mathbf{0}} \quad \min_{l=1,\ldots,L} \frac{(\mathbf{F}\mathbf{p})_l}{p_l} = \min_{\mathbf{p} \geq \mathbf{0}} \quad \max_{l=1,\ldots,L} \frac{(\mathbf{F}\mathbf{p})_l}{p_l}$$

- Optimal max-min SINR value: $1/\rho(\mathbf{F})$ and optimal power solution:[5] $\mathbf{p}^\star = \mathbf{x}(\mathbf{F})$ is Perron-Frobenius right eigenvector of $\mathbf{F}$

[4] J. M. Aein, "Power balancing in systems employing frequency reuse," COMSAT Tech. Rev., vol. 3, no. 2, 1973

[5] This must be one of the earliest applications of Perron-Frobenius right eigenvector on a global scale in satellite communications for a worldwide network linking 82 earth stations in 50 countries, decades before the debut of Google's Pagerank for linking webpages in the Internet.

# Max-Min SINR Problem

- $\max_{\mathbf{p}\geq\mathbf{0}} \min_{l=1,\ldots,L} \dfrac{\mathsf{SINR}_l(\mathbf{p})}{\beta_l}$ subject to $\mathbf{p} \in P$.[6]

- Power constraints $P$ can be any affine set, e.g., $\sum_l p_l \leq \bar{P}$ for downlink or $p_l \leq \bar{P} \quad \forall\, l$ for uplink.

- Can reformulate as GP and solve by interior point algorithm (but not distributed)

**Theorem 1.** *The optimal solution is such that the value* $\mathsf{SINR}_l/\beta_l$ *for all users are equal. The optimal weighted max-min* SINR *is given by*

$$\gamma^* = \frac{1}{\rho(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B})},$$

*where* $\mathbf{B} = \mathbf{F} + (1/\bar{P})\mathbf{v}\mathbf{1}^\top$. *Further, the optimal* $\mathbf{p}$, *denoted by* $\mathbf{p}^*$, *is* $t\mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B})$ *for some constant* $t > 0$.

[6] Homework: Problem 12.2 in Additional Exercises for Convex Optimization by S. Boyd and L. Vandenberghe, Solution: Wireless Network Optimization by Perron-Frobenius Theory, Chapter 3, C. W. Tan, FnT in Networking, 2015

# Distributed and Fast Max-min SINR Algorithm

- Distributed algorithm based on Nonlinear Perron-Frobenius Theory

- **Algorithm 1.**

  *1. Update power* $\mathbf{p}(k+1)$*:*

$$p_l(k+1) = \frac{\beta_l}{\mathsf{SINR}_l(\mathbf{p}(k))} p_l(k) \;\; \forall\, l.$$

  *2. Normalize* $\mathbf{p}(k+1)$*:*

$$p_l(k+1) \leftarrow p_l(k+1) / \sum_j p_j(k+1) \cdot \bar{P} \;\; \forall\, l.$$

- **Theorem 2.** *Starting from any initial point* $\mathbf{p}(0)$*,* $\mathbf{p}(k)$ *in Algorithm* $\boxed{1}$ *converges geometrically fast to* $\mathbf{x}(\mathrm{diag}(\boldsymbol{\beta})\mathbf{B})$ *(unique up to a scaling constant).*

# Min-Max Outage Probability

- Suppose we have channel fading, i.e., $G_{lj}$ is stochastic. Given a threshold $\beta_l$ for reliable communication, an outage occurs at the $l$th receiver whenever $\mathsf{SINR}_l(\mathbf{p}) < \beta_l$.

- Let the probability of outage be $P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l)$. Under Rayleigh fading, we have (prove it!):

$$P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l) = 1 - e^{\frac{-v_l \beta_l}{p_l}} \prod_{j=1}^{L} \left(1 + \frac{\beta_l F_{lj} p_j}{p_l}\right)^{-1}$$

- $\min_{\mathbf{p} \geq \mathbf{0}} \max_{l=1,\ldots,L} P(\mathsf{SINR}_l(\mathbf{p}) < \beta_l)$ subject to $\mathbf{p} \in P$.

- Design a distributed algorithm to solve this stochastic program

**Algorithm 2.** *Worst Outage Probability Minimization*

1. *Update power* $\mathbf{p}(k+1)$*:*

$$p_l(k+1) = -\log\left(1 - \phi_l(\mathbf{p}(k))\right) p_l(k) \qquad \forall\, l. \tag{2}$$

2. *Normalize* $\mathbf{p}(k+1)$*:*

$$\mathbf{p}(k+1) \leftarrow \frac{\mathbf{p}(k+1)\cdot \bar{P}}{\mathbf{a}^\top \mathbf{p}(k+1)} \ \text{ if } \mathcal{P} = \{\mathbf{p} \mid \mathbf{a}^\top \mathbf{p} \leq \bar{P}\}. \tag{3}$$

$$\mathbf{p}(k+1) \leftarrow \frac{\mathbf{p}(k+1)\cdot \bar{p}}{\max_{j=1,\ldots,L} p_j(k+1)} \ \text{ if } \mathcal{P} = \{\mathbf{p} \mid p_l \leq \bar{p}\,\forall\, l\}. \tag{4}$$

**Theorem 3.** *Starting from any initial point* $\mathbf{p}(0)$*,* $\mathbf{p}(k)$ *in Algorithm 2 converges geometrically fast to the optimal solution of Min-Max Outage Probability Minimization.*

# Nonlinear Perron-Frobenius Theory

- Find $(\check{\lambda}, \check{\mathbf{s}})$ in

$$\lambda \mathbf{s} = \mathbf{A}\mathbf{s} + \mathbf{b}, \quad \lambda \in \mathbb{R}, \quad \mathbf{s} \geq \mathbf{0}, \quad \|\mathbf{s}\| = 1,$$

  where $\mathbf{A}$ and $\mathbf{b}$ is a square irreducible nonnegative matrix and nonnegative vector, respectively and $\|\cdot\|$ a monotone vector norm.

- $(\check{\lambda}, \check{\mathbf{s}})$ is Perron-Frobenius eigenvalue-vector pair of $\mathbf{A} + \mathbf{b}\mathbf{c}_*{}^{\top}$, where

$$\mathbf{c}_* = \arg \max_{\|\mathbf{c}\|_* = 1} \rho(\mathbf{A} + \mathbf{b}\mathbf{c}^{\top}),$$

  where $\|\cdot\|_*$ is the dual norm of $\|\cdot\|$, and $\check{\mathbf{s}} = (\mathbf{A}\check{\mathbf{s}} + \mathbf{b})/\|\mathbf{A}\check{\mathbf{s}} + \mathbf{b}\|$.

V. D. Blondel, L. Ninove and P. Van Dooren, *An Affine Eigenvalue Problem on the Nonnegative Orthant*, Linear Algebra and its Applications, 2005

# Nonnegative Matrix Theory: Minimax Theorem

- **Theorem 4.** *Friedland-Karlin Inequality:*[7] *For any irreducible nonnegative matrix* $\mathbf{A}$,

$$\prod_l ((\mathbf{A}\mathbf{z})_l / z_l)^{x_l y_l} \geq \rho(\mathbf{A})$$

*for all strictly positive* $\mathbf{z}$*, where* $\mathbf{x}$ *and* $\mathbf{y}$ *are the Perron and left eigenvectors of* $\mathbf{A}$ *respectively. Equality holds in (5) if and only if* $\mathbf{z} = a\mathbf{x}$ *for some positive* $a$.

- Friedland-Karlin inequality via convex optimization of nonnegative matrices and inverse problems[8]

[7] Friedland, S.; Karlin, S. Some inequalities for the spectral radius of non-negative matrices and applications. ", Duke Mathematical Journal, Vol. 42, no. 3, pp 459–490, 1975.

[8] C. W. Tan, S. Friedland and S. H. Low, Nonnegative Matrix Inequalities and Their Application to Nonconvex Power Control Optimization, SIAM Journal on Matrix Analysis and Applications, Vol. 32, No. 3, pp. 1030-1055, 2011

- Donsker-Varadhan's variational principle (1975):

$$\max_{\boldsymbol{\lambda}\geq\mathbf{0},\mathbf{1}^\top\boldsymbol{\lambda}=1}\ \min_{\mathbf{p}\geq\mathbf{0}}\sum_l \lambda_l\frac{(\mathbf{A}\mathbf{p})_l}{p_l} = \min_{\mathbf{p}\geq\mathbf{0}}\max_{\boldsymbol{\lambda}\geq\mathbf{0},\mathbf{1}^\top\boldsymbol{\lambda}=1}\sum_l \lambda_l\frac{(\mathbf{A}\mathbf{p})_l}{p_l}$$

- Let $\mathbf{B}\in\mathcal{R}_+^{L\times L},\mathbf{m}\in\mathcal{R}_+^L$ be a given irreducible nonnegative matrix with positive diagonal elements and a positive probability vector, respectively. Then, there exists $\boldsymbol{\eta}\in\mathcal{R}^L$ such that $\mathbf{x}(\mathrm{diag}(e^{\boldsymbol{\eta}})\mathbf{B})\circ\mathbf{y}(\mathrm{diag}(e^{\boldsymbol{\eta}})\mathbf{B})=\mathbf{m}$. Furthermore, $\boldsymbol{\eta}$ is unique up to an addition of $t\mathbf{1}$. In particular, this $\boldsymbol{\eta}$ can be computed by solving the following convex optimization problem:[9]

$$\begin{array}{rl}\text{maximize} & \mathbf{m}^\top\boldsymbol{\eta}\\ \text{subject to} & \log\rho(\mathrm{diag}(e^{\boldsymbol{\eta}})\mathbf{B})\leq 0,\\ \text{variables:} & \boldsymbol{\eta}=(\eta_1,\ldots,\eta_L)^\top\in\mathcal{R}^L.\end{array}\tag{5}$$

[9] Can reformulate the Google's PageRank problem in the web or the max-min SINR problem in wireless network as this convex optimization problem

# Generalized Max-Min Utility Optimization

Consider a Generalized Max-min Utility Optimization for Wireless Networks formulated as follows:[10]

$$\text{maximize} \quad \min_{i=1,\ldots,L} u_i(\mathbf{p}) \tag{6a}$$

$$\text{subject to } \mathbf{g}(\mathbf{p}) \leq \bar{\mathbf{g}} \tag{6b}$$

$$\text{variables} : \mathbf{p} \tag{6c}$$

where $\mathbf{g}(\mathbf{p}) = [g_1(\mathbf{p}), \ldots, g_K(\mathbf{p})]^\top$ is the vector of constraint functions and $\bar{\mathbf{g}} = [\bar{g}_1, \ldots, \bar{g}_K]^\top$ is the vector of constraint values. Due to the monotonicity of the functions $\{g_k\}_{k=1}^K$, we shall refer to (6b) as the set of monotonic constraints.

[10] L. Zheng, Y. W. Hong, C. W. Tan, et al, Wireless Max-min Utility Fairness with General Monotonic Constraints by Perron-Frobenius Theory, IEEE Transactions on Information Theory, Vol. 62, No. 12, pp. 7283-7298, 2016

For arbitrarily general $u_i$ and $\{g_k\}_{k=1}^K$, solving ($\boxed{6}$) is in general difficult. Let us solve ($\boxed{6}$) for $u_i$ and $\{g_k\}_{k=1}^K$ taking certain forms. Specifically, consider a class of utility functions that satisfy the following assumptions (Competitive Utility Functions).

- *Positivity*: For all $i$, $u_i(\mathbf{p}) > 0$ if $\mathbf{p} > \mathbf{0}$ and, in addition, $u_i(\mathbf{p}) = 0$ if and only if $p_i = 0$.

- *Competitiveness*: For all $i$, $u_i$ is strictly increasing with respect to $p_i$ and is strictly decreasing with respect to $p_j$, for $j \neq i$, when $p_i > 0$.

- *Directional Monotonicity*: For $\lambda > 1$ and $p_i > 0$, $u_i(\lambda\mathbf{p}) > u_i(\mathbf{p})$, for all $i$.

Consider a class of monotonic constraints satisfying the following:

- *Strict Monotonicity*: For all $k$, $g_k(\mathbf{p}_1) > g_k(\mathbf{p}_2)$ if $\mathbf{p}_1 > \mathbf{p}_2$, and $g_k(\mathbf{p}_1) \geqq g_k(\mathbf{p}_2)$ if $\mathbf{p}_1 \geqq \mathbf{p}_2$.

- *Feasibility*: The set $\{\mathbf{p} > \mathbf{0} : \mathbf{g}(\mathbf{p}) \leqq \bar{\mathbf{g}}\}$ is non-empty.

- *Validity*: For any $\mathbf{p} > \mathbf{0}$, there exists $\lambda > 0$ such that $g_k(\lambda \mathbf{p}) \geqq \bar{g}_k$, for some $k$.

**Algorithm 3.** *Max-Min Utility Optimization under Monotonic Constraints*

1. *Update power vector* $\mathbf{p}(t+1)$*:*

$$p_i(t+1) = \frac{p_i(t)}{u_i(\mathbf{p}(t))} \left( \triangleq T_i(\mathbf{p}(t)) \right), \ \forall i. \qquad (7)$$

2. *Scale power vector* $\mathbf{p}(t+1)$*:*

$$\mathbf{p}(t+1) \leftarrow \frac{\mathbf{p}(t+1)}{\beta(\mathbf{p}(t+1))}. \tag{8}$$

**Algorithm 4.** *Computation of* $\beta$ *via Bisection Search*

*Initialization:*

*i) Set* $i \leftarrow 0$, $L \leftarrow 0$, *and* $U \leftarrow 2^i$.

*ii) If there exists* $k$ *such that* $g_k(\mathbf{p}/U) > \bar{g}_k$, *then increment* $i \leftarrow i+1$ *and set* $U \leftarrow 2^i$.

*iii) Repeat (ii) until* $g_k(\mathbf{p}/U) \leq \bar{g}_k$ *for all* $k$.

*Bisection Search*

1. *Set* $\beta \leftarrow (U+L)/2$. *If* $g_k(\mathbf{p}/\beta) > \bar{g}_k$ *for some* $k$, *then set* $L \leftarrow \beta$. *Otherwise, set* $U \leftarrow \beta$.

2. *Repeat until* $|U - L| < \epsilon$.

Alternatively, notice that, for $\mathbf{p} > \mathbf{0}$, finding $\beta(\mathbf{p})$ is equivalent to choosing the minimum value of $\beta' > 0$ such that $I_k(\beta') \triangleq \beta' g_k(\mathbf{p}/\beta')/\bar{g}_k \leqq \beta'$, for $k = 1, \ldots, K$. Thus, an efficient iterative algorithm can compute $\beta(\mathbf{p})$ as follows.

**Definition 1.** *A function* $I : \mathcal{R}_+ \rightarrow \mathcal{R}_+$ *is* $standard$ *if the following conditions are satisfied for all* $\beta > 0$*:*

1. *Monotonicity: If* $\beta^{(a)} \geqq \beta^{(b)}$, *then* $I(\beta^{(a)}) \geqq I(\beta^{(b)})$.

2. *Scalability: For* $\lambda > 1$, $\lambda I(\beta) > I(\lambda\beta)$.

For $\{I_k\}_{k=1}^K$ that are standard, the following algorithm computes $\beta(\mathbf{p})$ where $\beta(t)$ denotes the value of $\beta$ in the $t$-th iteration.

**Algorithm 5.** *Computation of $\beta$ via Fixed-Point Iteration*

1. *Set initial value $\beta(0) > 0$.*

2. *Set $\beta(t+1) \leftarrow \max_k I_k(\beta(t))$.*

3. *Repeat Step 2 until convergence.*

**Theorem 5.** *For given $\mathbf{p} > 0$ and for $\{g_k\}_{k=1}^K$ such that $I_k(\beta) \triangleq \beta g_k(\mathbf{p}/\beta)$ is standard, for all $k$, the following properties hold:*

1. *Algorithm 5 converges to a unique fixed point $\beta^\star$.*

2. *The fixed point $\beta^\star$ is equal to $\beta(\mathbf{p})$.*

# Sum Rate Maximization

- Wireless data rate (nats per channel use) is often a function of SINR, e.g., Shannon capacity formula $r_l = \log(1 + \text{SINR}_l)$

- Find $\mathbf{p}^\star = \arg \max_{\mathbf{0} \leq \mathbf{p} \leq \bar{\mathbf{p}}} \sum_l w_l \log(1 + \text{SINR}_l(\mathbf{p}))$ where $\mathbf{1}^\top \mathbf{w} = 1$[11]

- Two-User case:

$$\max \quad w_1 \log \left(1 + \frac{G_{11} p_1}{G_{12} p_2 + n_1}\right) + w_2 \log \left(1 + \frac{G_{22} p_2}{G_{21} p_1 + n_2}\right)$$

subject to: $\quad 0 \leq p_1 \leq \bar{p}_1, \;\; 0 \leq p_2 \leq \bar{p}_2$

[11] Homework: Problem 12.3 in Additional Exercises for Convex Optimization by S. Boyd and L. Vandenberghe, Solution: Wireless Network Optimization by Perron-Frobenius Theory, Chapter 5, C. W. Tan, FnT in Networking, 2015

# Summary and Further Reading

- Wireless Network Optimization: Exploiting Positivity, Monotonicity, Convexity, Perron-Frobenius Theory

- Nonnegative Matrix Inequalities and Their Application to Nonconvex Power Control Optimization, C. W. Tan, S. Friedland and S. H. Low, SIAM Journal on Matrix Analysis and Applications, 2011.

- Optimal Power Control in Interference Limited Fading Wireless Channels with Outage Probability Specifications, S. Kandukuri and S. Boyd, IEEE Trans. on Wireless Communications, 2002

- Wireless Network Optimization by Perron-Frobenius Theory, C. W. Tan, Foundations and Trends in Networking, 2015

## Wireless Network Optimization by Perron-Frobenius Theory

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A basic question in wireless networking is how to optimize the wireless network resource allocation for utility maximization and interference management. How can we overcome interference to efficiently optimize fair wireless resource allocation, under various stochastic constraints on quality of service demands? Network designs are traditionally divided into layers. How does fairness permeate through layers? Can physical layer innovation be jointly optimized with network layer routing control? How should large complex wireless networks be analyzed and designed with clearly-defined fairness using beamforming?

*Wireless Network Optimization by Perron-Frobenius Theory* provides a comprehensive survey of the models, algorithms, analysis, and methodologies using a Perron-Frobenius theoretic framework to solve wireless utility maximization problems. This approach overcomes the notorious non-convexity barriers in these problems, and the optimal value and solution of the optimization problems can be analytically characterized by the spectral property of matrices induced by nonlinear positive mappings. It can even solve several previously open problems in the wireless networking literature.

This survey will be of interest to all researchers, students and engineers working on wireless networking.

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now
the essence of knowledge