Mathematics and Games

Julia Robinson Mathematics Festival

GLOBAL MATH PROJECT

Uplifting Mathematics for All

COMPUTER SCIENCE CHALLENGE 電腦科學大挑戰 2018

Gathering 4 Gardner PRESENTS Celebration of Mind

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Ambassador of Global Math Project (GMP)

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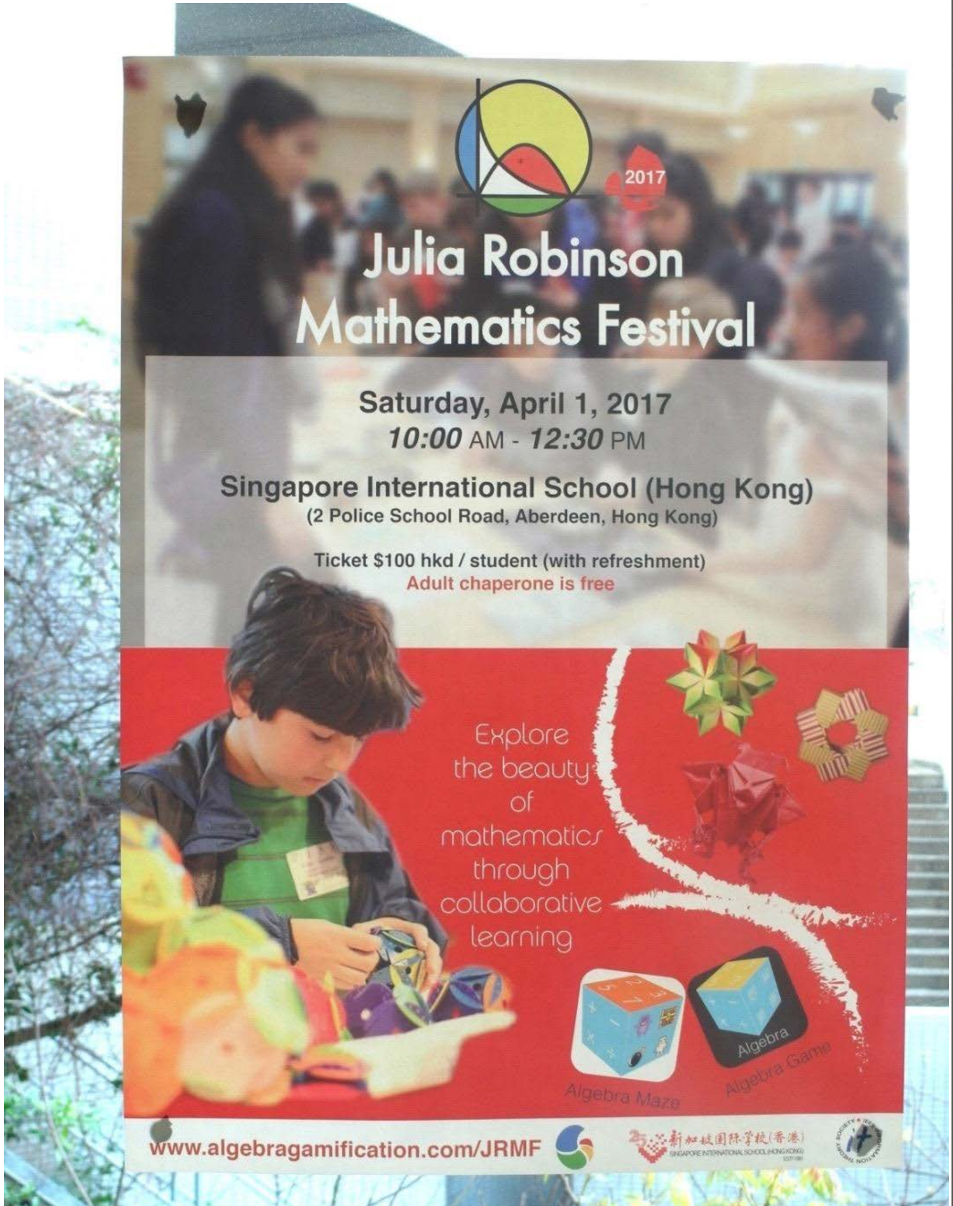
Mathematics and Games

1) Learn problem-solving skills and advanced mathematics

2) Learn math behind fun games and puzzles with applications to Computer Science

3) Algebra Game, Word Ladders, Bridg-It Game, Magic Squares


2017
Julia Robinson
Mathematics Festival
Saturday, April 1, 2017
10:00 AM - 12:30 PM
Singapore International School (Hong Kong)
(2 Police School Road, Aberdeen, Hong Kong)
Ticket $100 hkd / student (with refreshment)
Adult chaperone is free
Explore the beauty of mathematics through collaborative learning
Algebra
Algebra Maze
Algebra Game
www.algebragamification.com/JRMF
新加坡国际学校(香港)


Julia Robinson
2018
Mathematics Festival
March 24, 2018, Saturday
10:00 AM - 12:30 PM
Singapore International School (Hong Kong)
2 Police School Road, Wong Chuk Hang, Hong Kong
Ticket $100 hkd / Student (with Refreshment)
Adult Chaperone is Free
V-E+F=2
Explore the beauty of mathematics through collaborative learning
A
B
$a^{p-1} \equiv 1 \pmod p$
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Google Play
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新加坡国际学校(香港)
American Institute of Mathematics

Problem-Solving Tips for Student

Do something — get hands dirty

Start Simple

Organisation

Find Patterns

Draw a picture

Archimedes Society

MSRI
Mathematical Sciences
Research Institute

BERKELEY CITY COLLEGE

Not on the Test: The Pleasures and Uses of Mathematics—A Lecture Series in 2013–14

The Archimedes Society of the MATHEMATICAL SCIENCES RESEARCH INSTITUTE and BERKELEY CITY COLLEGE
present

Video Games for Mathematics

KEITH DEVLIN, Stanford University and InnerTube Games

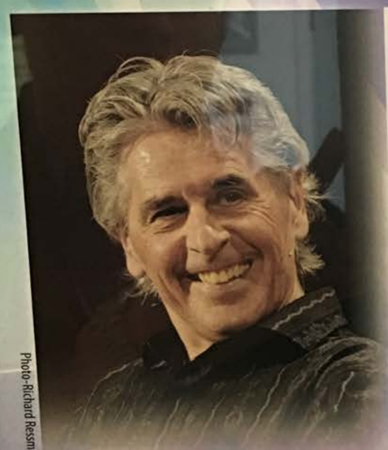


Photo-Richard Ressman

Keith Devlin will show how casual games that provide representations of mathematics enable children (and adults) to learn basic mathematics by "playing," in the same way we learn music by learning to play the piano.

Dr. Devlin is a mathematician at Stanford University, a co-founder and executive director of the university's H-STAR institute, a co-founder of the Stanford Media X research network, and a senior researcher at CSLI. His current research is focused on the use of different media to teach and communicate mathematics to diverse audiences. In this connection, he is a co-founder and president of an educational video games company, InnerTube Games. And, he is known to National Public Radio listeners of *Weekend Edition* as "the Math Guy."

Wednesday, October 9, 7:00 pm to 8:15 pm

Berkeley City College Auditorium • 2050 Center Street, Berkeley
(between Shattuck Ave. & Milvia St., near the Downtown Berkeley BART station)

FREE ADMISSION!
To register, go to http://tinyurl.com/KeithDevlin

Sponsored by
SIMONS FOUNDATION

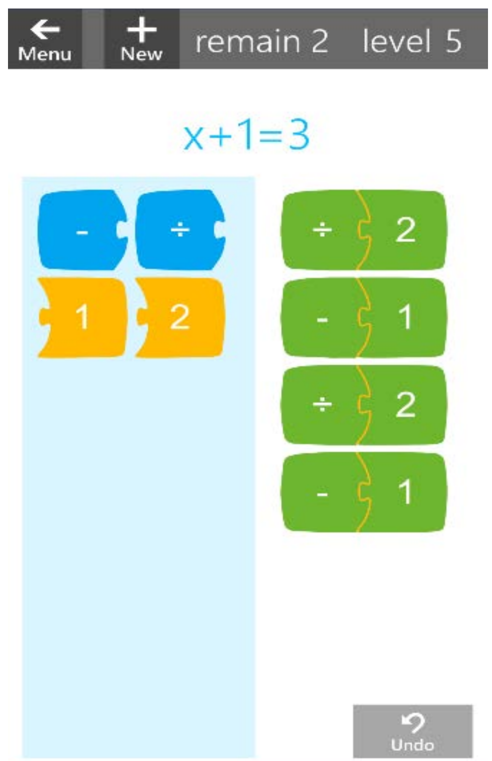
Design: www.mbohbot.com

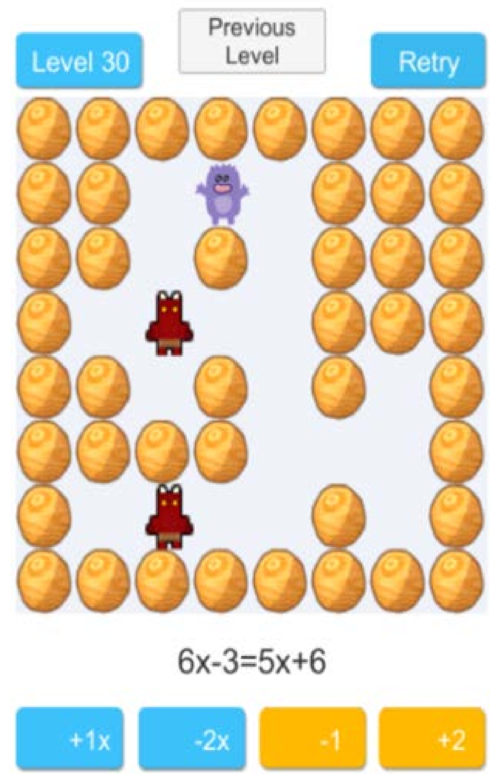
Algebra Games to Learn Mathematics

http://www.algebragamification.com

1

Algebra





Mathematics Gamification =

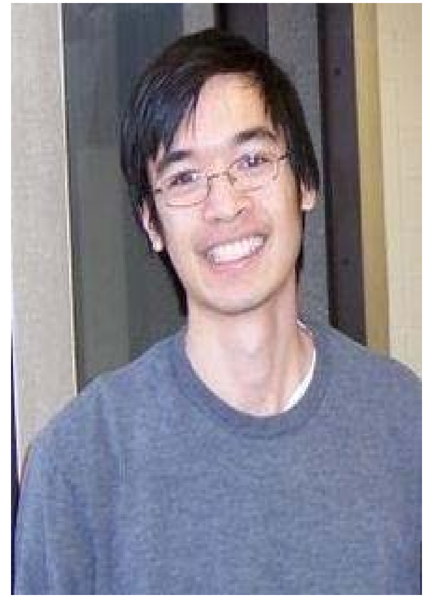
Mathematics

+

Puzzle Game-like Instantiations

Computing Technologies

The set of <u>problem-solving skills needed to solve algebra problems</u> is somewhat similar to the set of skills needed to <u>solve puzzle-type computer games</u>, in which a certain limited set of moves must be applied in a certain order to achieve a desired result. **-Terence Tao** (2012)



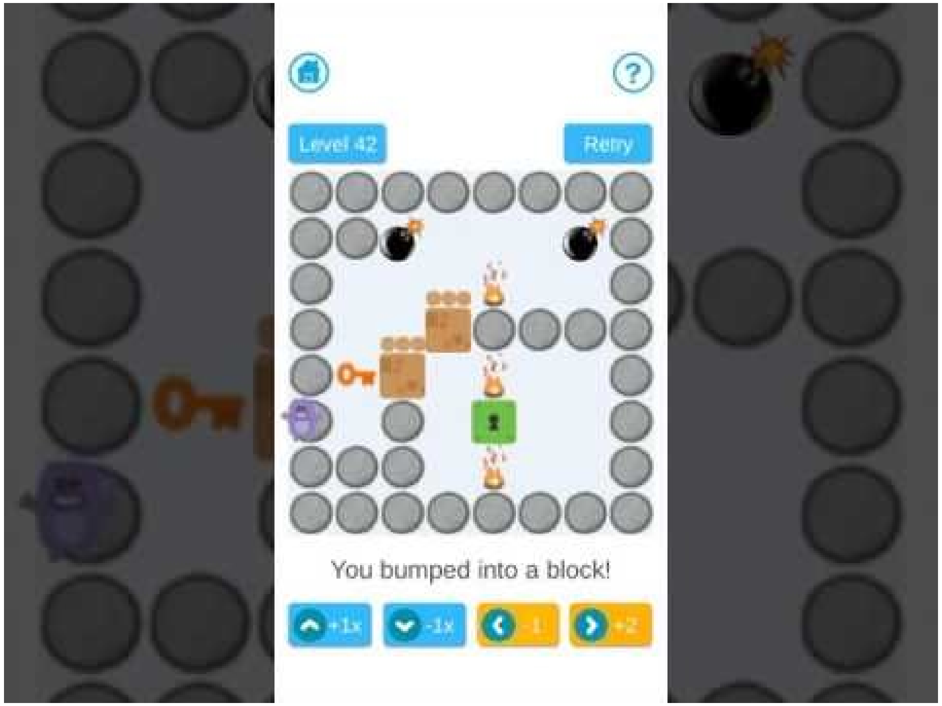
Terence Tao

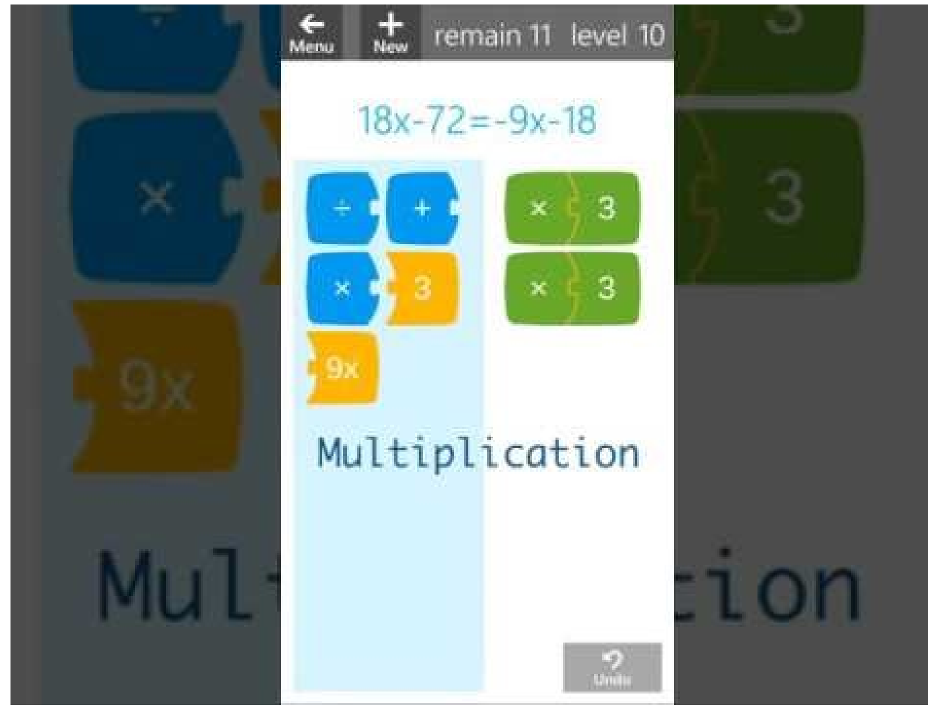
Professor of Mathematics at University of California Los Angeles Field Medalist

"Mozart of Math"

10yr old Tao with Paul Erdős in 1985

Online Tutorials of Algebra Games





Get it on Google play

Available on the App Store

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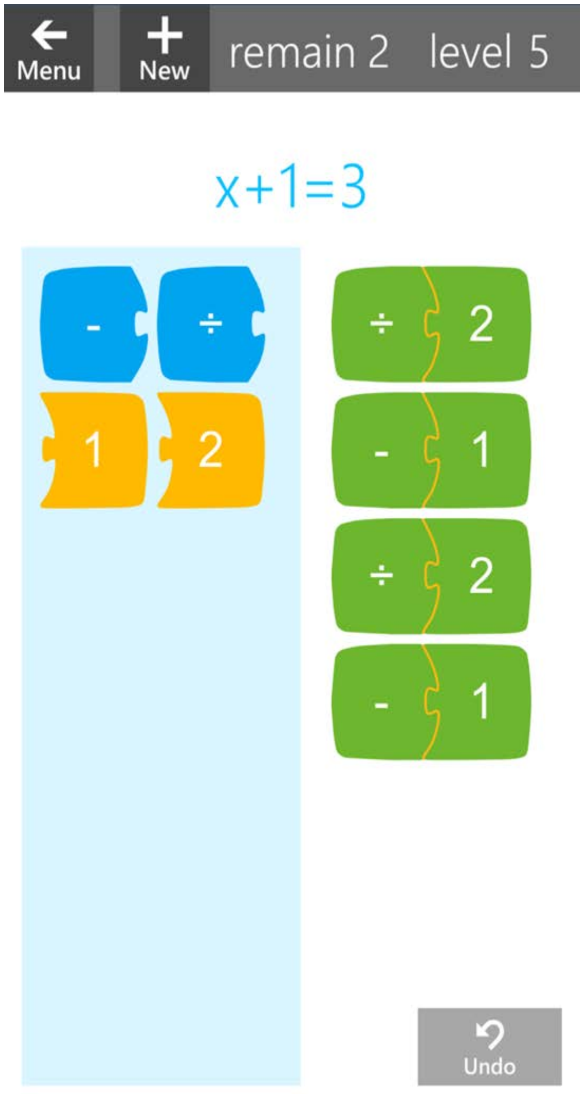
Available on the App Store

Example

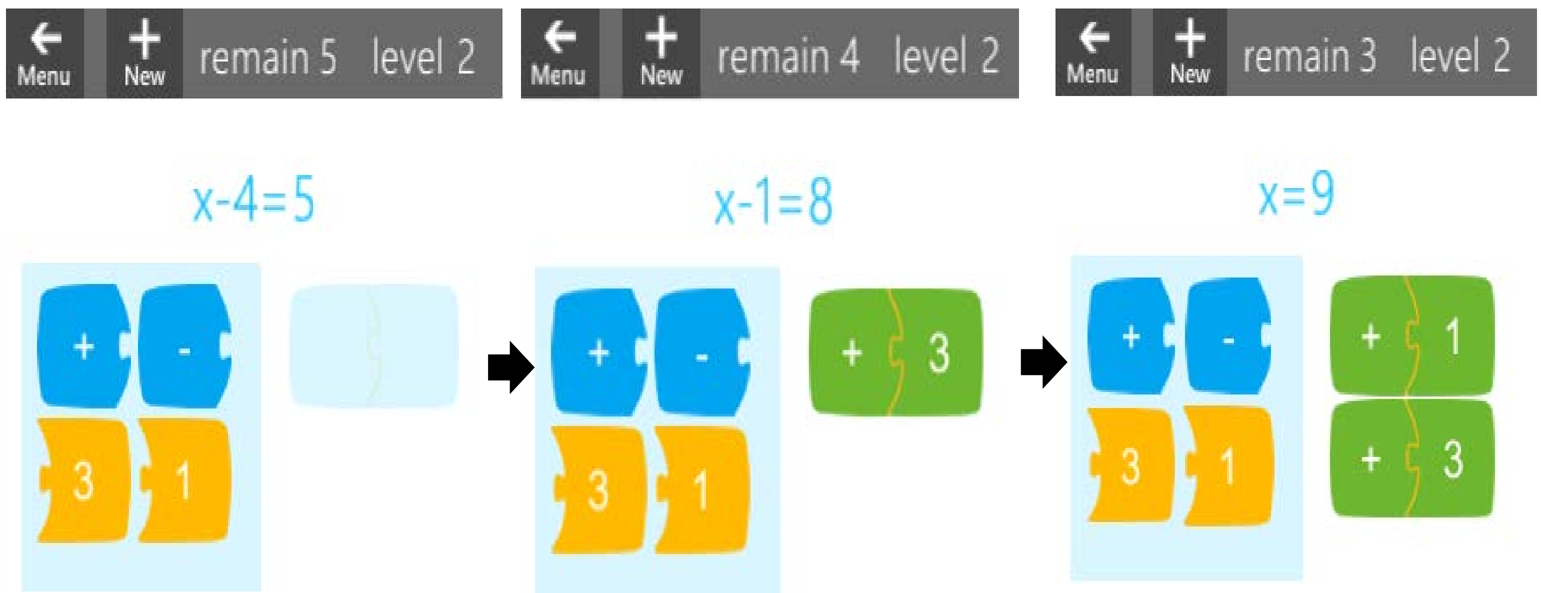


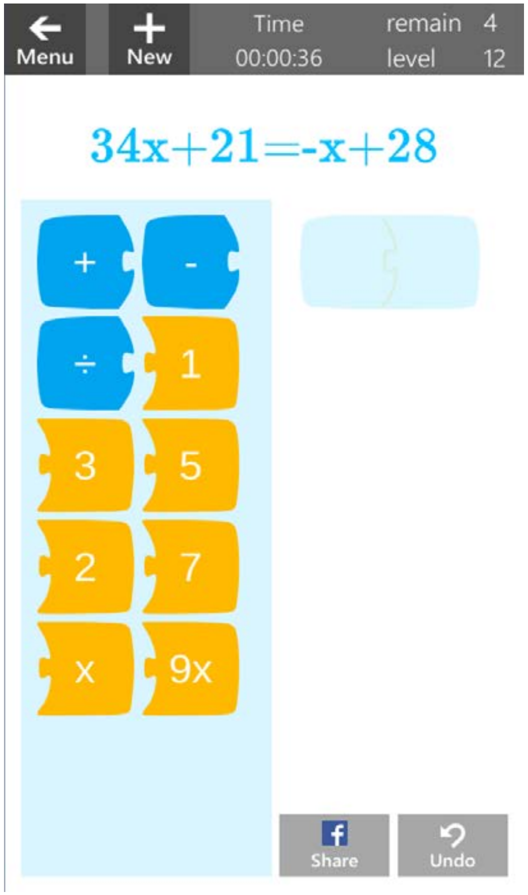
Only 2 simple steps:

1. Pair the operator clue and number clue by drag and drop to change the equation step by step.
2. Complete the level when the puzzle state is “**x = solution_value**” and the remaining number of moves allowed is zero or more (indicated by **remain**).

Algebra Game

$$x - 4 = 5 \xrightarrow{\{+,-,1,3\}} x = 9$$

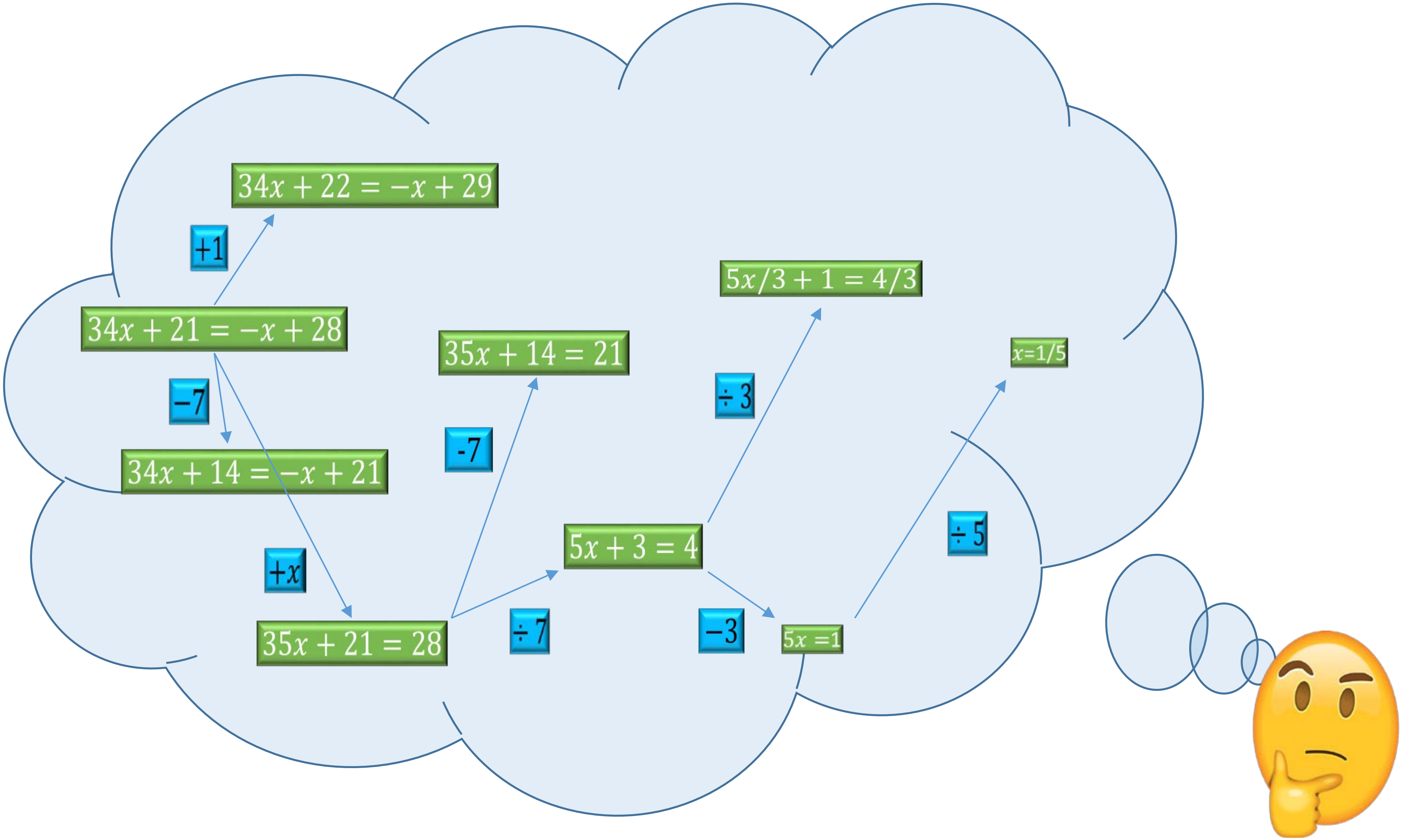




Suppose we don't allow multiplication and division by x-terms.

Then, there are 130,321 combinations of given operators !

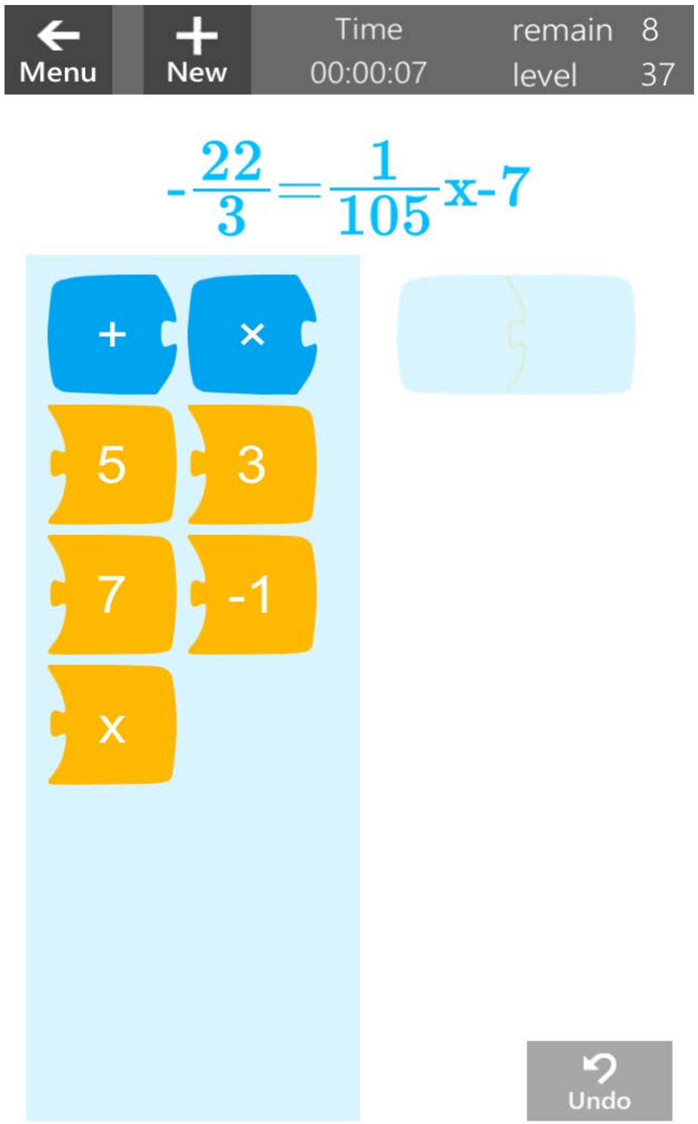
What's your strategy to reach the required equation $x = \frac{1}{5}$ in four moves?



$34x + 22 = -x + 29$
$+1$
$34x + 21 = -x + 28$
-7
$34x + 14 = -x + 21$
$+x$
$35x + 21 = 28$
$35x + 14 = 21$
-7
$\div 7$
$5x + 3 = 4$
$5x/3 + 1 = 4/3$
$\div 3$
-3
$5x = 1$
$\div 5$
$x=1/5$

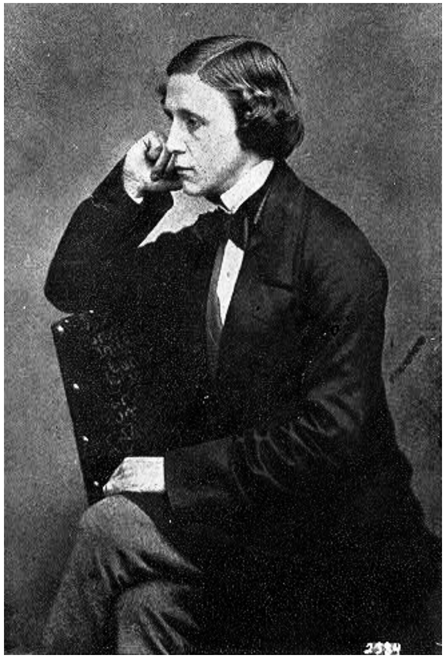
Mental Exercise !

Can you reach the required equation $x = -35$ in seven moves?



Word Ladder Game

A word ladder puzzle begins with two words, and to solve the puzzle one must find a chain of other words to link the two, in which two adjacent words (that is, words in successive steps) differ by one letter. Lewis Carroll invented the game on Christmas day in 1877



From HEAD to TAIL:

H E A D → H E A L → T E A L → T E L L

→ T A L L → T A I L

Five moves needed.

Can you come up with less than five moves? How many possible solutions?

The objective of the game is to find a path with the least number of moves

Turn APE to MAN

Word Ladder Challenge

Driving HORSE into FIELD

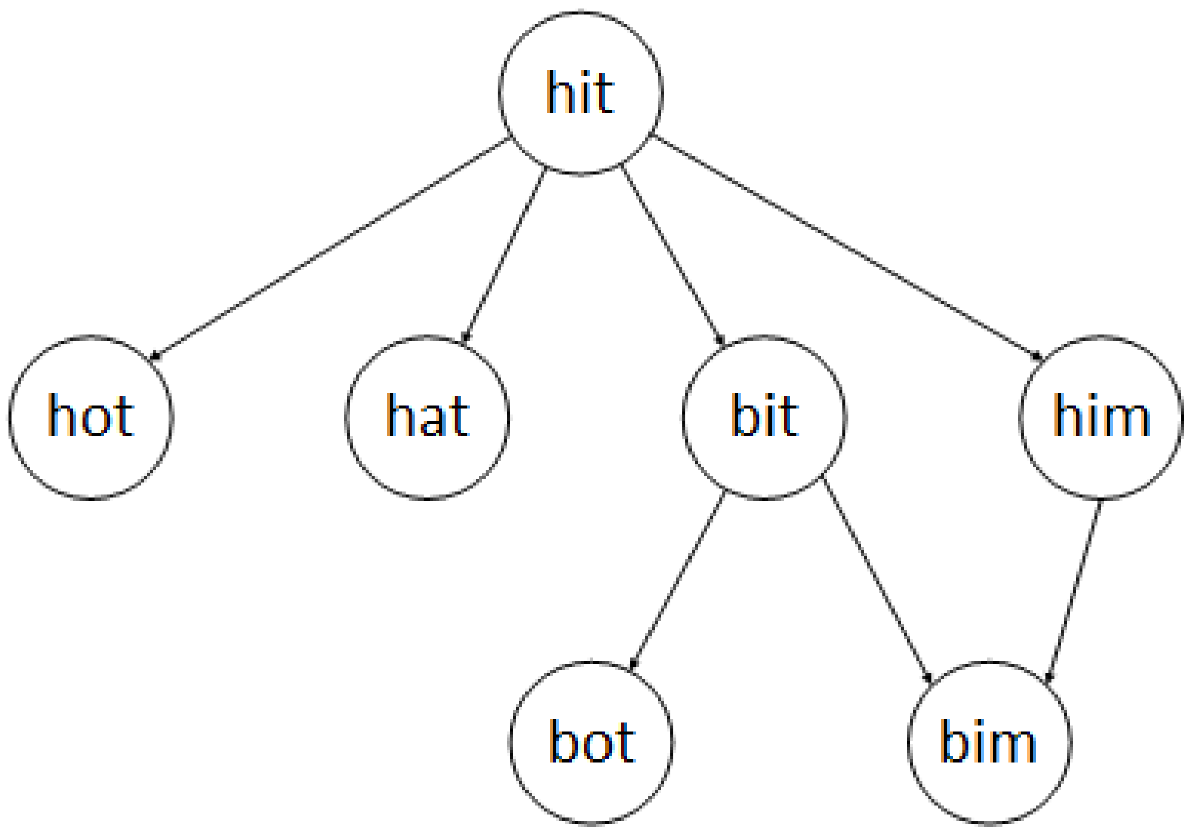
In Lewis Carroll's day, the problem remained unsolved. In last few years, this must have been accomplished using some modern words unknown in Victorian times. Can you solve it?

Graphs of Word Ladder Game

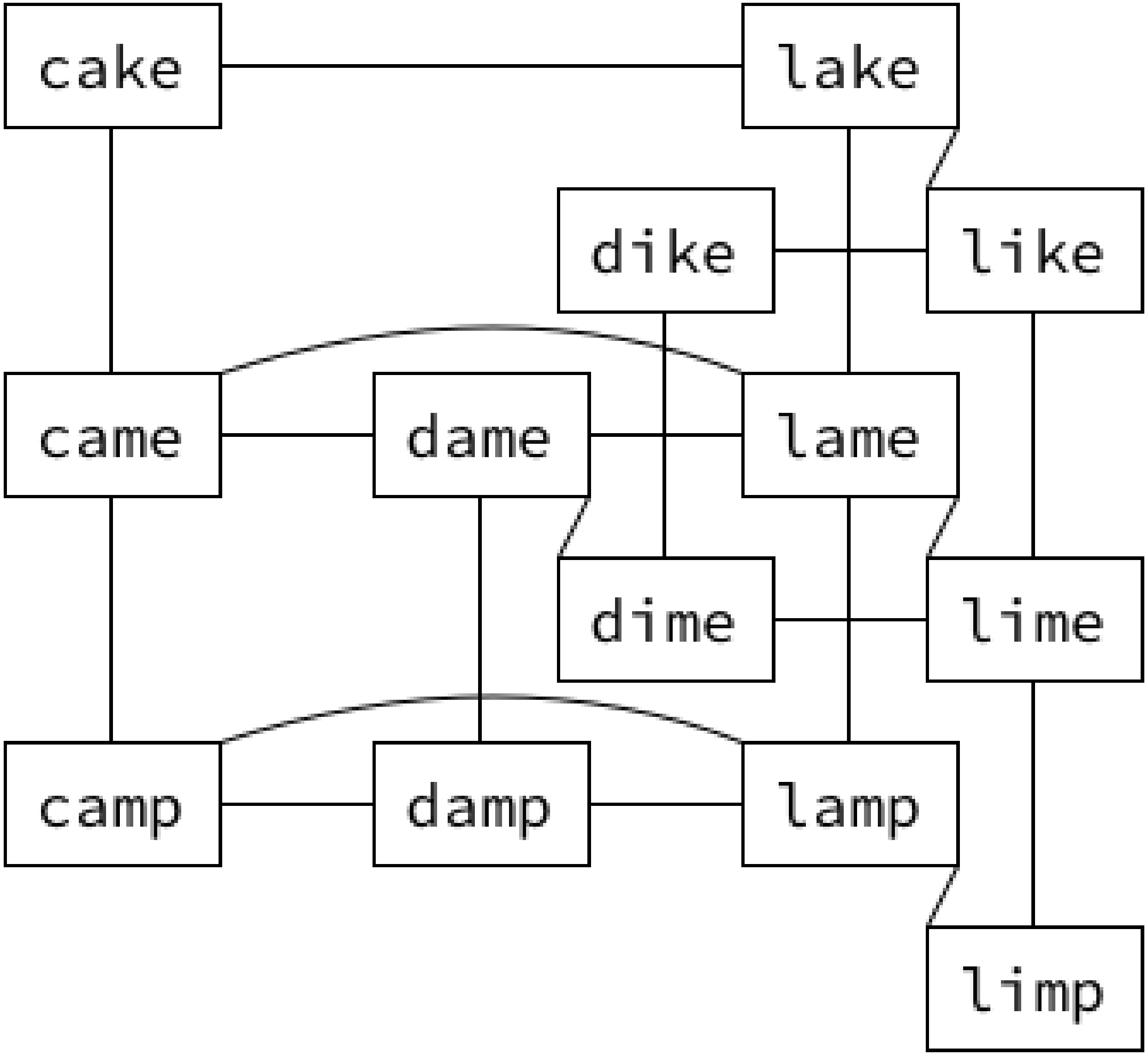
A graph is an important mathematical object:

Graph vertex or node: model a state of a game

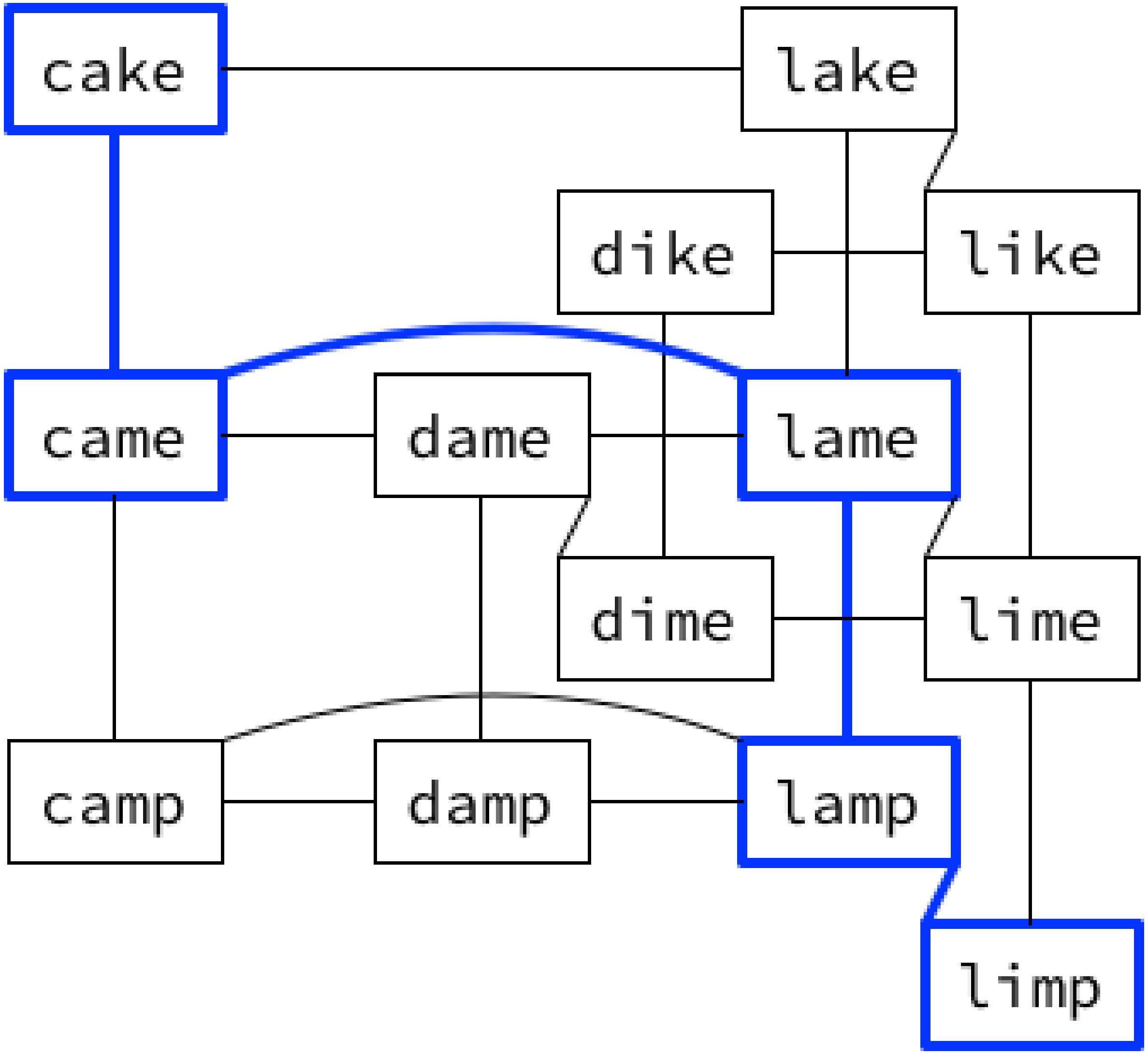
Graph edge: model transition or relationship



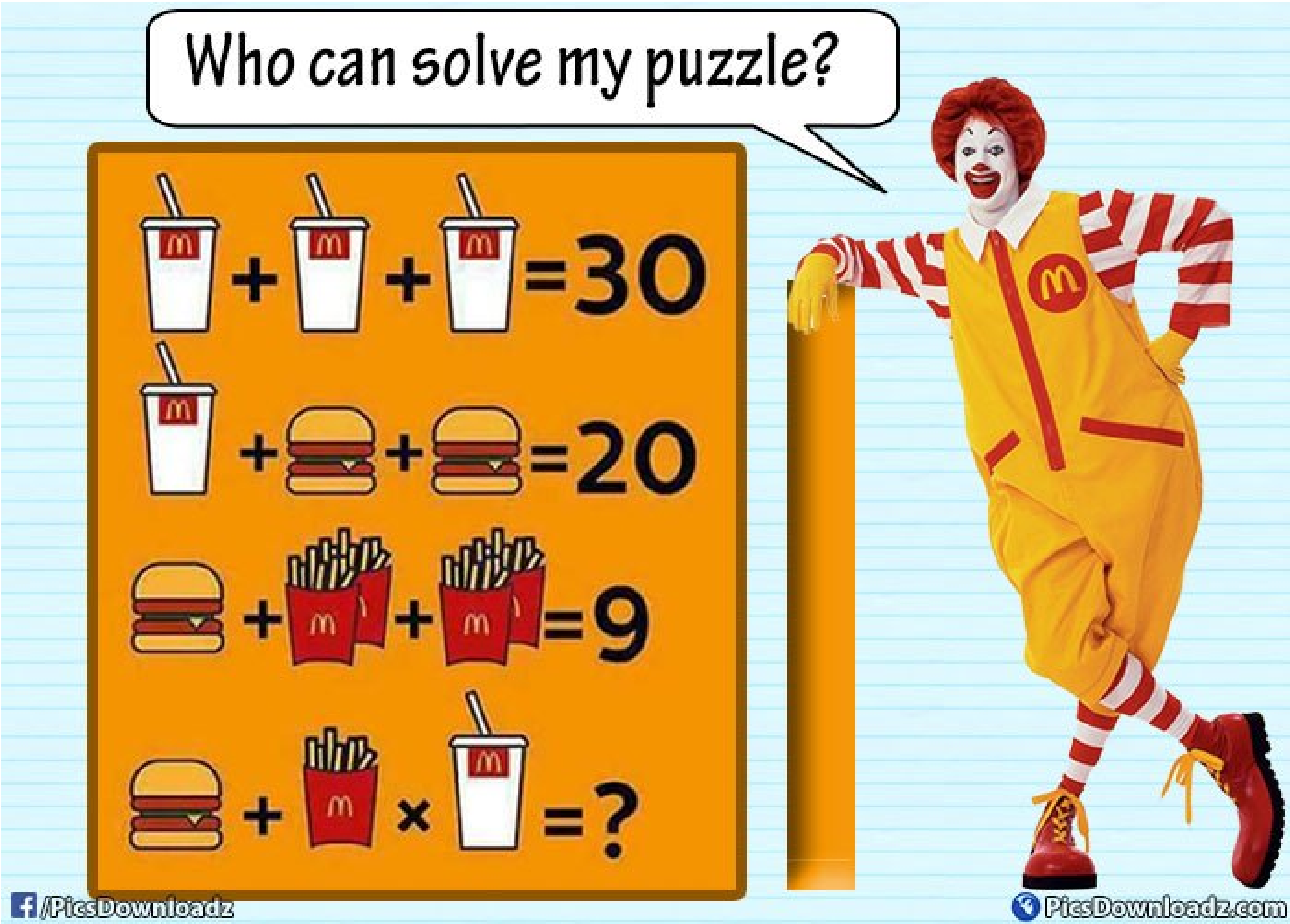
Graphs of Word Ladder Game



Graphs of Word Ladder Game



A Viral Math Puzzle



What if I Ask This?

In words, a hamburger and a soft-drink costs \$9 and \$2 respectively. How many hamburgers and soft-drinks can I buy to spend all \$30 in my pocket?

Can I spend all \$31 in my pocket if the price of a hamburger shoots up to \$10?

Diophantine equation

A Diophantine equation is a polynomial equation whose solutions are restricted to integers. These types of equations are named after the ancient Greek mathematician, Diophantus. A linear Diophantine equation is a first degree equation of this type. In the digital age, Diophantine analysis is very important in the study of public-key cryptography, for example.

Diophantine equations are important when a problem requires a solution in whole amounts (i.e., positive or negative integers).

Solving Diophantine equation

Travis is purchasing beverages for an upcoming party. He has $68 to spend. He can purchase packs of cans for $12, or smaller packs of bottles for $8.00. How many ways are there for him to purchase beverages if he spends all of his money?

Let c be the number of packs of cans he purchases and let b be the number of packs of bottles he purchases. The goal is to find the non-negative solutions to

$$12c + 8b = 68.$$

First note that there is a common factor, 4. Dividing out this common factor gives

$$3c + 2b = 17.$$

17 is not divisible by 3 nor 2, so he must purchase some combination of cans and bottles. Begin by finding a solution in which he purchases the maximum number of cans. He could purchase at most 5 packs of cans. This gives

$$15 + 2b = 17,$$

which leaves him with just enough money to purchase 1 pack of bottles. Now consider how this solution could be altered to find more solutions. Consider if he only bought 4 packs of cans:

$$12 + 2b = 17.$$

This equation gives no integer solution for b, so this is not possible. One can observe that the number of packs of cans must decrease in increments of 2. Meanwhile, the number of packs of bottles will increase in increments of 3. The ordered pair solutions (c, b) are listed below:

$$(5, 1), \quad (3, 4), \quad (1, 7).$$

Therefore, if Travis spends all his money, there are 3 ways he could purchase beverages for the party. $\square$

Solving Diophantine equation

Can I spend all $9 to get the soft-drinks and french fries?
Give two mathematical proofs.
Hint: Odd + Even = Odd, i.e., Use the concept of parity which is important in algebra

Solving Diophantine equation 

Initial Solution to a Diophantine Equation

You may have observed from the examples above that finding solutions to linear Diophantine equations involves finding an **initial solution**, and then altering that solution in some way to find the remaining solutions. The process of finding this initial solution isn't always as straightforward as the examples above. Fortunately, there is a formal process to finding an initial solution.

First, it is important to recognize when solutions exist. Recall the previous example in which there were no solutions. There was a common factor between the coefficients of the variables, but the constant term was not divisible by this factor. This observation is generalized with the Bézout's Identity:

Bézout's Identity:

Let a and b be non-zero integers and let $d = \gcd(a, b)$. Then there exist integers x and y that satisfy

$$ax + by = d.$$

Furthermore, there exist integers x and y that satisfy

$$ax + by = n$$

if and only if $d \mid n$.

One can determine if solutions exist or not by calculating the GCD of the coefficients of the variables, and then determining if the constant term can be divided by that GCD.

EXAMPLE

Find all integers solutions to the equation

$$14x + 91y = 53.$$

First calculate $\gcd(14, 91) = 7$. Then, observe that $7 \nmid 53$. Therefore, by Bézout's Identity, there are no integer solutions to the equation. $\square$

Solving Diophantine equation

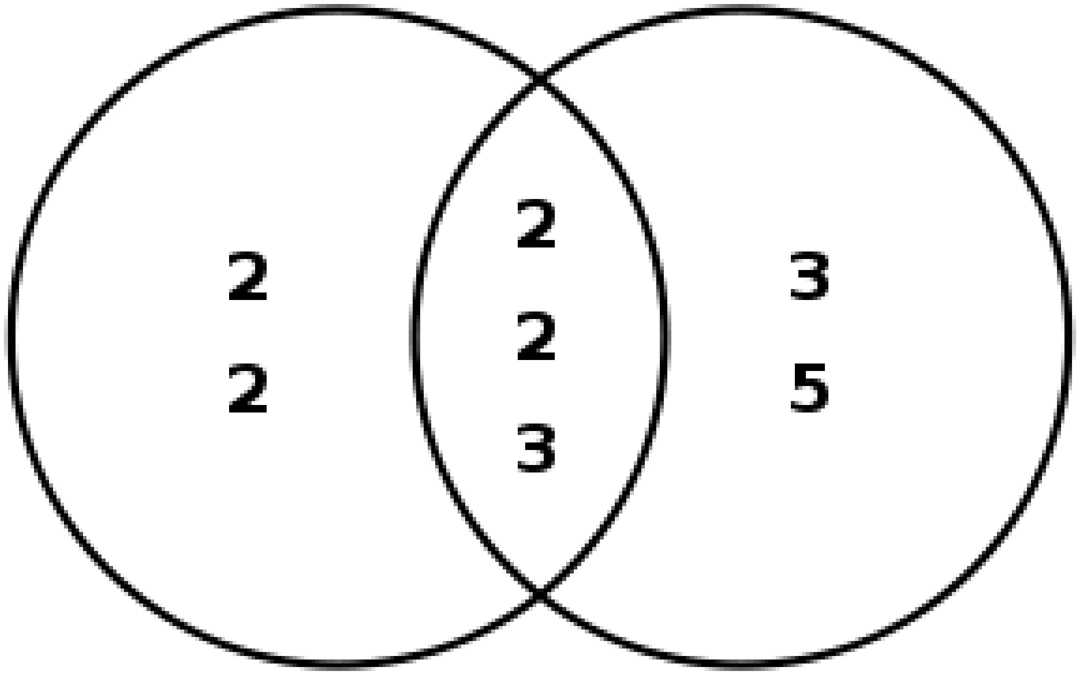
Method for computing the initial solution to a linear Diophantine equation in 2 variables:

Given an equation $ax + by = n$,

- Use the Euclidean algorithm to compute $\gcd(a, b) = d$, taking care to record all steps.
- Determine if $d \mid n$. If not, then there are no solutions.
- Reformat the equations from the Euclidean algorithm.
- Using substitution, go through the steps of the Euclidean algorithm to find a solution to the equation $ax_i + by_i = d$.
- The initial solution to the equation $ax + by = n$ is the ordered pair $\left(x_i \cdot \frac{n}{d}, y_i \cdot \frac{n}{d}\right)$.

Solving Diophantine equation

The Euclid Algorithm computes the GCD of two given integers, example below shows a Venn Diagram that illustrates the GCD of 48 and 180 is 12.



Euclid Algorithm for GCD

```
int euclid(int a, int b)
  { if (a == 0)
         return b;
    return euclid(b%a,a);
  }
```

```
int euclid(int a, int b)
  { if (a == b)
         return b;
    if (a > b)
        return euclid(a-b,b);
     else
        return euclid (b-a,a);
  }
```

What is the GCD of 12344321 and 34566543?

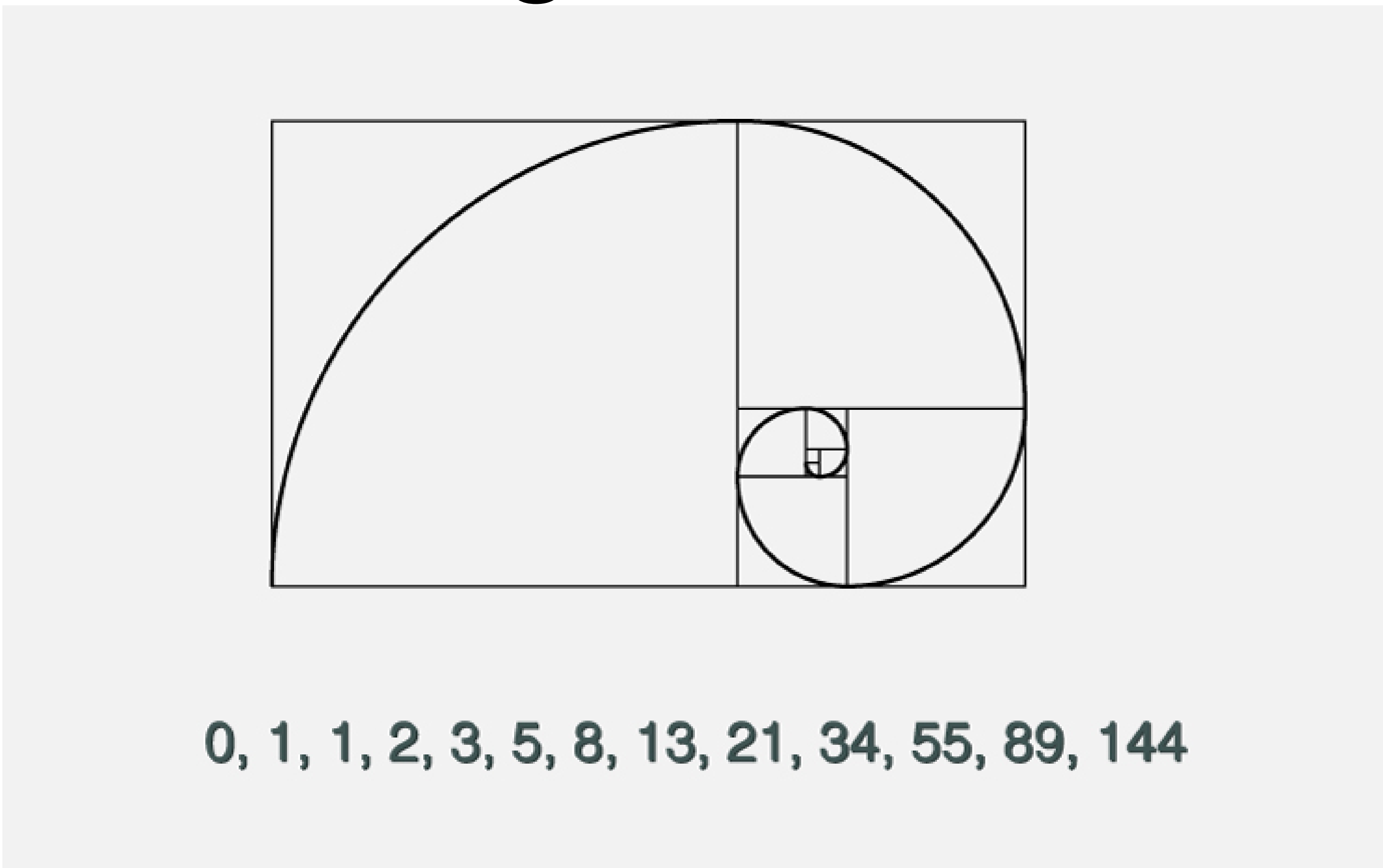
What is the GCD of 56788765 and 34566543?

What do you observe about the GCD of palindrome numbers? Give a proof.

A palindrome number is a number that remains the same when its digits are reversed. A palindrome is a word, phrase, or sequence that reads the same backwards as forwards, e.g. madam, noon.

Can you combine the idea of palindrome in Lewis Carroll's Word Ladder game?!

Euclid Algorithm for GCD



What is the GCD of a pair of neighboring Fibonacci numbers?
Use Bezout’s Identity (see previous slides) to prove your answer.

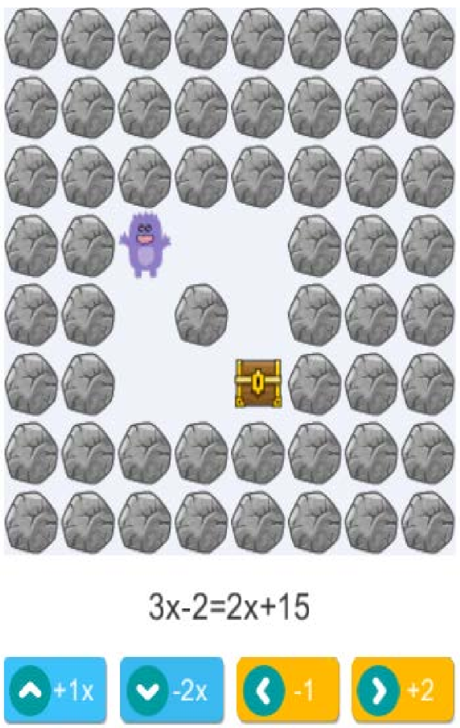
Algebra Game: Perspective from Number Theory

We now apply what we have learned to analyze the Algebra Game!

$$x - b' = d' \xrightarrow{\{+,-,q,r\}} x = b' + d'$$

This is equivalent to find two integers c_1 and c_2 such that $qc_1 + rc_2 = b' + d'$. By using the Euclidean g.c.d Algorithm, we can find two integers ${c_1}'$ and ${c_2}'$ such that $qc_1' + rc_2' = g.c.d(q,r)$. This implies $c_1 = (b' + d'){c_1}'$ and $c_2 = (b' + d'){c_2}'$, if $g.c.d(q,r) = 1$. In particular, this problem is solvable for any b' if and only if $g.c.d(q,r) = 1$.

Algebra Maze

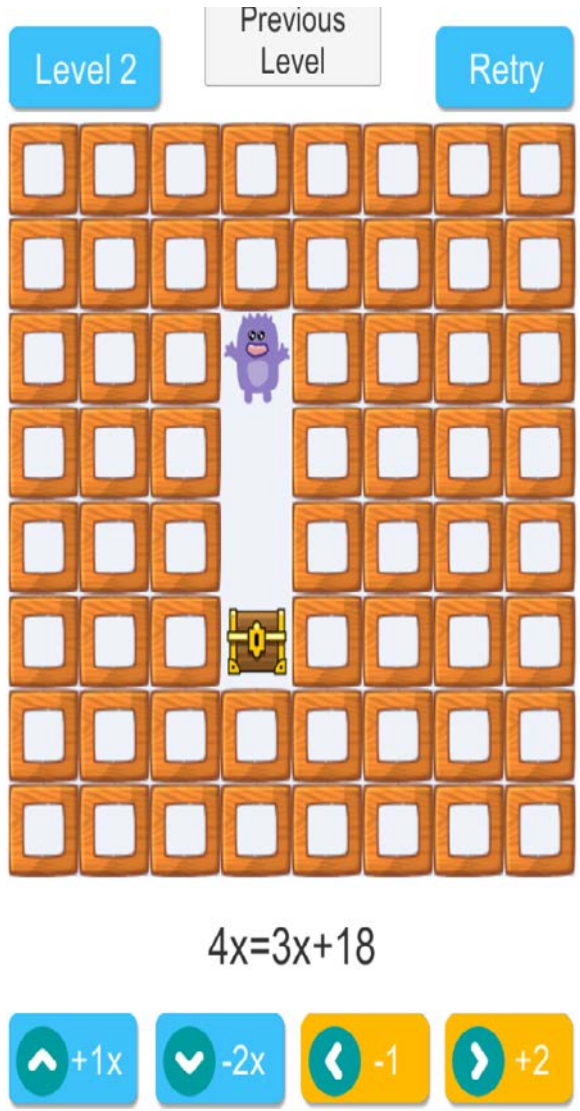


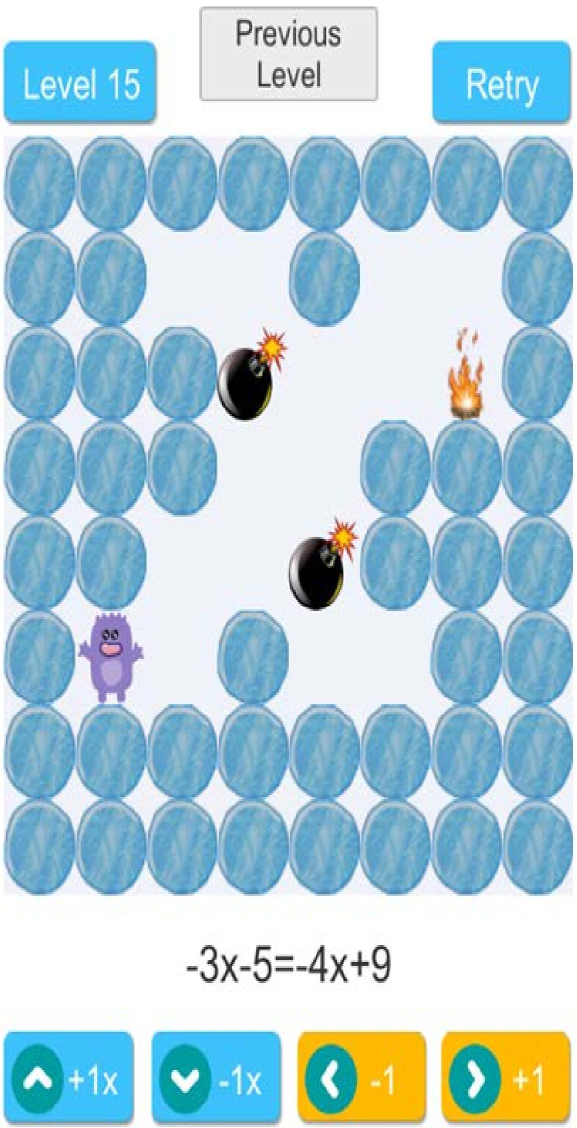
:Rick

Goal: Help Rick to find the treasure.

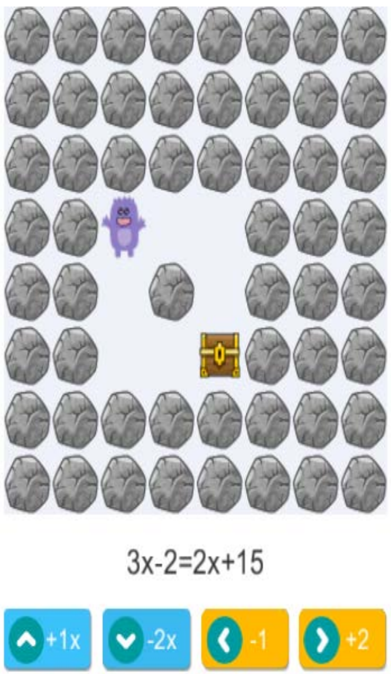
Once the equation is in the desired form **“$x =$some numerical value”,** Rick will get the treasure.

Maze version to resolve Algebra equations





From Algebra-solving to Maze-solving

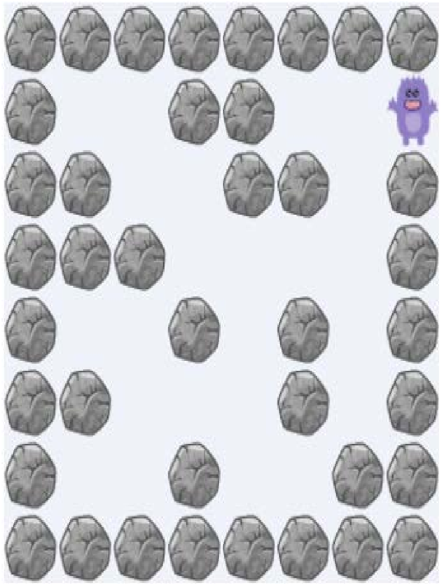


: Move rightward two cells, and the equation add 2 on both sides.

: Move leftward one cell, and the equation minus 1 on both sides.

: Move upward one cell, and the equation add 1x on both sides.

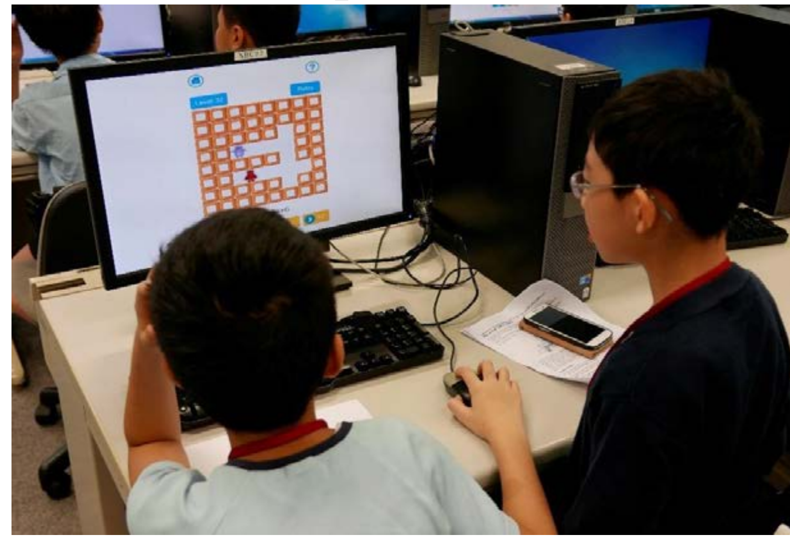
: Move downward two cells, and the equation minus 2x on both sides.



$4x+6=3x+24$

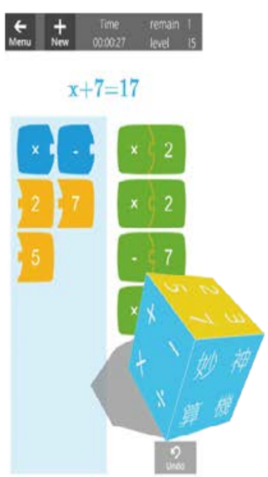
Where is the hidden treasure chest?

Computer Science Challenge 2016, 2017, 2018 (eSport for STEM)



COMPUTER SCIENCE CHALLENGE 電腦科學大挑戰 2018

CityU
香港城市大學
City University of Hong Kong



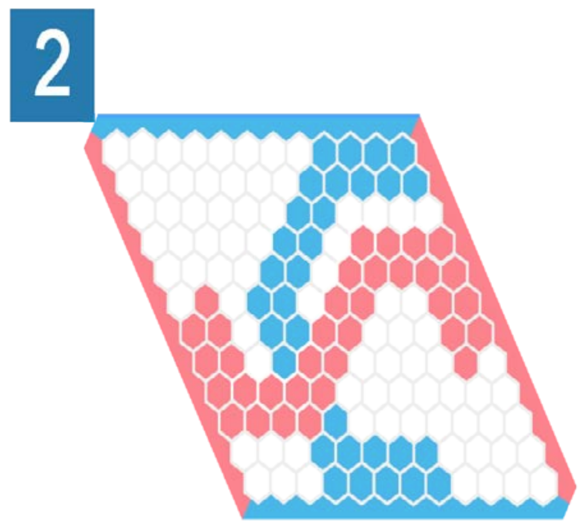
Computer Science Challenge 2016-2018 (https://cschallenge.cs.cityu.edu.hk)




Artificial Intelligence in Game

Play against Artificial Intelligence (AI) to place a connected path of bridges from one side of the board to the opposite side while blocking the opponent from doing the same. A strategic game that is easy to play for all ages!

Independently invented in the 1950-60s by the mathematician David Gale, thus known as the Game of Gale, the mathematician John Nash and also the mathematician Claude Shannon, when it was known as the Shannon Switching Game. The game has a deep mathematical root in graph theory and is challenging for users of all age groups.

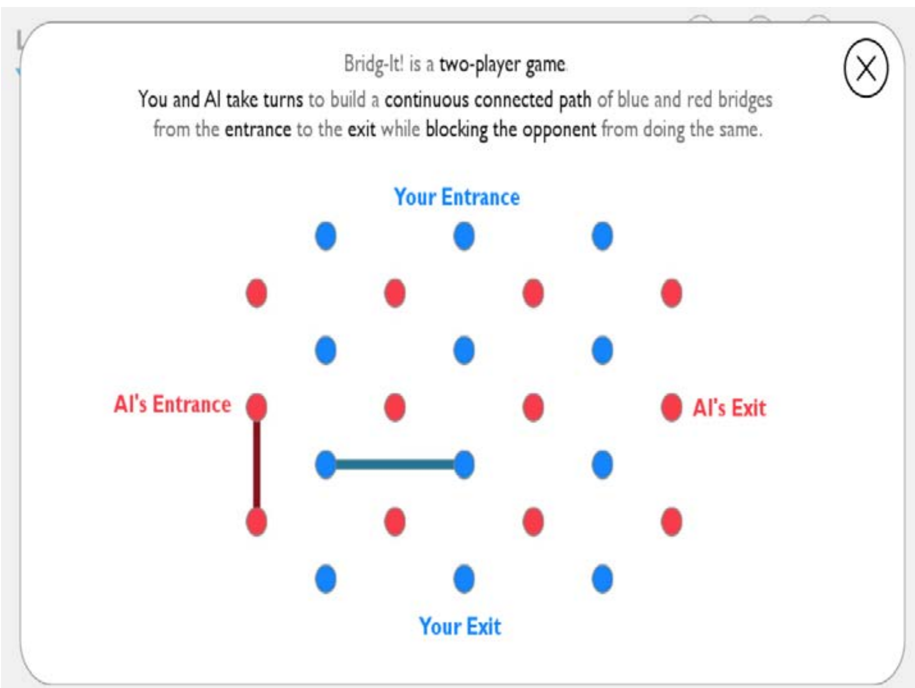


GAME

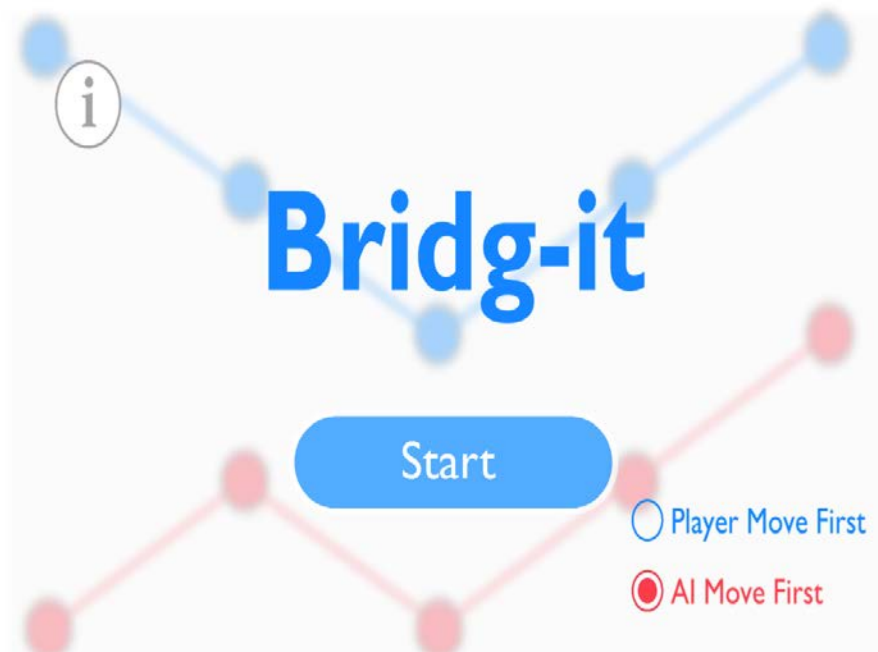
Brid**g**-It Game

You and AI take turns to build a continuous connected path of blue and red bridges from the entrance to the exit while blocking the opponent from doing the same.

Screen 1: Introduction



Screen 2: Menu – Select either AI or Player moves first



Game On!

http://game.algebragamification.com/csc2018/2/

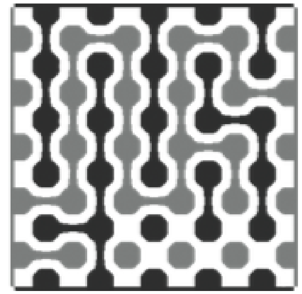
Bridg-It

Brambles start to barricade

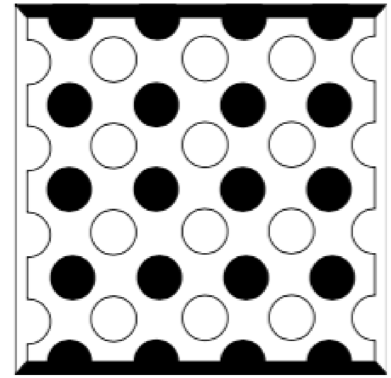
The tried-and-tested path

Fight our way through thorns across

Before we face their wrath!



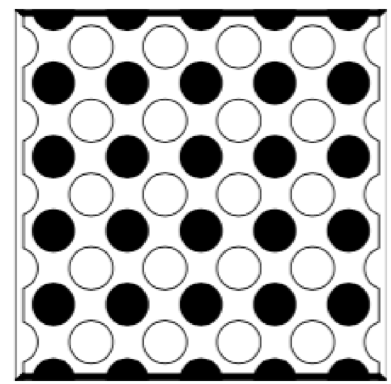
In this game of Bridg-It, the gray player wins.



4 × 4 board

Two players take turns to join the borders of their colors. In each turn, the player connects two adjacent circular nodes in the up-down or left-right direction, but not along diagonals. No overlapping is allowed. The player who bridges his/her borders the most quickly wins.

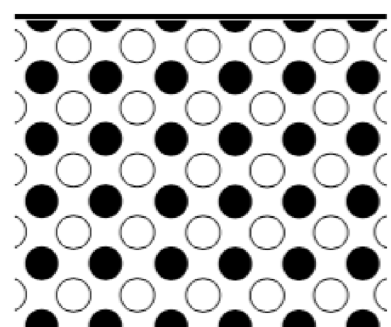
1. Play the game on the printout 4×4, 5×5 and 6×6 boards provided. Which player wins, the first or the second?
2. What is the winning strategy? (That is, what techniques enabled you or your partner to win?)
3. Can there be a tie? Why? Does the winning strategy depend on the size of the board?



5 × 5 board

Download the **JRMF** app and locate **Bridg-It** inside. Do Q. 4-5, with your device as the second player.

4. Play the game on the 7×7 and 8×8 (or larger) boards inside the app. Which player wins, the first or the second?
5. How would you know you are losing?
6. What could be a reason *diagonal* connections are not allowed?



Homework!

1) Give as many solutions as you can for Lewis Carroll's Word Ladder game for "Driving HORSE into FIELD"

2) Is it possible to measure exactly 4 liters of water using a 3-liter jug and a 5-liter jug?

3) Mary wants to form 97 cents in postage stamps using only 12-cent stamps and 9-cent stamps. How many of these stamps can she buy?

4) Use the Euclid algorithm to find the GCD of 1001 and 2001?

5) Give a proof about the GCD of palindrome numbers.

6) Design a Lewis Carroll's Word Ladder game to go from one palindrome word to another palindrome word.

Mathematics and Games

1) Learn problem-solving skills and advanced mathematics
2) Learn math behind fun games and puzzles with applications to Computer Science
3) Learn how to create math games for mobile apps
4) Learn software programming to build math games that we will use at Computer Science Challenge 2019 and Julia Robinson Mathematics Festival 2019!