Multiuser Detection of Alamouti Signals

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Abstract— **In a MIMO multiple-access channel where users employ Space-Time Block Codes (STBC), interference cancellation can be used to suppress co-channel interference and recover the desired signal of each user at the receiver. Leveraging the special properties of Alamouti matrices, we first show that spatial multiplexing of Alamouti signals retains the space-time diversity gain of Alamouti signaling using our proposed low-complexity Alamouti BLAST-MMSE (A-BLAST) Algorithm. Next, in contrast to traditional transmit diversity that focuses on STBC construction at the transmitter, this paper looks at transmit diversity from the perspective of the receiver. In other words, the receiver gets to choose the STBC's, which are favourable to the channel assuming a fixed BLAST receive algorithm. In a multiuser MAC setting, we first present a systematic methodology to exploit different decomposition structure in Alamouti matrices, each with different tradeoff between performance and decoding complexity using possibly different MIMO receive algorithms. We then demonstrate that the notion of angles (the inner product of two quaternionic vectors) between multiuser channels determines the performance of MIMO receive algorithms. As an application of the general theory, we transform the decoding problem for several types of Quasi-Orthogonal STBC (QOSTBC) into multiuser detection of virtual Alamouti users. Building upon our A-BLAST Algorithm, we propose new algorithms for decoding single-user and multiuser QOSTBC. In particular, we show that bit error probability is a function of the quaternionic angle between virtual users (for a single user) or multiple users. This angle varies with the type of QOSTBC and leads to a new form of adaptive modulation called code diversity, where feedback instructs the transmitter how to choose from a plurality of codes.**

Index Terms— Space-Time Block Code (STBC), Alamouti STBC, Quasi-Orthogonal STBC, quaternions, interference cancellation, BLAST, Decorrelator, Minimum Mean Squared Error (MMSE), Maximum Likelihood (ML), adaptive modulation, code diversity.

I. INTRODUCTION

Multiple transmit and receive antennas can increase the transmission rate and improve the reliability of wireless systems by using Multiple-Input Multiple-Output (MIMO) techniques. One such technique is Space-Time Block Coding (STBC) that improves transmission reliability by correlating signals across different transmit antennas [1], [19]. At the receiver, space-time signal processing starts with a system of linear equations where signals are multiplied by channel gains. The value of algebraic structure to the construction of STBC is to transfer correlation from the transmitter to the receiver. For example, orthogonal signaling at the transmitter enables orthogonal separation of signals at the receiver with Maximum Likelihood (ML) decoding complexity that is constant, i.e., independent of the signal constellation size [19]. Diversity gain of STBC measures the link level advantage over a single path from the transmitter to the receiver. Of particular note is the Alamouti scheme that employs two transmit antennas to achieve the maximal diversity gain of two and a rate of one symbol per time slot [1]. The Alamouti code is the only orthogonal STBC that achieves full rate using complex modulation scheme, i.e., it is the only STBC that achieves the full diversity gain without needing to sacrifice data rate [19].

In coherent multiuser detection, optimal error-rate performance is achieved by the joint ML detector [21]. However, since the computational complexity of ML detection increases exponentially as a function of bandwidth efficiency (measured in bits per channel use [21]), it is expedient to consider decoding strategies with lower complexity. An analysis of different MIMO receive algorithms that provides a great deal of geometric insight can be found in [10]. In particular, the MIMO detection problem can be viewed as finding the closest lattice point to a given point, and is known to be NP-hard. Effective and fast heuristics, which are often attractive (when the number of users is large), can be implemented by interference nulling and cancellation strategies [21]. The key advantage of interference suppression is an increase in spatial multiplexing gain or system capacity, since it is now possible to support many users on the same channel.

A celebrated interference suppression scheme considered in [10] is the Bell Laboratories Layered Space-Time (BLAST) system [6], which is mathematically equivalent to the decision feedback equalizer (DFE) [21]. Decisions in a DFE can be obtained in many different ways by using different MIMO receive algorithms. In particular, the BLAST receiver combines a MIMO receive algorithm with optimal ordered successive interference cancellation. One variant is the Vertical BLAST (V-BLAST) algorithm [6], [10]. The BLAST receiver has a lower decoding complexity than joint ML decoding, but its performance is limited by decoding error propagation [6], [10]. Since the whole BLAST architecture is used primarily for spatial multiplexing, the diversity gain of each user (BLAST layer) does not increase. Thus, its performance can be affected by poor reliability. One way to improve reliability in BLAST for each user is to use a STBC such as the Alamouti code, which increases the diversity gain and also mitigates the problem of error propagation in BLAST [2], [3], [5], [11], [15], [18].

For a multiuser setting, it was first shown in [4], [13], [14] that special structures of the Alamouti signal can be exploited to suppress co-channel interference while decoding the bit stream of the desired user. This paper generalizes these previous work by providing a mathematical framework to design multiuser detection algorithms in BLAST based on quaternions. Using the Alamouti code as a building block, we develop different ways of decoupling transmission signals (by exploiting the algebraic structure of quaternion) at the

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receiver, which are then decoded using BLAST. To illustrate our methodology, we present a systematic methodology to exploit different decomposition for a multiuser system that employs Quasi-Orthogonal STBC (QOSTBC), each with different tradeoff between performance and decoding complexity using possibly different MIMO receive algorithms. We show that Alamouti signaling not only increases the overall diversity gain and hence the resultant performance of multiuser decoding, but the Alamouti structure also yields simple decoding algorithms. Thus, spatial multiplexing of Alamouti signals retains the space-time diversity gain of Alamouti signaling with low-complexity. In particular, we propose a BLAST receive algorithm for Alamouti signals based on the Minimum Mean Squared Error (MMSE) filter, which we call the Alamouti BLAST-MMSE (A-BLAST) Algorithm.

We also demonstrate that the distribution of quaternionic angles (the normalized inner product of two quaternionic vectors[1]) between multiuser channels is an important geometric parameter that determines the performance of various MIMO receive algorithms including the A-BLAST Algorithm. Error probability is a function of the quaternionic angle between users, and this varies with the type of STBC. Given Channel State Information (CSI), the receiver is able to identify the code that maximizes the post processing signal-to-noise ratio SNR (or implicitly the Joules per bit). This leads to a new form of adaptive modulation where feedback is used to instruct the transmitter how to choose from a plurality of possible codes. Our example shows that given a set of equivalent codes, ML decoding of a particular code can be inferior (both in terms of error probability and decoding complexity) to a combination of adaptive modulation and suboptimal decoding. The improvement in performance is associated with a new form of diversity, which we call *code diversity*.

This paper is organized as follows. In Section II, we describe the Alamouti system model for users with two transmit antennas and analyze the performance of the MMSE detector with successive interference cancellation. In Section III, we introduce briefly the BLAST algorithms for multiuser detection. In Section IV, we give an analysis of the two-user detection problem. In Section V, we introduce the A-BLAST Algorithm. In Section VI, we extend our system model to users with four transmit antennas, each employing QOSTBC, and illustrate how code diversity is used systematically in conjunction with different decomposition of QOSTBC and the A-BLAST Algorithm. We compare the performance of our scheme with previous work in Section VII.

The following notation is used. Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and italics denote scalars. The super-scripts $(\cdot)^T$, $(\cdot)^*$, $(\cdot)^\dagger$ and $(\cdot)^+$ denote transpose, complex conjugate, Hermitian conjugate and pseudoinverse respectively.

II. System model

In this section, we consider a i.i.d. Rayleigh fading Multiple Access Channel (MAC) with N synchronous co-channel users, each employing two transmit antennas and a single receiver employing N receive antennas. Each user employs the 2×2 Alamouti space-time block code. Figure 1 shows the system architecture. The baseband received signal is given by

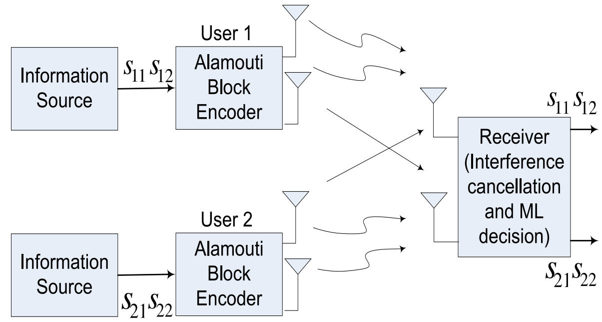

Fig. 1. Successive interference cancellation and decoding in a two-user MAC system, where each user transmits an Alamouti signal.

$$\mathbf{r} = \sum_{i=1}^{N} \mathbf{H}_i \mathbf{s}_i + \mathbf{n}, \tag{1}$$

where $\mathbf{r}$ has entries $\mathbf{r}_i = [r_{i1} \; -r_{i2}^*]^T$ with r_{i1} and r_{i2} being the signals received at the ith receive antenna over two consecutive symbol periods for $i = 1, \ldots, N$, $\mathbf{s}_i = [s_{i1} \;\; s_{i2}]^T$ is the codeword transmitted by the ith user over two consecutive symbol periods, $\mathbf{H}_i = [\mathbf{H}_{1i}^T \; \ldots \mathbf{H}_{Ni}^T]^T = [\mathbf{h}_{2i} \; \mathbf{h}_{2i+1}]$ are the channel matrices with $\mathbf{H}_{li}$ being the channel matrix from the ith user to the lth receive antenna.[2] Each $\mathbf{H}_{li}$ in $\mathbf{H}_i$ has the structure of a 2×2 quaternion [1], [4]:

$$\mathbf{H}_{li} = \begin{bmatrix} h_{li,1} & h_{li,2} \\ -h_{li,2}^* & h_{li,1}^* \end{bmatrix} \triangleq \mathsf{Q}(h_{li,1}, h_{li,2}), \tag{2}$$

where $h_{li,j}, j = 1, 2$ for all $i, l \in \{1, \ldots, N\}$, are zero mean complex Gaussian random variables with variance 1. Let $\mathbf{H}_{(i)} = [\mathbf{H}_1 \; \ldots \; \mathbf{H}_i]$ and $\mathbf{r}_{(i)}$ denote the channel sub-matrix and the received signal vector (of dimension $2i$) for the first i users respectively. So, $\mathbf{H} = [\mathbf{H}_1 \; \ldots \; \mathbf{H}_N]$ and $\mathbf{r}_N = \mathbf{r}$. We define $\|\mathbf{H}_{li}\|$ as the normalized Frobenius norm of the 2×2 matrix $\mathbf{H}_{li}$, i.e., $\|\mathbf{H}_{li}\|^2 = \mathrm{Tr}(\mathbf{H}_{li}^\dagger \mathbf{H}_{li})/2$. We denote the norm of a quaternion vector $\mathbf{H}_i$ by $\|\mathbf{H}_i\| = \sqrt{\sum_{l=1}^{N} \|\mathbf{H}_{li}\|^2}$.

In addition, we assume that the vector $\mathbf{n}$ is a circularly symmetric complex Gaussian random variable with zero mean and covariance matrix $E[\mathbf{n}\mathbf{n}^\dagger] = \sigma^2 \mathbf{I}$, where $E[\cdot]$ stands for the expectation operator and $\mathbf{I}$ is the identity matrix of appropriate size. If E_s is the total energy radiated by all transmit antennas in a given time slot, then $E_s/2$ is the energy per code symbol for Alamouti signaling. When the signal constellation consists of equally spaced points on the unit circle, no scaling is necessary. We also define the pre-processing SNR of the signal by the expression $\mathsf{snr} = E_s/\sigma^2$.

We first look at the performance analysis of BLAST with MMSE detection for a particular user ordering (starting from

[1]The quaternionic angle can be interpreted as a measure of correlation between channels.

[2]Note $\mathbf{H}_i$ is a quaternionic column vector since its entries are Alamouti matrix (quaternion), and $(\mathbf{H})_i$ denotes the ith quaternionic column vector of the matrix $\mathbf{H}$.

the Nth user with channel matrix $\mathbf{H}_N$). An optimal user ordering scheme is given in Section V (see the A-BLAST Algorithm). Assume that the users indexed from N to $i+1$ have been detected successfully and cancelled from the received signal $\mathbf{r}$ (to obtain $\mathbf{r}_{(i)}$). The receive covariance matrix $\mathbf{M}_i$ of the ith user is given by

$$\mathbf{M}_i = E[\mathbf{r}_{(i)}\mathbf{r}_{(i)}^\dagger] = \mathbf{H}_{(i)}\mathbf{H}_{(i)}^\dagger + \frac{1}{\mathsf{snr}}\mathbf{I} = \sum_{j=1}^{i}\mathbf{H}_j\mathbf{H}_j^\dagger + \frac{1}{\mathsf{snr}}\mathbf{I}. \quad (3)$$

The following result gives the MMSE performance of detecting the two symbols of the ith user.

Theorem 1: The nulling vectors of the MMSE detector for the first and second symbol of the ith user are given, respectively, by

$$\mathbf{w}_{i,1} = \mathbf{M}_i^{-1}\mathbf{h}_{2i} \quad \text{and} \quad \mathbf{w}_{i,2} = \mathbf{M}_i^{-1}\mathbf{h}_{2i+1}. \quad (4)$$

Furthermore, the post-processing SNR when decoding the first symbol and the second symbol of the ith user are given, respectively, by

$$\mathsf{snr}_{i,1} = \mathbf{h}_{2i}^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{h}_{2i} \quad \text{and} \quad \mathsf{snr}_{i,2} = \mathbf{h}_{2i+1}^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{h}_{2i+1}. \quad (5)$$

Proof: First, we have

$$\begin{aligned}\mathbf{w}_{i,1} &= (\mathbf{M}_i - \mathbf{h}_{2i+1}\mathbf{h}_{2i+1}^\dagger)^{-1}\mathbf{h}_{2i}\\ &= \left(\mathbf{M}_i^{-1} + \frac{\mathbf{M}_i^{-1}\mathbf{h}_{2i+1}\mathbf{h}_{2i+1}^\dagger\mathbf{M}_i^{-1}}{1-\mathbf{h}_{2i+1}^\dagger\mathbf{M}_i^{-1}\mathbf{h}_{2i+1}}\right)\mathbf{h}_{2i} = \mathbf{M}_i^{-1}\mathbf{h}_{2i},\end{aligned}$$

where we use the matrix inversion lemma in the second equality and the last equality is due to applying the following result in [4]:

$$\mathbf{h}_i^\dagger\mathbf{M}_i^k\mathbf{h}_j = 0, \quad i \neq j \quad (6)$$

for any integer exponent k. Similarly, $\mathbf{w}_{i,2} = (\mathbf{M}_i - \mathbf{h}_{2i}\mathbf{h}_{2i}^\dagger)^{-1}\mathbf{h}_{2i+1}$ can be simplified to $\mathbf{w}_{i,2} = \mathbf{M}_i^{-1}\mathbf{h}_{2i+1}$, thus proving (4).

Next, using the MMSE nulling vectors in (4), the post-processing SNR of the first symbol of the ith user can be written as

$$\begin{aligned}\mathsf{snr}_{i,1} &= \frac{(\mathbf{w}_{i,1}^\dagger\mathbf{h}_{2i})(\mathbf{h}_{2i}^\dagger\mathbf{w}_{i,1})}{\mathbf{w}_{i,1}^\dagger(\mathbf{M}_i - \mathbf{h}_{2i}\mathbf{h}_{2i}^\dagger)\mathbf{w}_{i,1}} = \frac{\mathbf{h}_{2i}^\dagger\mathbf{M}_i^{-1}\mathbf{h}_{2i}}{1-\mathbf{h}_{2i}^\dagger\mathbf{M}_i^{-1}\mathbf{h}_{2i}}\\ &\overset{(a)}{=} \mathbf{h}_{2i}^\dagger\left(\mathbf{M}_i - \mathbf{h}_{2i}\mathbf{h}_{2i}^\dagger\right)^{-1}\mathbf{h}_{2i}\\ &\overset{(b)}{=} \mathbf{h}_{2i}^\dagger\left(\mathbf{M}_{i-1}^{-1} + \frac{\mathbf{M}_{i-1}^{-1}\mathbf{h}_{2i+1}\mathbf{h}_{2i+1}^\dagger\mathbf{M}_{i-1}^{-1}}{1-\mathbf{h}_{2i+1}^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{h}_{2i+1}}\right)\mathbf{h}_{2i}\\ &\overset{(c)}{=} \mathbf{h}_{2i}^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{h}_{2i}, \quad (7)\end{aligned}$$

where we use the matrix inversion lemma in (a) and (b), and (c) is due to

$$\mathbf{h}_{2i}^\dagger\mathbf{M}_{i-1}^k\mathbf{h}_{2i+1} = 0 \quad (8)$$

for any integer exponent k and quaternionic column vector $[\mathbf{h}_{2i}\;\mathbf{h}_{2i+1}]$. (8) is implied by (6) and the fact that (3) can be defined recursively as $\mathbf{M}_i = \mathbf{M}_{i-1} + \mathbf{H}_i\mathbf{H}_i^\dagger$ for $1 \leq i \leq N$ and $\mathbf{M}_0 = (1/\mathsf{snr})\mathbf{I}$. Similarly, the post-processing SNR of the second symbol of the ith user can be obtained as $\mathsf{snr}_{i,2} = \mathbf{h}_{2i+1}^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{h}_{2i+1}$, thus proving (5). ■

By combining $\mathbf{w}_{i,1}$ and $\mathbf{w}_{i,2}$ in (4), the quaternionic MMSE nulling vector for the ith user can be written as

$$\mathbf{W}_i = \begin{bmatrix}\mathbf{w}_{i,1}^\dagger\\ \mathbf{w}_{i,2}^\dagger\end{bmatrix} = \left[\mathbf{h}_{2i}\;\mathbf{h}_{2i+1}\right]^\dagger\mathbf{M}_i^{-1} = \mathbf{H}_i^\dagger\mathbf{M}_i^{-1} = \mathbf{P}_i\mathbf{H}_i^\dagger, \quad (9)$$

where $\mathbf{P}_i = (\mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)} + (1/\mathsf{snr})\mathbf{I})^{-1}$. The estimated signal $\hat{s}_i$ is then given by $\hat{s}_i = \mathbf{W}_i\mathbf{r}_{(i)}$.

Remark 1: The first and second symbol of each user do not interfere with each other, because the MMSE filters of the first and second symbol are orthogonal as $\mathbf{w}_{i,1}^\dagger\mathbf{w}_{i,2} = 0$ for all i. Only $\mathbf{P}_i$ (that includes effects of the interfering signals, if any) is needed to compute (9).

Remark 2: At each block transmission time, the post-processing average SNR per symbol of the ith user can be given by the square root of the sum of squares of the first and second symbols, i.e., $\|\mathbf{H}_i^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{H}_i\| = \sqrt{\mathsf{snr}_{i,1}^2 + \mathsf{snr}_{i,2}^2}$.

In the following section, we first introduce the V-BLAST algorithm for detecting Alamouti signals. We then continue to examine the role of interference in (9) from a geometric point of view for a two-user system in Section IV.

III. V-BLAST Algorithm

V-BLAST first decodes the "strongest" signal (or layer), i.e., the signal that has the largest post-processing SNR. It then cancels the effect of this particular detected signal from the received signal, and proceeds to decode the "strongest" of the remaining signals [6], [10]. This "divide and conquer" approach minimizes error propagation from one iteration to the next. We first present the general V-BLAST algorithm for MMSE detection and interference cancellation. We then propose our Alamouti BLAST-MMSE (A-BLAST) algorithm that exploits the algebraic structure of Alamouti signal to achieve lower decoding complexity in Section V.

A. V-BLAST for Spatial Multiplexing

We assume the model in Section II, where each user transmits an Alamouti signal. An estimate of the transmitted signal $\mathbf{s}$ using the MMSE detector is given by [10]:

$$\hat{\mathbf{s}} = \begin{bmatrix}\mathbf{H}\\ \frac{1}{\sqrt{\mathsf{snr}}}\mathbf{I}\end{bmatrix}^+\begin{bmatrix}\mathbf{r}\\ \mathbf{0}\end{bmatrix}. \quad (10)$$

We denote the first N columns of the pseudo-inverse in (10) by $\mathbf{H}_{inv}$. Following [10], the V-BLAST with MMSE algorithm for Alamouti signal can be given as follows.

Algorithm 1: (V-BLAST with MMSE)

1) Initialize $i = N$. Set $\mathbf{r}_{(i)} = \mathbf{r}$.
2) Find $\mathbf{P}_i = \left(\mathbf{H}_{(i)}^\dagger\mathbf{H}_{(i)} + (1/\mathsf{snr})\mathbf{I}\right)^{-1}$ and $\mathbf{H}_{inv}$.
3) Find the index with the smallest diagonal entry in $\mathbf{P}_i$ and reorder the entries of $\mathbf{s}$ (and thus $\mathbf{H}_{(i)}$) so that the user with this index is the last (ith).
4) Compute the MMSE estimate $\hat{\mathbf{s}}_i$ of the ith Alamouti user in (10) by $\hat{\mathbf{s}}_i = (\mathbf{H}_{inv})^\dagger\mathbf{r}_{(i)}$ or $\hat{\mathbf{s}}_i = \mathbf{P}_i\mathbf{H}_{(i)}^\dagger\mathbf{r}_{(i)}$.
5) Cancel the effect of $\hat{\mathbf{s}}_i$, and consider the reduced-order detection problem using

$$\mathbf{r}_{(i-1)} = \mathbf{r}_{(i)} - \mathbf{H}_i\hat{\mathbf{s}}_i.$$

6) Set $i = i-1$, and go to Step 2).

In comparison to V-BLAST scalar signal decoding in [6], two symbols of a particular user are detected in parallel at each iteration of Algorithm 1 without inter-symbol interference (cf. Theorem 1). However, the complexity of Algorithm 1 is dominated by the matrix inversion and pseudo-inversion in Step 2) and 4) respectively. To avoid these operations, an algorithm based on the square root $\mathbf{P}_i^{1/2}$ of the MMSE covariance matrix was proposed in [7] and [22], where we denote $\mathbf{P}_i^{1/2}$ as the square root of $\mathbf{P}_i$ when the (reduced) channel matrix is $\mathbf{H}_{(i)}$. By exploiting the Alamouti structure that will be discussed in the following Section IV, we propose a more efficient BLAST-MMSE receive algorithm that applies exclusively to Alamouti signals in Section V with lower complexity than Algorithm 1 or other faster implementation such as the square root algorithm in [7], [22].

IV. Two-user Alamouti BLAST-MMSE Detection

In this section, we consider the system model in Section II and obtain a closed-form solution to the BLAST-MMSE detector for two Alamouti users using two receive antennas. Consider the channel matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{H}_1 & \mathbf{H}_2 \end{bmatrix}, \tag{11}$$

where $\mathbf{H}_{ij}$ and $\mathbf{H}_i$ for $i,j \in \{1,2\}$ are Alamouti matrices and quaternionic column vectors respectively. Denote

$$\langle \mathbf{H}_1, \mathbf{H}_2 \rangle = \mathbf{H}_{11}^\dagger \mathbf{H}_{12} + \mathbf{H}_{21}^\dagger \mathbf{H}_{22}, \tag{12}$$

which is an Alamouti matrix, and can be interpreted as the inner product of $\mathbf{H}_1$ and $\mathbf{H}_2$. Thus, we have $\langle \mathbf{H}_1, \mathbf{H}_1 \rangle = \|\mathbf{H}_1\|^2 \mathbf{I}_2 = (\|\mathbf{H}_{11}\|^2 + \|\mathbf{H}_{21}\|^2)\mathbf{I}_2$, where $\|\mathbf{H}_{ij}\|^2 \mathbf{I}_2 = \mathbf{H}_{ij}^\dagger \mathbf{H}_{ij}$ for $i,j \in \{1,2\}$.

We note that (12) is the unnormalized parameter $\boldsymbol{\lambda}$ in [17], which is defined as

$$\boldsymbol{\lambda} = \frac{\langle \mathbf{H}_1, \mathbf{H}_2 \rangle}{\|\mathbf{H}_1\| \|\mathbf{H}_2\|}, \tag{13}$$

and is shown in [17] to govern the performance of joint detection, which depends on the receive algorithms, e.g., ML or Zero-Forcing (ZF). Note that $\langle \mathbf{H}_1, \mathbf{H}_2 \rangle^\dagger \langle \mathbf{H}_1, \mathbf{H}_2 \rangle = \|\langle \mathbf{H}_1, \mathbf{H}_2 \rangle\|^2 \mathbf{I}_2$. In this paper, we call $\|\boldsymbol{\lambda}\|$ the quaternionic angle. Now, $\|\boldsymbol{\lambda}\|$ can be interpreted as a metric that measures the amount of cross interference between the two channels $\mathbf{H}_1$ and $\mathbf{H}_2$.

Next, we form the following augmented channel matrix:

$$\mathbf{H}_{aug} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \\ \multicolumn{2}{c}{\frac{1}{\sqrt{\mathsf{snr}}}\mathbf{I}_4} \end{bmatrix}. \tag{14}$$

We now show that $\mathbf{H}_{aug}$ in (14) can be decomposed into Alamouti block matrices by the following QR factorization, which motivates a geometric perspective to our A-BLAST Algorithm in Section V through the role of the quaternionic angle.

Lemma 1: The augmented channel matrix $\mathbf{H}_{aug}$ can be decomposed as:

$$\mathbf{H}_{aug} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \boldsymbol{\Lambda} \tilde{\mathbf{R}} = \begin{bmatrix} \mathbf{Q}_1 \\ \mathbf{Q}_2 \end{bmatrix} \mathbf{R}, \tag{15}$$

where

$$\boldsymbol{\Lambda} = \begin{bmatrix} 1/a\mathbf{I}_2 & \mathbf{0}_2 \\ \mathbf{0}_2 & 1/b\mathbf{I}_2 \end{bmatrix}, \; \tilde{\mathbf{R}} = \begin{bmatrix} a\mathbf{I}_2 & a\mathbf{X} \\ \mathbf{0}_2 & b\mathbf{I}_2 \end{bmatrix},$$

$$\mathbf{Q}_1 = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} - \mathbf{H}_{11}\mathbf{X} \\ \mathbf{H}_{21} & \mathbf{H}_{22} - \mathbf{H}_{21}\mathbf{X} \end{bmatrix}, \; \mathbf{Q}_2 = \begin{bmatrix} \frac{1}{\sqrt{\mathsf{snr}}}\mathbf{I}_2 & -\frac{1}{\sqrt{\mathsf{snr}}}\mathbf{X} \\ \mathbf{0}_2 & \frac{1}{\sqrt{\mathsf{snr}}}\mathbf{I}_2 \end{bmatrix},$$

$$\mathbf{X} = \frac{\langle \mathbf{H}_1, \mathbf{H}_2 \rangle}{\|\mathbf{H}_1\|^2 + 1/\mathsf{snr}}, \; a = \sqrt{\|\mathbf{H}_1\|^2 + 1/\mathsf{snr}},$$

$$b = \sqrt{\|\mathbf{H}_2\|^2 + 1/\mathsf{snr} - \frac{\|\langle \mathbf{H}_1, \mathbf{H}_2 \rangle\|^2}{\|\mathbf{H}_1\|^2 + 1/\mathsf{snr}}}. \tag{16}$$

Proof: Lemma 1 is proved using the reduced QR factorization in [9] (See [9], Appendix A, pp. 732), and associating the first quaternionic column of $[\mathbf{Q}_1^T \; \mathbf{Q}_2^T]^T$ with those of $\mathbf{H}_{aug}$. ■

Remark 3: Both a and b are nonnegative real scalars. In particular, to show that b is real, we have $b^2 = \|\mathbf{H}_2\|^2 + 1/\mathsf{snr} - \|\langle \mathbf{H}_1, \mathbf{H}_2 \rangle\|^2 / (\|\mathbf{H}_1\|^2 + 1/\mathsf{snr}) \geq 0$, where the last inequality follows from the 'Cauchy Schwartz inequality' for Alamouti matrices:

$$\|\mathbf{H}_2\|^2 \|\mathbf{H}_1\|^2 \geq \|\langle \mathbf{H}_1, \mathbf{H}_2 \rangle\|^2, \tag{17}$$

which can be proved by observing that $\mathbf{H}_{aug}^\dagger \mathbf{H}_{aug}$ is positive semidefinite. (17) implies that $\|\boldsymbol{\lambda}\| \leq 1$ and thus b in (16) is real and nonnegative.

Our next result shows a particular decomposition of the MMSE error covariance matrix $\mathbf{P}_2$, which facilitates computing $\mathbf{P}_2$ without matrix inversion.

Theorem 2: For the two-user system, we have

$$\mathbf{P}_2 = \tilde{\mathbf{R}}^{-1} \left(\tilde{\mathbf{R}}^{-1} \right)^\dagger, \quad \text{where} \quad \tilde{\mathbf{R}}^{-1} = \begin{bmatrix} 1/a\mathbf{I}_2 & -1/b\mathbf{X} \\ \mathbf{0}_2 & 1/b\mathbf{I}_2 \end{bmatrix}. \tag{18}$$

Furthermore, $\mathbf{P}_2$ can be decomposed as

$$\mathbf{P}_2 = \left(\tilde{\mathbf{R}}^{-1} \right)_2 \left(\tilde{\mathbf{R}}^{-1} \right)_2^\dagger + \left[\begin{array}{c|c} \left(\mathbf{H}_1^\dagger \mathbf{H}_1 + (1/\mathsf{snr})\mathbf{I}_2 \right)^{-1} & \mathbf{0}_2 \\ \hline \mathbf{0}_2 & \mathbf{0}_2 \end{array} \right]. \tag{19}$$

Proof: From Lemma 1, the quaternionic column vectors of the matrix $\mathbf{Q} = \begin{bmatrix} \mathbf{Q}_1^T & \mathbf{Q}_2^T \end{bmatrix}^T \boldsymbol{\Lambda}$ in (15) are orthonormal, i.e., $\mathbf{Q}^\dagger \mathbf{Q} = \mathbf{I}_4$. Thus, $\mathbf{P}_2 = (\mathbf{H}_{aug}^\dagger \mathbf{H}_{aug})^{-1} = \tilde{\mathbf{R}}^{-1}(\tilde{\mathbf{R}}^{-1})^\dagger$, which means $\mathbf{P}_2^{1/2} = \tilde{\mathbf{R}}^{-1}$, since $\tilde{\mathbf{R}}^{-1}$ is clearly a unique Cholesky factor of $\mathbf{P}_2$. Next, $\tilde{\mathbf{R}}^{-1}$ in (18) can be obtained by applying the matrix inversion formula on Alamouti block matrices [9], [16]:

$$\begin{bmatrix} \mathbf{A} & \mathbf{D} \\ \mathbf{0}_2 & \mathbf{B} \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}^{-1} & -\mathbf{A}^{-1}\mathbf{D}\mathbf{B}^{-1} \\ \mathbf{0}_2 & \mathbf{B}^{-1} \end{bmatrix} \tag{20}$$

and the fact that upper triangular Alamouti block matrices must have multiples of $\mathbf{I}_2$ along the diagonal. Note that the algebraic structure of the Alamouti matrix (closure under addition, multiplication and taking inverses) implies that (20) is true for any Alamouti block matrices. Lastly, it is easy to verify using (18) that (19) holds. ■

V. Low Complexity Multiuser Alamouti BLAST-MMSE (A-BLAST) Algorithm

Although there exist several BLAST algorithms based on the factorization of the MMSE error covariance matrix $\mathbf{P}_i$ for all i (for example, the square root algorithm in [7], [22]), the algebraic Alamouti structure can be exploited in decomposing $\mathbf{P}_i$ to realize a low-complexity BLAST-MMSE algorithm that provides further insight to decoding performance from a geometric point of view. In particular, Theorem 2 shows that peeling off two users can be done jointly with detection as $\mathbf{P}_2$ can be decomposed into a sum of two matrices: the first matrix (a quaternionic dyad) and the second matrix in (18) performs detection and peeling respectively.

Generalizing Theorem 2 to the N-user case, we give a recursive construction of the MMSE nulling quaternionic vectors , which will be subsequently used in our A-BLAST Algorithm.

Theorem 3: For the N-user system, $\mathbf{P}_i$ can be decomposed as

$$\mathbf{P}_i = (1/c_i^2)\mathbf{z}_i\mathbf{z}_i^\dagger + \left[\begin{array}{c|c} \mathbf{P}_{i-1} & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_2 \end{array}\right], \qquad (21)$$

where $c_i = \|\mathbf{H}_i\|^2 + 1/\mathsf{snr} - \|\mathbf{P}_{i-1}^{1/2}\mathbf{H}_{(i-1)}^\dagger\mathbf{H}_i\|^2$ and $\mathbf{z}_i = [-(\mathbf{P}_{i-1}\mathbf{H}_{(i-1)}^\dagger\mathbf{H}_i)^T \quad \mathbf{I}_2]^T$ for $i = 2, \ldots, N$, and $\mathbf{P}_1 = (1/a^2)\mathbf{I}_2$.

The nulling quaternionic vector for the ith Alamouti user is then given by

$$(1/c_i^2)[-\mathbf{H}_i^\dagger\mathbf{H}_{(i-1)}\mathbf{P}_{i-1} \quad \mathbf{I}_2] \cdot \mathbf{H}_{(i)}^\dagger. \qquad (22)$$

Proof: Using the inverse formula for the recursive triangulation of block matrices in [9] (See [9], Appendix A, pp. 727), and let $\mathbf{z}_i = [-(\mathbf{P}_{i-1}\mathbf{H}_{(i-1)}^\dagger\mathbf{H}_i)^T \ \mathbf{I}_2]^T$, $\mathbf{P}_i$ can be written as

$$\mathbf{P}_i = \mathbf{z}\Delta_i^{-1}\mathbf{z}^\dagger + \left[\begin{array}{c|c} \mathbf{P}_{i-1} & \mathbf{0} \\ \hline \mathbf{0}^T & \mathbf{0}_2 \end{array}\right], \qquad (23)$$

where $\Delta_i = \mathbf{H}_i^\dagger\mathbf{H}_i + (1/\mathsf{snr})\mathbf{I}_2 - \mathbf{H}_i^\dagger\mathbf{H}_{(i-1)}\mathbf{P}_{i-1}\mathbf{H}_{(i-1)}^\dagger\mathbf{H}_i$ is the Schur complement of $\mathbf{P}_{i-1}^{-1}$ in $\mathbf{P}_i^{-1}$ [9]. Now, since Δ_i is an Alamouti matrix and, more specifically, a multiple of the identity matrix, (23) simplifies to (21). Next, the optimal nulling quaternionic vector of the ith Alamouti user is given by the ith (last) quaternionic row of $\mathbf{P}_i\mathbf{H}_{(i)}^\dagger$. Using $\mathbf{P}_i$ given by (23), the nulling quaternionic vector simplifies to (22). ∎

Using Theorem 3, the following result can be used to simplify Step 3) in Algorithm 1.

Corollary 1: The index of the diagonal block entry of $\mathbf{P}_i$ in Theorem 3 with the smallest Frobenius norm is given by $\arg\max_{1\le l\le i} c_l^2$.

Proof: The index of the diagonal block entry of $\mathbf{P}_2$ in Theorem 3 with the smallest Frobenius norm corresponds to the user that has the highest post-processing average SNR, which can be given by the square root of the sum of squares of the first and second symbols, i.e., $\|\mathbf{H}_i^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{H}_i\|$ (cf. Remark 2).

Next, we can expand $\mathbf{H}_i^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{H}_i$ as $\mathbf{H}_i^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{H}_i = \mathbf{H}_i^\dagger(\mathsf{snr}\mathbf{I} - \mathsf{snr}\mathbf{H}_{(i-1)}\mathbf{P}_{i-1}\mathbf{H}_{(i-1)}^\dagger)\mathbf{H}_i = \mathsf{snr}\|\mathbf{H}_i\|^2\mathbf{I} - \mathsf{snr}\mathbf{H}_i^\dagger\mathbf{H}_{(i-1)}\mathbf{P}_{i-1}\mathbf{H}_{(i-1)}^\dagger\mathbf{H}_i$. But, this implies $\mathbf{H}_i^\dagger\mathbf{M}_{i-1}^{-1}\mathbf{H}_i = \mathsf{snr}\Delta_i - \mathbf{I}$, where Δ_i is given in (23). Since $\|\Delta_i\| = c_i^2$, $\max_{1\le l\le i}\|\mathbf{H}_l^\dagger\mathbf{M}_{l-1}^{-1}\mathbf{H}_l\|$ is thus equivalent to $\max_{1\le l\le i} c_l^2$. ∎

Our proposed A-BLAST Algorithm, which is based on Theorem 3 and Corollary 1, is given as follows.

Algorithm 2: (A-BLAST Algorithm)

1) Initialize $i = N$. Set $\mathbf{r}_{(i)} = \mathbf{r}$.
2) Compute recursively c_j and $\mathbf{P}_j$ for $j = 1, \ldots, N$ using (21) in Theorem 3.
3) Reorder the entries of $\mathbf{s}$ so that the entry corresponding to $\arg\max_{1\le l\le i} c_l^2$ is the last (ith).
4) The nulling quaternionic vector for the ith Alamouti signal is given by
$$\mathbf{W}_i = (1/c_i^2)[-\mathbf{H}_i^\dagger\mathbf{H}_{(i-1)}\mathbf{P}_{i-1} \quad \mathbf{I}_2] \cdot \mathbf{H}_{(i)}^\dagger. \qquad (24)$$
5) Obtain $\hat{\mathbf{s}}_i = \mathbf{W}_i\mathbf{r}_{(i)}$ using (24), cancel the effect of $\hat{\mathbf{s}}_i$, and consider the reduced-order detection problem using
$$\mathbf{r}_{(i-1)} = \mathbf{r}_{(i)} - \mathbf{H}_i\hat{\mathbf{s}}_i. \qquad (25)$$
6) Set $i = i-1$, and go to Step 3), but now use $\mathbf{P}_{i-2}$ (from the second summand of (21)).

In comparison to Algorithm 1, the key step that reduces complexity is Step 2) in the A-BLAST Algorithm as no matrix inversion or pseudo-inversion is required. Peeling off users and detection are done jointly at Step 3) and Step 4) respectively.

We note that, similar to the square root decomposition of $\mathbf{P}_i$ in [22], the quaternionic square root of $\mathbf{P}_i$ in (21) can be given as

$$\mathbf{P}_i^{1/2} = \left[\begin{array}{c|c} \dfrac{\mathbf{P}_{i-1}^{1/2}}{(\mathbf{0})_{(i-1)}^T} & \left(\mathbf{P}_i^{1/2}\right)_i \end{array}\right] \qquad (26)$$

for $i = 2, \ldots, N$, where $\mathbf{P}_{i-1}^{1/2}$ is an Alamouti upper triangular block matrix, and the last quaternion of $(\mathbf{P}_i^{1/2})_i$ is a multiple of the identity matrix. In the following example, we illustrate the quaternionic square root and Theorem 3 using the two-user case in Section IV.

Example 1: From the two-user case in Section IV, we see that $\mathbf{P}_2^{1/2} = \tilde{\mathbf{R}}^{-1}$, which is given by

$$\left[\begin{array}{c|c} 1/a\mathbf{I}_2 & -1/b\mathbf{X} \\ \hline \mathbf{0}_2 & 1/b\mathbf{I}_2 \end{array}\right]. \qquad (27)$$

Using the A-BLAST Algorithm, the quaternionic MMSE detector for the second user (the user that satisfies $\arg\max_{l=\{1,2\}} c_l^2$) is thus given by (note that $c_2 = b$):

$$(1/b^2)[-\mathbf{X}^\dagger \ \ \mathbf{I}_2]\begin{bmatrix} \mathbf{H}_1^\dagger \\ \mathbf{H}_2^\dagger \end{bmatrix}. \qquad (28)$$

Next, the detected signal of the second user is again modulated and cancelled from the received signal. At the next iteration of the A-BLAST Algorithm, we proceed to Step 3), but we see that the upper left block matrix $1/a\mathbf{I}_2$ in (27) is the square root of the matrix $\mathbf{P}_1 = \left(\mathbf{H}_1^\dagger\mathbf{H}_1 + (1/\mathsf{snr})\mathbf{I}_2\right)^{-1}$, which we can readily use to detect the signal of the first user. We also derive the matrix $\mathbf{R}$ for the three-user case in the appendix. After decoding the third user, the matrix $\mathbf{P}_2^{1/2}$ obtained from the upper left sub-matrix of $\mathbf{P}_3^{1/2}$ is given by the two-user case.

We conclude that the distribution of quaternionic angles (cf. (13)) is an important geometric parameter that determines the performance of the A-BLAST Algorithm (from the quaternionic dyad of (21) in Theorem 3). It leads to a property of a plurality of QOSTBC, which we call *code diversity* that is described in the following section.

VI. Applications

In this section, we first examine space-time block code construction for single user that can be decomposed into a virtual i.i.d. Rayleigh fading MAC with virtual users transmitting Alamouti signals. We then demonstrate the value of Alamouti signal detection using the A-BLAST Algorithm with *code diversity*. Finally, we extend our results to a multiuser setting.

A. Quasi-Orthogonal Space Time Block Codes

We consider a single user system with 4 transmit antennas and 1 receive antenna, and the following QOSTBC [20]:

$$\mathbf{C} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{A} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix}. \tag{29}$$

The received signals at four consecutive symbol periods can be written as

$$\begin{aligned} r_1 &= h_1x_1 + h_2x_2 + h_3x_3 + h_4x_4 + n_1, \\ r_2 &= -h_1x_2^* + h_2x_1^* - h_3x_4^* + h_4x_3^* + n_2, \\ r_3 &= h_1x_3 + h_2x_4 + h_3x_1 + h_4x_2 + n_3, \\ r_4 &= -h_1x_4^* + h_2x_3^* - h_3x_2^* + h_4x_1^* + n_4. \end{aligned} \tag{30}$$

Filtering at the receiver transforms (30) to

$$\begin{bmatrix} r_1+r_3 \\ r_2+r_4 \end{bmatrix} = \begin{bmatrix} x_1+x_3 & x_2+x_4 \\ -(x_2+x_4)^* & (x_1+x_3)^* \end{bmatrix} \begin{bmatrix} h_1+h_3 \\ h_2+h_4 \end{bmatrix} + \begin{bmatrix} n_1+n_3 \\ n_2+n_4 \end{bmatrix}$$

and

$$\begin{bmatrix} r_1-r_3 \\ r_2-r_4 \end{bmatrix} = \begin{bmatrix} x_1-x_3 & x_2-x_4 \\ -(x_2-x_4)^* & (x_1-x_3)^* \end{bmatrix} \begin{bmatrix} h_1-h_3 \\ h_2-h_4 \end{bmatrix} + \begin{bmatrix} n_1-n_3 \\ n_2-n_4 \end{bmatrix},$$

which is equivalent to

$$\begin{bmatrix} \hat{r}_1 \\ -\hat{r}_2^* \\ \hat{r}_3 \\ -\hat{r}_4^* \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{H}_{11} & \mathbf{H}_{11} \\ \hline \mathbf{H}_{21} & -\mathbf{H}_{21} \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} \hat{n}_1 \\ -\hat{n}_2^* \\ \hat{n}_3 \\ -\hat{n}_4^* \end{bmatrix},$$

where

$$\mathbf{H}_{11} = \mathsf{Q}(\hat{h}_1, \hat{h}_2), \qquad \mathbf{H}_{21} = \mathsf{Q}(\hat{h}_3, \hat{h}_4) \tag{31}$$

and

$$\begin{aligned} (\hat{r}_1,\hat{r}_2,\hat{r}_3,\hat{r}_4) &= (r_1+r_3, r_2+r_4, r_1-r_3, r_2-r_4), \\ (\hat{h}_1,\hat{h}_2,\hat{h}_3,\hat{h}_4) &= (h_1+h_3, h_2+h_4, h_1-h_3, h_2-h_4), \\ (\hat{n}_1,\hat{n}_2,\hat{n}_3,\hat{n}_4) &= (n_1+n_3, n_2+n_4, n_1-n_3, n_2-n_4). \end{aligned} \tag{32}$$

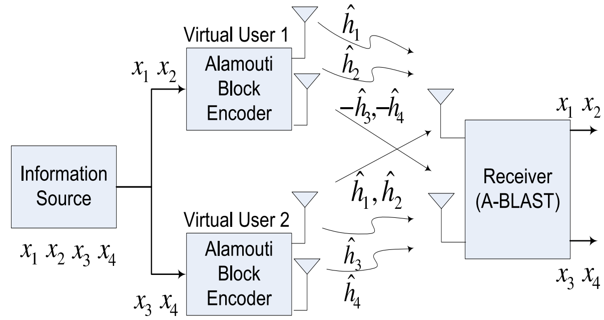

Fig. 2. Decomposition of a single-user system using the QOSTBC in (29) with one receive antenna into a virtual Alamouti multiuser system whose virtual channel gains $\hat{h}_1, \ldots, \hat{h}_4$ and corresponding virtual noise powers are defined in (32). The A-BLAST Algorithm is used to detect the signals $x_1, \ldots, x_4$ at the receiver.

Interestingly, (31) can be interpreted as a virtual two-user system with two receive antennas (cf. Section IV). Here, (x_1, x_2) and (x_3, x_4) are interpreted as the first and second signals of virtual users respectively. Figure 2 illustrates this virtual multiuser system decomposed from the QOSTBC in (29). The quaternionic angle of this virtual multiuser system is given by

$$\begin{aligned} \|\boldsymbol{\lambda}_1\| &= \left\| 1 - \frac{2\,\|\mathbf{H}_{21}\|^2}{\|\mathbf{H}_{11}\|^2 + \|\mathbf{H}_{21}\|^2} \right\| && (33) \\ &= \frac{|h_1^*h_3 + h_3^*h_1 + h_2^*h_4 + h_4^*h_2|}{\sum_j |h_j|^2}. && (34) \end{aligned}$$

Similarly, by using a sub-matrix of the form

$$\begin{bmatrix} x_1 & x_2 \\ x_2 & x_1 \end{bmatrix}, \tag{35}$$

we can also consider different types of QOSTBC with the following block matrix configurations [8]:

$$\mathbf{C} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{B}^\dagger & \mathbf{A}^\dagger \end{bmatrix} \quad \text{or} \quad \mathbf{C} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^\dagger & -\mathbf{A}^\dagger \end{bmatrix}, \tag{36}$$

which also allows decomposition into virtual Alamouti signals (after an appropriate change of variables similar to (31), i.e., let $(\hat{r}_1,\hat{r}_2,\hat{r}_3,\hat{r}_4) = (r_1+r_2, r_3+r_4, r_1-r_2, r_3-r_4)$ and so on). The quaternionic angle of these virtual systems can be computed as

$$\|\boldsymbol{\lambda}_2\| = \frac{|h_1^*h_2 + h_2^*h_1 + h_3^*h_4 + h_4^*h_3|}{\sum_j |h_j|^2}. \tag{37}$$

Lastly, applying an appropriate change-of-variables technique similar to (31), i.e., $(\hat{r}_1,\hat{r}_2,\hat{r}_3,\hat{r}_4) = (r_1+r_4, r_2+r_3, r_1-r_4, r_2-r_3)$, the QOSTBC:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & x_4^* & -x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ x_4 & x_3 & x_2 & x_1 \end{bmatrix} \tag{38}$$

leads to the quaternionic angle

$$\|\boldsymbol{\lambda}_3\| = \frac{|h_1^*h_4 + h_4^*h_1 + h_2^*h_3 + h_3^*h_2|}{\sum_j |h_j|^2}. \tag{39}$$

Now, the QOSTBC given by (29), (36) and (38) has essentially the same performance curve (since they have the same diversity of order 2). In addition, if a QOSTBC given by (29), (36) or (38) is used, each of the two virtual users in the respective decomposition sees the same channel gain magnitude $2\sum_j |h_j|^2$, e.g., virtual user 1 in (31) has channel gain given by $\|[\mathbf{H}_{11}^T \ \mathbf{H}_{21}^T]^T\|^2 = 2\sum_j |h_j|^2$. This implies no ordering in the virtual Alamouti users is required in the A-BLAST Algorithm. On the other hand, as shown by (34), (37) and (39), each of these different virtual Alamouti systems has a different quaternionic angle for a given channel realization. This fact will be exploited in Section VI-B. Assuming the h_l's are complex Gaussian distributed, as shown by (33), $\|\boldsymbol{\lambda}_1\|$ is a function of $\|\mathbf{H}_{21}\|^2/(\|\mathbf{H}_{11}\|^2 + \|\mathbf{H}_{21}\|^2)$, which is beta distributed with parameters $\alpha = 2$ and $\beta = 2$. Now, $\|\boldsymbol{\lambda}_2\|$ and $\|\boldsymbol{\lambda}_3\|$ also have similar distribution functions to $\|\boldsymbol{\lambda}_1\|$. More precisely, the density function of $\|\boldsymbol{\lambda}_t\|$ for $t = 1, 2$ and 3, respectively, in (34), (37) and (39) is given by

$$f_{\|\boldsymbol{\lambda}_t\|}(y) = \frac{3}{2}(1 - y^2), \ \ 0 \le y \le 1, \ \forall\, t. \tag{40}$$

B. QOSTBC with Adaptive Modulation

The different decomposition of QOSTBC into Alamouti signals in Section VI-A can be extended to multiple antennas at the receiver (with an increased diversity order), but gives different tradeoffs between performance and decoding complexity depending on the receiver. This leads systematically to a new form of diversity by adaptive modulation, in which a code is chosen at each block tranmission time from a family of space-time block codes at the receiver to maximize a given performance metric. We call this *code diversity*. Code diversity can be implemented in many different ways, e.g., in our paper, it is based on characterizing the quaternionic angle properties of a family of QOSTBC's, wherein one code is chosen at each block transmission time, and, in [12], a user-control parameter is implanted within a fixed QOSTBC code and rotated accordingly to increase diversity at each block transmission time.

Based on the QOSTBC quaternionic angle characterization in Section VI-A, we propose a QOSTBC adaptive modulation scheme that uses a small number of bits as feedback from the receiver to the transmitter at each block transmission time. Define $\mathcal{S}_1 = \{1, 2\}$, $\mathcal{S}_2 = \{1, 3\}$, $\mathcal{S}_3 = \{2, 3\}$ and $\mathcal{S}_4 = \{1, 2, 3\}$. For example, $\mathcal{S}_1$ is the set that contains the QOSTC in (29) and (36) and so on. Initially, we select one of these sets $\mathcal{S}_k$ where $k \in \{1, 2, 3, 4\}$, and thus determines the number of feedback bits required, i.e., we need one-bit feedback if $k = \{1, 2, 3\}$ or two-bit feedback if $k = 4$. At each block transmission time, the receiver selects a QOSTBC with a quaternionic angle that satisfies

$$\|\boldsymbol{\lambda}\| = \min_{t \in \mathcal{S}_k} \|\boldsymbol{\lambda}_t\|, \tag{41}$$

and informs the transmitter about this choice by feedback. Selecting the QOSTBC with the minimum quaternionic angle maximizes the post-processing SNR per symbol of the A-BLAST Algorithm at each block transmission time. This is due to the fact that each virtual user sees an equivalent channel gain when the QOSTBC is given by (29), (36) or (38) with an equivalent representation given by (31). Therefore, maximizing c_2^2 (cf. Corollary 1) is equivalent to (41). It is interesting to note that enabling a receiver to choose the STBC it wants translates to power saving at the transmitter.

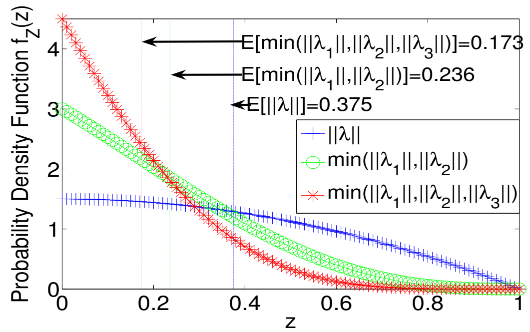

Fig. 3. Probability density function of a) quaternionic angle of QOSTBC in (29), (36) or (38), all given by (40), b) quaternionic angle of QOSTBC with 1-bit adaptive modulation using $\mathcal{S}_1$ in (41), and c) quaternionic angle of QOSTBC with 2-bit adaptive modulation using $\mathcal{S}_4$. The vertical line indicates the mean quaternionic angle for each of the scheme a)-c).

Figure 3 plots the probability density function of the quaternionic angle distribution for the three different schemes, namely a) use only one QOSTBC from (29), (36) or (38) for all block transmission, b) use 1-bit feedback to select one QOSTBC from two fixed QOSTBC in (34), (37) or (39) according to (41) at each block transmission, and c) use 2-bit feedback to select one QOSTBC from three QOSTBC in (34), (37) or (39) at each block transmission. We see from Figure 3 that the mean quaternionic angle decreases by more than 37 and 53 percent with 1-bit and 2-bit feedback respectively. Figure 3 translates differences in expected quaternionic angles to differences in performance, and Figure 4 shows BER performance as a function of snr (see Section VII for more performance evaluation).

C. Multiuser QOSTBC Adaptive Modulation

The A-BLAST Algorithm with adaptive modulation can be generalized to a multiuser setting, where each user employs more than two transmit antennas and all users have an equal number of transmit antennas. At the receiver, the number of receive antennas M_r must be equal to or greater than the number of users so that the signals of all users can be separated using successive interference cancellation [14]. In particular, we illustrate BLAST-MMSE detection with adaptive modulation for the case where each user employs four transmit antennas and uses the same QOSTBC, e.g., either (29), (36) or (38), at each block transmission time synchronously. In general, the diversity order for each user is $2 \times M_r$, because the diversity order for the QOSTBC (29), (36) or (38) is 2 for a receiver with a single antenna.

We illustrate the case for two users that use QOSTBC given by (29) at a particular block transmission time and when there are two receive antennas only. This means a diversity order 4 to each user. Suppose r_{ij} and n_{ij} are, respectively, the signal received and noise at antenna j in time slot i, and h_{ij} (g_{ij}) is the channel gain for the desired (interfering) user associated with the path from transmit antenna i to receive antenna j. Consider the following system at the first receive antenna:

$$\begin{bmatrix}\hat{r}_{11}\\ \hat{r}_{21}\\ \hat{r}_{31}\\ \hat{r}_{41}\end{bmatrix} = \underbrace{\underbrace{\begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_1\\ \tilde{\mathbf{H}}_1 & -\tilde{\mathbf{H}}_1\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\\ c_4\end{bmatrix}}_{\text{intra-user detection}} + \begin{bmatrix}\mathbf{G}_1 & \mathbf{G}_1\\ \tilde{\mathbf{G}}_1 & -\tilde{\mathbf{G}}_1\end{bmatrix}\begin{bmatrix}s_1\\ s_2\\ s_3\\ s_4\end{bmatrix}}_{\text{inter-user detection}} + \left[\hat{n}_{11}\ \hat{n}_{21}\ \hat{n}_{31}\ \hat{n}_{41}\right]^T, \quad (42)$$

and the following system at the second receive antenna:

$$\begin{bmatrix}\hat{r}_{12}\\ \hat{r}_{22}\\ \hat{r}_{32}\\ \hat{r}_{42}\end{bmatrix} = \underbrace{\underbrace{\begin{bmatrix}\mathbf{H}_2 & \mathbf{H}_2\\ \tilde{\mathbf{H}}_2 & -\tilde{\mathbf{H}}_2\end{bmatrix}\begin{bmatrix}c_1\\ c_2\\ c_3\\ c_4\end{bmatrix}}_{\text{intra-user detection}} + \begin{bmatrix}\mathbf{G}_2 & \mathbf{G}_2\\ \tilde{\mathbf{G}}_2 & -\tilde{\mathbf{G}}_2\end{bmatrix}\begin{bmatrix}s_1\\ s_2\\ s_3\\ s_4\end{bmatrix}}_{\text{inter-user detection}} + \left[\hat{n}_{12}\ \hat{n}_{22}\ \hat{n}_{32}\ \hat{n}_{42}\right]^T, \quad (43)$$

where

$$\begin{bmatrix}\hat{r}_{11}\\ \hat{r}_{21}\\ \hat{r}_{31}\\ \hat{r}_{41}\end{bmatrix} = \begin{bmatrix}r_{11}+r_{31}\\ -r_{21}^*-r_{41}^*\\ r_{11}-r_{31}\\ -r_{21}^*+r_{41}^*\end{bmatrix}, \quad \begin{bmatrix}\hat{r}_{12}\\ \hat{r}_{22}\\ \hat{r}_{32}\\ \hat{r}_{42}\end{bmatrix} = \begin{bmatrix}r_{12}+r_{32}\\ -r_{22}^*-r_{42}^*\\ r_{12}-r_{32}\\ -r_{22}^*+r_{42}^*\end{bmatrix},$$

$$\begin{bmatrix}\hat{n}_{11}\\ \hat{n}_{21}\\ \hat{n}_{31}\\ \hat{n}_{41}\end{bmatrix} = \begin{bmatrix}n_{11}+n_{31}\\ -n_{21}^*-n_{41}^*\\ n_{11}-n_{31}\\ -n_{21}^*+n_{41}^*\end{bmatrix}, \quad \begin{bmatrix}\hat{n}_{12}\\ \hat{n}_{22}\\ \hat{n}_{32}\\ \hat{n}_{42}\end{bmatrix} = \begin{bmatrix}n_{12}+n_{32}\\ -n_{22}^*-n_{42}^*\\ n_{12}-n_{32}\\ -n_{22}^*+n_{42}^*\end{bmatrix},$$

and

$$\mathbf{H}_j = \mathsf{Q}(h_{1j}+h_{3j}, h_{2j}+h_{4j}), \mathbf{G}_j = \mathsf{Q}(g_{1j}+g_{3j}, g_{2j}+g_{4j}),$$
$$\tilde{\mathbf{H}}_j = \mathsf{Q}(h_{1j}-h_{3j}, h_{2j}-h_{4j}), \tilde{\mathbf{G}}_j = \mathsf{Q}(g_{1j}-g_{3j}, g_{2j}-g_{4j}). \quad (44)$$

We note that other virtual Alamouti systems with similar structures to (42)-(43) can also be obtained by applying an appropriate change-of-variable technique, as in Section VI-A, to the QOSTBC in (36) and (38). We thus denote $\mathbf{H}_j(t), \mathbf{G}_j(t), \tilde{\mathbf{H}}_j(t), \tilde{\mathbf{G}}_j(t)$ as the matrices in (44) for $t = 1, 2, 3$, respectively, generated by the QOSTBC in (29), (36) and (38).

In addition to inter-user cancellation (using the A-BLAST Algorithm), our A-BLAST scheme consists of an intra-user cancellation scheme based on the virtual two-user Alamouti systems, as in Section VI-B, at each user (also using the A-BLAST Algorithm). With QOSTBC adaptive modulation, a set $\mathcal{S}_k, k = \{1, 2, 3, 4\}$ is first selected. Then, each receive antenna computes the minimum quaternionic angle using $\mathcal{S}_k$. For example, in (42), the first receive antenna computes

$$\min\left\{\min_{t\in\mathcal{S}_k}\frac{|\|\mathbf{H}_1(t)\|^2-\|\tilde{\mathbf{H}}_1(t)\|^2|}{\|\mathbf{H}_1(t)\|^2+\|\tilde{\mathbf{H}}_1(t)\|^2}, \min_{t\in\mathcal{S}_k}\frac{|\|\mathbf{G}_1(t)\|^2-\|\tilde{\mathbf{G}}_1(t)\|^2|}{\|\mathbf{G}_1(t)\|^2+\|\tilde{\mathbf{G}}_1(t)\|^2}\right\} \quad (45)$$

and, in (43), the second receive antenna computes:

$$\min\left\{\min_{t\in\mathcal{S}_k}\frac{|\|\mathbf{H}_2(t)\|^2-\|\tilde{\mathbf{H}}_2(t)\|^2|}{\|\mathbf{H}_2(t)\|^2+\|\tilde{\mathbf{H}}_2(t)\|^2}, \min_{t\in\mathcal{S}_k}\frac{|\|\mathbf{G}_2(t)\|^2-\|\tilde{\mathbf{G}}_2(t)\|^2|}{\|\mathbf{G}_2(t)\|^2+\|\tilde{\mathbf{G}}_2(t)\|^2}\right\}. \quad (46)$$

The index for the minimum quaternionic angle in (45) and (46) may be different at each receive antenna, so we select the minimum quaternionic angle among all these receive antennas, and broadcasts its index to all users by feedback.

Now, instead of the virtual Alamouti system decomposition as in (42)-(43), we describe another virtual Alamouti system decomposition (an 8×1 system) with three other detection approaches:

$$\begin{bmatrix}\hat{r}_{11}\\ \hat{r}_{21}\\ \hat{r}_{31}\\ \hat{r}_{41}\\ \hat{r}_{12}\\ \hat{r}_{22}\\ \hat{r}_{32}\\ \hat{r}_{42}\end{bmatrix} = \underbrace{\left[\begin{array}{cc|cc}\mathbf{H}_1 & \mathbf{0}_2 & \mathbf{G}_1 & \mathbf{0}_2\\ \mathbf{0}_2 & \tilde{\mathbf{H}}_1 & \mathbf{0}_2 & \tilde{\mathbf{G}}_1\\ \hline \mathbf{H}_2 & \mathbf{0}_2 & \mathbf{G}_2 & \mathbf{0}_2\\ \mathbf{0}_2 & \tilde{\mathbf{H}}_2 & \mathbf{0}_2 & \tilde{\mathbf{G}}_2\end{array}\right]}_{\left[\begin{array}{c|c}\mathbf{H}_D & \mathbf{G}_D\end{array}\right]}\begin{bmatrix}c_1+c_3\\ c_2+c_4\\ c_1-c_3\\ c_2-c_4\\ s_1+s_3\\ s_2+s_4\\ s_1-s_3\\ s_2-s_4\end{bmatrix} + \begin{bmatrix}\hat{n}_{11}\\ \hat{n}_{21}\\ \hat{n}_{31}\\ \hat{n}_{41}\\ \hat{n}_{12}\\ \hat{n}_{22}\\ \hat{n}_{32}\\ \hat{n}_{42}\end{bmatrix}. \quad (47)$$

The first approach is to apply QR decomposition first to (47), and, at the inter-user level, successive interference cancellation using the ZF receiver is used to obtain an estimate for the 8×1 signal in (47) [11]. In [11], these estimates are subsequently recombined to obtain two separate QOSTBC decoding problems, where each user is decoded using the ML pairwise decoding scheme, i.e., ML decoding over subsets of two symbols [8]. Though [11] only considers the QOSTBC in (29), this approach also applies to QOSTBC in (36) and (38).

The second approach is the decorrelator with adaptive modulation. By exploiting the algebraic structure of the Alamouti code, the following linear filter (having quaternions as block matrix elements):

$$\mathbf{W} = \left[\begin{array}{cc|cc} \multicolumn{2}{c|}{\mathbf{I}_4} & \begin{array}{c}-\mathbf{G}_1\mathbf{G}_2^{-1}\\ \mathbf{0}_2\end{array} & \begin{array}{c}\mathbf{0}_2\\ -\tilde{\mathbf{G}}_1\tilde{\mathbf{G}}_2^{-1}\end{array}\\ \hline \begin{array}{c}-\mathbf{H}_2\mathbf{H}_1^{-1}\\ \mathbf{0}_2\end{array} & \begin{array}{c}\mathbf{0}_2\\ -\tilde{\mathbf{H}}_2\tilde{\mathbf{H}}_1^{-1}\end{array} & \multicolumn{2}{c}{\mathbf{I}_4}\end{array}\right] \quad (48)$$

can be used to transform the problem of joint detection in (47) into two separate problems of detection with each user employing the QOSTBC in (29). Note (48) is obtained by extending the multiuser decorrelator in [4] to the virtual Alamouti systems in (47). Now, a receive stratgey better than the first approach is to combine the decorrelator and adaptive modulation. Specifically, using the Bayesian detection analysis in [17] for ML detection on (47), the post-processing SNR per symbol in using $\mathbf{W}$ in (48) is a function of the quaternionic angle of the virtual Alamouti system in (47) given by $\boldsymbol{\lambda}_D = 1/(\|\mathbf{H}_D\|\|\mathbf{G}_D\|)\mathbf{H}_D^\dagger\mathbf{G}_D$. A reasonable approach to maximize the post-processing SNR per symbol at each block transmission time is to find the QOSTBC with minimum $\|\boldsymbol{\lambda}_D\|$

or equivalently,

$$\min_{t\in\mathcal{S}_k}(\|\mathbf{H}_1(t)^\dagger\mathbf{G}_1(t)+\mathbf{H}_2(t)^\dagger\mathbf{G}_2(t)\|^2+\|\tilde{\mathbf{H}}_1(t)^\dagger\tilde{\mathbf{G}}_1(t)+ \tilde{\mathbf{H}}_2(t)^\dagger\tilde{\mathbf{G}}_2(t)\|^2)/((\|\mathbf{H}_1(t)\|^2+\|\mathbf{H}_2(t)\|^2)(\|\tilde{\mathbf{H}}_1(t)\|^2+\|\tilde{\mathbf{H}}_2(t)\|^2) \cdot(\|\mathbf{G}_1(t)\|^2+\|\mathbf{G}_2(t)\|^2)(\|\tilde{\mathbf{G}}_1(t)\|^2+\|\tilde{\mathbf{G}}_2(t)\|^2))$$

The third approach that has similar complexity as the previous two but with better performance is to apply the BLAST-MMSE with interference cancellation and adaptive modulation. In particular, we apply the A-BLAST Algorithm in Section V to each of the following systems (formed by equations at the first and second antenna):

$$\begin{bmatrix}\hat{r}_{11}\\ \hat{r}_{21}\\ \hat{r}_{12}\\ \hat{r}_{22}\end{bmatrix}=\begin{bmatrix}\mathbf{H}_1 & \mathbf{G}_1\\ \mathbf{H}_2 & \mathbf{G}_2\end{bmatrix}\begin{bmatrix}c_1+c_3\\ c_2+c_4\\ s_1+s_3\\ s_2+s_4\end{bmatrix}+\begin{bmatrix}\hat{n}_{11}\\ \hat{n}_{21}\\ \hat{n}_{12}\\ \hat{n}_{22}\end{bmatrix}\tag{50}$$

and

$$\begin{bmatrix}\hat{r}_{31}\\ \hat{r}_{41}\\ \hat{r}_{32}\\ \hat{r}_{42}\end{bmatrix}=\begin{bmatrix}\tilde{\mathbf{H}}_1 & \tilde{\mathbf{G}}_1\\ \tilde{\mathbf{H}}_2 & \tilde{\mathbf{G}}_2\end{bmatrix}\begin{bmatrix}c_1-c_3\\ c_2-c_4\\ s_1-s_3\\ s_2-s_4\end{bmatrix}+\begin{bmatrix}\hat{n}_{31}\\ \hat{n}_{41}\\ \hat{n}_{32}\\ \hat{n}_{42}\end{bmatrix}.\tag{51}$$

An adaptive modulation strategy that maximizes the post processing SNR per symbol at each block transmission time in using the A-BLAST Algorithm is to minimize the sum of squares of the quaternionic angles in the two virtual Alamouti systems given by (50) and (51). In particular, at each block transmission time, we select the QOSTBC that satisfies

$$\min_{t\in\mathcal{S}_k}\frac{\|\mathbf{H}_1(t)^\dagger\mathbf{G}_1(t)+\mathbf{H}_2(t)^\dagger\mathbf{G}_2(t)\|^2}{(\|\mathbf{H}_1(t)\|^2+\|\mathbf{H}_2(t)\|^2)(\|\mathbf{G}_1(t)\|^2+\|\mathbf{G}_2(t)\|^2)}+\frac{\|\tilde{\mathbf{H}}_1(t)^\dagger\tilde{\mathbf{G}}_1(t)+\tilde{\mathbf{H}}_2(t)^\dagger\tilde{\mathbf{G}}_2(t)\|^2}{(\|\tilde{\mathbf{H}}_1(t)\|^2+\|\tilde{\mathbf{H}}_2(t)\|^2)(\|\tilde{\mathbf{G}}_1(t)\|^2+\|\tilde{\mathbf{G}}_2(t)\|^2)}.$$

We show in the next section that, in a multiuser environment, the A-BLAST Algorithm achieves comparable performance to the scheme in [11], but at a much lower complexity. Furthermore, with adaptive modulation using 1 or 2 bits for feedback, the A-BLAST Algorithm outperforms the scheme in [11].

VII. Numerical Results

We first consider a single user with four transmit antennas and one receive antenna. We compare the performance of ML pairwise decoding (our baseline algorithm), A-BLAST with adaptive modulation (selects QOSTBC in (29), (36) or (38)) and A-BLAST using QOSTBC in (38) only. We assume 4-QAM signal constellations with unit energy. Figure 4 shows BER performance as a function of snr. The A-BLAST Algorithm performs within 0.5dB of ML pairwise decoding, yet decoding complexity is much lower. With adaptive modulation (1-bit feedback), A-BLAST provides a 1dB improvement over ML pairwise decoding with 1-bit feedback. The incremental performance improvement from a second bit of feedback is marginal. This illustrates the value of a small number of feedback bits for the adaptive modulation scheme.

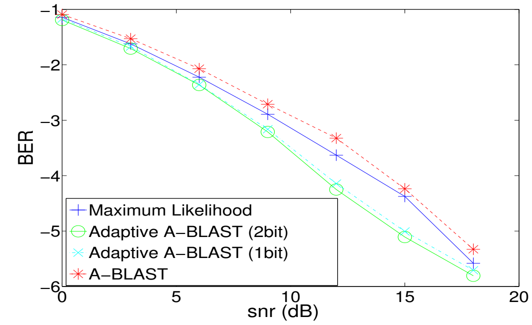

Fig. 4. Performance comparison of the ML pairwise decoding, the A-BLAST Algorithm with adaptive modulation (1-bit and 2-bit feedback) and the standalone A-BLAST Algorithm. The y-axis shows the Bit Error Rate (BER).

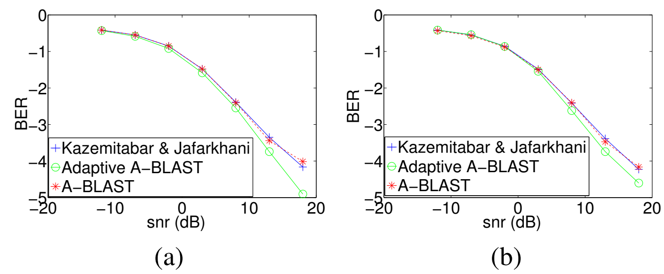

Fig. 5. Performance comparison of the ML pairwise decoding, A-BLAST Algorithm with adaptive modulation and the standalone A-BLAST Algorithm of a two-user MAC system where each user has four transmit antennas and employs QOSTBC. The receiver is equipped with two receive antennas. (a) BER vs. snr at receive antenna 1 (b) BER vs. snr at receive antenna 2.

Next, we consider a two-user MAC system where each user has four transmit antennas and employs QOSTBC. The receiver is equipped with two receive antennas, and employs successive interference cancellation to separate the two users. We compare the multiuser interference cancellation scheme using QOSTBC in [11] with our A-BLAST Algorithm with and without adaptive modulation. Fig. 5 shows that the A-BLAST Algorithm without adaptive modulation is comparable to the scheme in [11]. On the other hand, the A-BLAST Algorithm with adaptive modulation outperforms the scheme in [11] by at least 1dB in the high snr regime at both receive antennas.

VIII. Conclusions

We presented solutions to the multiuser detection problem of Alamouti signals over a i.i.d. Rayleigh fading multiple access channel. By emphasizing on decoders with low complexity, we enlarged the scope of prior algorithms for BLAST receivers, and developed different MIMO receive algorithms for Alamouti signals. In a multiuser MAC setting where each user has four transmit antennas and employs QOSTBC, we presented a systematic methodology to exploit different decomposition structure in Alamouti block matrices, each with different tradeoff between performance and decoding complexity using possibly different MIMO receive algorithms. As a particular illustration, we proposed a low-complexity Alamouti BLAST-MMSE (A-BLAST) receive algorithm. The performance of our A-BLAST Algorithm is determined by the quaternionic angle (the inner product of two quaternionic vectors) between the desired Alamouti signal and interference. In combination with our A-BLAST Algorithm, this led us to introduce a new adaptive modulation strategy that we called code diversity for single-user point-to-point system or multiuser MAC system. For a family of quasi-orthogonal STBC codes, we proposed adaptive modulation where feedback using a small number of bits instructs the transmitter how to choose from a plurality of codes based on the quaternionic angles. In a single user setting, we demonstrated that the advantage of code diversity is power saving at the transmitter, and an added advantage in a multiuser setting is to minimize the co-channel interference in the system.

APPENDIX

For the three-user case where $\mathbf{H} = [\mathbf{H}_1\ \mathbf{H}_2\ \mathbf{H}_3]$, by setting the first quaternionic column of $\mathbf{Q}$ in the QR factorization to be $\mathbf{H}_1$, $\mathbf{R}$ is given by

$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_2 & \mathbf{X}' & \mathbf{X}'' \\ \mathbf{0}_2 & \mathbf{I}_2 & \mathbf{X}''' \\ \mathbf{0}_2 & \mathbf{0}_2 & \mathbf{I}_2 \end{bmatrix}, \tag{52}$$

where $\mathbf{X}' = \langle \mathbf{H}_1, \mathbf{H}_2 \rangle / (\|\mathbf{H}_1\|^2 + 1/\mathsf{snr})$, $\mathbf{X}'' = \langle \mathbf{H}_1, \mathbf{H}_3 \rangle / (\|\mathbf{H}_1\|^2 + 1/\mathsf{snr})$ and $\mathbf{X}''' = (\mathbf{I}_2 - \mathbf{X}')^\dagger \langle \mathbf{H}_2, \mathbf{H}_3 \rangle / b^2$ with b given by (16) in Lemma 1.

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