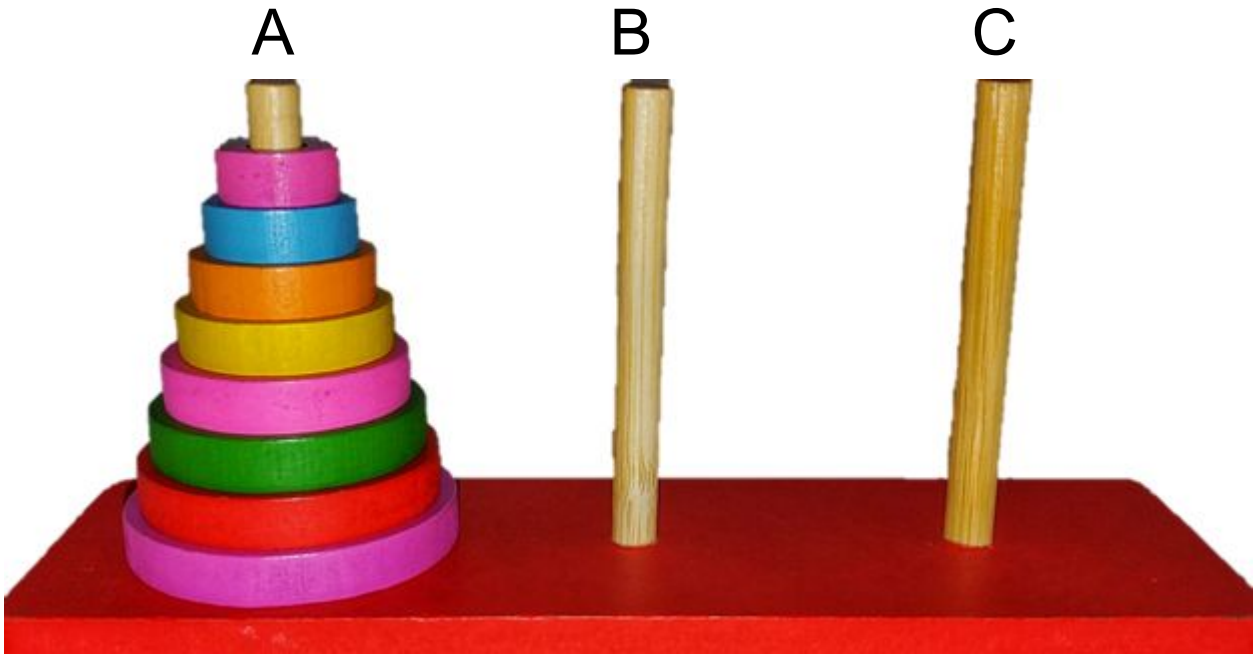
Julia Robinson Mathematics Festival

Tower of Hanoi



Legend has it that in Hanoi there is a tower of 64 disks of different sizes, initially all stacked on peg A as shown, and a group of monks working tirelessly to move the disks to peg C. They can only move one disk at a time, and never may a larger disk be placed on top of a smaller one. When the monks complete their task, the legend says, the world will end.

Assuming the monks start now and can move one disk per second, uninterruptedly, how soon may the world end?

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SFMC SFSU

Intermediate

1. How many moves will it take to complete moving all disks if there is only one disk?

2. How many moves will it take if there are two disks?

3. What is the minimum number of moves that it will take for three disks?

4. What is the minimum number of moves that it will take for four disks?

5. Generalize: What is the minimum number of moves it will take for *n* disks?

6. If the monks never make a mistake, and can move one disk every second, 24 hours per day, how many years will it take for them to complete their tower of 64 disks?

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Advanced

7. Now repeat the previous problems, but with an additional rule: Every disk must be moved either to or from peg C, never directly between pegs A and B.

8. Show that, in solving the previous exercise, you must encounter every legal arrangement of disks on the three pegs.

9. Let Q_n be the number of moves required to move a tower of *n* disks from peg A to peg B with all moves being "forward" (that is, A to B, or B to C, or C to A). Similarly, let R_n be the number of moves required to move a tower of *n* disks from peg B to peg A with all moves being forward. Prove that

$$Q_n = \begin{cases} 0, & \text{if } n = 0 \\ 2R_{n-1}, & \text{if } n > 0 \end{cases} \qquad R_n = \begin{cases} 0, & \text{if } n = 0 \\ Q_n + Q_{n-1} + 1, & \text{if } n > 0. \end{cases}$$

10. A Double Tower of Hanoi contains 2*n* disks of *n* different sizes, two of each size. How many moves does it take to move a double tower from peg A to peg B (under the usual one-disk-at-a-time rule of the initial problem)?

11. Generalize: What if you have a given number of repeats of each peg, such as 1 of the smallest, 2 of the next smallest, 3 of the next smallest, and 5 of the biggest disk?