Optimal Charging of Electric Vehicles in Smart Grid: Characterization and Valley-filling Algorithms

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Outline

1. Motivation
2. Problem Formulation
3. Our Results/Contribution
   - Decomposition
   - Characterization of Optimal Solution
   - Algorithms
4. Numerical Results

Motivation

- Increasing popularity of EV
  - Reduce CO_2
  - Reduce dependence of fossil fuel
- Possible Impact
  - Uncoordinated charging + high penetration $\rightarrow$ grid overload
  - Coordinated charging is beneficial
    - Save infrastructure investment
    - Reduce operation cost
- Objective
  - Study optimal coordinated EV charging
  - Produce real time control algorithm under demand uncertainty

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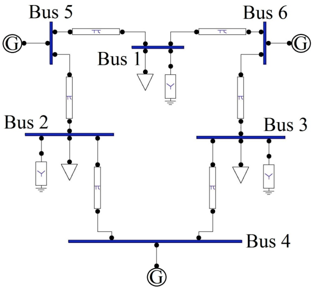
Optimal Power Flow Problem

- Objective
  - Generation cost
  - Power loss
- Constraints
  - Power constraints
  - Line constraints
  - Voltage constraints



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- However, OPF only captures stable state
- Need to capture time-varying EV charging power

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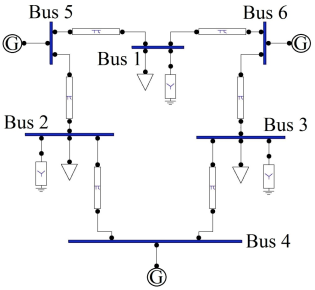

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Joint OPF-EV Charging Problem

- Consider two types of load
  - Price-inelastic load, e.g., lighting
  - Price-elastic load, e.g., EV
- Augment OPF with extra time dimension
- Add EV charging constraints
- Consider two different scenarios
  - Offline – no uncertainty
  - Online – uncertain future demand

System Model

- Vector of bus voltages $\mathbf{v}[t] = (V_1[t], \ldots, V_n[t]) \in \mathbb{C}^n$
- Vector of current injection $\mathbf{i}[t] = (I_1[t], \ldots, I_n[t]) \in \mathbb{C}^n$
- Let $\mathbf{Y}$ be the admittance matrix, by Kirchoff's Law

$$\mathbf{i}[t] = \mathbf{Y}\mathbf{v}[t]$$

- Power injected at bus k

$$\begin{aligned}(\mathbf{p}[t]_k - \mathbf{p}[t]_k^d) + (\mathbf{q}[t]_k - \mathbf{q}[t]_k^d)j &= V_k[t] I_k[t]^* \\ = (e_k^* \mathbf{v})(e_k^* \mathbf{i})^* &= \mathrm{Tr}\{\mathbf{v}[t]\mathbf{v}[t]^* Y^* e_k e_k^*\}\end{aligned}$$

- Introduce auxillary variable $W[t] = \mathbf{v}[t]\mathbf{v}[t]^*$

System Model

- Power Constraints
  - $P_k^{\min} \leq (\mathbf{p}[t])_k \leq P_k^{\max}$
  - $Q_k^{\min} \leq (\mathbf{q}[t])_k \leq Q_k^{\max}$
  - $\text{Trace}\{\mathbf{W}[t]\mathbf{Y}^* e_k e_k^*\} = (\mathbf{p}[t])_k - ((\tilde{\mathbf{p}}[t])_k + (\hat{\mathbf{p}}[t])_k) + ((\mathbf{q}[t])_k - (\tilde{\mathbf{q}}[t])_k)\mathrm{j}$
- Voltage Constraints
  - $(V_k^{\min})^2 \leq \mathbf{W}[t]_{kk} \leq (V_k^{\max})^2$
- Line Constraints
  - $(\mathbf{W}[t]_{ll} - \mathbf{W}[t]_{lm})\mathbf{Y}_{lm}^* \leq S_{lm}^{\max}$
- SDP constraints
  - $\mathbf{W}[t] \succeq 0$
  - $\text{rank}(W[t]) = 1$ (non-convex!)

System Model

- Objective is to minimize the sum of total generation cost and EV charging cost
- Joint OPF-EV Charging Problem can be written as

$$\min_{\{\mathbf{W}[t],\hat{\mathbf{p}}[t]\}} \sum_{t=1}^{T-1} \sum_{k\in\mathcal{N}} f_k((\mathbf{p}[t])_k) + \sum_{t=1}^{T-1} \sum_{k\in\mathcal{N}} \alpha_k(\hat{\mathbf{p}}[t])_k$$

such that all constraints (power, voltage, line, SDP) are satisfied

Assumptions

- State of the system is discrete
- OPF has zero duality gap$\rightarrow$ rank relaxation is exact.
  - True for all distribution network [1]
  - [1] $\rightarrow$ zero gap for Joint OPF-EV charging [2]
- EV charging schedule is known
- EV charging cost is time-invariant

Joint OPF-EV Charging Problem

Challenges

- OPF is non-convex, so is the augmented problem
- Lots of control variables (with long time horizon)
- (Online case) Uncertain future demand

Approach

- Solve SDP relaxation of OPF - exact for most network [1]
- Decompose
- Characterize optimal offline solution, follow the characterization online

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Decomposing Joint OPF-EV Charging Problem

- Decouple EV scheduling ($\tilde{\mathbf{p}}[t]$) from power dispatching ($W[t]$)
- Define function

$$F(\hat{\mathbf{p}}[t] + \tilde{\mathbf{p}}[t]) = \min_{W[t]} \left(\sum_{k \in \mathcal{N}} f((\mathbf{p}[t]))_k \right)$$

s.t.all network constraints are satisfied

Decomposing Joint OPF-EV Charging Problem

- Define nested optimization problem

$$\begin{aligned}\min_{\hat{\mathbf{p}}} \quad & \sum_{t=1}^{T-1} F(\tilde{\mathbf{p}}[t] + \hat{\mathbf{p}}[t]) + \alpha[t]^\top \hat{\mathbf{p}}[t] \\ \text{s. t.} \quad & \sum_{t=1}^{T-1} \hat{\mathbf{p}}[t] = \mathbf{c} \\ & \underline{\mathbf{r}}[t] \leq \hat{\mathbf{p}}[t] \leq \bar{\mathbf{r}}[t]\end{aligned}$$

- Can we solve $\hat{\mathbf{p}}^*$ without knowing W^*?
- . . . Yes

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- ... Yes

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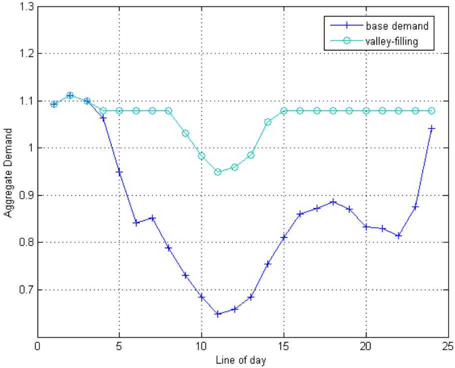
Characterization of Optimal Solution

- The nested optimzation problem has some nice structure
- (Theorem 1) $F : \mathbb{R}^N \to \mathbb{R}$ is convex
- Proof Sketch
  - F is a parameterized OPF problem
  - By zero duality gap, F look at the convex dual
  - $\tilde{\mathbf{p}}[t] + \hat{\mathbf{p}}[t]$ appears only in the dual objective function

Characterization of Optimal Solution

- (Theorem 2) Optimal EV charing profile is *valley-filling*
- We call a charging profile *valley-filling*, if there exists $\mathbf{a} \in \mathbb{R}^N$, such that $\hat{\mathbf{p}}[t] = [\mathbf{a} - \tilde{\mathbf{p}}[t]]_{\underline{\mathbf{r}}}^{\bar{\mathbf{r}}}$



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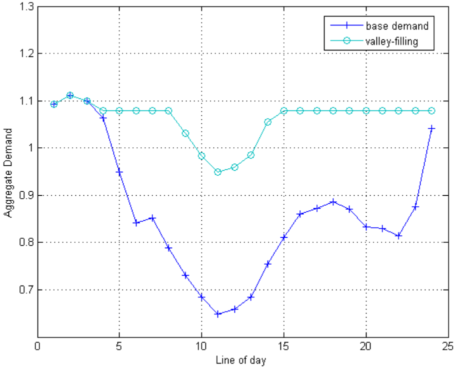

Characterization of Optimal Solution

Theorem 2 Proof Sketch

- Follow intuition of Jensen's inequality
- Use a swapping argument
  - If there exists an optimal non valley-filling profile
  - There are finite swapping steps to make the profile valley-filling
  - Each step will not increase the objective value

Characterization of Optimal Solution

Valley-filling : $\hat{\mathbf{p}}[t] = [\mathbf{a} - \tilde{\mathbf{p}}[t]]_{\underline{\mathbf{r}}[t]}^{\bar{\mathbf{r}}[t]}$

- Given the valley level $\mathbf{a}$, determining $\hat{\mathbf{p}}^*[t]$ can be done in $\mathbf{O}(\mathbf{1})$ time.
- How to find $\mathbf{a}$?
- Note that by conservation of energy

$$\sum_{t=1}^{T-1} \hat{\mathbf{p}}[t] = g(\mathbf{a}) = \sum_{t=1}^{T-1} [\mathbf{a} - \tilde{\mathbf{p}}[t]]_{\underline{\mathbf{r}}[t]}^{\bar{\mathbf{r}}[t]} = c$$

- $g(\mathbf{a})$ is a continuously increasing function of $\mathbf{a}$
- Bisection algorithm

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Offline Algorithm for Joint OPF-EV Charging

- Solve $\mathbf{a}$ using bisection
- For each t, $\hat{\mathbf{p}}[t] = [\mathbf{a} - \tilde{\mathbf{p}}[t]]_{\underline{\mathbf{r}}[t]}^{\bar{\mathbf{r}}[t]}$
- Find $W[t]$ by solving OPF with total demand $\tilde{\mathbf{p}}[t] + \hat{\mathbf{p}}[t]^*$

Remark

- Once $\hat{\mathbf{p}}^*$ is solved, Joint OPF-EV Charging problem become decoupled
- Complexity reduces from SDP with $O((|\mathcal{N}| + |\mathcal{L}|)(T))$ variables to T SDPs with $O(|\mathcal{N}| + |\mathcal{L}|)$ variables

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Online Scenario

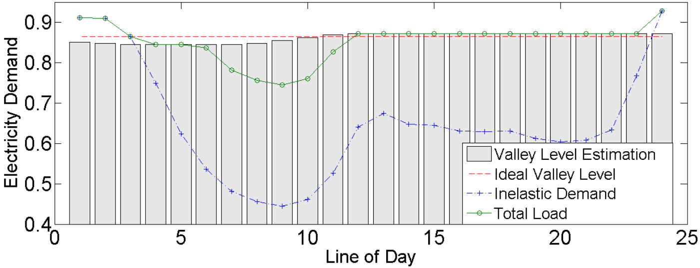
Challenge of the online scenario: $\tilde{\mathbf{p}}[t]$ is not known until time t

- Cannot use bisection to solve for $\mathbf{a}$

Strategy : Estimate and adjust

- Estimate $\mathbf{a}'[1]$ be the average of total demand
- If $\tilde{\mathbf{p}}[t] + \hat{\mathbf{p}}[t]$ does not reach $\mathbf{a}'[t]$, adjust $\mathbf{a}'[t+1]$ in the opposite direction

Illustration of Dynamic Valley Level Adjustment



- The aggregate charging profile (green line) is valley-filling
- The valley level $A'[t]$ (grey bar) is adjusting dynamically

Online Algorithm for Joint OPF-EV Charging

1. Estimate $\mathbf{a}'[1]$ using historical data
2. $\hat{\mathbf{p}}[t] = [\mathbf{a}'[t] - \tilde{\mathbf{p}}[t]]_{\underline{\mathbf{r}}[t]}^{\bar{\mathbf{r}}[t]}$
3. Find $W[t]$ by solving OPF with total demand $\tilde{\mathbf{p}}[t] + \hat{\mathbf{p}}[t]$
4. Update $\mathbf{a}'[t+1] = \mathbf{a}'[t] + (\mathbf{a}'[t] - \tilde{\mathbf{p}}[t] - \hat{\mathbf{p}}[t])/(T-1)$
5. Repeat until $t = T$

Online Algorithm for Joint OPF-EV Charging

Remarks:

1. (Theorem 3)The online algorithm will always produce a feasible EV charging schedule.
2. The online algorithm have natural decentralized implementation.
   - At beginning of each t, utility broadcast valley level $A'[t]$
   - Each EV determines its charging rate base on $A'[t]$
   - Utility gather $\hat{\mathbf{p}}[t]$ to calculate next $A'[t+1]$
3. In practice, the online algorithm produce charging profiles very close to the offline. optimal

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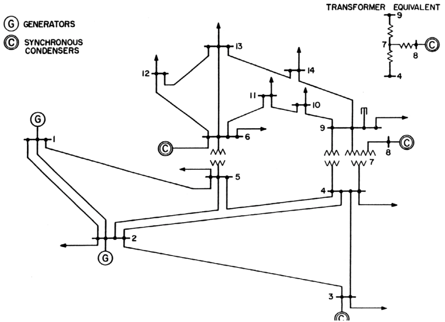
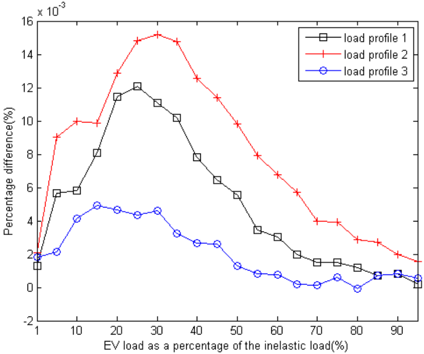
Test cases

- IEEE 14-bus test archive
- SCE residential load for demand variation



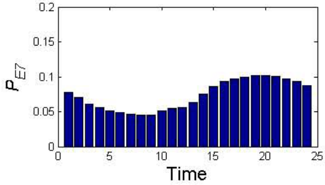
Effect of EV penetration

- Define percentage difference = $(p^*_{\text{online}} - p^*_{\text{offline}})/p^*_{\text{online}}$

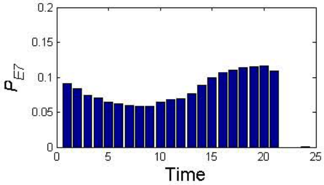


- Differenece to optimal is very small $\sim 10^{-5}$
- Approaches 0 as penetration goes to 100%

Effect of imperfect first estimation



(a) Offline solution for EV no. 7



(b) Online solution for EV no. 7

- The charging profiles are still similar
- An initial over-estimation cause EV to charge quicker than optimal
- Finish charging two time slots before deadline

Table: Performance Comparison

T	SDP Optimization	Offline Algorithm	Online Algorithm
6	6.02 s	5.84 s	5.87 s
12	13.11 s	11.63 s	11.56 s
24	31.47 s	22.87 s	22.81 s
48	84.05 s	45.75 s	45.67 s
96	262.55 s	87.48 s	87.36 s

- Both online and offline algorithms scale linearly to T.
- Gain from the decomposition approach becomes more significant as T increases.

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Summary

In this work, we

- Studied the Joint OPF-EV Charging Problem
- Characterized the optimal EV charging profile
- Proposed an online decentralized algorithm

- The following remains interesting for further investigation
  - Stochastic arrival of EVs
  - Real time varying EV charing cost

For Further Reading I

- J. Lavaei and S. Low.
  Zero duality gap for optimal power flow problem.
  IEEE Trans. Power Syst. vol. 27, no. 1, pp. 92-107, Feb. 2012.
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  Proc. IEEE Power and Energy Society General Meeting, pp. 1-6, Jul. 2011.