Convex Relaxation and Decomposition in Large Resistive Power Networks with Energy Storage

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Motivation

Optimal Power Flow

- First proposed by Carpentier in 1962,
- Minimize the generation cost/the transmission loss subject to physical laws and bounds
- Nonlinear and nonconvex

Existing solutions

- Linearization and approximation
  - small angle approximation: Christie et al 2000
  - other relaxations: Conejo et al 1998, Aguado et al 2001 and so on
- Relaxation and reformulation
  - bus injection model: SDP relaxation, Lavaei et al 2010, 2012
  - branch flow model: SOCP relaxation, Farivar et al 2011
- Decomposition
  - DC power network: uniqueness discussion and dual decomposition algorithms, Tan et al SmartGridComms 2012

Motivation

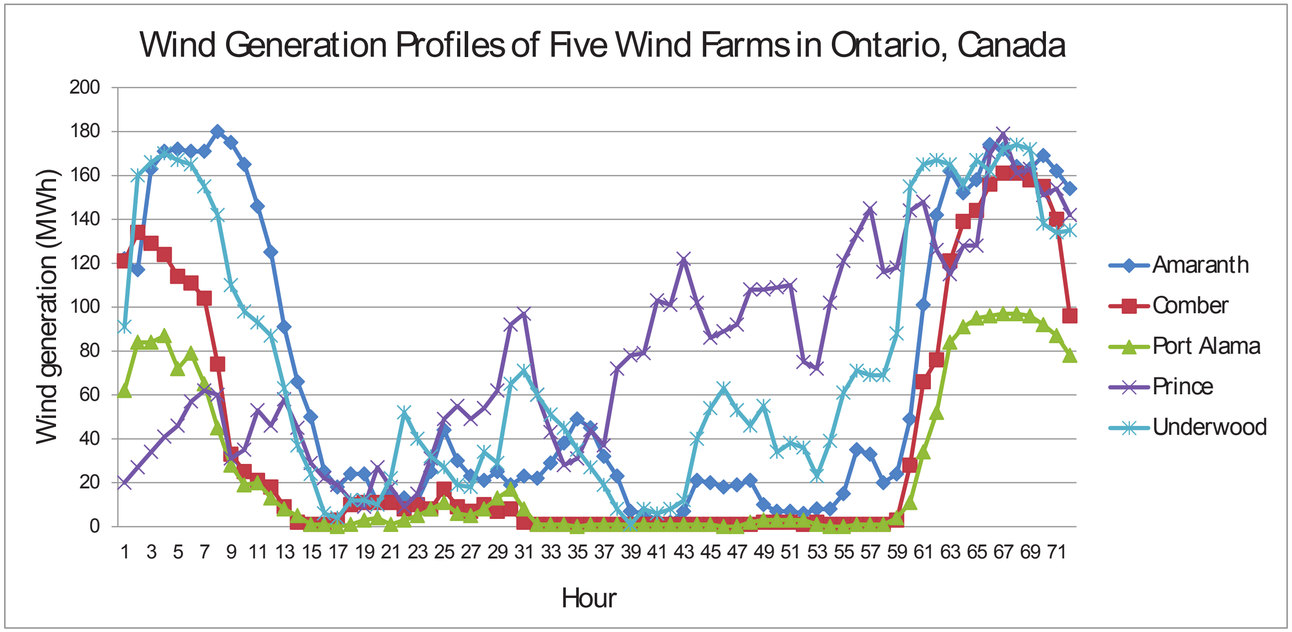
Resistive power network

Source: EPRI

- Resistive power network: only has DC power
- Practically important and promising in smart grids
- Renewable energy produces the DC power
- Examples: charging station for electric vehicles, naval ships, industrial systems

Motivation

Renewable energy



The wind power generation in five wind farms in Ontario from January 14, 2013 to January 16, 2013. The data is available from the website of IESO.

- Intermittent
- Difficult to predict and harness

One efficient solution: incorporate the energy storage

Motivation

Problem features

- Nonconvex Quadratic-Constrained Quadratic Programming (QCQP)
- NP-hard in general
- SDP relaxation can be tight (Lavaei and Low TPS 2012)

Key Challenges

- Any tight convex relaxation better than SDP?
- How to resolve the coupling over space and time?
- How to decompose this coupled problem?

System Model

- Resistive power network with N buses, where a bus is either a generation bus $(i \in \mathcal{G})$ or a demand bus $(i \in \mathcal{D})$
- Topology by a graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of buses and $\mathcal{E}$ is the set of transmission lines
- Ω_i represents the set of buses connecting to bus i
- $\mathbf{Y}$ is the system admittance matrix and the line admittance satisfies $Y_{ij} = Y_{ji} \in \mathbb{R}_+$, if $(i,j) \in \mathcal{E}$; and $Y_{ij} = Y_{ji} = 0$, otherwise
- $\mathbf{V}(t)$ and $\mathbf{I}(t)$ denote the voltage vector $(V_i(t))_{i \in \mathcal{N}}$ and current vector $(I_i(t))_{i \in \mathcal{N}}$ at $t = 1, \ldots, T$, respectively
- $\mathbf{b}(t)$ represents battery amount vector $(b_i(t))_{i \in \mathcal{N}}$
- $\mathbf{r}(t)$ represents the charge rate (positive) or discharge rate (negative) vector $(r_i(t))_{i \in \mathcal{N}}$

System Model

- Nodal power constraint: $V_i(t)I_i(t) \le \bar{p}_i \xrightarrow{\mathbf{I}(t)=\mathbf{Y}\mathbf{V}(t)}$

$$\mathbf{V}(t)^\top \mathbf{Y}_i \mathbf{V}(t) \le \bar{p}_i(t) - r_i(t) \ \forall i \in \mathcal{N}, \ \forall t = 1, \dots, T \quad (1)$$

  - $\mathbf{Y}_i = \frac{1}{2}(\mathbf{E}_i\mathbf{Y} + \mathbf{Y}\mathbf{E}_i)$, $\mathbf{E}_i = \mathbf{e}_i\mathbf{e}_i^\top \in \mathbb{R}^{n \times n}$, $\mathbf{e}_i$ is the standard basis vector in $\mathbb{R}^n$
  - $\bar{p}_i(t) > 0$ represents the generator capacity, if bus i is a generator bus
  - $\bar{p}_i(t) < 0$ represents the minimum demand, if bus i is a demand bus
  - Assume load over-satisfaction (Lavaei and Low 2012)
- Nodal voltage constraint:

$$\underline{\mathbf{V}} \le \mathbf{V}(t) \le \overline{\mathbf{V}} \ \ \forall t = 1, \dots, T \quad (2)$$

- Battery constraint:

$$b_i(t+1) \le b_i(t) + r_i(t) \ \ \forall i \in \mathcal{D}, \ \forall t = 1, \dots, T \quad (3)$$

  - Take into account the fact that charge loss or storage leakage can happen in a general battery model

System Model

- Initial condition:

$$b_i(1) = B_i^0 \;\; \forall i \in \mathcal{D},\; \forall t = 1, \ldots, T \tag{4}$$

- Battery capacity constraint:

$$0 \leq b_i(t) \leq B_i \;\; \forall i \in \mathcal{N},\; \forall t = 1, \ldots, T+1 \tag{5}$$

- Charge/Discharge rate constraint:

$$\underline{r_i} \leq r_i(t) \leq \bar{r_i} \;\; \forall i \in \mathcal{N},\; \forall t = 1, \ldots, T \tag{6}$$

- In summary, we have:

$$\mathbb{V} := \{\mathbf{V}(t) | \mathbf{V}(t) \text{ satisfies } (1), (2)\}$$

$$\mathbb{B} := \{(\mathbf{b}(t), \mathbf{r}(t)) | (\mathbf{b}(t), \mathbf{r}(t)) \text{ satisfies } (3) - (6)\}$$

Problem Formulation

- Objective: $\sum_{t=1}^{T} \mathbf{V}(t)^\top \mathbf{Y} \mathbf{V}(t) + \sum_{i \in \mathcal{N}} \sum_{t=1}^{T+1} h_i(t)$
  - $h_i(t) = \alpha_i(B_i - b_i(t))$, i.e., the penalty is proportional to the deviation from the capacity (Chandy et al 2010)

This dynamic OPF problem is formulated as:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{t=1}^{T} \mathbf{V}(t)^\top \mathbf{Y} \mathbf{V}(t) + \sum_{i \in \mathcal{N}} \sum_{t=1}^{T+1} h_i(t) \\
\text{subject to} \quad & \mathbf{V}(t) \in \mathbb{V}, (\mathbf{b}(t), \mathbf{r}(t)) \in \mathbb{B}, \\
\text{variables:} \quad & \mathbf{V}(t), \mathbf{b}(t), \mathbf{r}(t).
\end{aligned}
\tag{7}
$$

Dynamic nonconvex QCQP

SOCP Convex Relaxation

Introduce auxiliary variables $X_i(t) = V_i^2(t)$ and $W_{ij}(t) = V_i(t)V_j(t)$ $\forall i \in \mathcal{N}$, $\forall (i,j) \in \mathcal{E}$, $t = 1, \ldots, T$, then we have:

$$
\begin{aligned}
\text{minimize} \quad & \sum_{i\in\mathcal{N}} \left(\sum_{t=1}^{T} \sum_{j\in\Omega_i} Y_{ij}(X_i(t) - W_{ij}(t)) + \sum_{t=1}^{T+1} h_i(t) \right) \\
\text{subject to} \quad & \sum_{j\in\Omega_i} Y_{ij}(X_i(t) - W_{ij}(t)) \leq \overline{p}_i(t) - r_i(t) \;\forall t = 1, \ldots, T, \\
& X_i(t)X_j(t) = W_{ij}^2(t) \;\forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T, \\
& \underline{V}_i^2 \leq X_i(t) \leq \overline{V}_i^2 \;\forall i \in \mathcal{N}, \forall t = 1, \ldots, T, \\
& (\mathbf{b}(t), \mathbf{r}(t)) \in \mathbb{B}, \\
\text{variables:} \quad & \mathbf{X}(t), \mathbf{W}(t), \mathbf{b}(t), \mathbf{r}(t),
\end{aligned}
\tag{8}
$$

where $\mathbf{X}(t)$ and $\mathbf{W}(t)$ represent $(X_i(t))_{i\in\mathcal{N}}$ and $(W_{ij}(t))_{(i,j)\in\mathcal{E}}$, respectively.

SOCP Convex Relaxation

We relax the nonconvex constraint in the SOCP formulation:

$$X_i(t)X_j(t) \geq W_{ij}^2(t) \ \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T, \tag{9}$$

which can be rewritten as equivalent to the SOCP constraints:

$$\left\| \begin{array}{c} 2W_{ij}(t) \\ X_i(t) - X_j(t) \end{array} \right\|_2 \leq X_i(t) + X_j(t) \ \forall (i,j) \in \mathcal{E}, \forall t = 1, \ldots, T \tag{10}$$

Theorem 1. *Solving the SOCP convex relaxation of (8) with the constraint (10) yields the optimal solution to (7)*

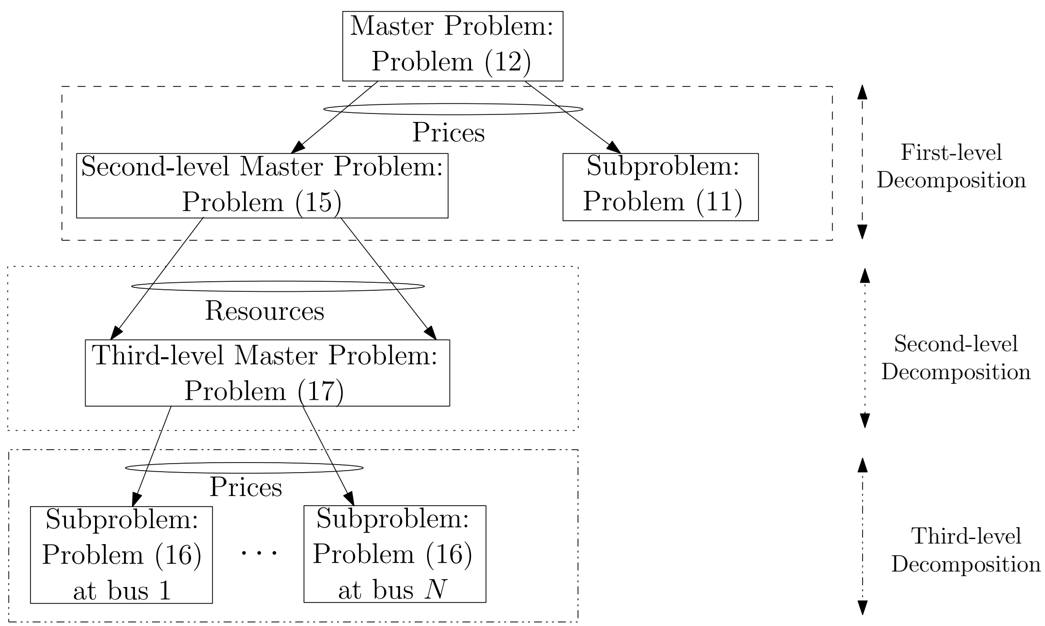
Two key remarks on the proof:

- The objective function has to be monotonically increasing in $\mathbf{X}(t)$ and decreasing in $\mathbf{W}(t)$
- Variables corresponding to the energy storage part (i.e., $\mathbb{B}$) are linear in (7) and do not appear in the SOCP constraints (10)

Indirect Decomposition

Observations

- The SOCP relaxation is tight regardless of the network topology (mesh or tree networks)
- The SOCP relaxation in (8) is convex
- The Lagrange duality gap of (7) is zero



Indirect Decomposition

First-level decomposition

We first apply the partial dual decomposition to (7):

$$\begin{aligned} \text{minimize} \quad & \mathbf{V}(t)^\top \mathbf{Y}\mathbf{V}(t) + \sum_{i\in\mathcal{N}} \lambda_i(t)(\mathbf{V}(t)^\top \mathbf{Y}_i \mathbf{V}(t)) \\ \text{subject to} \quad & \underline{\mathbf{V}} \le \mathbf{V}(t) \le \overline{\mathbf{V}}, \\ \text{variables:} \quad & \mathbf{V}(t). \end{aligned} \tag{11}$$

Due to the zero duality gap, each decomposed subproblem at each t can be solved with Algorithm 1 (Tan et al 2012):

Compute voltage $\mathbf{V}(t)$:

$$V_i^{k+1}(t) = \max\left\{\underline{V}_i, \min\left\{\overline{V}_i, \sum_{j\in\Omega_i} B_{ij}(t)V_j^k(t)\right\}\right\},$$

$\forall i \in \mathcal{N}$, where $B_{ij}(t) = \frac{2Y_{ij}+\lambda_i(t)Y_{ij}+\lambda_j(t)Y_{ij}}{2(1+\lambda_i(t))\sum\limits_{j\in\Omega_i} Y_{ij}}, \forall (i,j) \in \mathcal{E}$.

Indirect Decomposition

First-level decomposition

The primary master dual problem is given by:

$$\begin{array}{rl} \text{maximize} & \sum_{t=1}^{T} g(\boldsymbol{\lambda}(t)) + \boldsymbol{\lambda}(t)^\top(\mathbf{r}(t) - \bar{\mathbf{p}}(t)) \\ \text{subject to} & \boldsymbol{\lambda}(t) \geq \mathbf{0} \;\; \forall t = 1, \ldots, T, \\ \text{variables:} & \boldsymbol{\lambda}(t), \end{array} \tag{12}$$

where $g(\boldsymbol{\lambda}(t))=\mathbf{V}^*(t)^\top\mathbf{Y}\mathbf{V}^*(t)+\sum_{i\in\mathcal{N}}\lambda_i(t)(\mathbf{V}^*(t)^\top\mathbf{Y}_i\mathbf{V}^*(t))$.

Due to the uniqueness of (11) (Tan et al 2012), we have the following gradient updates:

$$\lambda_i^{k+1}(t)=[\lambda_i^k(t)+\beta(\mathbf{V}^*(t)^\top\mathbf{Y}_i\mathbf{V}^*(t)-\overline{p}_i(t)+r_i(t))]^+ \; \forall i \in \mathcal{N},$$

where β is a stepsize and $[\cdot]^+$ denotes the projection onto the nonnegative orthant.

Indirect Decomposition

Second-level decomposition

Consider the decomposed problem corresponding to the energy storage:

$$\begin{array}{rl} \text{minimize} & \displaystyle\sum_{t=1}^{T+1} \boldsymbol{\alpha}^\top(\mathbf{B}-\mathbf{b}(t)) + \sum_{t=1}^{T} \boldsymbol{\lambda}(t)^\top \mathbf{r}(t) \\ \text{subject to} & (\mathbf{b}(t), \mathbf{r}(t)) \in \mathbb{B}, \\ \text{variables:} & \mathbf{b}(t), \mathbf{r}(t). \end{array} \tag{13}$$

We apply the primal decomposition to obtain the second-level subproblem:

$$\begin{array}{rl} \text{minimize} & \displaystyle\sum_{t=1}^{T+1} \boldsymbol{\alpha}^\top(\mathbf{B}-\mathbf{b}(t)) + \sum_{t=1}^{T} \boldsymbol{\lambda}(t)^\top \mathbf{r}(t) \\ \text{subject to} & \mathbf{b}(t) \in \mathbb{B}, \\ \text{variables:} & \mathbf{b}(t), \end{array} \tag{14}$$

where $\mathbf{r}(t)$ is fixed in $\mathbb{B}$.

Indirect Decomposition

Second-level decomposition

For the second-level master problem, we have:

$$\begin{array}{rl} \text{minimize} & f^*(\mathbf{r}(t)) \\ \text{subject to} & \underline{\mathbf{r}} \leq \mathbf{r}(t) \leq \bar{\mathbf{r}} \;\; \forall t = 1, \ldots, T, \\ \text{variables:} & \mathbf{r}(t), \end{array} \tag{15}$$

where $f^*(\mathbf{r}(t))$ is the optimal value of (13) for given $\mathbf{r}(t)$ $\forall t = 1, \ldots, T$. By the subgradient method, we can get the following update on $r_i(t)$:

$$r_i^{k+1}(t) = [r_i^k(t) - \theta(\lambda_i^*(t) - \mu_i^*(t))]_{\underline{r}_i}^{\overline{r}_i} \; \forall i \in \mathcal{N},$$

where $\mu_i(t)$ is the dual variable corresponding to (3), θ is a stepsize and $[\cdot]_y^x$ denotes the projection onto the closed set $[y, x]$ (x and y are the parameters).

Indirect Decomposition

Third-level decomposition

Decompose (14) by partial dual decomposition, we have:

$$\begin{array}{rl} \text{minimize} & \displaystyle\sum_{t=1}^{T+1} \alpha_i(B_i - b_i(t)) + \sum_{t=1}^{T} \mu_i(t)(b_i(t+1) - b_i(t)) \\ \text{subject to} & 0 \le b_i(t) \le B_i, b_i(1) = B_i^0 \;\; \forall t = 1, \dots, T+1, \\ \text{variables:} & b_i(t). \end{array} \tag{16}$$

Then, the third-level master dual problem is:

$$\begin{array}{rl} \text{maximize} & \displaystyle\sum_{t=1}^{T} \left(\sum_{i \in \mathcal{N}} g_i(\boldsymbol{\mu}(t)) + (\boldsymbol{\lambda}(t) - \boldsymbol{\mu}(t))^\top \mathbf{r}(t) \right) + \boldsymbol{\alpha}^\top (\mathbf{B} - \mathbf{b}^*(T+1)) \\ \text{subject to} & \boldsymbol{\mu}(t) \ge \mathbf{0} \;\; \forall t = 1, \dots, T, \\ \text{variables:} & \boldsymbol{\mu}(t), \end{array} \tag{17}$$

where $g_i(\boldsymbol{\mu}(t)) = \alpha_i(B_i - b_i^*(t)) + \mu_i(t)(b_i^*(t+1) - b_i^*(t))$ in (16). Similarly, the subgradient update at each $t = 1, \dots, T$ is given by:

$$\mu_i^{k+1}(t) = [\mu_i^k(t) - \gamma(b_i^*(t+1) - b_i^*(t) - r_i^*(t))]^+ \; \forall i \in \mathcal{N}.$$

where γ is a stepsize.

Indirect Decomposition

Jointly Optimal Power Flow and Energy Storage (Algorithm 2):

1. Set the stepsizes $\beta, \theta, \gamma \in (0, 1)$.
2. Calculate the battery amount:

$$b_i^{\ell+1}(t) = \text{arg}\min[\sum_{k=1}^{T+1} \alpha_i(B_i - b_i(k)) + \sum_{k=1}^{T} \mu_i^{\ell}(t)(b_i(k+1) - b_i(k))]_0^{B_i}$$

   $\forall i \in \mathcal{N}$ and $\forall t = 1, \ldots, T+1$, subject to $b_i(1) = B_i^0$.

3. Compute:

$$\mu_i^{\ell+1}(t) = [\mu_i^{\ell}(t) - \gamma(b_i^{\ell+1}(t+1) - b_i^{\ell+1}(t) - r_i^{\tau}(t))]^+$$

   $\forall i \in \mathcal{N}$ and $\forall t = 1, \ldots, T$.

4. Compute:

$$r_i^{\tau+1}(t) = [r_i^{\tau}(t) - \theta(\lambda_i^{l}(t) - \mu_i^{\ell}(t))]_{\underline{r}_i}^{\overline{r}_i}$$

   $\forall i \in \mathcal{N}$ and $\forall t = 1, \ldots, T$.

5. Run Algorithm 1 to get $\mathbf{V}^k(t)$, $\forall t = 1, \ldots, T$.

6. Compute:

$$\lambda_i^{l+1}(t) = [\lambda_i^l(t) + \beta(\mathbf{V}^k(t)^\top \mathbf{Y}_i \mathbf{V}^k(t) - \overline{p}_i(t) + r_i^\tau(t))]^+$$

$\forall i \in \mathcal{N}$, $\forall t = 1, \ldots, T$.

Update β, θ and γ until convergence.

Remark 1. *Algorithm 2 can converge fast to the optimal solution by properly choosing the stepsize. The proposed algorithm is carried out in a distributed manner, because each bus only communicates with its local one-hop neighbors*

Numerical Example

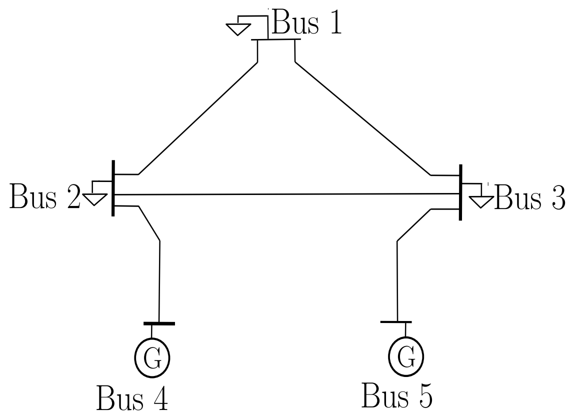
Efficiency of SOCP relaxation vs SDP relaxation

Table 1: Comparison of the average computation time of the SOCP and the SDP relaxation (in Seconds)

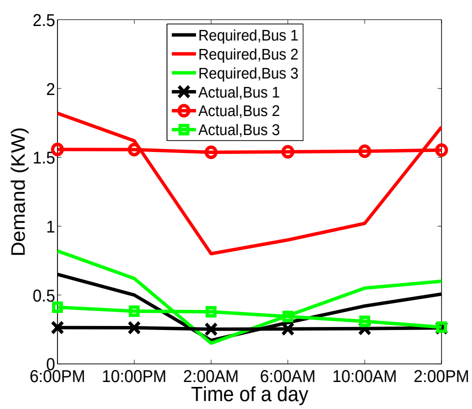
Systems	SOCP relaxation	SDP relaxation (Lavaei and Low 2012)
14-bus	0.22	0.22
30-bus	0.34	0.56
57-bus	0.51	2.41
118-bus	1.24	14.48
500-bus	3.20	639.91

Numerical Example

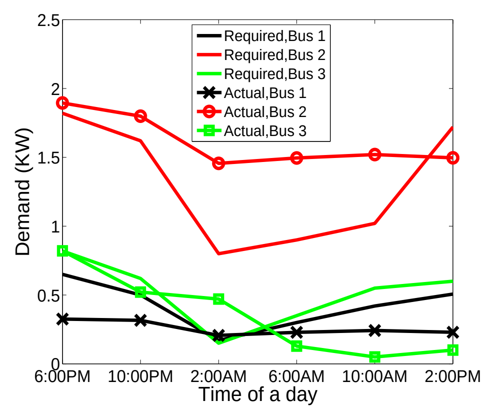
Illustration of load curve smoothing in energy storage



A 5-bus system (Glover et al, Chapter 6, pp.327). In this system, we assume that each bus has a battery attached to it



(a)



(b)

Actual and required demand under different α_i $\forall i = 1, 2, 3$. (a) $\alpha_i = 0.01$ (b) $\alpha_i = 0.1$

Numerical Example

Algorithm performance

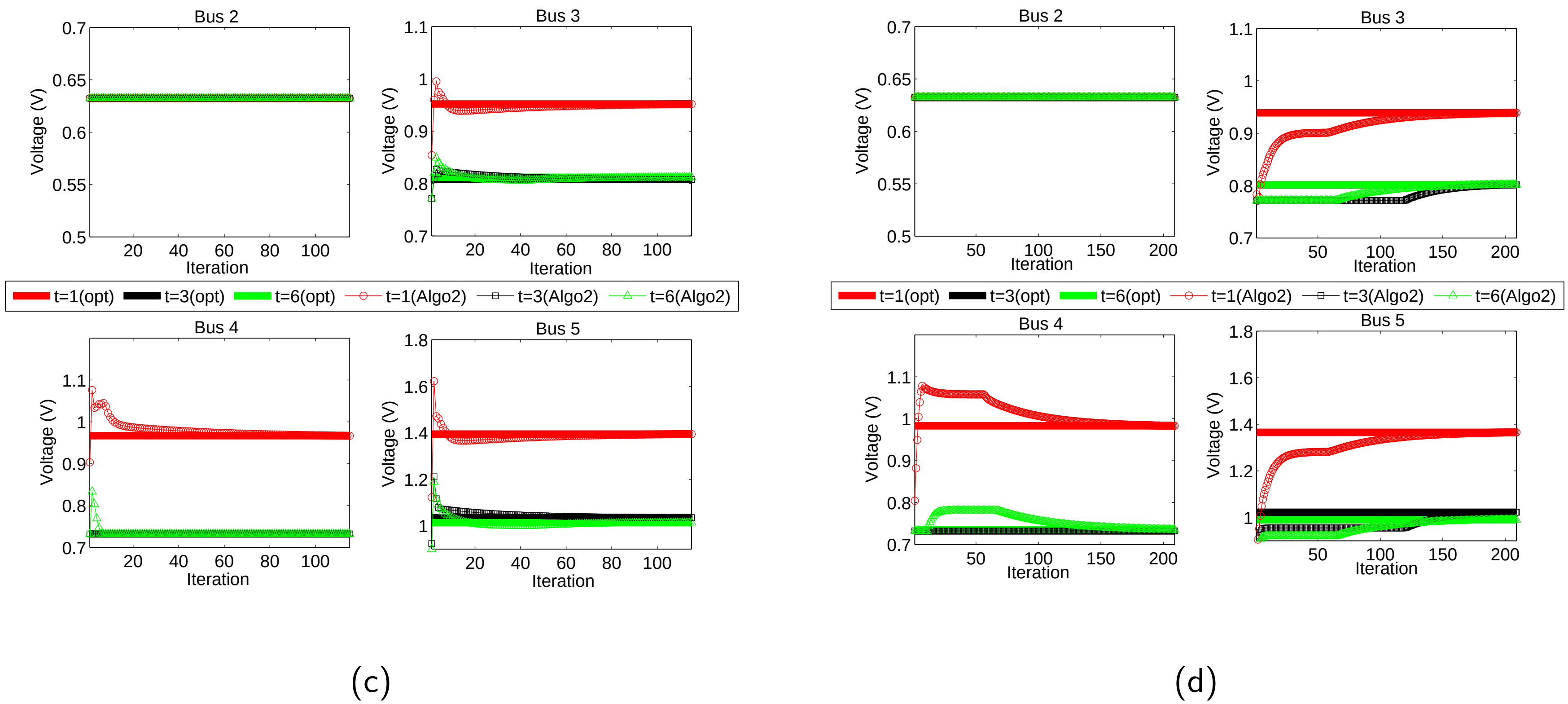


Illustration of convergence of Algorithm 2 in the 5-bus system with the initial: c) set close to the optimal solution and d) set further away from the optimal solution

Summary

- Resistive power network OPF with energy storage as a time-dependent QCQP

- SOCP convex relaxation computationally more efficient than state-of-the-art SDP

- Indirect dual-dual decomposition algorithms to unravel coupling over time and space

- Different decomposition leads to various interpretation of energy storage prices

Thank You

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