Solving Nonconvex Power Control Problems in Wireless Networks: Low SIR Regime and Distributed Algorithms

Chee Wei Tan, Daniel P. Palomar, Mung Chiang
Department of Electrical Engineering, Princeton University

***Abstract*— In wireless cellular networks that are interference-limited, a variety of power control problems can be formulated as nonlinear optimization with a system-wide objective subject to many QoS constraints from individual users. Previous work have been done in the high SIR regime by solving these problems with nonlinear objectives and constraints as geometric programs. However, in the medium to low SIR regime, these problems cannot be transformed into tractable convex optimization problems. This paper makes two contributions: (1) In the low SIR regime, we propose a method with centralized computation to obtain the globally optimal solution by solving a *series* of geometric programs. (2) While efficient and robust algorithms have been extensively studied for centralized solutions of geometric programs, distributed algorithms have not been investigated before this paper. We present a systematic method of distributed algorithms for power control based on geometric programs in high SIR regime. These two contributions can be readily combined to distributively solve nonlinear power control problems in general SIR regime.**

I. Introduction

As wireless networks support an increasingly wide variety of applications, *e.g.*, voice, data, video, Quality of Service (QoS) provisioning has become an important issue. Due to the broadcast nature of radio transmission, data rates and other QoS metrics in wireless networks are affected by signal interference from other users. The Signal to Interference Ratio (SIR) is often used to capture the effect of both co-channel and adjacent channel interference. Power control can be used to control SIR, and in doing so, indirectly control the QoS seen by users in the network. While a system-wide goal must be supported by the network, individual users' QoS requirements must also be satisfied. This tradeoff between user-centric constraints and a network centric objective can be captured in an optimization formulation.

Traditionally, optimal power control problems (*e.g.*, reviewed in [1]) are often formulated as minimizing a *linear* objective function, such as the total power, and the methods are not general enough to support a diverse set of *nonlinear* system objective and QoS constraints. Many QoS metrics are nonlinear functions of SIR, which is in turn a nonlinear function of transmit powers. It may appear that many power control problems involving QoS metrics are not efficiently solvable. Recently in [2], [3], [4], it is shown that a variety of such power control problems can be formulated as *convex* optimization with a system-wide objective, *e.g.*, maximizing total system throughput or worst case user throughput, under many QoS constraints from individual users *e.g.*, on data rate, delay, and outage probability, through the Geometric Programming (GP) framework. GP can be used to efficiently compute a variety of resource allocation in wireless network, and efficiently determine the feasibility of user requirements by returning either a feasible set of solutions or a certificate of infeasibility. It also leads to an effective admission control and admission pricing method.

However, there are two main limitations in the GP-based power control methods. The first limitation is the high SIR assumption used to render some nonconvex QoS problems tractable by GP, and the second limitation is the need for centralized computation. This paper shows how to overcome both limitations in GP-based power control.

The paper is organized as follows. In section II, we describe the system model of a cellular network. In section III, we describe GP-based power control. In section IV, we solve the nonconvex problem of power control in low SIR regime through a *series* of GP. In section V, we show how special structures in GP and its Lagrange dual problem lead to distributed algorithms, which can also be used to solve any standard GP problem.

II. System Model

Due to space limit, we focus only on cellular networks. The results are readily extendable to ad hoc networks, as in [2], [3]. We consider a single-cell cellular network with one base station and n mobile users. The setup has n *logical* links (equivalently, users or transceiver pairs). Transmit powers for each user are denoted by $P_1, \dots, P_n$. Under Rayleigh fading, the power received from transmitter j at receiver i is given by $G_{ij}F_{ij}P_j$ where $G_{ij} \geq 0$ represents the path gain and is often modeled as proportional to $d_{ij}^{-\gamma}$ where d_{ij} is distance and γ is the power fall-off factor. We also let G_{ij} encompass antenna gain and coding gain. The numbers F_{ij} model Rayleigh fading and are assumed to be independent with unit mean. The distribution of the received power from transmitter j at receiver i is exponential with mean value $\mathbf{E}\left[G_{ij}F_{ij}P_j\right] = G_{ij}P_j$. The SIR for the receiver on logical link i is:

$$\mathsf{SIR}_i = \frac{P_i G_{ii} F_{ii}}{\sum_{j \neq i}^{N} P_j G_{ij} F_{ij} + n_i} \quad (1)$$

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where n_i is the noise for receiver i.

The constellation size M used by a link can be closely approximated for MQAM modulations as follows: $M = 1 + \frac{-\phi_1}{\ln(\phi_2 \text{BER})}\mathsf{SIR}$, where BER is the bit error rate and ϕ_1, ϕ_2 are constants that depend on the modulation type. Defining $K = \frac{-\phi_1}{\ln(\phi_2 \text{BER})}$ leads to an expression of the data rate R_i on the ith link as a function of the SIR:

$$R_i = \frac{1}{T}\log_2(1 + K\mathsf{SIR}_i), \tag{2}$$

which can be approximated as $R_i = \frac{1}{T}\log_2(K\mathsf{SIR}_i)$ when $K\mathsf{SIR}$ is much larger than 1. High SIR assumption enables the GP method to turn an apparently nonconvex problem into a convex, thus computationally tractable, problem. This approximation is reasonable either when the signal level is much higher than the interference level or when the spreading gain is large. For notational simplicity in the rest of this paper, we redefine G_{ii} as K times the original G_{ii}, thus absorbing constant K into the definition of SIR.

III. Geometric Programming-based power control

It is shown in [2], [3] that in high SIR regime, many nonlinear QoS constraints can be solved using GP-based power control. There are two equivalent forms of GP: standard form and convex form. We first define a monomial as a function $f : \mathbf{R}^n_{++} \to \mathbf{R}$:

$$f(\mathbf{x}) = d x_1^{a^{(1)}} x_2^{a^{(2)}} \dots x_n^{a^{(n)}}$$

where the multiplicative constant $d \geq 0$ and the exponential constants $a^{(j)} \in \mathbf{R}, j = 1, 2, \dots, n$. A sum of monomials, indexed by k below, is called a posynomial:

$$f(\mathbf{x}) = \sum_{k=1}^{K} d_k x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_n^{a_k^{(n)}}.$$

where $d_k \geq 0$, $k = 1, 2, \dots, K$, and $a_k^{(j)} \in \mathbf{R}$, $j = 1, 2, \dots, n, k = 1, 2, \dots, K$.

Minimizing a posynomial subject to posynomial upper bound inequality constraints and monomial equality constraints is called GP in standard form:

$$\begin{array}{ll} \text{minimize} & f_0(\mathbf{x}) \\ \text{subject to} & f_i(\mathbf{x}) \leq 1, \ \ i = 1, 2, \dots, m, \\ & h_l(\mathbf{x}) = 1, \ \ l = 1, 2, \dots, M \end{array} \tag{3}$$

where f_i, $i = 0, 1, \dots, m$, are posynomials: $f_i(\mathbf{x}) = \sum_{k=1}^{K_i} d_{ik} x_1^{a_{ik}^{(1)}} x_2^{a_{ik}^{(2)}} \dots x_n^{a_{ik}^{(n)}}$, and h_l, $l = 1, 2, \dots, M$ are monomials: $h_l(\mathbf{x}) = d_l x_1^{a_l^{(1)}} x_2^{a_l^{(2)}} \dots x_n^{a_l^{(n)}}$.

GP in standard form is not a convex optimization problem, because posynomials are not convex functions. However, with a logarithmic change of the variables and multiplicative constants: $y_i = \log x_i, b_{ik} = \log d_{ik}, b_l = \log d_l$, we can turn it into the following equivalent problem in $\mathbf{y}$:

$$\begin{array}{ll} \text{minimize} & p_0(\mathbf{y}) = \log \sum_{k=1}^{K_0} \exp(\mathbf{a}_{0k}^T \mathbf{y} + b_{0k}) \\ \text{subject to} & p_i(\mathbf{y}) = \log \sum_{k=1}^{K_i} \exp(\mathbf{a}_{ik}^T \mathbf{y} + b_{ik}) \leq 0, \ \ \forall i, \\ & q_l(\mathbf{y}) = \mathbf{a}_l^T \mathbf{y} + b_l = 0, \ \ l = 1, 2, \dots, M. \end{array} \tag{4}$$

This is referred to as GP in convex form, which is a convex optimization problem since it can be verified that the log-sum-exp function is convex, and the equality constraint functions are affine.

Figure 1 summarizes the approach of GP-based power control for general SIR regime. In high SIR regime, it is known that we need to solve only *one* GP. The first contribution of this paper (in the next section) shows that, in medium to low SIR regime, we can solve truly nonconvex power control problems that cannot be turned into convex formulation through a *series* of GPs.

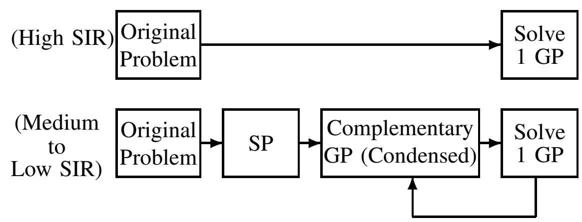

Fig. 1. Geometric Programming-based power control for general SIR regime. The High SIR regime was studied in [2], [3] and the Medium to Low SIR regime is tackled in Section IV of this paper.

IV. Transmit power allocation in low SIR regime

A major limitation in GP-based power control in [2], [3] is the high SIR assumption. When SIR is not much larger than 0dB, the approximation of $\log(1 + \mathsf{SIR})$ as $\log \mathsf{SIR}$ does not hold. Unlike SIR, which is an inverted posynomial, $1 + \mathsf{SIR}$ is not an inverted posynomial. Instead, $\frac{1}{1+\mathsf{SIR}}$ is a ratio between two posynomials. To overcome this issue, GP can be extended to Signomial Programming (SP): minimizing a signomial subject to upper bound inequality constraints on signomials, where a signomial is a sum of monomials, possibly with *negative* multiplicative coefficients:

$$s(\mathbf{x}) = \sum_{i=1}^{N} c_i g_i(\mathbf{x})$$

where $\mathbf{c} \in \mathbf{R}^N$ and $g_i(\mathbf{x})$ are monomials. SP is more general than GP since it allows the use of signomials in lieu of posynomials.

A. Condensation methods

In contrast to GP, an SP is much more difficult to be solved for an global optimum since it cannot be rewritten as a convex optimization problem. We will first convert an SP into a Complementary GP, which allows upper bound constraints on the ratio between two posynomials, and then apply a monomial approximation iteratively to obtain a series of GPs. This is called the condensation method (see [2] and references therein).

The first step of conversion from an SP into a Complementary GP is trivial. An inequality in SP of the following form

$$f_{i1}(\mathbf{x}) - f_{i2}(\mathbf{x}) \leq 1,$$

where f_{i1}, f_{i2} are posynomials, is clearly equivalent to

$$\frac{f_{i1}(\mathbf{x})}{1 + f_{i2}(\mathbf{x})} \leq 1.$$

Now we have two choices to make a monomial approximation: (1) the *single condensation* method, based on approximating the denominator $1 + f_{i2}(\mathbf{x})$ with a monomial but leaving the numerator $f_{i1}(\mathbf{x})$ as a posynomial (this results in a GP approximation of an SP), and (2) the *double condensation* method, based on making a monomial approximation for both the denominator $1 + f_{i2}(\mathbf{x})$ and numerator $f_{i1}(\mathbf{x})$ (this results in a linear program approximation of an SP).

However, one approximation is not enough and a series of approximations needs to be carried out. Iterations, indexed by k, can then be carried out: start with a given feasible $\mathbf{x}^k$ and perform the monomial approximation (as described with details in the next subsection), then the approximated convex problem can be solved, obtaining $\mathbf{x}^{k+1}$ that becomes the starting point for the next iteration.

There are many ways to make a monomial approximation of a posynomial. One possibility [2] is based on the following simple inequality: arithmetic mean is greater than or equal to geometric mean, *i.e.*,

$$\sum_i \alpha_i v_i \geq \prod_i v_i^{\alpha_i},$$

where $\mathbf{v} \succ 0$ and $\boldsymbol{\alpha} \succeq 0,\ \mathbf{1}^T\boldsymbol{\alpha} = 1$. Letting $u_i = \alpha_i v_i$, we can write this basic inequality as

$$\sum_i u_i \geq \prod_i \left(\frac{u_i}{\alpha_i}\right)^{\alpha_i}.$$

Let $\{u_i(\mathbf{x})\}$ be the monomial terms in a posynomial $f(\mathbf{x}) = \sum_i u_i(\mathbf{x})$. A lower bound inequality on posynomial $f(\mathbf{x})$ can now be approximated by an upper bound inequality on the following monomial:

$$\prod_i \left(\frac{u_i(\mathbf{x})}{\alpha_i}\right)^{-\alpha_i}. \tag{5}$$

This approximation is in the conservative direction because the original constraint is now tightened. There are many choices of $\boldsymbol{\alpha}$. One possibility is to let

$$\alpha_i = u_i(\mathbf{x})/f(\mathbf{x}),\ \ \forall i,$$

which obviously satisfies the condition that $\boldsymbol{\alpha} \succ 0$ and $\mathbf{1}^T\boldsymbol{\alpha} = 1$. Given an $\boldsymbol{\alpha}$ for each lower bound posynomial inequality, a standard form GP can be obtained based on the above geometric mean approximation.

B. Applications to power control

GP-based power control problems in medium to small SIR regimes become signomial programs, which can be solved by single or double condensation method. We discuss an application of the single condensation method first. Consider a representative problem formulation: maximizing total system throughput in a cellular wireless network subject to user rate and outage probability constraints:

$$\begin{array}{ll} \text{maximize} & R_{system}(\mathbf{P}) \\ \text{subject to} & R_i(\mathbf{P}) \geq R_{i,min}, \forall i, \\ & P_{o,i}(\mathbf{P}) \leq P_{o,i,max}, \forall i, \\ & P_i \leq P_{i,max}, \forall i, \end{array} \tag{6}$$

which is explicitly written out as:

$$\begin{array}{ll} \text{minimize} & \prod_{i=1}^{N} \frac{1}{1+\mathsf{SIR}_i} \\ \text{subject to} & (2^{TR_{i,min}} - 1)\frac{1}{\mathsf{SIR}_i} \leq 1, \forall i, \\ & (\mathsf{SIR}_{th})^{N-1}(1 - P_{o,i,max}) \prod_{i \neq k}^{N} \frac{G_{ik}P_k}{G_{ii}P_i} \leq 1, \forall i, \\ & P_i(P_{i,max})^{-1} \leq 1, \forall i, \end{array} \tag{7}$$

where $R_{i,min}$ is the minimum required data rate for each user, SIR_{th} denotes the threshold for SIR outage, $P_{o,i,max}$ is the maximum outage probability, and $P_{i,max}$ is the maximum power constraint. The variables are $\mathbf{P}$. The outage probability can be interpreted as the fraction of time the i-th logical link expreiences an outage due to fading, and a derivation of the above outage probability constraint can be found in [4].

All the constraints are posynomials. However, the objective is not a posynomial, but a ratio between two posynomials. This power control problem is a Complementary GP, and can be solved by condensation method by solving a series of GPs. Specifically, we have the following single-condensation algorithm:

STEP 0: Choose an initial feasible $\mathbf{P}$.
STEP 1: Evaluate the denominator posynomial of the (7) objective function with the given $\mathbf{P}$.
STEP 2: Compute for each term i in this posynomial,

$$\alpha_i = \frac{\text{value of } i\text{-th term in posynomial}}{\text{value of posynomial}}.$$

STEP 3: Condense the denominator posynomial of the (7) objective function into a monomial using (5) with weights α_i.
STEP 4: Solve the resulting GP using an interior point method.
STEP 5: Go to STEP 1 using $\mathbf{P}$ of STEP 4.
STEP 6: Terminate the k-th loop if $\parallel \mathbf{P}^{(k)} - \mathbf{P}^{(k-1)} \parallel \leq \epsilon$ where ϵ is the error tolerance for exit condition.

As condensing the objective in the above problem gives us an underestimate of the objective value, each GP in the condensation iteration tries to improve the accuracy of the approximation to a particular minimum in the original feasible region.

Example 1. We consider a cellular wireless network with 3 users. Let $T = 10^{-6}$s, $G_{ii} = 1.5$, and generate $G_{ij}, i \neq j$, as independent random variables uniformly distributed between 0 and 0.3. Threshold SIR is $\mathsf{SIR}_{th} = -10$dB, and minimal data rate requirements are 100 kbps, 600 kbps and 1000 kbps for logical links 1, 2 and 3 respectively. Maximal outage probabilities are 0.01 for all links, and maximal transmit powers are 3mW, 4mW and 5mW for link 1, 2 and 3 respectively.

For each instance of SP in (7), we pick a random initial feasible power vector $\mathbf{P}$ uniformly between 0 and $\mathbf{P}_{max}$. Fig. 2 compares the maximized total network throughput achieved over five hundred sets of experiments with different initial vectors. With (single) condensation method, SP converges to different optima over the entire set of experiments, achieving (or coming very close to) the global optimum at 5290 bps 96% of the time and a local optimum at 5060 bps 4% of the time, thus very likely to converge to or very close to the global optimum. The number of GP iterations required by condensation method over the same set of experiments is 15 GPs if an extremely tight exit condition is picked: $\epsilon = 1 \times 10^{-10}$. This average can be substantially reduced by using a larger ϵ, *e.g.*, increasing ϵ to 1×10^{-2} requires on the average 4 GPs.

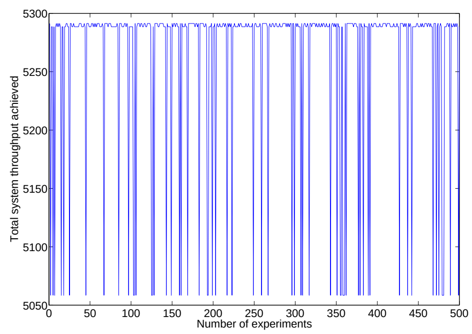

Fig. 2. Maximized total system throughput achieved by (single) condensation method for 500 different initial feasible vectors (Example 1).

The optimum of power control produced by condensation method may be a local one. The following heuristics of solving a series of SPs (each in turn solved through a series of GPs) can be further applied to help find the global optimum. After the original SP (7) is solved, a slightly modified SP is formulated and solved:

$$\begin{array}{ll} \text{minimize} & t \\ \text{subject to} & \prod_{i=1}^{N} \frac{1}{1+\mathsf{SIR}_i} \leq t, \\ & t \leq \frac{t_0}{\beta}, \\ & \text{Same set of constraints as problem (7)} \end{array} \tag{8}$$

where β is a constant slightly larger than 1. At each iteration of a modified SP, the previous computed optimum value is set to constant t_0 and the modified problem (8) is solved to yield an objective value that is better than the objective value of the previous SP by at least β. The auxiliary variable t is introduced so as to turn the problem formulation into SP in $(\mathbf{P}, t)$.

Example 2. The above heuristics is applied to the instances of Example 1 where solving SP returns a locally optimal power allocation, and is found to indeed obtain the globally optimal solution by solving 1 or 2 additional SPs (8).

We have discussed a power control problem (7) where only the objective function needs to be condensed. The method is also applicable if some constraint functions are signomials and need to be condensed. For example, consider the case of differentiated services where a user expects to obtain a predicted QoS relatively better than the other users. We may have a proportional delay differentiation model where a user who pays more tariff obtains a delay proportionally lower compared to users who pay less.

Packet traffic entering the base station at the transmitter of logical link i is assumed to be Poisson with parameter λ_i and to have an exponentially distributed length with parameter Γ. Using the model of an $M/M/1$ queue, the total packet arrival rate at queue i is Λ_i. The expected delay $\bar{D}_i$ can be written as

$$\bar{D}_i = \frac{1}{\Gamma R_i(\mathbf{P}) - \Lambda_i}. \tag{9}$$

Then for a particular delay ratio between any user i and j, σ_{ij}, we have $\frac{\overline{D}_i}{\overline{D}_j} = \sigma_{ij}$, which, by (9), is equivalent to

$$\frac{1+\mathsf{SIR}_j}{(1+\mathsf{SIR}_i)^{\sigma_{ij}}} = 2^{(\lambda_j - \sigma_{ij}\lambda_i)T/\Gamma}. \tag{10}$$

The denominator on the left hand side is a ratio between posynomials raised to a positive power. Double condensation method can be used to solve the proportional delay differentiation problem because the function on the left hand side can be condensed to a monomial, and a monomial equality constraint is allowed in GP.

Example 3. We consider the wireless cellular network in Example 1 with an additional constraint $\frac{D_1}{D_3} = 1$. The arrival rates of each user at base station is measured and input as network parameters into (10). Fig. 3 and 4 show the convergence towards satisfying all the QoS constraints including the DiffServ constraint. As shown on the figures, the convergence is fast, with the power allocations very close to the optimal power allocation by the 8-th GP iteration. Table I summarizes the optimizers for Examples 1, 2, and 3 using $\epsilon = 1 \times 10^{-10}$.

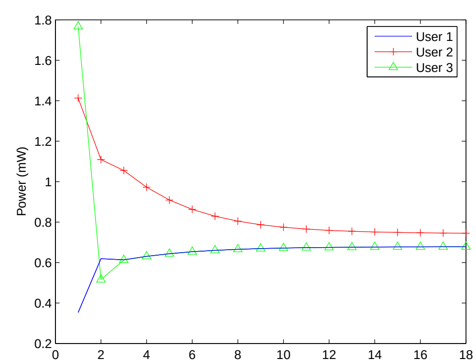

Fig. 3. Numerical example: Convergence of power variables (Example 3).

V. DISTRIBUTED ALGORITHMS FOR GP-BASED POWER CONTROL

The second limitation for GP based power control is the need for centralized computation if a GP is solved by interior point methods. The GP formulations of power control problems can be solved by a new general method of distributed algorithm for GP presented here. The basic idea is that each

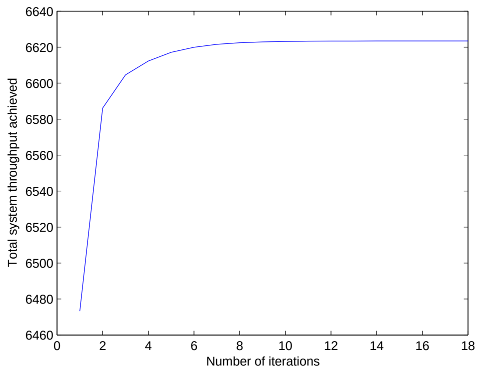

Fig. 4. Numerical example: Convergence of total system throughput (Example 3).

Method	System throughput	P_1^*	P_2^*	P_3^*
Exhaustive search	6626 kbps	0.65	0.77	0.79
Single condensation	6626 kbps by solving 17 GPs	0.65	0.77	0.78
Exhaustive search	6624 kbps	0.68	0.75	0.68
Added DiffServ constraint	6624 kbps by solving 17 GPs	0.68	0.75	0.68

TABLE I

ROW 2 SHOWS EXAMPLE 1 USING SINGLE CONDENSATION ON SYSTEM THROUGHPUT ONLY, AND ROW 1 SHOWS THE OPTIMAL SOLUTIONS FOUND BY EXHAUSTIVE SEARCH. ROW 4 SHOWS EXAMPLE 3 WITH AN ADDITIONAL DIFFSERV CONSTRAINT $D_1/D_3 = 1$, AND ROW 3 SHOWS THE OPTIMAL SOLUTIONS FOUND BY EXHAUSTIVE SEARCH.

user solves its own local optimization problem and the coupling among users is taken care of by message passing among the users. Specifically, we use a dual decomposition method to decompose a GP into smaller subproblems whose solutions are jointly and iteratively coordinated by the use of dual variables. The key innovation is to introduce auxiliary variables and to add extra equality constraints, thus transferring the coupling in the objective to coupling in the constraints, which can be solved by introducing 'consistency pricing'. We illustrate this idea through an unconstrained minimization problem followed by an application of the technique to wireless network power control.

A. General method

Suppose we have the following unconstrained standard form GP in $\mathbf{x} \succ 0$:

$$\text{minimize} \quad \textstyle\sum_i f_i(x_i, \{x_j\}_{j \in I(i)}) \tag{11}$$

where x_i denotes the local variable of the ith user, $\{x_j\}_{j\in I(i)}$ denote the coupled variables from other users, and f_i is either a monomial or posynomial. Making a change of variable $y_i = \log x_i, \forall i$, we obtain

$$\text{minimize} \quad \textstyle\sum_i f_i(e^{y_i}, \{e^{y_j}\}_{j \in I(i)}).$$

We now rewrite the problem by introducing auxiliary variables y_{ij} for the coupled arguments and additional equality constraints to enforce consistency:

$$\begin{array}{ll} \text{minimize} & \sum_i f_i(e^{y_i}, \{e^{y_{ij}}\}_{j \in I(i)}) \\ \text{subject to} & y_{ij} = y_j, \forall j \in I(i), \forall i. \end{array} \tag{12}$$

Each ith user controls the local variables $(y_i, \{y_{ij}\}_{j \in I(i)})$. Next, the Lagrangian of (12) is formed as

$$L(\{y_i\}, \{y_{ij}\}; \{\gamma_{ij}\}) = \sum_i L_i(y_i, \{y_{ij}\}; \{\gamma_{ij}\})$$

$$\text{where} \quad L_i(y_i, \{y_{ij}\}; \{\gamma_{ij}\}) = f_i(e^{y_i}, \{e^{y_{ij}}\}_{j \in I(i)}) + \left(\sum_{j: i \in I(j)} \gamma_{ji}\right) y_i - \sum_{j \in I(i)} \gamma_{ij} y_{ij}.$$

The minimization of the Lagrangian with respect to the primal variables $(\{y_i\}, \{y_{ij}\})$ can be done simultaneously (in a parallel fashion) by each user. In the more general case where the original problem (11) is constrained, the additional constraints can be included in the minimization at each L_i.

In addition, the following master dual problem has to be solved to obtain the optimal dual variables or consistency prices $\{\gamma_{ij}\}$:

$$\max_{\{\gamma_{ij}\}} \quad g(\{\gamma_{ij}\}) \tag{13}$$

where $g(\{\gamma_{ij}\}) = \sum_i \min_{y_i, \{y_{ij}\}} L_i(y_i, \{y_{ij}\}; \{\gamma_{ij}\})$.

Note that the transformed primal problem (12) is convex, and the duality gap is zero under mild conditions; hence the Lagrange dual problem indeed solves the original standard GP problem. A simple way to solve the maximization in (13) is with the gradient-based update for the consistency prices:

$$\gamma_{ij}(t+1) = \gamma_{ij}(t) + \delta(t)(y_j(t) - y_{ij}(t)). \tag{14}$$

Appropriate choice of the stepsize $\delta(t) > 0$ leads to stability and convergence of the dual algorithm [2].

Summarizing, the ith user has to: i) minimize the function L_i in (13) involving only *local* variables upon receiving the updated dual variables $\{\gamma_{ji}, j : i \in I(j)\}$ (note that $\{\gamma_{ij}, j \in I(i)\}$ are local dual variables), and ii) update the local consistency prices $\{\gamma_{ij}, j \in I(i)\}$ with (14).

B. Applications to power control

As an illustrative example, we maximize the total system throughput in the high SIR regime with constraints local to each user. If we directly applied the general distributed approach described in the last subsection, the resulting algorithm would not be very practical since it would require knowledge by each user of the interfering channels and interfering transmit powers, which would translate into a large amount of message passing. To obtain a practical distributed solution, we can leverage the structures of power control problems at hand, and instead keep a local copy of each of the *effective received powers* $P_{ij}^R = G_{ij} P_j$ and write the problem as follows (using the log change of variable $\tilde{P} = \log P$):

$$\begin{array}{ll} \text{minimize} & \sum_i \log\left(G_{ii}^{-1}\exp(-\tilde{P}_i)\left(\sum_{j\neq i}\exp(\tilde{P}_{ij}^R)+\sigma^2\right)\right) \\ \text{subject to} & \tilde{P}_{ij}^R = \tilde{G}_{ij}+\tilde{P}_j, \\ & \text{Constraints local to each user.} \end{array} \tag{15}$$

The partial Lagrangian is

$$\begin{aligned} L &= \sum_i \log\left(G_{ii}^{-1}\exp(-\tilde{P}_i)\left(\sum_{j\neq i}\exp(\tilde{P}_{ij}^R)+\sigma^2\right)\right) \\ &\quad + \sum_i\sum_{j\neq i}\gamma_{ij}\left(\tilde{P}_{ij}^R - \left(\tilde{G}_{ij}+\tilde{P}_j\right)\right), \end{aligned}$$

from which the dual variable update is found as

$$\begin{aligned} \gamma_{ij}(t+1) &= \gamma_{ij}(t)+\delta(t)\left(\tilde{P}_{ij}^R - \left(\tilde{G}_{ij}+\tilde{P}_j\right)\right) \\ &= \gamma_{ij}(t)+\delta(t)\left(\tilde{P}_{ij}^R - \log G_{ij}P_j\right). \end{aligned}$$

Each user has to minimize the following Lagrangian (with respect to the primal variables) subject to the local constraints:

$$\begin{aligned} L_i = &\log\left(G_{ii}^{-1}\exp(-\tilde{P}_i)\left(\sum_{j\neq i}\exp(\tilde{P}_{ij}^R)+\sigma^2\right)\right) \\ &+\sum_{j\neq i}\gamma_{ij}\tilde{P}_{ij}^R - \left(\sum_{j\neq i}\gamma_{ji}\right)\tilde{P}_i. \end{aligned}$$

where the local variables are $\tilde{P}_i, \left\{\tilde{P}_{ij}^R\right\}_j; \{\gamma_{ij}\}_j$.

Some practical observations are in order:

- For the minimization of the local Lagrangian, each user only needs to know the term $\left(\sum_{j\neq i}\gamma_{ji}\right)$ involving the dual variables from the interfering users, which requires message passing.
- For the dual variable update, each user needs to know the effective received power from each of the interfering users $P_{ij}^R = G_{ij}P_j$ for $j \neq i$, which in practice may be estimated from the received messages, hence no explicit message passing is required for this.

With this approach we have avoided the need to know all the interfering channels G_{ij} and the powers used by the interfering users P_j. However, each user still needs to know the consistency prices from the interfering users via some message passing. This communication complexity can be reduced in practice by ignoring the messages from links that are physically far apart, leading to suboptimal distributed heuristics.

Example 4. We apply the distributed algorithm to solve the power control problem (15) for three logical links with $G_{ij} = 0.2, i \neq j$, $G_{ii} = 1, \forall i$, maximal transmit powers of 6mW, 7mW and 7mW for link 1, 2 and 3 respectively. Figure 5 shows the convergence of the dual objective function which is also the global optimal total throughput of the links. Figure 6 shows the convergence of the two auxiliary variables in link 1 and 3 towards the optimal solutions.

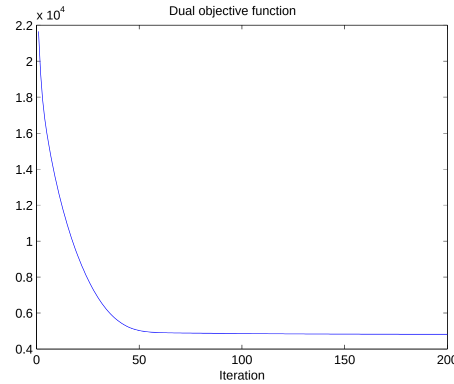

Fig. 5. Convergence of the dual objective function through distributed algorithm (Example 4).

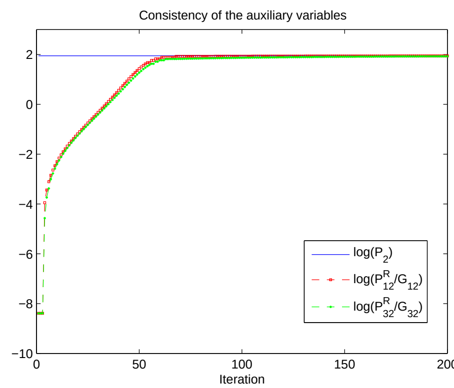

Fig. 6. Convergence of the consistency constraints through distributed algorithm (Example 4).

VI. Conclusion

GP-based power control has recently been developed to compute globally optimal power allocation in high-SIR regime for a variety of nonlinear objectives and constraints. In this paper, we present two contributions that overcome the current bottlenecks of GP power control. First, in the low SIR regime, signomial programming (with centralized computation) is presented to obtain the global optimal solution by solving a series of geometric programs. Two, we present a systematic theory of distributed algorithms for GP power control in high SIR regime using dual decomposition methods. Numerical methods show that the convergence to the global optimal solutions using these methods is fast. These two contributions can be readily combined to distributively obtain a globally optimal solution for general SIR regime, which is also applicable to Digital Subscriber Line (DSL) spectrum management.

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