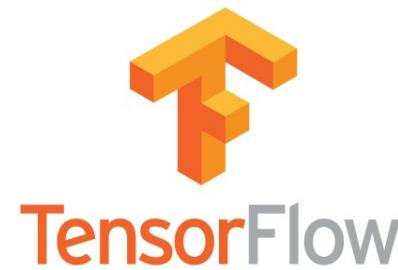
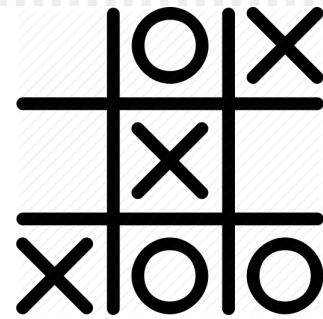
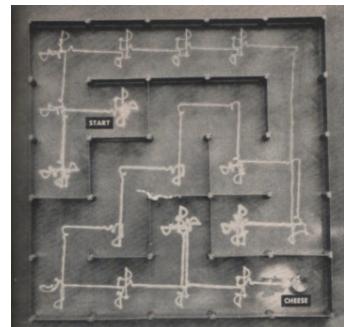
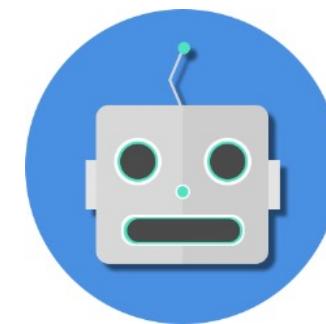
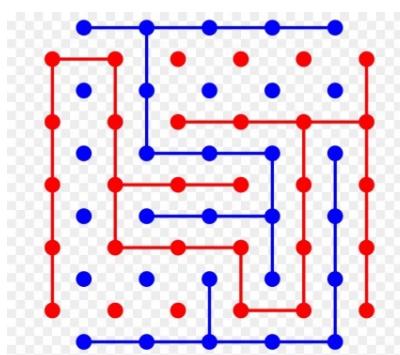
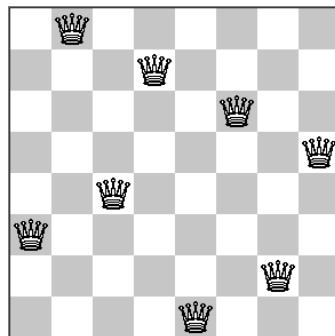


# Artificial Intelligence:

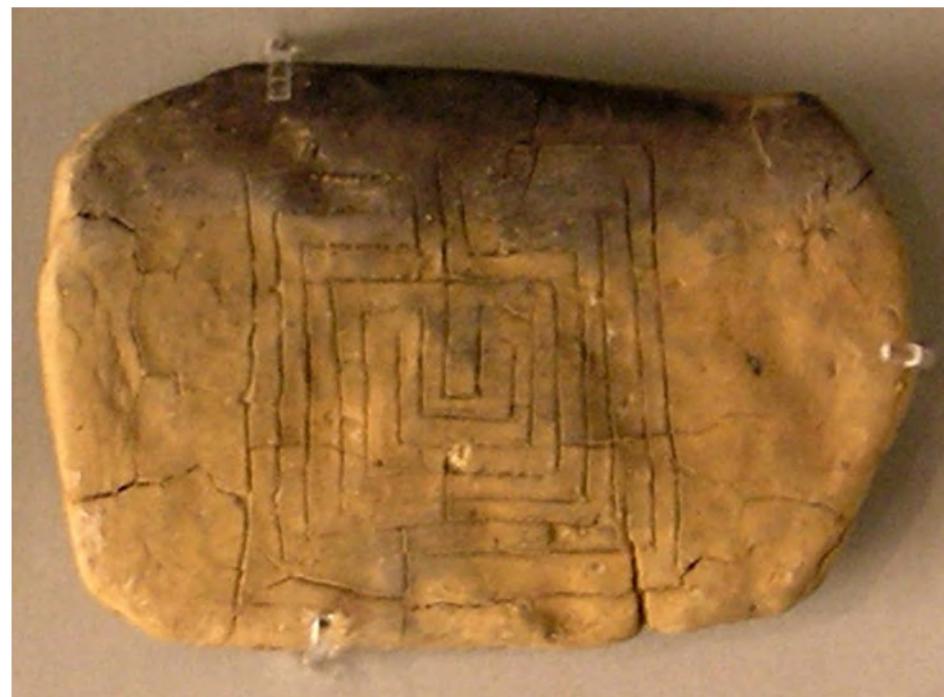
## Past, Present and Future



Chee Wei Tan

# Maze Games

Mazes capture human's imagination since time immemorial



<http://mathworld.wolfram.com/Maze.html>

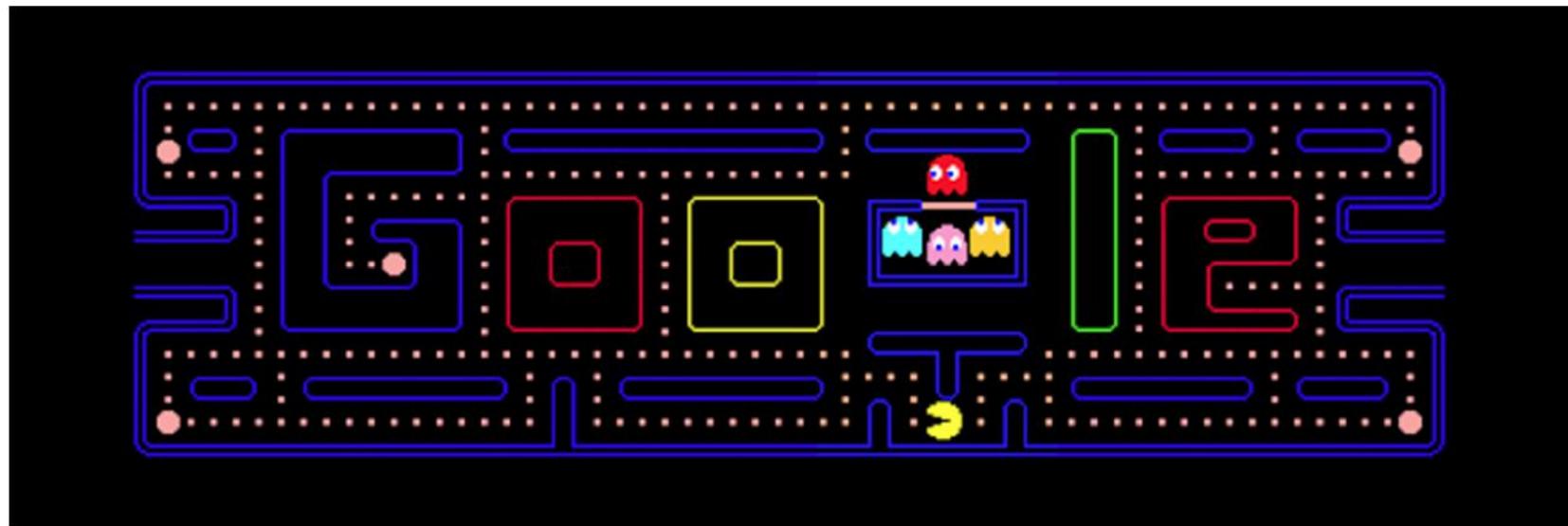
2000 Year Old Labyrinth Uncovered in India Shows Same Pattern as a Greek Maze from 1200 BC

<https://www.ancient-origins.net/news-history-archaeology/2000-year-old-labyrinth-uncovered-india-shows-same-pattern-greek-maze-020474>

# Maze Games

A popular classic game released by Namco in 1980

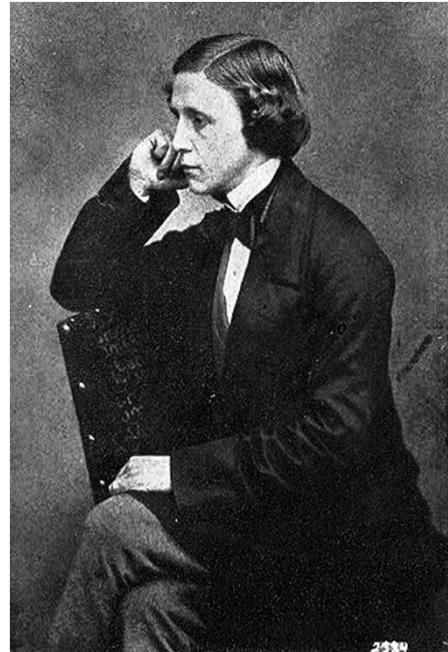
The player navigates Pac-Man through a maze with no dead ends. The objective of the game is to accumulate as many points as possible by eating dots, fruits, and blue ghosts. When all of the dots in a stage are eaten, that stage is completed, and the player will advance to the next.



Type **Pac Man** in Google search bar and play!

# Word Ladder Game

A word ladder puzzle begins with two words, and to solve the puzzle one must find a chain of other words to link the two, in which two adjacent words (that is, words in successive steps) differ by one letter. Lewis Carroll invented the game on Christmas day in 1877



[https://en.wikipedia.org/wiki/Lewis\\_Carroll](https://en.wikipedia.org/wiki/Lewis_Carroll)

# Word Ladder Game

From HEAD to TAIL:

H E A D → H E A L → T E A L → T E L L → T A L L  
→ T A I L

Five moves needed. Can you come up with fewer moves?  
How many possible solutions?

APE to MAN



# Word Ladder Challenge

Driving HORSE into FIELD

In Lewis Carroll's day, the problem remained unsolved. In last few years, this must have been accomplished using some modern words unknown in Victorian times. Can you solve it?

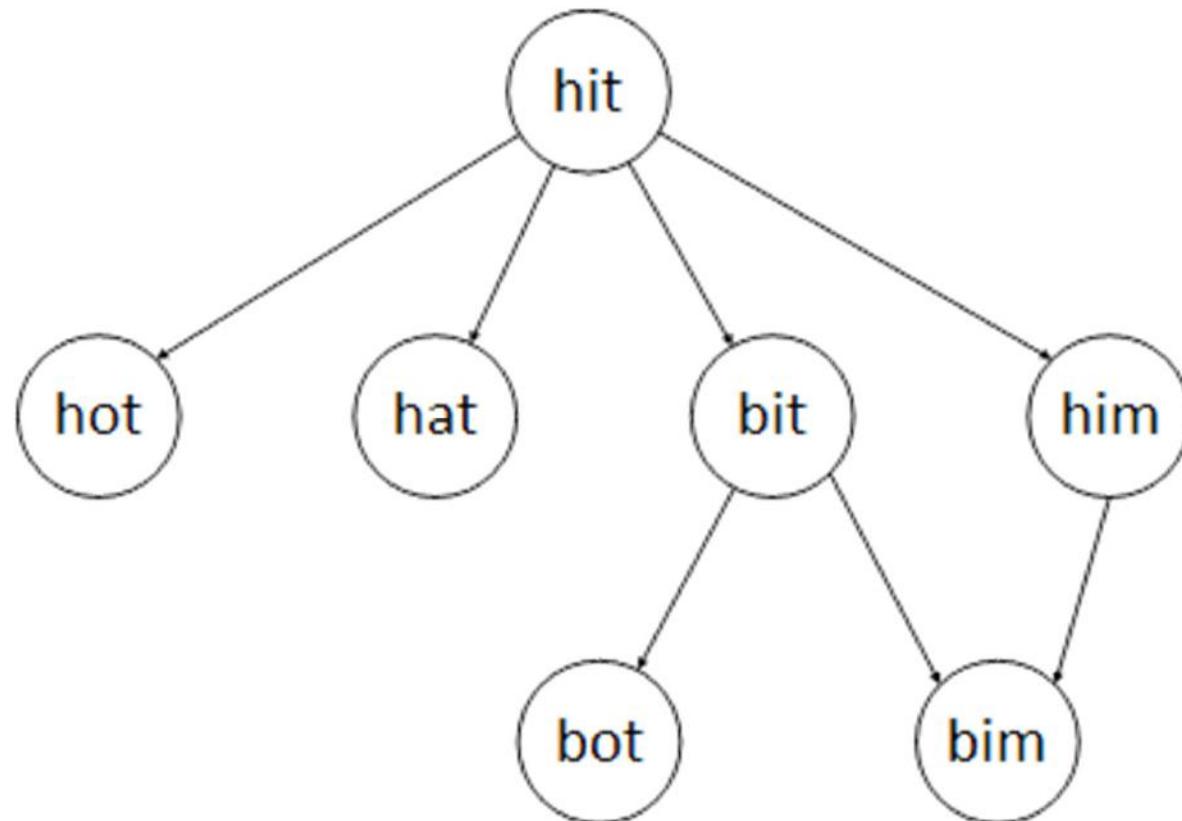


# Graph of Word Ladder Game

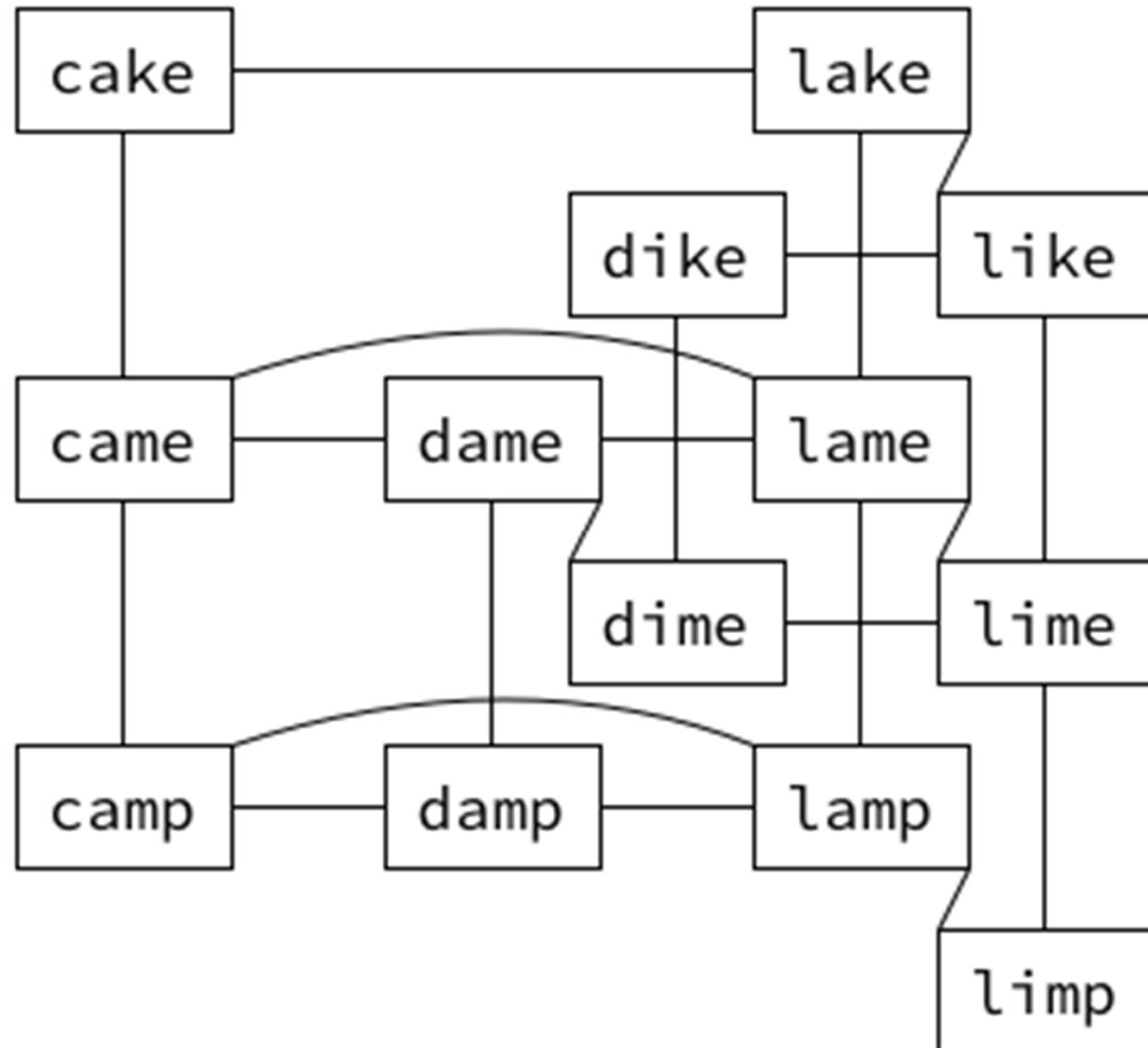
A graph is an important mathematical object:

Graph vertex or node: model a state of a game

Graph edge: model transition or relationship

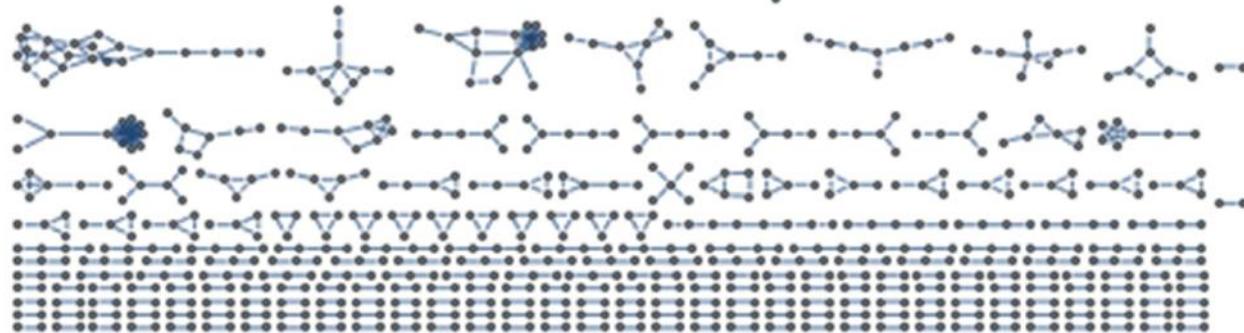
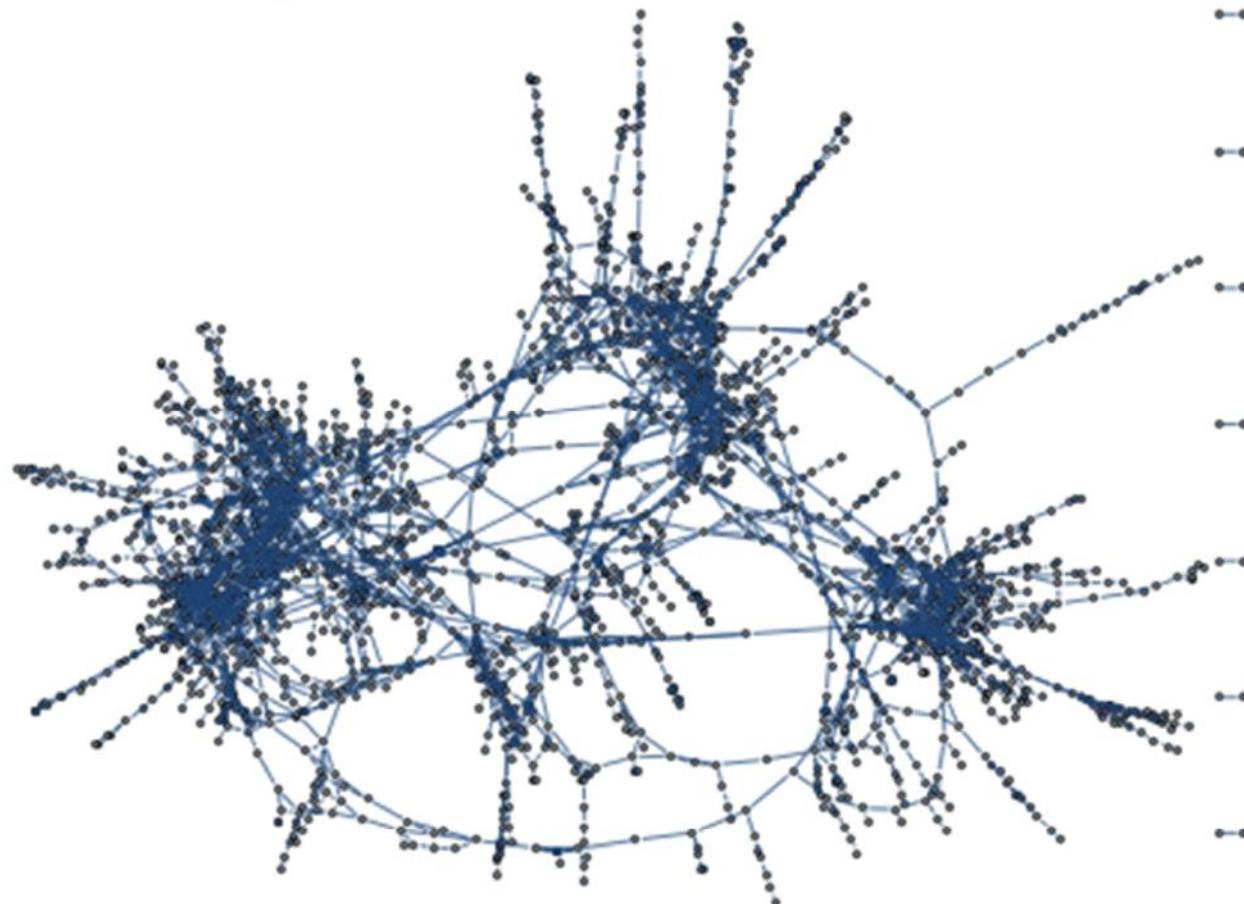


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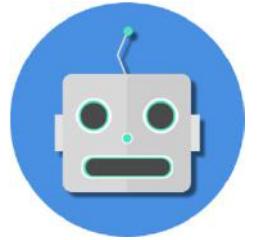
# Graph of Word Ladder Game

Out[4]=



<https://blog.wolfram.com/2012/01/11/the-longest-word-ladder-puzzle-ever>

Donald Knuth investigated Word Ladder game in 60's and found 817 five-letter words that are *aloof* (completely unconnected). Can you find any of these?



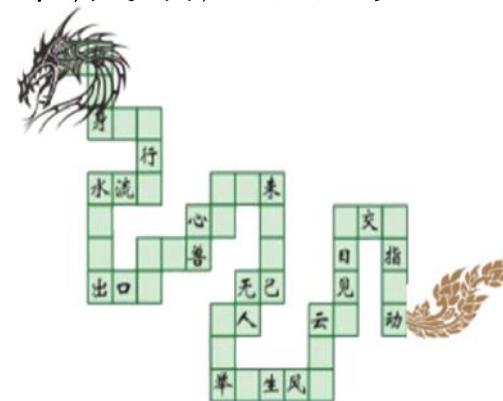
# Word Ladder Game

Play Word Ladder Game in Nemo Bot!

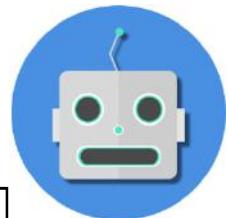
Chinese Variant  
成語接龍 Rule

1. You pick an idiom whose first letter matches the last letter of the idiom that comes directly before it

(你所接成语前面的一个字,必须和上一个成语后面的一个字相同或者音相同才可以)



# Word Ladder Game



由 "不" 連接到  
"断"

Nemo Bot  
智能Messenger在線中

成语使用方式说明。  
下一个成语的字头接上一个成语的字尾，例如：快心消遣—直到现场。  
在帮助的步骤内输入连接词吧。  
Start the game now?

Sure!  
No, thanks

開始新的中 - 选山  
准备好了吗？  
如果失败的话可以连络提示的：  
我要看提示  
我要看答案  
由 "不" 連接到以新替是  
禁记吧  
太棒了我不坏

由 "不" 連接到 "断"  
Step 0:  
Step 1:  
Step 2:  
Step 3:  
Step 4: 断

不善之教	不由自主
不破不立	不改其乐
不舍昼夜	不可言长

不改其乐



不改其乐  
拼音：bù gǎi qí lè  
释义：不改变原有的快乐。指处于困苦的境况仍然很快乐。  
出处：例句《论语雍也》：「一箪食，一瓢饮，在陋巷，人不堪其忧，回也不改其乐。」  
(例子) 你赞赏他的乐还不够，～。(朱自清《论吃饭》)

由 "不" 連接到 "断"  
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乐山乐水	乐透安乐
拼音：yuè shān yuè shuǐ	拼音：yuè tóu ān lè
释义：乐，爱好，喜欢。有人喜爱山，有人喜爱水，比如各人的爱好不同。 出处：例句《论语雍也》：「知者乐山，仁者乐水。」 ～，气节高尚。(宋·程子集外书卷二)	释义：形容安逸或做事千净利落。

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水木清华	水火无情
拼音：shuǐ mù qīng huá	拼音：shuǐ huǒ wú qíng
释义：清华，秀美，雅致。有人喜爱山，有人喜爱水，比如各人的爱好不同。 出处：例句《论语雍也》：「知者乐山，仁者乐水。」 ～，气节高尚。(宋·程子集外书卷二)	释义：形容凶恶或做事千净利落。

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水秀山明	明白了解
拼音：shuǐ xiù shān míng	拼音：míng bai liǎo dé
释义：山光明媚，水色秀丽，形容风景优美。 出处：例句宋·黄庭坚《惠山源墨妙院碑》：「量泉数脉流，是湖南，山明水秀。」	释义：形容对事或对事理解透彻。

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释义：指面对紧要的时侯不能犹豫，果断地作出决断。 出处：例句《史记项羽本纪》：「当断不断，反受其乱。」 投失之太阴，望又失之太柔，～，反受其乱。(苏轼·许广文·民风俗说) 第八十一回	释义：指面对紧要的时侯不能犹豫，果断地作出决断。

由 "不" 連接到 "断"  
step 0: 不改其乐  
step 1: 乐山乐水  
step 2: 水秀山明  
step 3: 明白了解  
step 4:  
step 5: 断

当机立断	当机立断
拼音：dāng jī lì duàn	拼音：dāng jī lì duàn
释义：指面对紧要的时侯不能犹豫，果断地作出决断。 出处：例句《史记项羽本纪》：「当断不断，反受其乱。」 投失之太阴，望又失之太柔，～，反受其乱。(苏轼·许广文·民风俗说) 第八十一回	释义：指面对紧要的时侯不能犹豫，果断地作出决断。

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当机立断	当机立断
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step 3: 明白了解  
step 4:  
step 5: 断

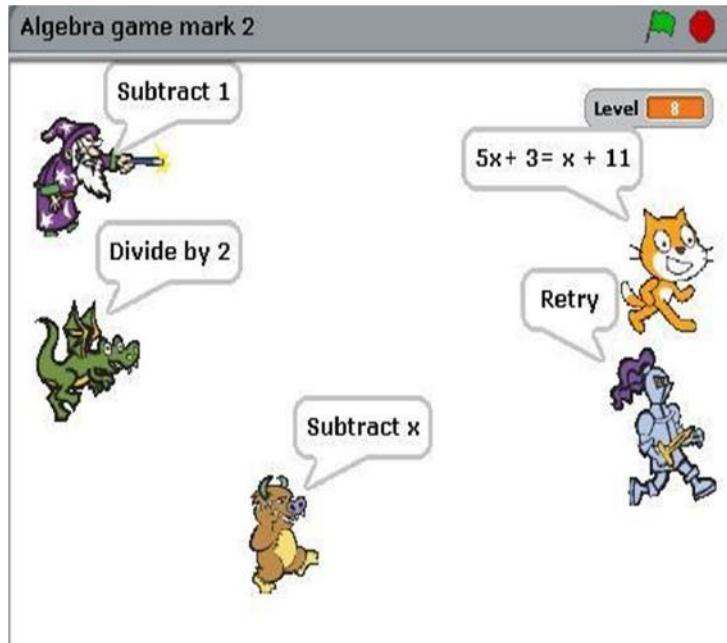
当机立断	当机立断
拼音：dāng jī lì duàn	拼音：dāng jī lì duàn
释义：指面对紧要的时侯不能犹豫，果断地作出决断。 出处：例句《史记项羽本纪》：「当断不断，反受其乱。」 投失之太阴，望又失之太柔，～，反受其乱。(苏轼·许广文·民风俗说) 第八十一回	释义：指面对紧要的时侯不能犹豫，果断地作出决断。

由 "不" 連接到 "断"  
step 0: 不改其乐  
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step 2: 水秀山明  
step 3: 明白了解  
step 4:  
step 5: 断

当机立断	当机立断

<tbl\_r cells="2" ix="1" maxcspan="1"

# Algebra Maze

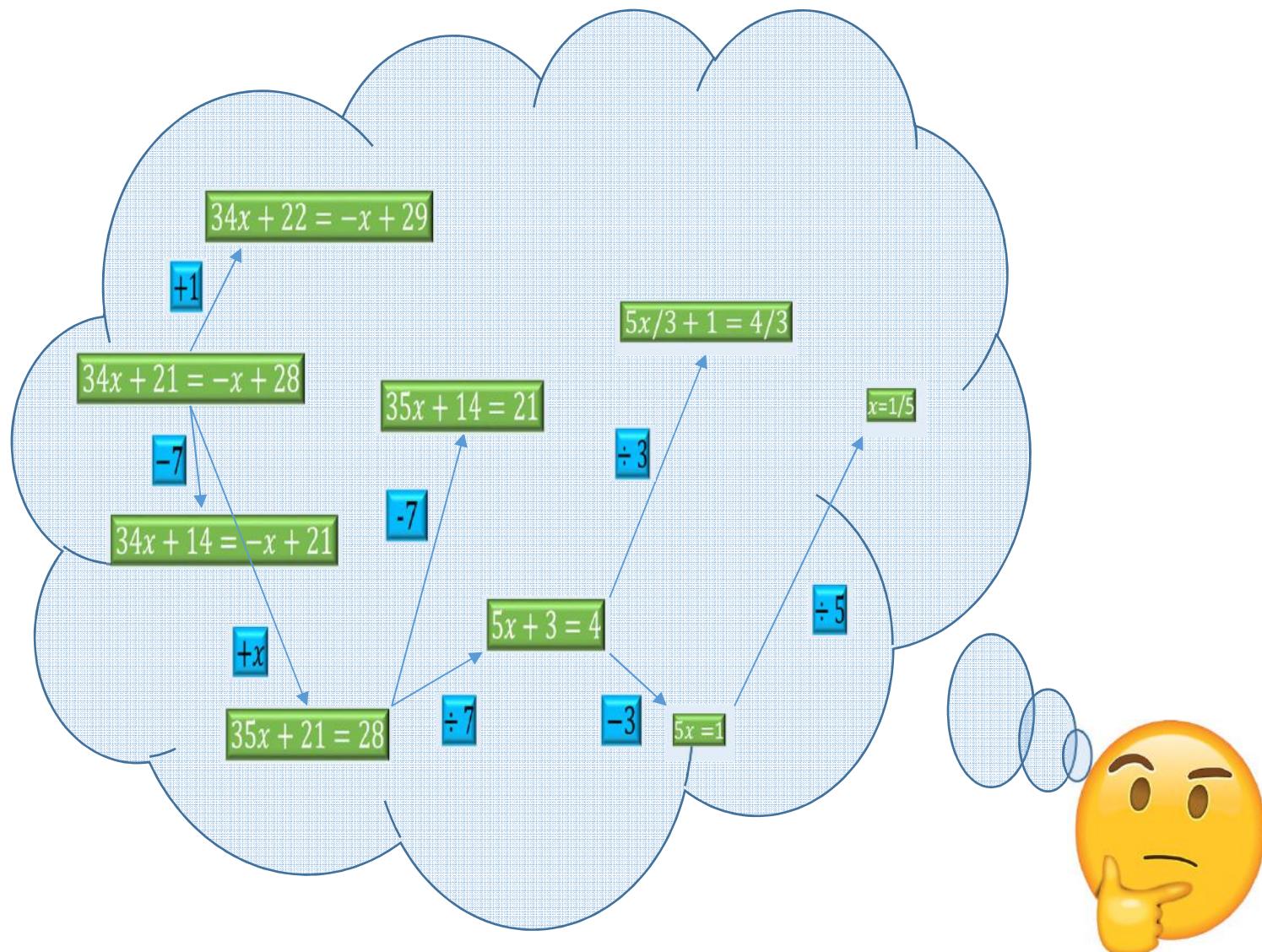


**Terence Tao**

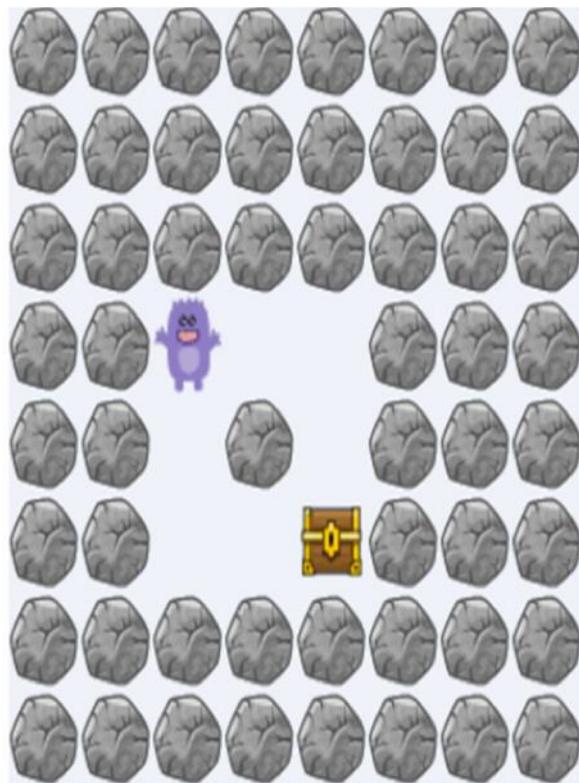
Professor of  
Mathematics at  
UCLA, Field Medalist

- The set of problem-solving skills needed to solve algebra problems is somewhat similar to the set of skills needed to *solve puzzle-type computer games*, in which a certain limited set of moves must be applied in a certain order to achieve a desired result. - **Terence Tao** (2012)
- Tan CW, Yu PD, Lin L, *Teaching Computational Thinking Using Mathematics Gamification in Computer Science Game Tournaments*. Springer (2019)  
[https://link.springer.com/chapter/10.1007/978-981-13-6528-7\\_10](https://link.springer.com/chapter/10.1007/978-981-13-6528-7_10)

# Algebra Maze



# Algebra Maze



$$3x - 2 = 2x + 15$$



:Rick

Goal: Help Rick to find the treasure.

Once the equation is in the desired form " **$x = \text{some numerical value}$** ", Rick will get the treasure.



# Algebra Maze



: Move rightward two cells, and the equation add 2 on both sides.



: Move leftward one cell, and the equation minus 1 on both sides.



: Move upward one cell, and the equation add 1x on both sides.



: Move downward two cells, and the equation minus 2x on both sides.



$$x-3=0$$



$$4x=3x+18$$



# Algebra Maze

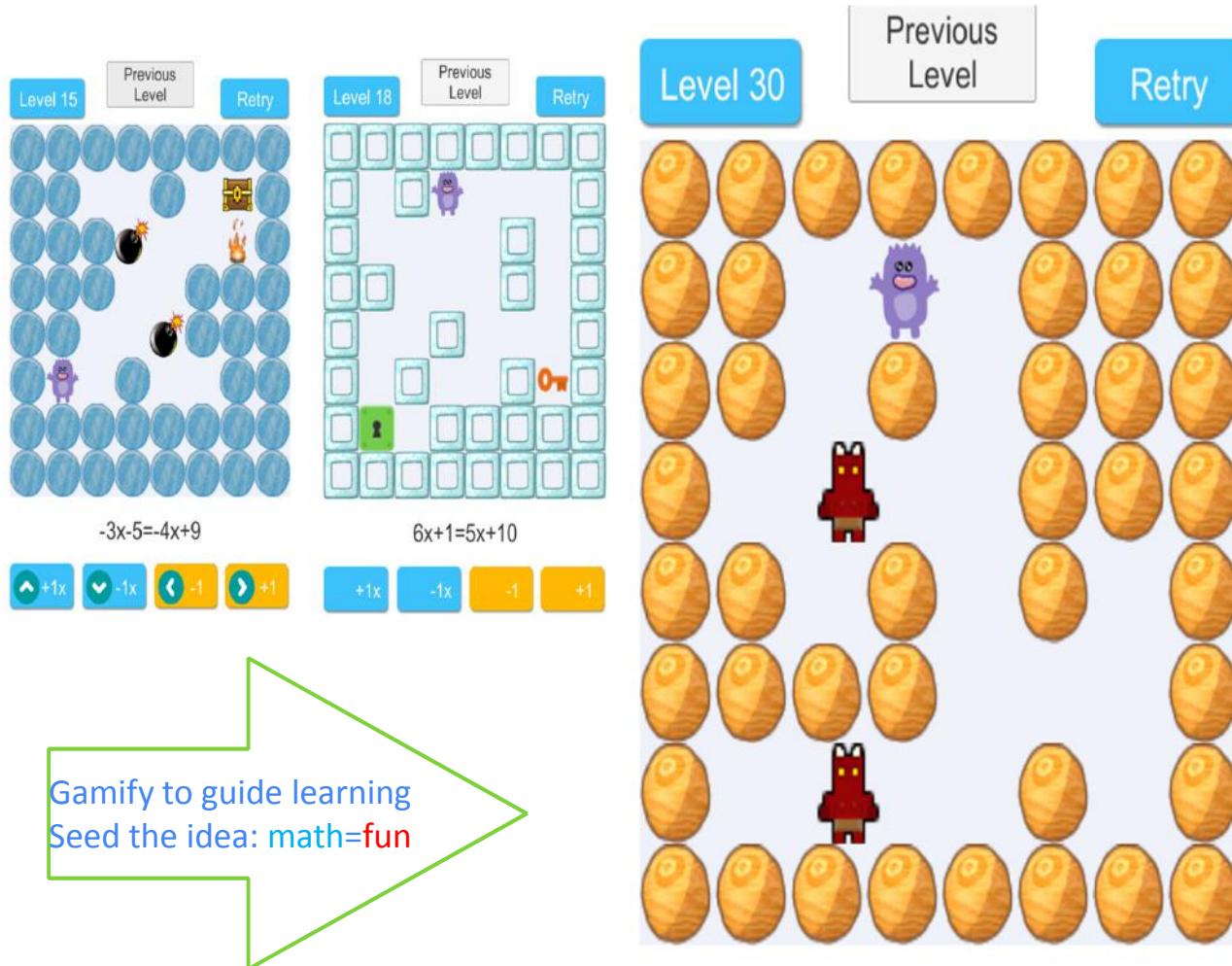


Where is the  
hidden  
treasure  
chest?

$$4x+6=3x+24$$

- +1x
- 3x
- 1
- +3

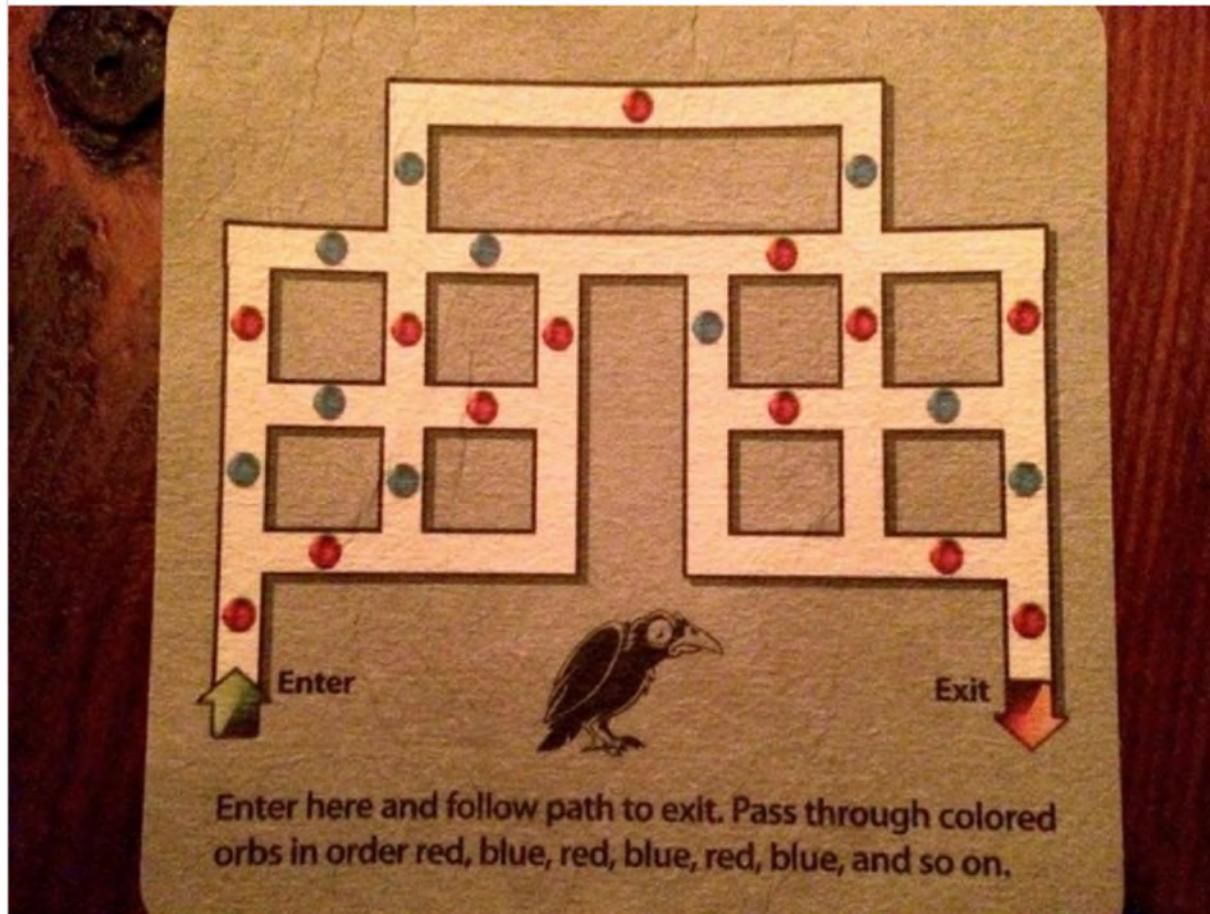
# Algebra Maze



19

# Maze Game in a Pub

Share ↗



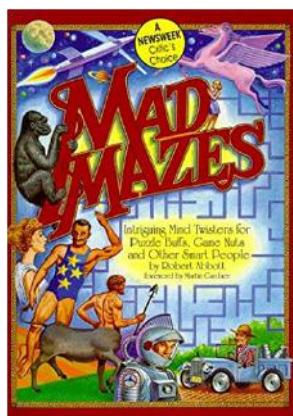
Found this on a coaster at a pub. Is it possible or are they just screwing with the drunks?

• 19,418

 imgur

<https://www.businessinsider.com/bar-puzzle-drunk-people-2015-12>

# Logic Maze

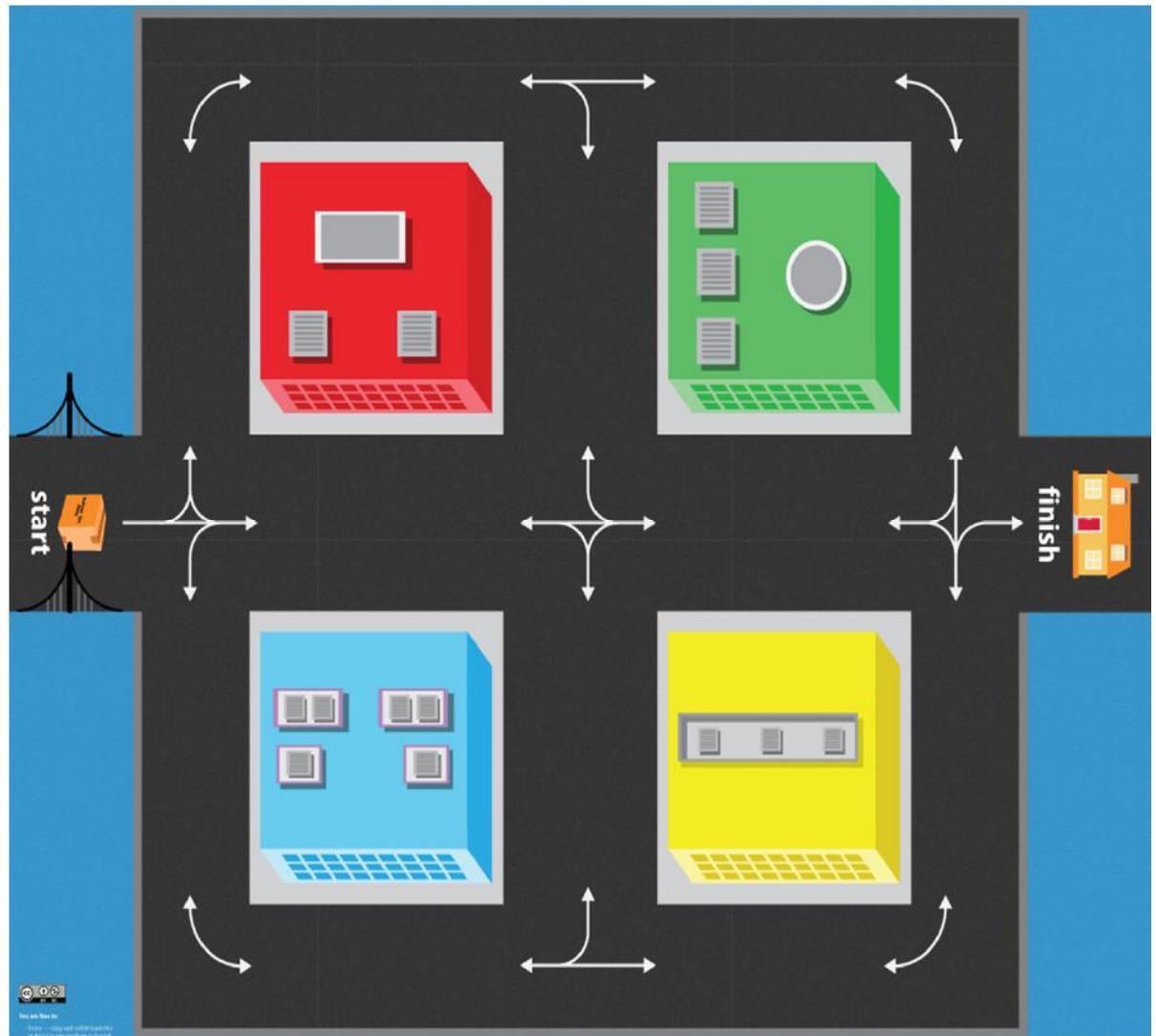


- **Robert Abbott** (1933-2018) was an American game inventor, sometimes referred to by fans as "The Official Grand Old Man of Card Games". He invented **Logic Mazes**, with the first one published in 1962.
- Logic mazes are logical puzzles framed in a maze setting with special rules (sometimes including multiple states of the maze or navigator). A ruleset can be basic (such as "you cannot make left turns") or complex.

[https://en.wikipedia.org/wiki/Robert\\_Abbott\\_\(game\\_designer\)](https://en.wikipedia.org/wiki/Robert_Abbott_(game_designer))

# Logic Maze: Challenge 1

- Travel along the roads from start to finish to deliver a package to Julia.
- At each intersection follow one of the arrows. You can turn in a certain direction only when there is a curved line in that direction, and you can go straight only when there is a straight line.
- **U-turns are not allowed.**



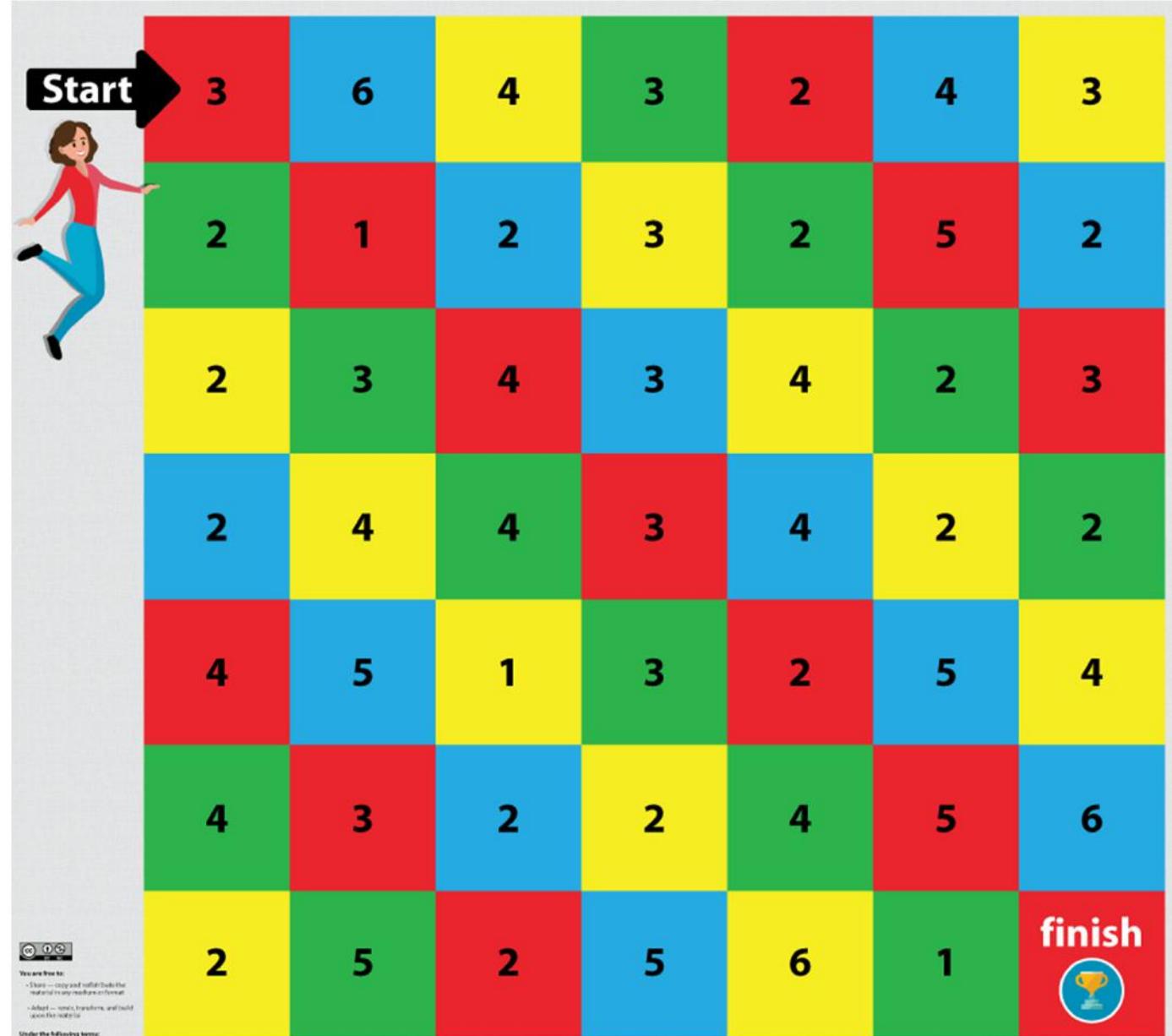
# Logic Maze: Challenge 2



- Travel from start to finish. When you reach a red sign ●, you must turn left or right. You can't continue straight. U-turns aren't permitted

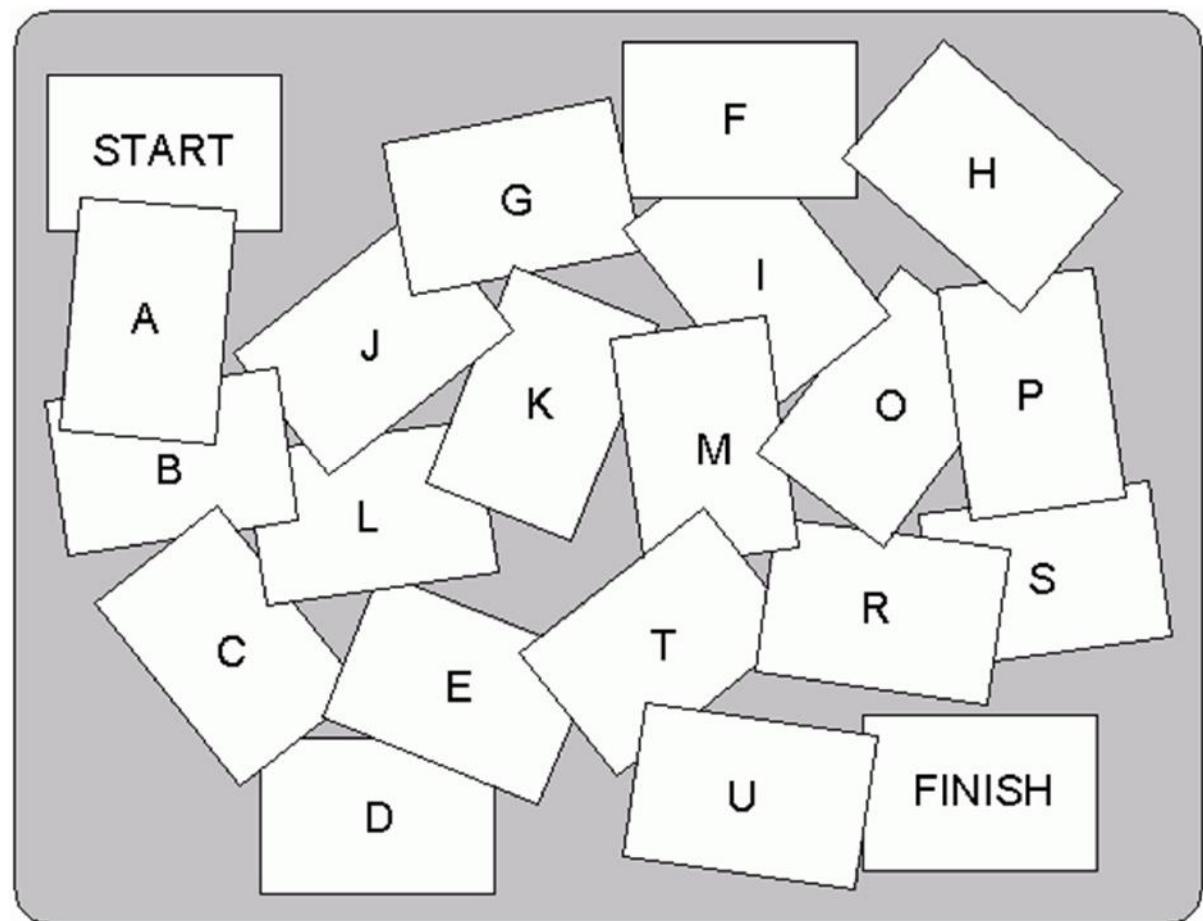
# Logic Maze: Challenge 3

- Help Julia get from Start to finish: begin on the square in the upper left.
- Make a series of jumps that will take you to the square marked finish.
- The number on each square indicates how far you move—horizontally or vertically (your entire move must be horizontal or vertical), not diagonally—when you bounce off the square.



# Logic Maze: Challenge 4

- Let's look at a Up-and-Down Maze
- Suppose you have a maze setting like the below set of cards, and that going from card to card required the magic elvish rope. In addition, suppose that the rope could never go up twice in a row, or down twice in a row.
- **Up and down have to alternate in your navigation.**



# Backward Reasoning

We Start With the Customer and  
We Work Backwards

-- Jeff Bezos on Amazon's success



We learn whatever skills we need to service the customer. We build whatever technology we need to service the customer. The second thing is, we are inventors, so you won't see us focusing on "me too" areas. We like to go down unexplored alleys and see what's at the end. Sometimes they're dead ends. Sometimes they open up into broad avenues and we find something really exciting. And then the third thing is, we're willing to be long-term-oriented, which I think is one of the rarest characteristics. If you look at the corporate world, a genuine focus on the long term is not that common. But a lot of the most important things we've done have taken a long time.

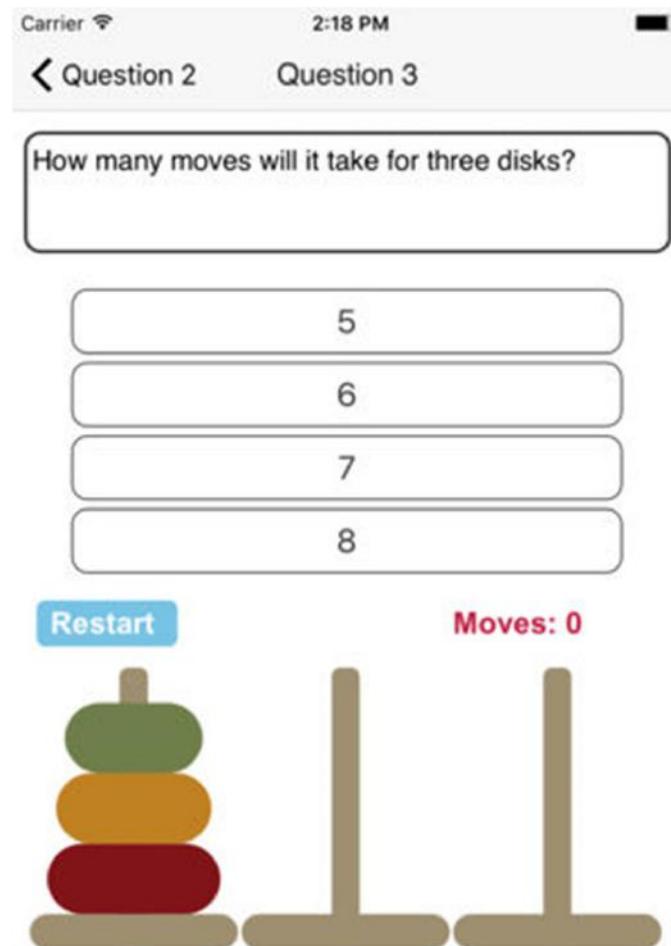
<https://slate.com/news-and-politics/2009/12/jeff-bezos-on-amazon-s-success.html>

# Backward Reasoning

- Imagine you have already solved the problem you are trying to solve. Work backwards from your solution to the starting point of your problem. **Backward reasoning** is also known as **backward induction** in mathematics or **retrograde analysis** in chess.
- Working backwards is suitable for problems, where some information has not been provided at the beginning of the problem. It helps to start with the answer and work methodically backwards to fill in the missing information. You can even uncover new solution through this process.
- This strategy is useful in dealing with problems that require a sequence of decisions to be made (**multi-stage decision process**). The decisions occur one after the other and each stage is affected by what comes next and what was decided before.

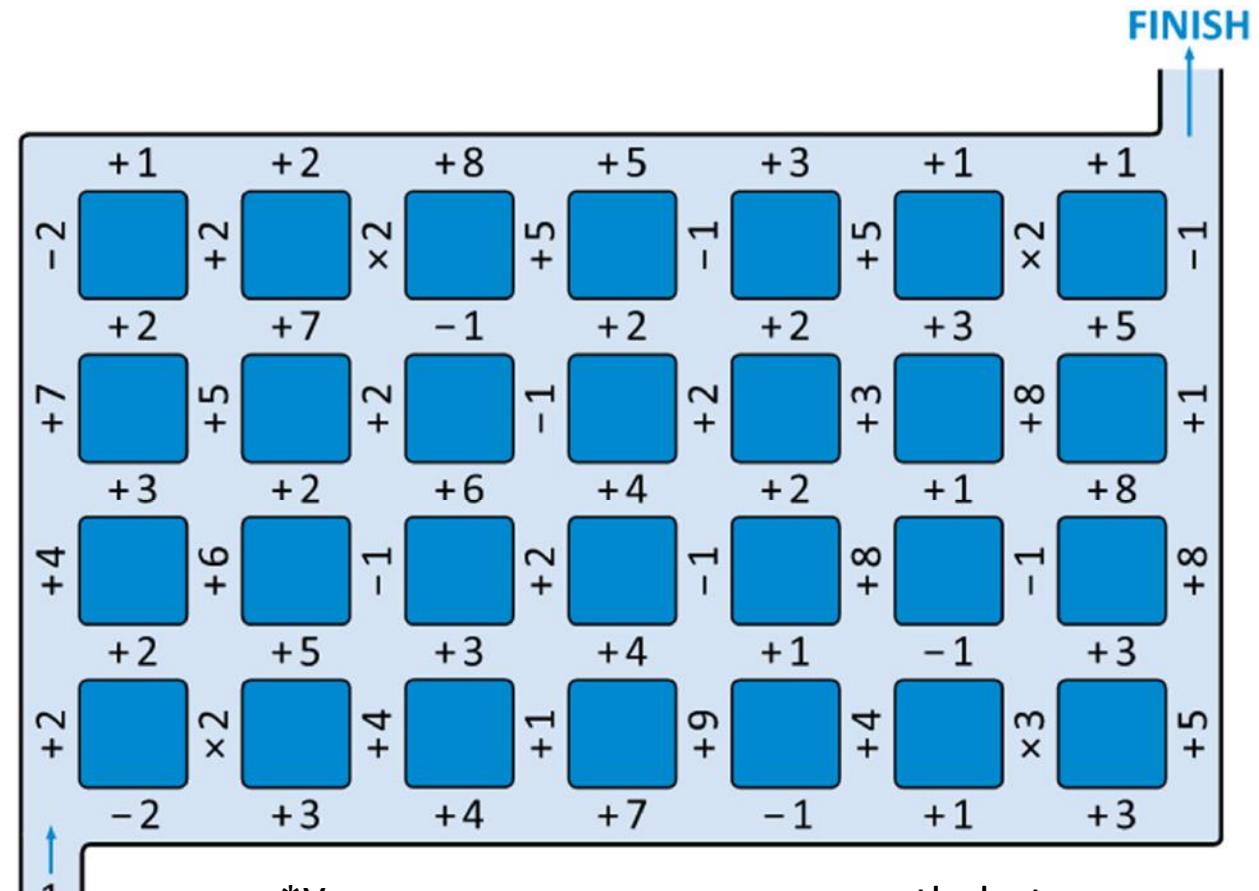
# Backward Reasoning

- Water lilies double in area every 24 hours. At the beginning of the summer, there is one water lily on a lake. It takes 60 days for the lake to become covered with water lilies. On what day is the lake half covered?
- Find the minimal number of steps for 4 disks in the game of Tower of Hanoi.



# Backward Reasoning: Example

Find the route\* that will earn the most points.



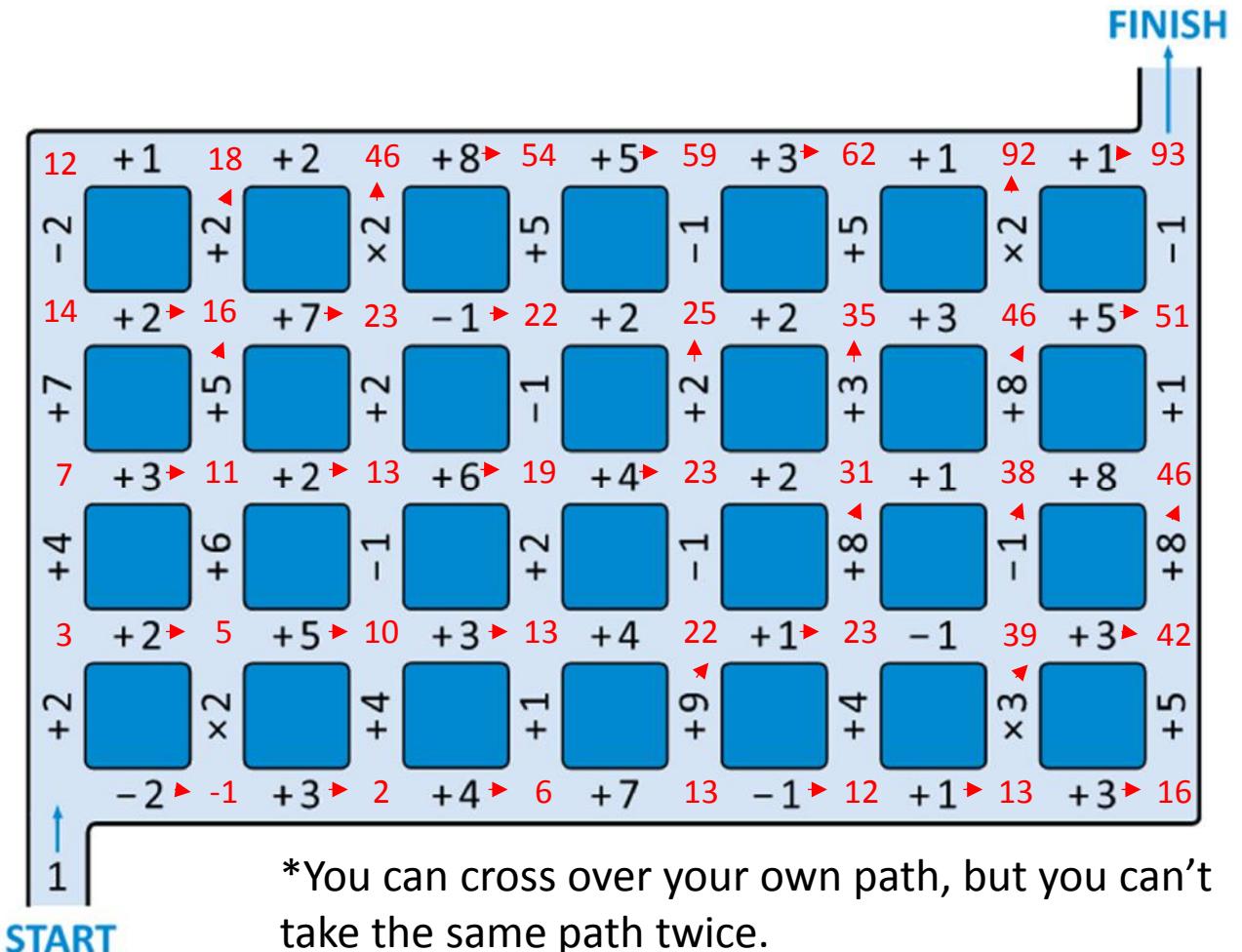
START

\*You can cross over your own path, but you can't take the same path twice.

# Backward Reasoning: Example

Find the route\* that will earn the most points.

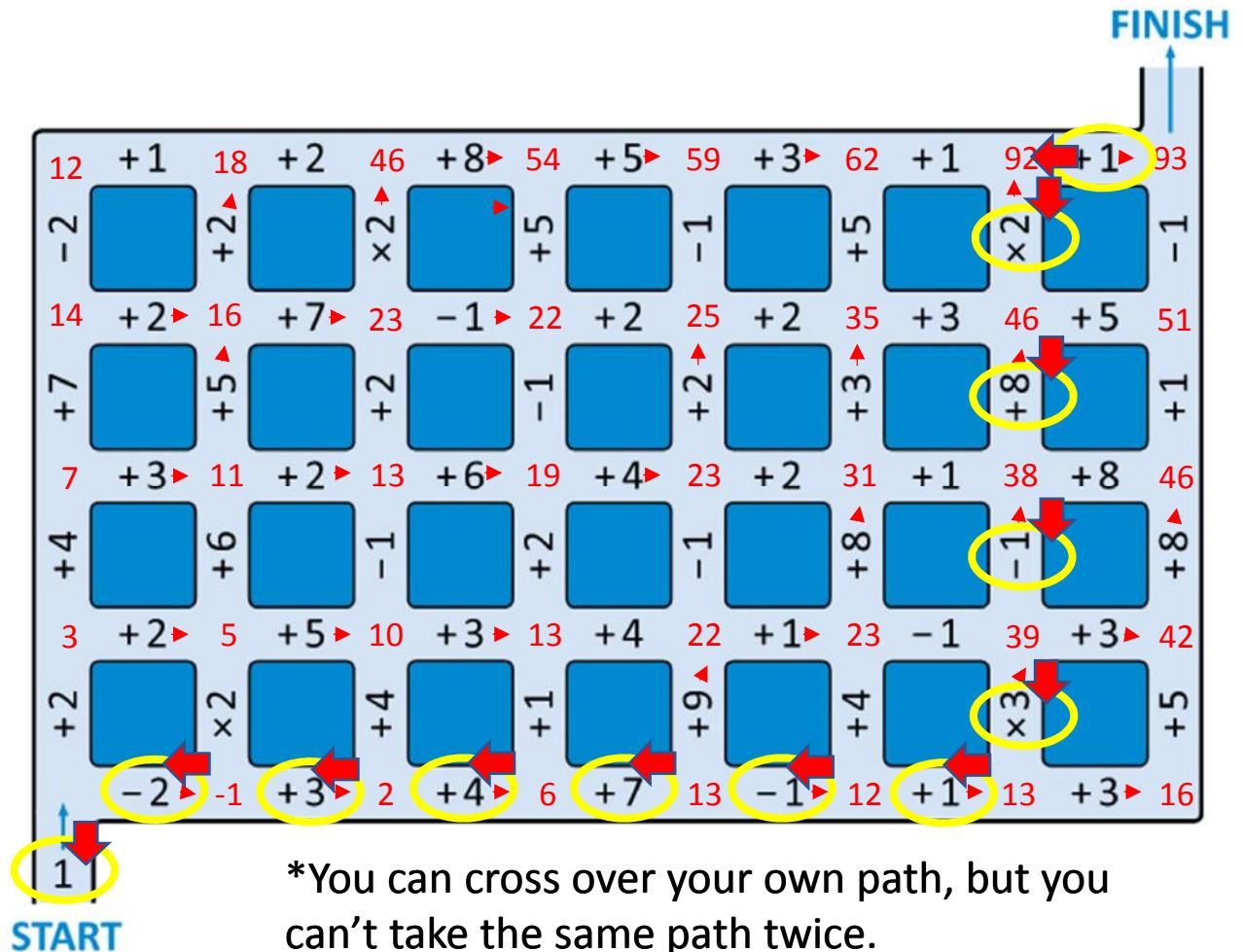
Find out all the crossroad value first.



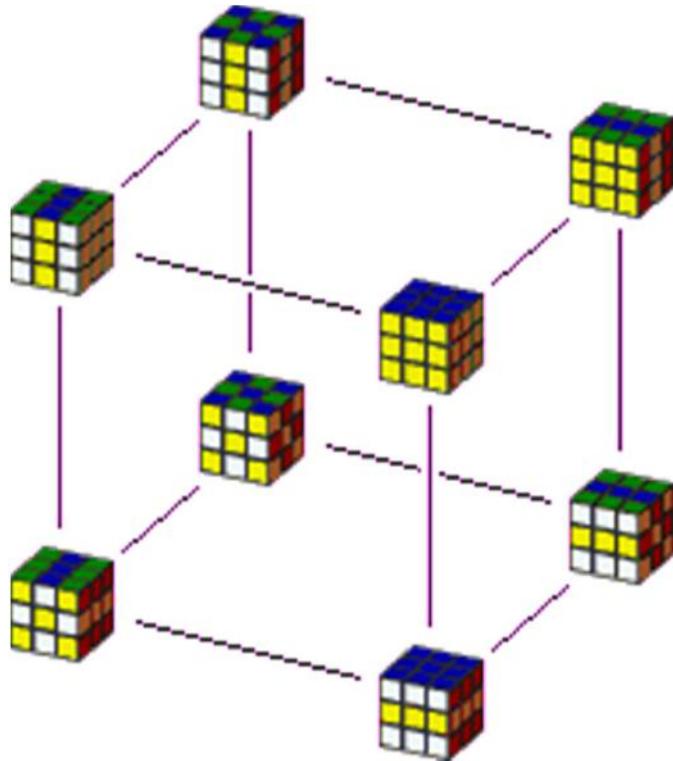
# Backward Reasoning: Example

Find the route\* that will earn the most points.

## Working backward to find the route.



# Rubik Cube as a Maze



The Rubik Cube graphs are Cayley graphs – an important special case of graphs that encodes the abstract structure of a group.

<http://mathworld.wolfram.com/RubiksCube.html>

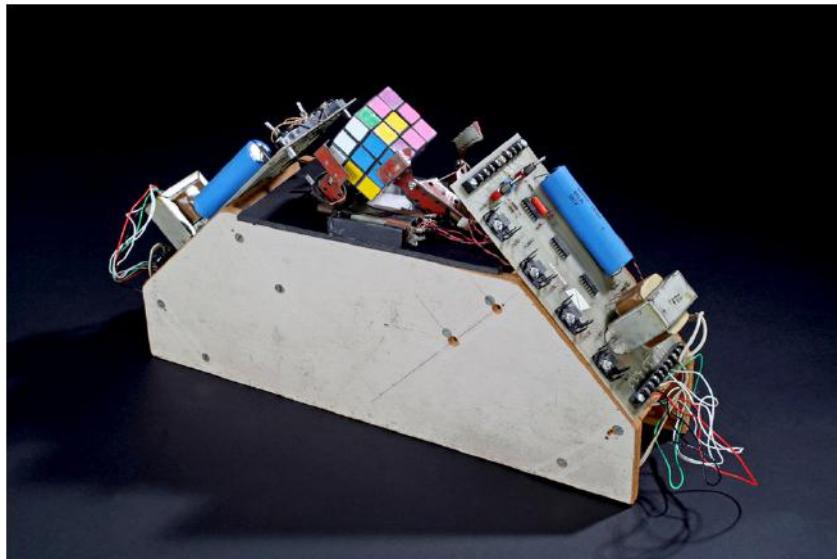


**[Ernő Rubik](#)** (1944 –) is a Hungarian inventor and professor of architecture. He invented the Rubik Cube in 1974. “And it was at that moment that I came face to face with the Big Challenge: *What is the way home?*”

Rubik was in even worse shape than his disciples. *He didn't know if the problem could be solved. Perhaps there was only one sure way to get back to the start: by exactly retracing every step he had taken.*”

<http://www.puzzlesolver.com/puzzle.php?id=29;page=15>

# Rubik Cube and AI: Shannon's Machine



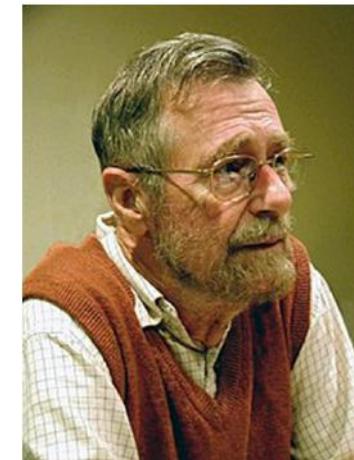
- Shannon's Rubik cube manipulator in 1984
- See Shannon's poem A Rubric on Rubik's Cubics:  
*Forty-three quintillion plus Problems Rubik posed for us. Numbers of this awesome kind Boggle even Sagan's mind.*
- *There are  $8! 12!/2 \times 3^8/3 \times 2^{12}/2 = 43,252,003,274,489,856,000$  possible arrangements of the cube.*
- <https://blogs.scientificamerican.com/cross-check/poetic-masterpiece-of-claude-shannon-father-of-information-theory-published-for-the-first-time/>

MIT Museum

<https://webmuseum.mit.edu/detail.php?module=objects&type=related&kv=75599>

# Maze-Solving: Graph Algorithms

- Maze-solving is a task of finding a desirable path from a given vertex to another desired vertex in a graph
- The *shortest path* may be desirable due to modeling considerations such as costs, efficiency or demonstrating skills
- Given a graph and two vertices (start and finishing), finding the *shortest path* from one to the other was conceived in 1956 by Dijkstra while giving a computer demo
- Backward reasoning underlies Dijkstra's algorithm



**Edsger W. Dijkstra** (1930-2002)

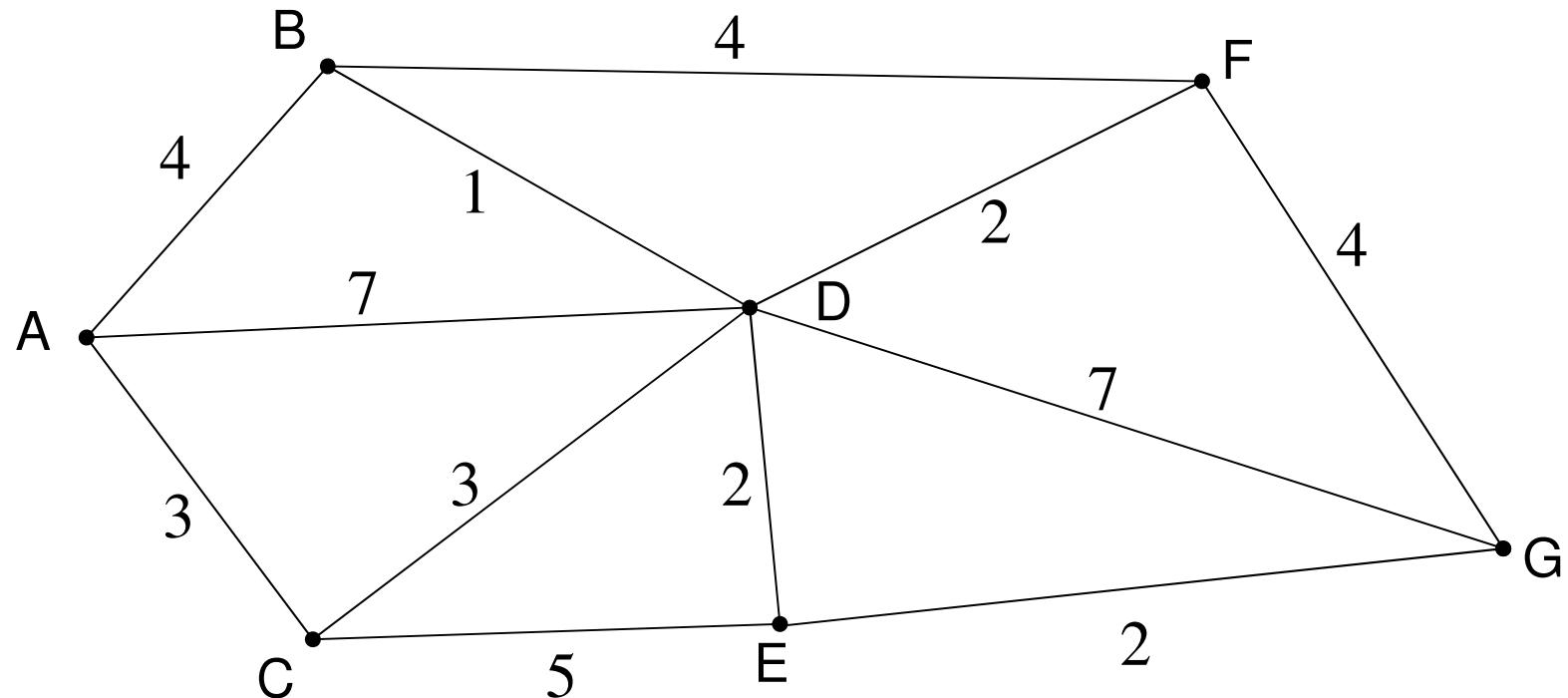
One of the most influential figures of computing science's founding generation, Dijkstra was a theoretical physicist whose career was a computer programmer. His ideas lay the foundations for the birth and development of the professional discipline of **software engineering**. And he gave his name to one of the most famous algorithms in graph theory.

[https://en.wikipedia.org/wiki/Dijkstra%27s\\_algorithm](https://en.wikipedia.org/wiki/Dijkstra%27s_algorithm)

[https://en.wikipedia.org/wiki/Edsger\\_W.\\_Dijkstra](https://en.wikipedia.org/wiki/Edsger_W._Dijkstra)

# Dijkstra's Algorithm

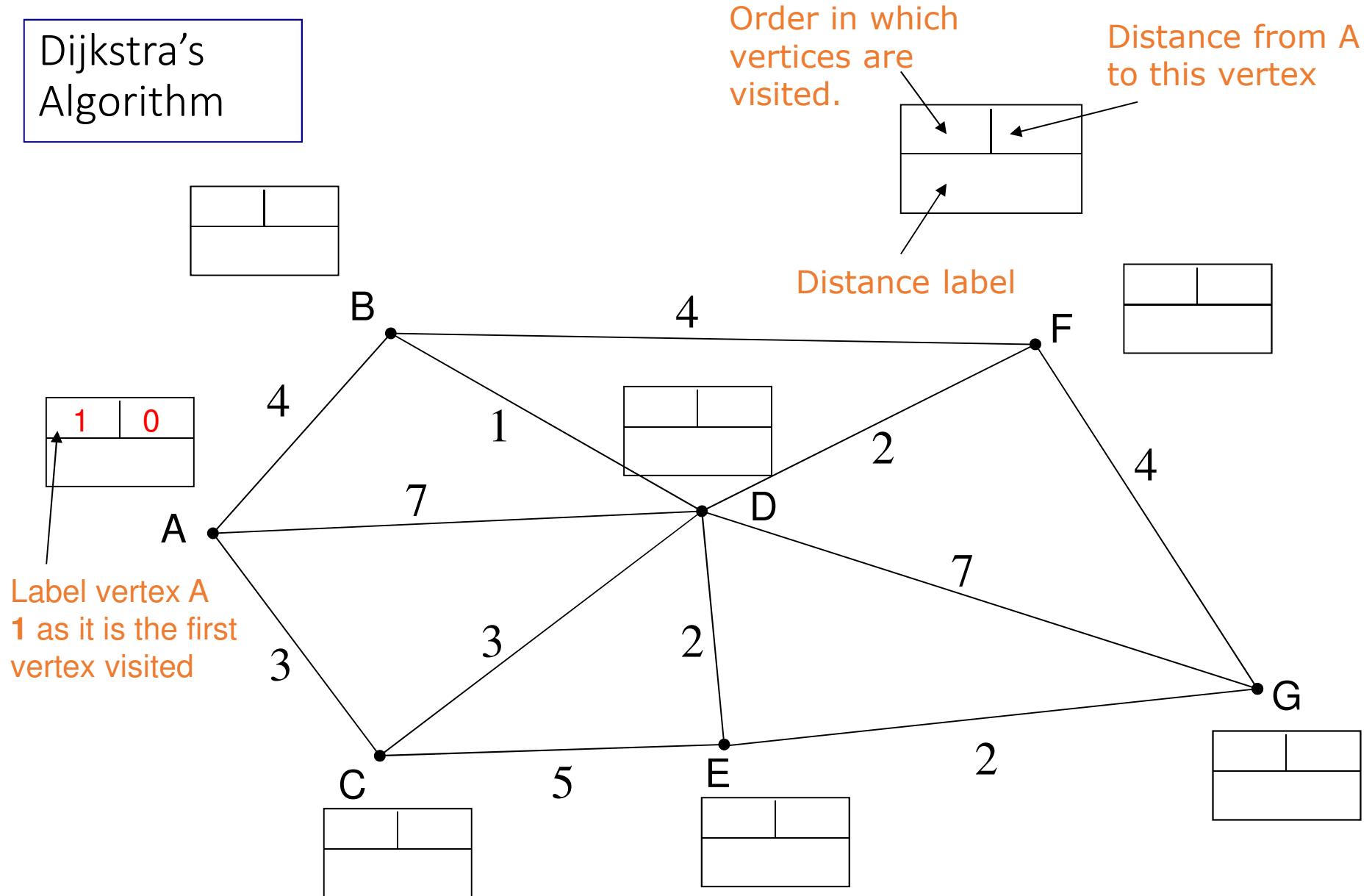
Find the shortest path from a given start vertex to a finishing vertex in the network. We will find the shortest path from A to G by backward reasoning



# Dijkstra's Algorithm

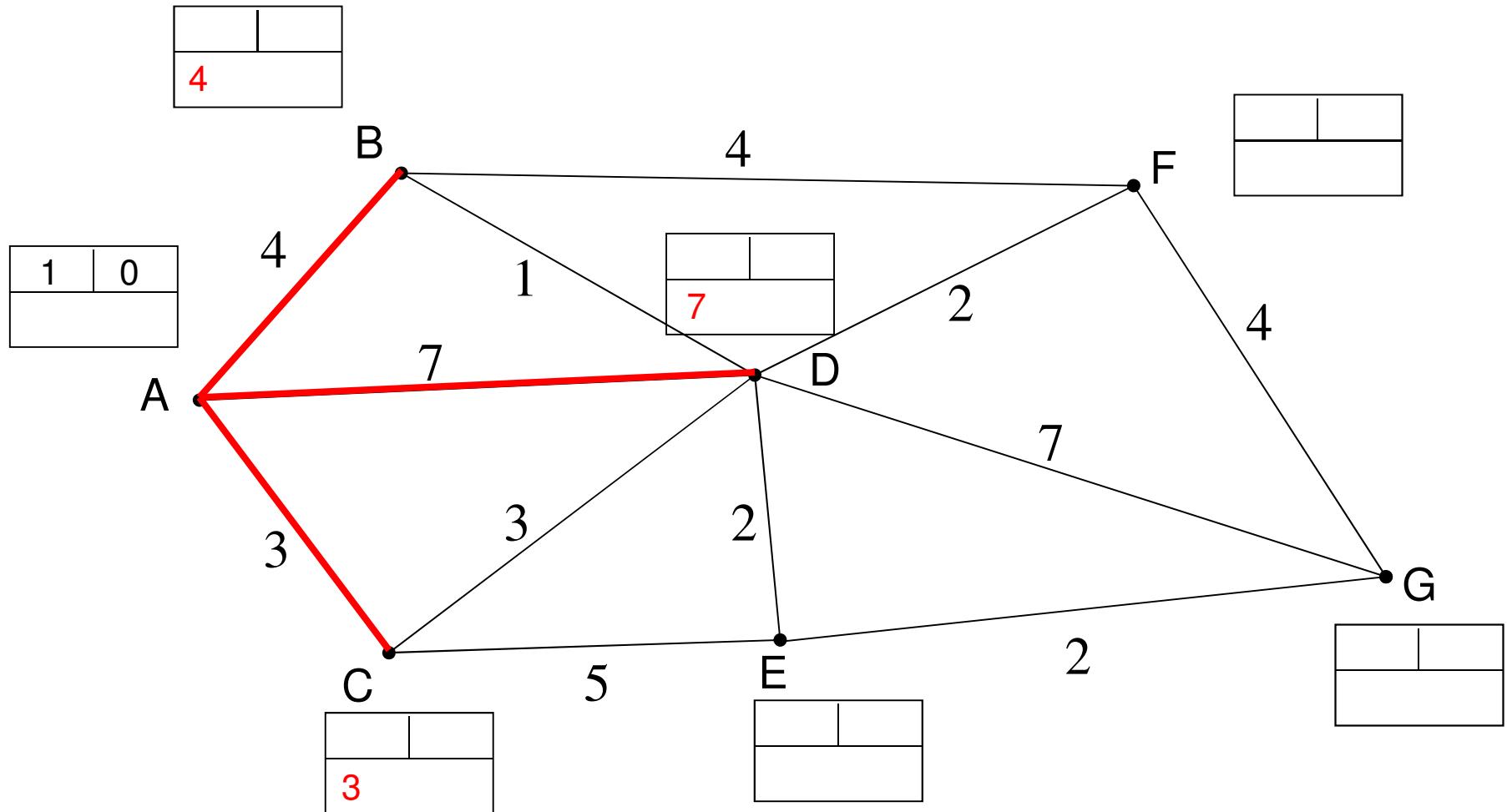
1. Initialize the start vertex with distance label 0 and “visited order” 1
- 2 Assign temporary distance labels to all the vertices that can be reached directly from the start vertex
- 3 Select the vertex with the smallest temporary distance label and make this distance label permanent. Update this vertex as “visited” with a “visited order” index incremented by one.
- 4 Put temporary distance labels on each neighboring one-hop vertex from the vertex you have just made permanent. The temporary distance label is equal to the sum of the permanent distance label plus the connecting edge value. Replace an existing temporary distance label at a vertex only if this new sum is smaller.
- 5 Go to Step 3.
- 6 Repeat until the finishing vertex has a permanent label.
- 7 To find the shortest paths(s), trace back from the end vertex to the start vertex.

## Dijkstra's Algorithm

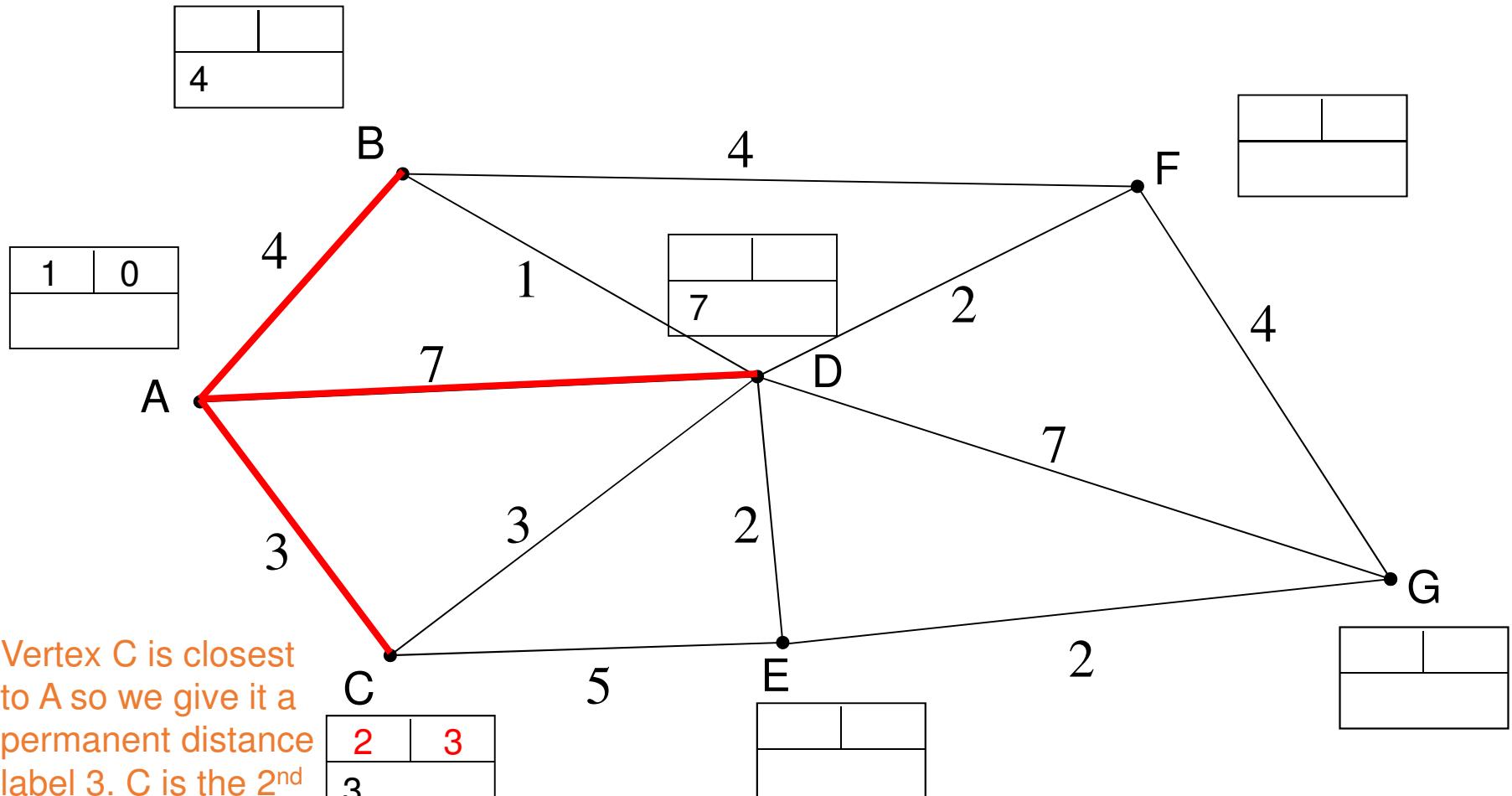


## Dijkstra's Algorithm

We update each vertex adjacent to A with a 'working value' for its distance from A.



## Dijkstra's Algorithm

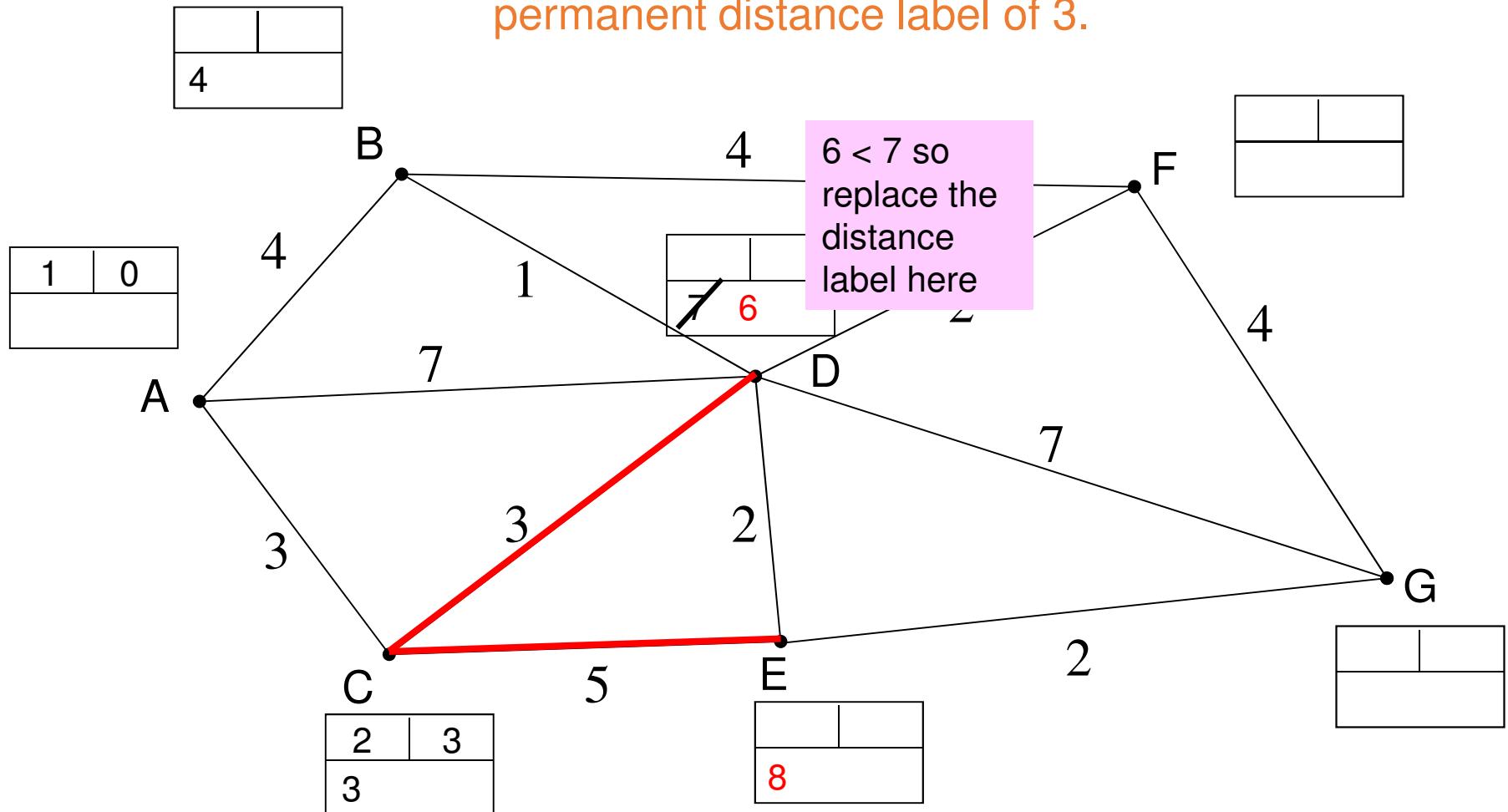


Vertex C is closest to A so we give it a permanent distance label 3. C is the 2<sup>nd</sup> vertex to be visited and permanently labelled.

2	3
3	

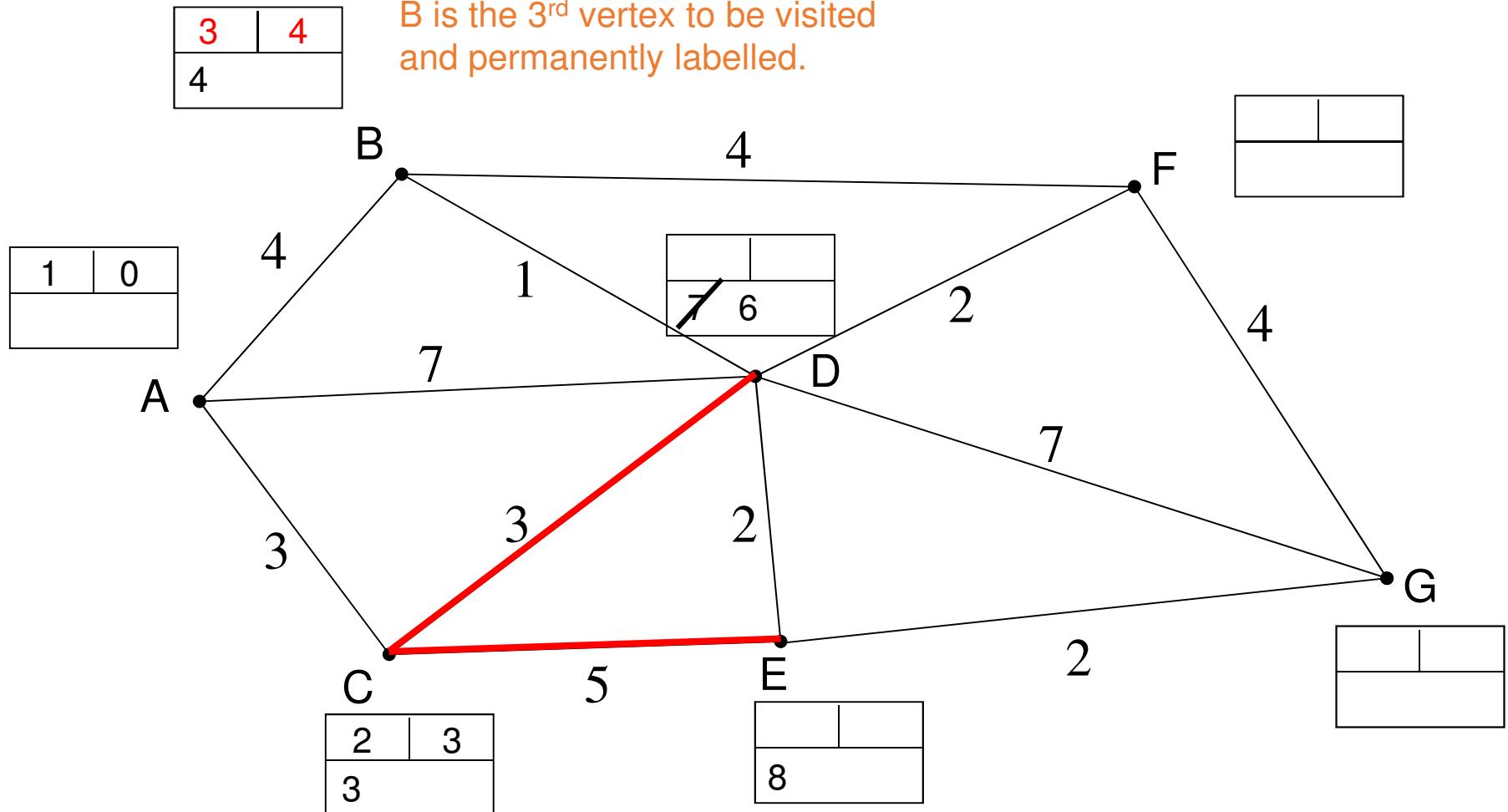
## Dijkstra's Algorithm

We update each vertex adjacent to C with a 'working distance value' for its total distance from A, by adding its distance from C to C's permanent distance label of 3.



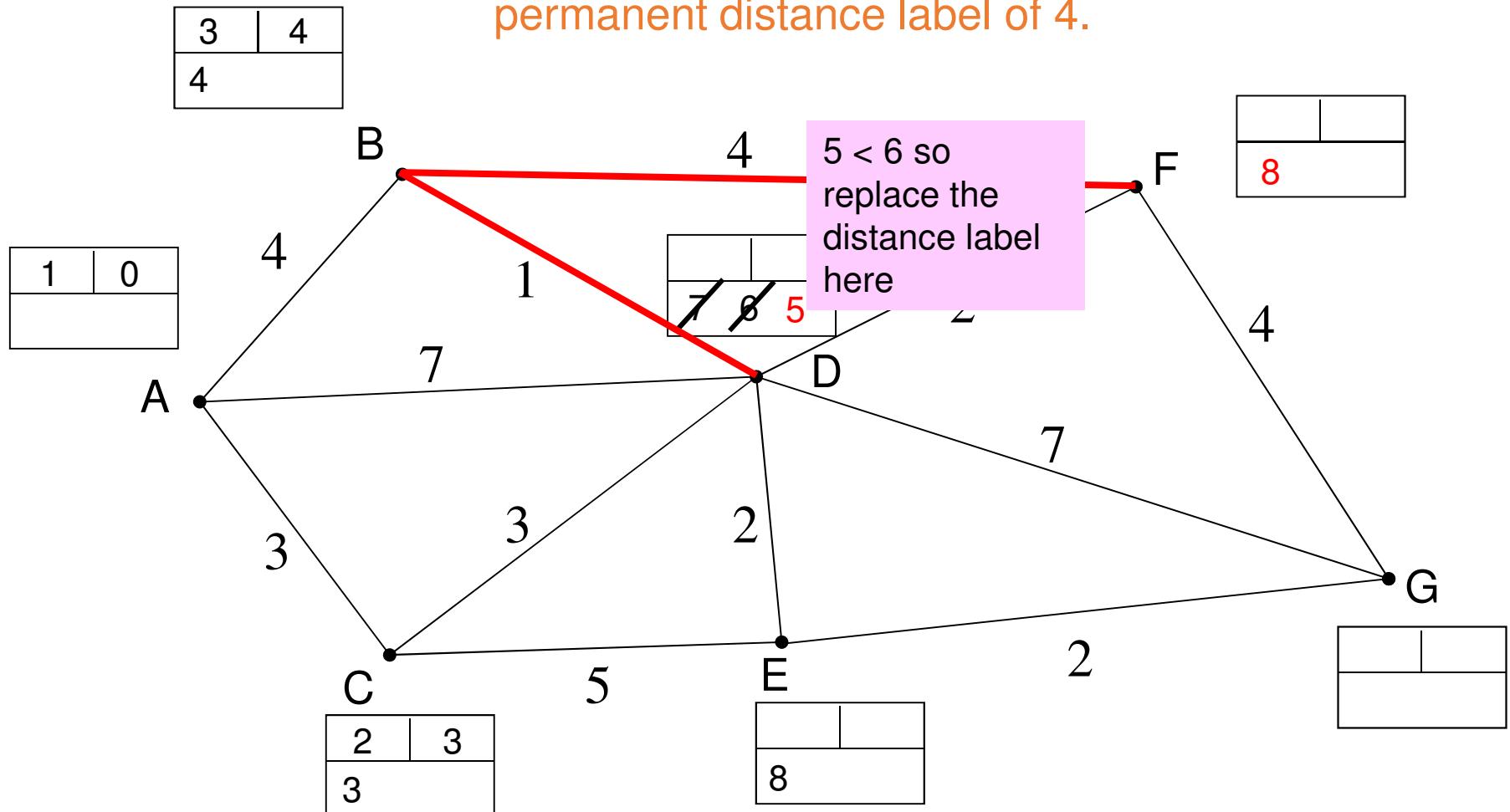
# Dijkstra's Algorithm

The vertex with the smallest temporary distance label is B, so make this label permanent. B is the 3<sup>rd</sup> vertex to be visited and permanently labelled.



## Dijkstra's Algorithm

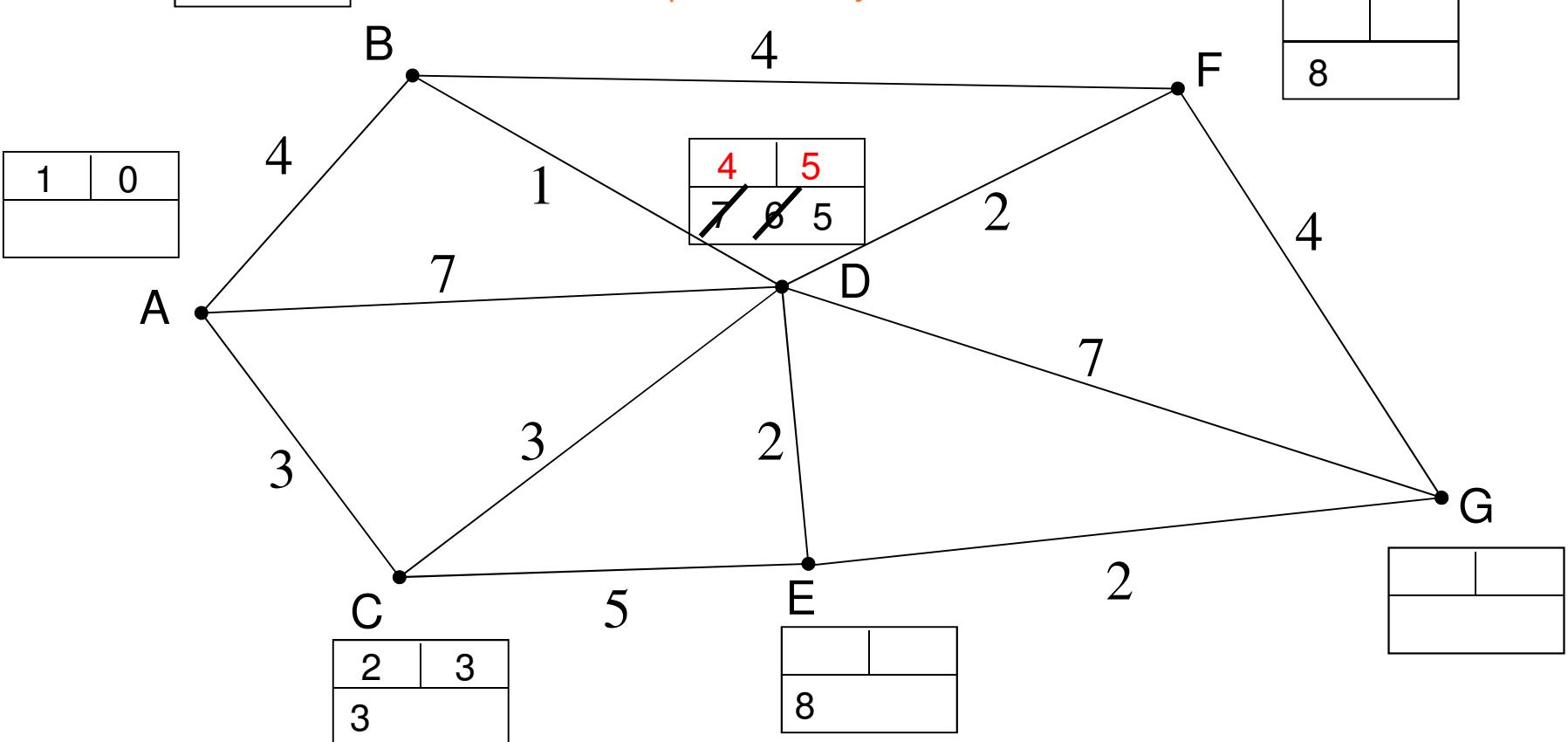
We update each vertex adjacent to B with a 'working distance value' for its total distance from A, by adding its distance from B to B's permanent distance label of 4.



## Dijkstra's Algorithm

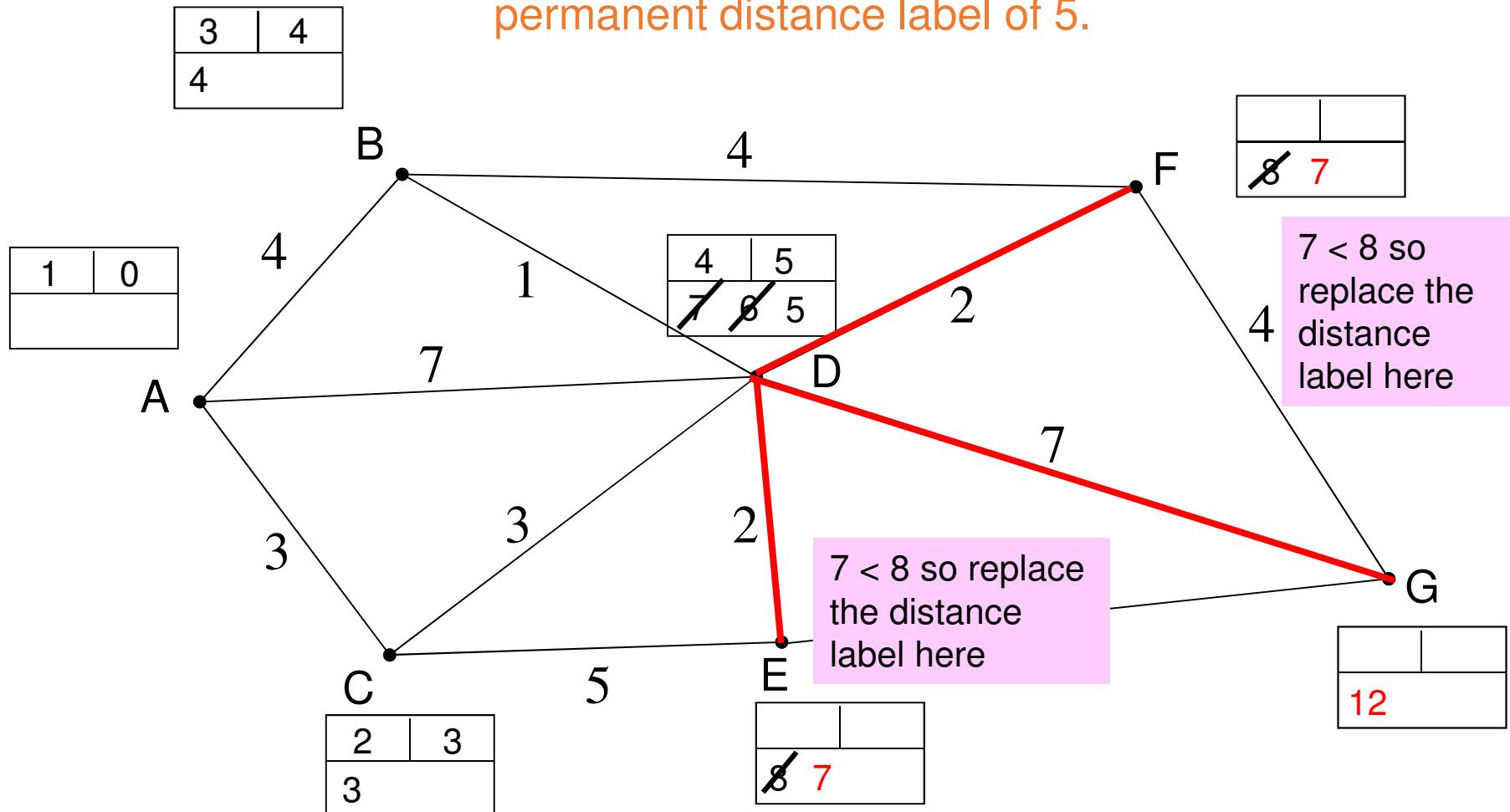
3	4
4	

The vertex with the smallest temporary distance label is D, so make this label permanent. D is the 4<sup>th</sup> vertex to be visited and permanently labelled.

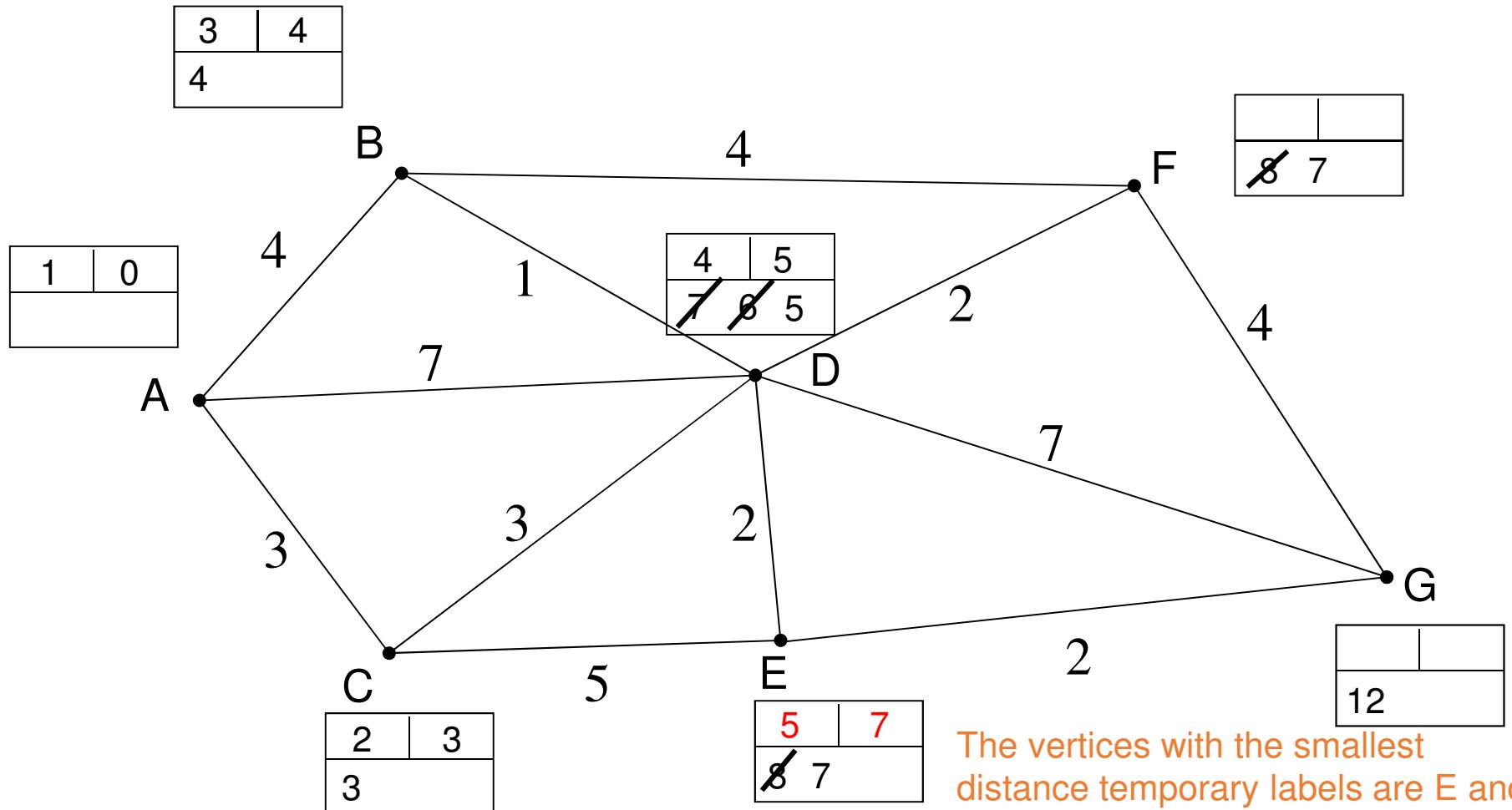


## Dijkstra's Algorithm

We update each vertex adjacent to D with a 'working distance value' for its total distance from A, by adding its distance from D to D's permanent distance label of 5.



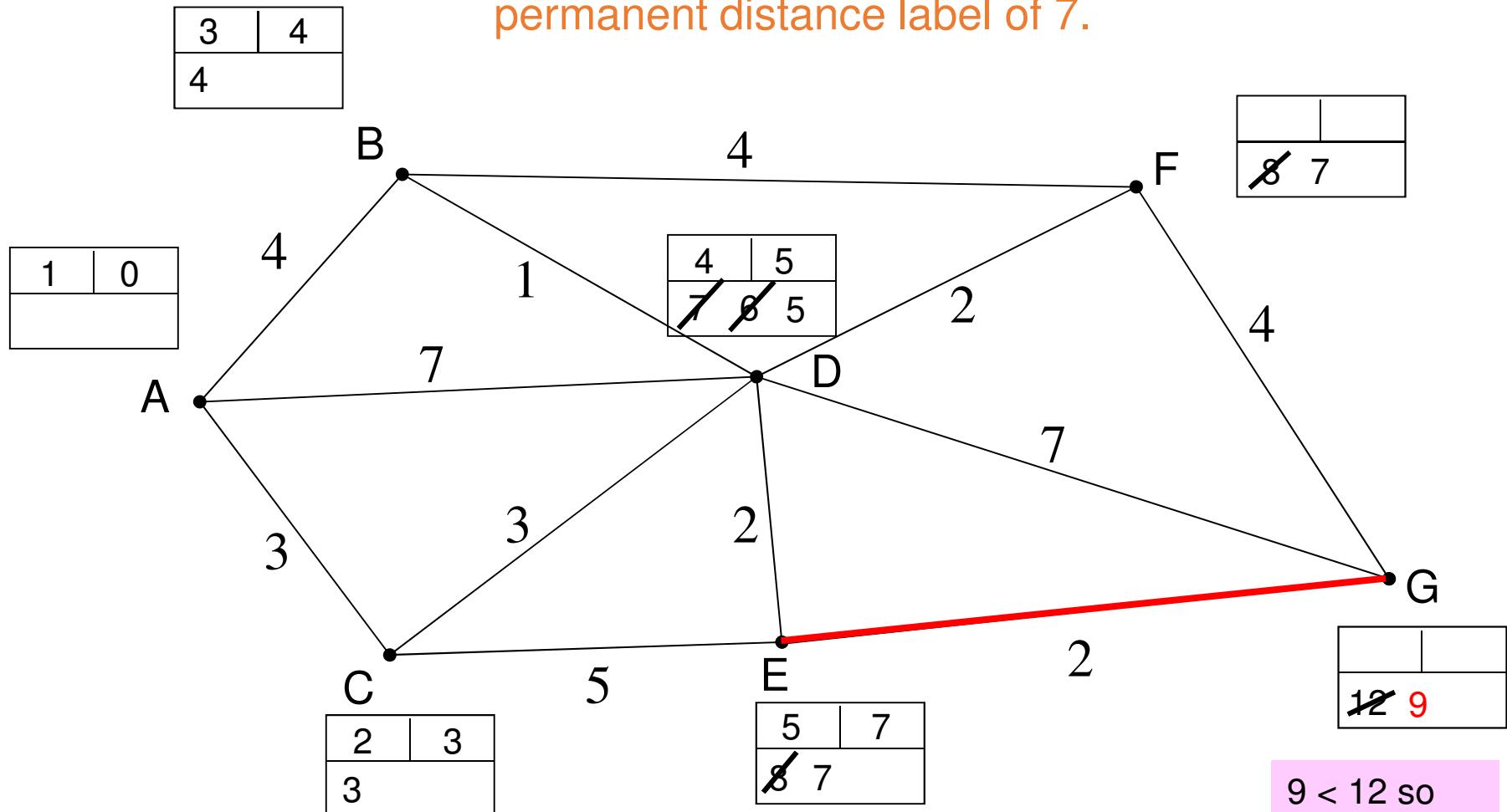
# Dijkstra's Algorithm



The vertices with the smallest distance temporary labels are E and F, so choose one and make the label permanent. E is chosen - the 5<sup>th</sup> vertex to be visited and permanently labelled.

## Dijkstra's Algorithm

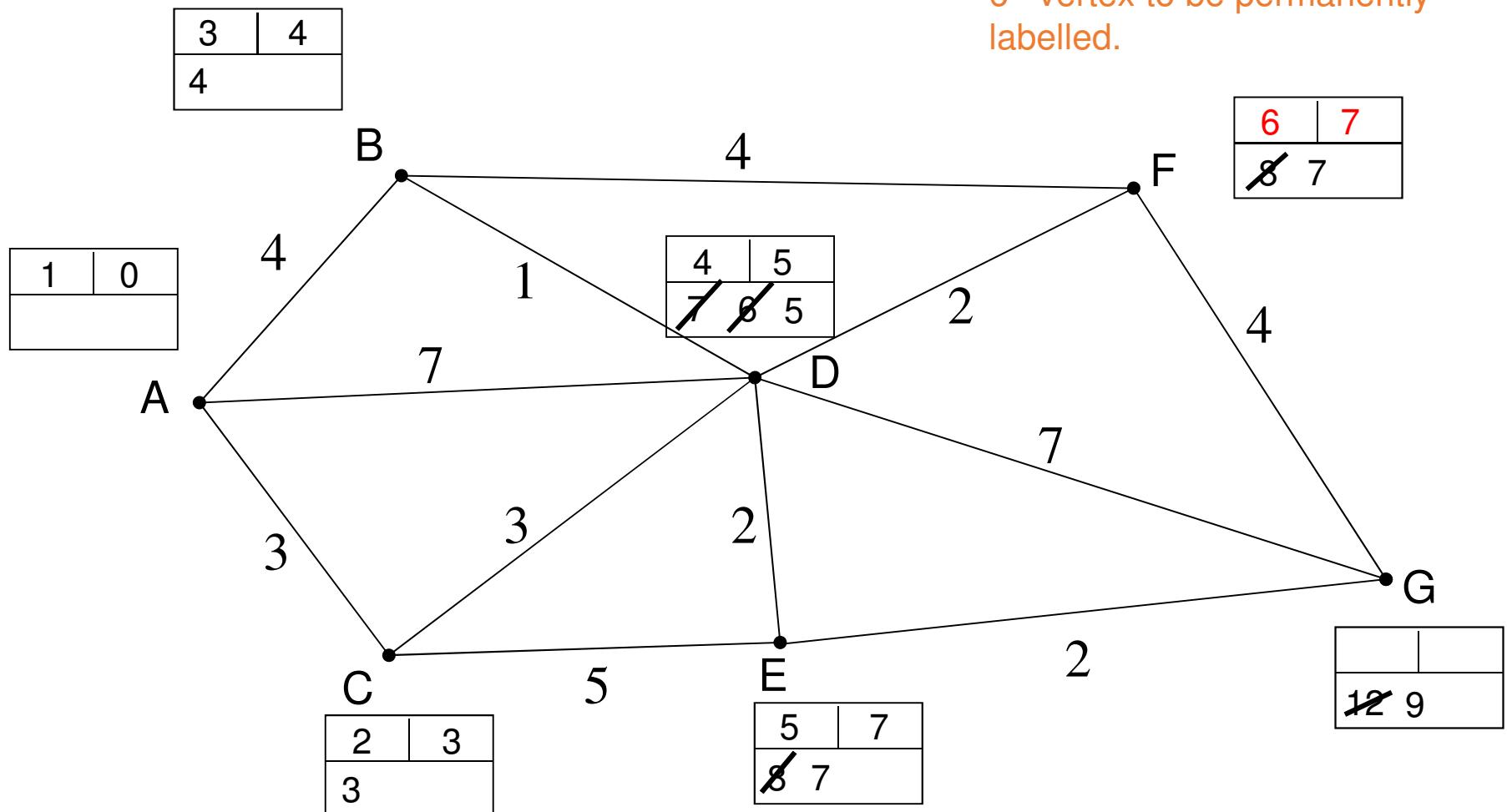
We update each vertex adjacent to E with a 'working distance value' for its total distance from A, by adding its distance from E to E's permanent distance label of 7.



9 < 12 so  
replace the  
distance  
label here

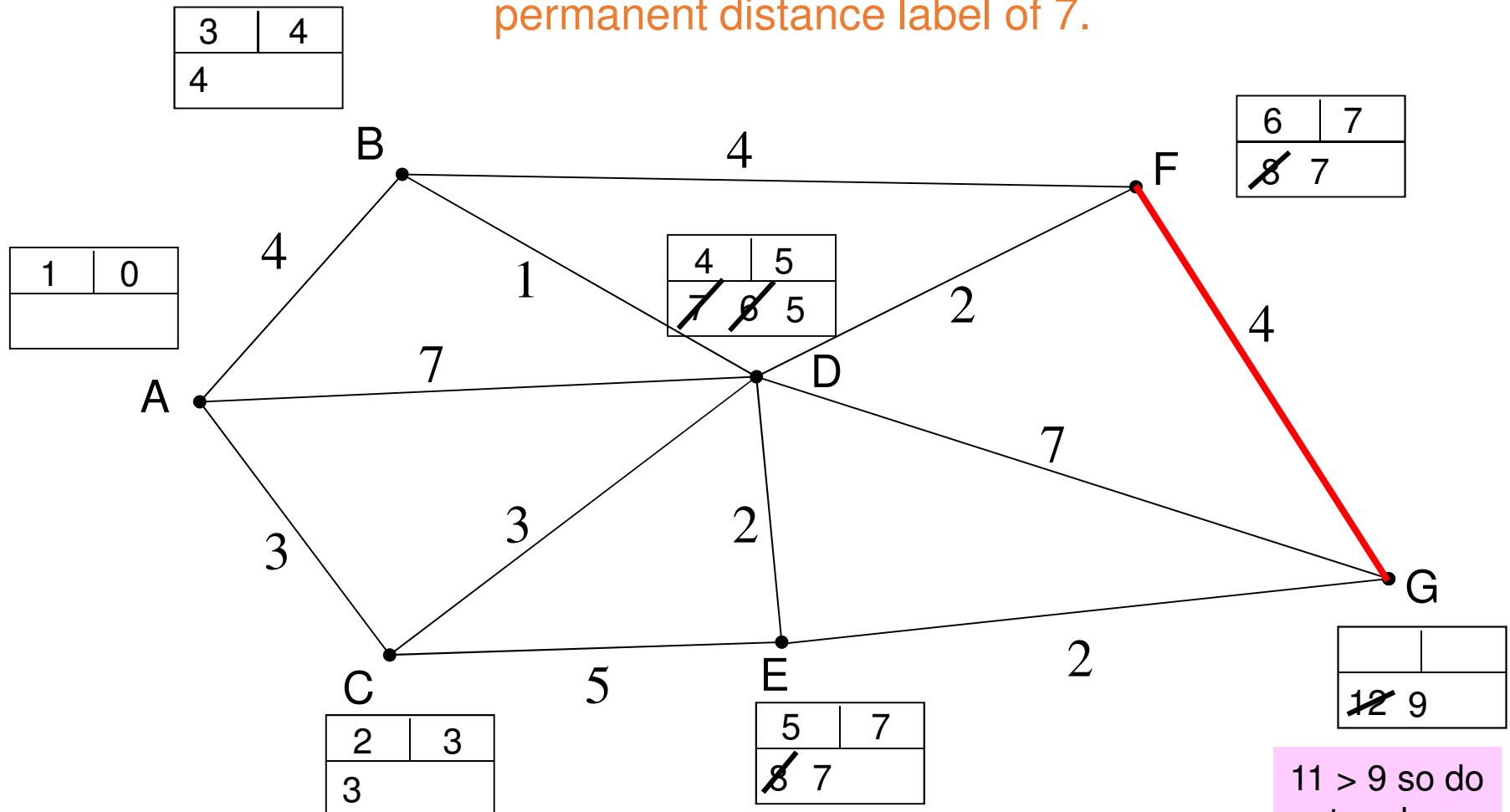
## Dijkstra's Algorithm

The vertex with the smallest temporary label is F, so make this label permanent. F is the 6<sup>th</sup> vertex to be permanently labelled.



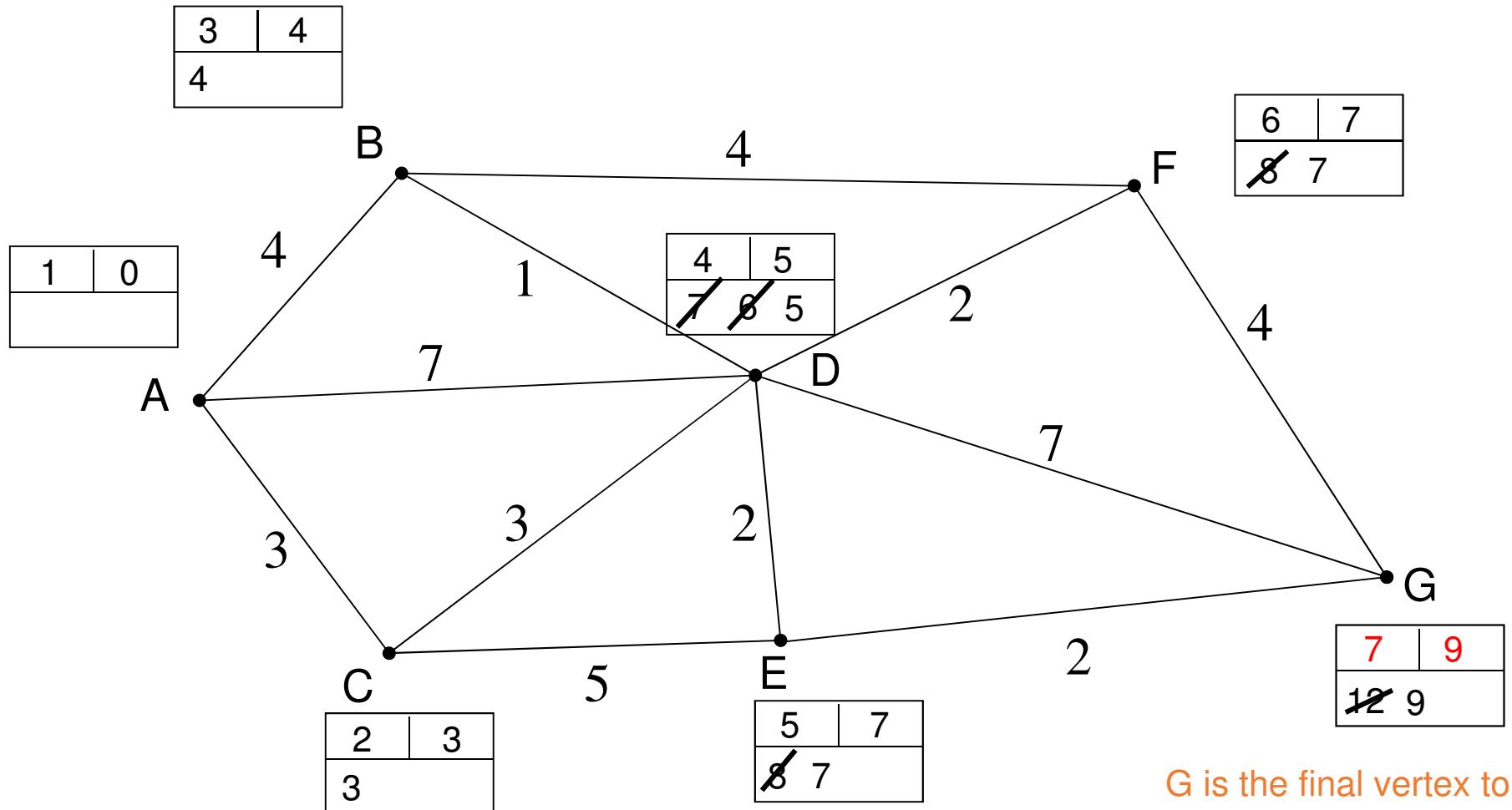
## Dijkstra's Algorithm

We update each vertex adjacent to F with a 'working distance value' for its total distance from A, by adding its distance from F to F's permanent distance label of 7.



11 > 9 so do  
not replace  
the distance  
label here

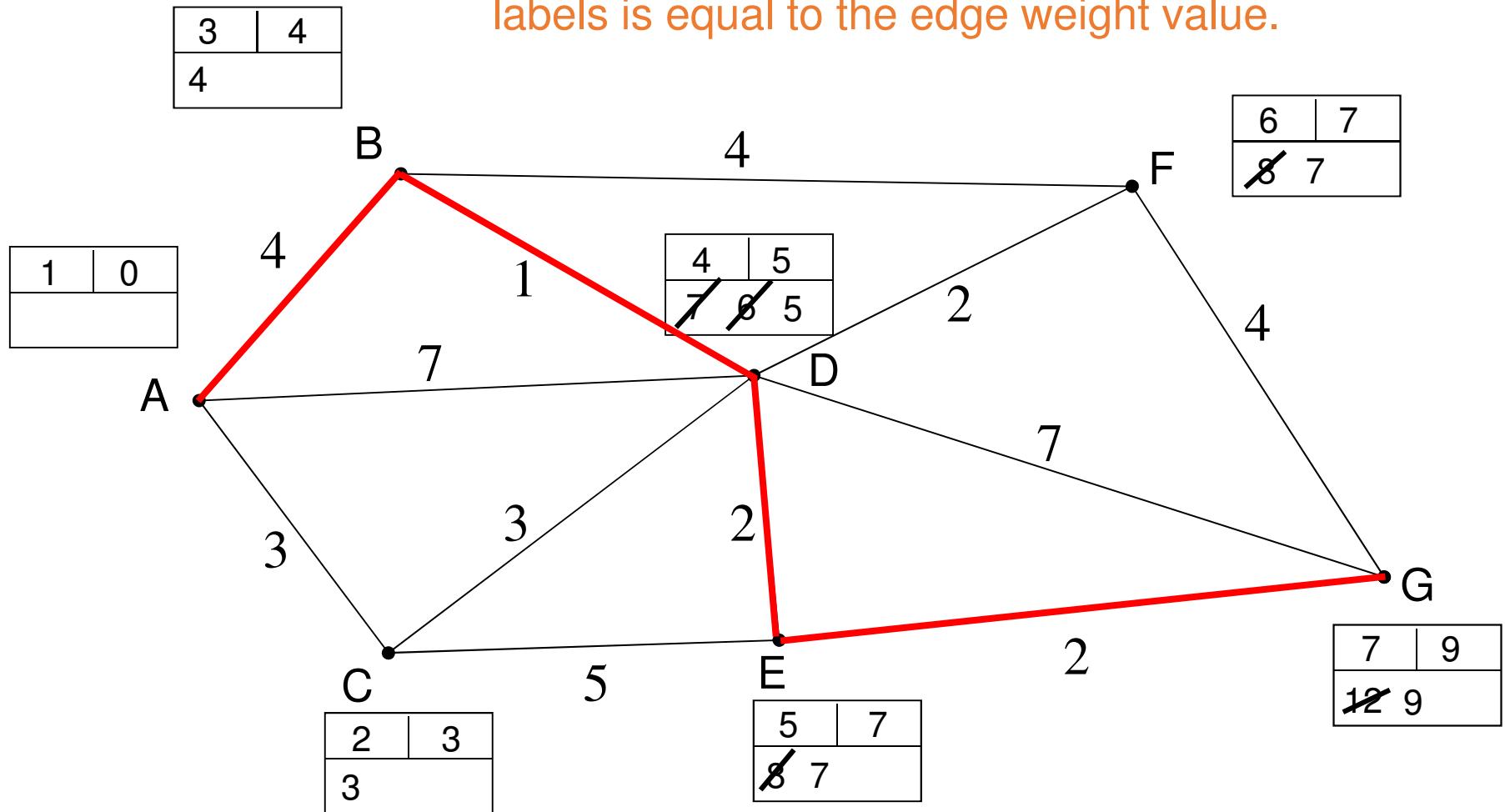
## Dijkstra's Algorithm



G is the final vertex to be visited and permanently labelled.

## Dijkstra's Algorithm

To find the shortest path from A to G, start from G and work backwards, choosing edges for which the difference between the permanent distance labels is equal to the edge weight value.

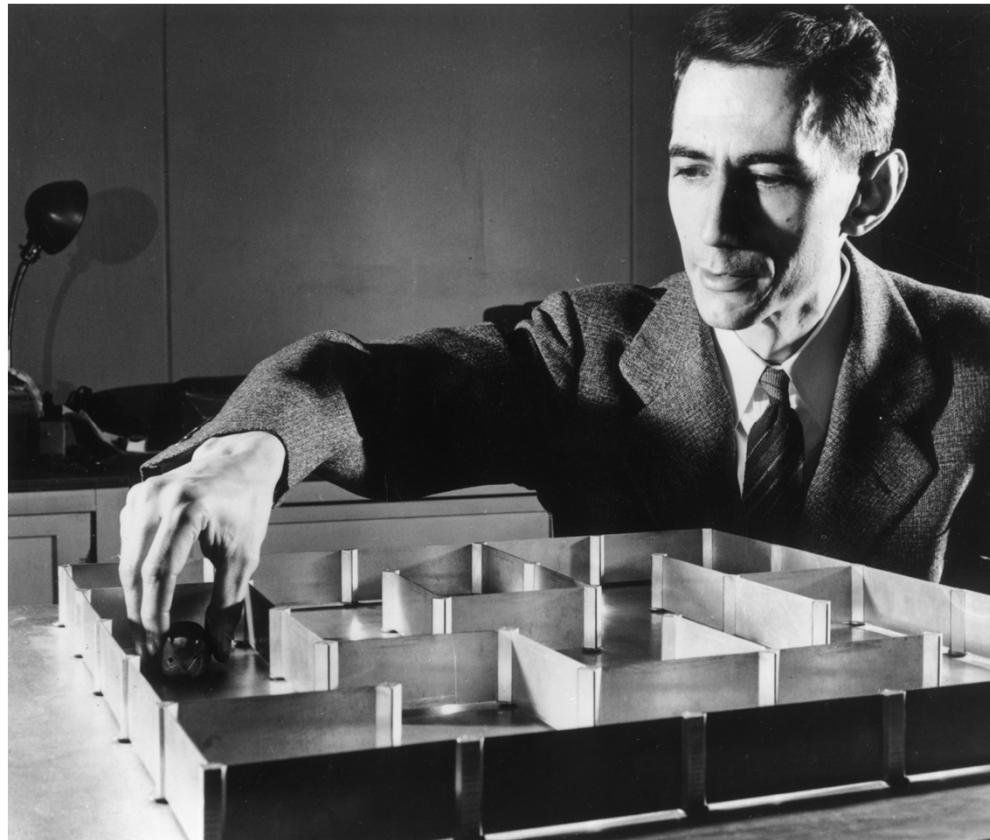


The shortest path is ABDEG, with length 9.

# Theseus: Shannon's Mouse-in-Maze

Theseus (1952) AT&T Bell Labs

Video demo by Shannon: <https://www.youtube.com/watch?v=nS0luYZd4fs>



<https://time.com/4311107/claudeshannon-100-years/>

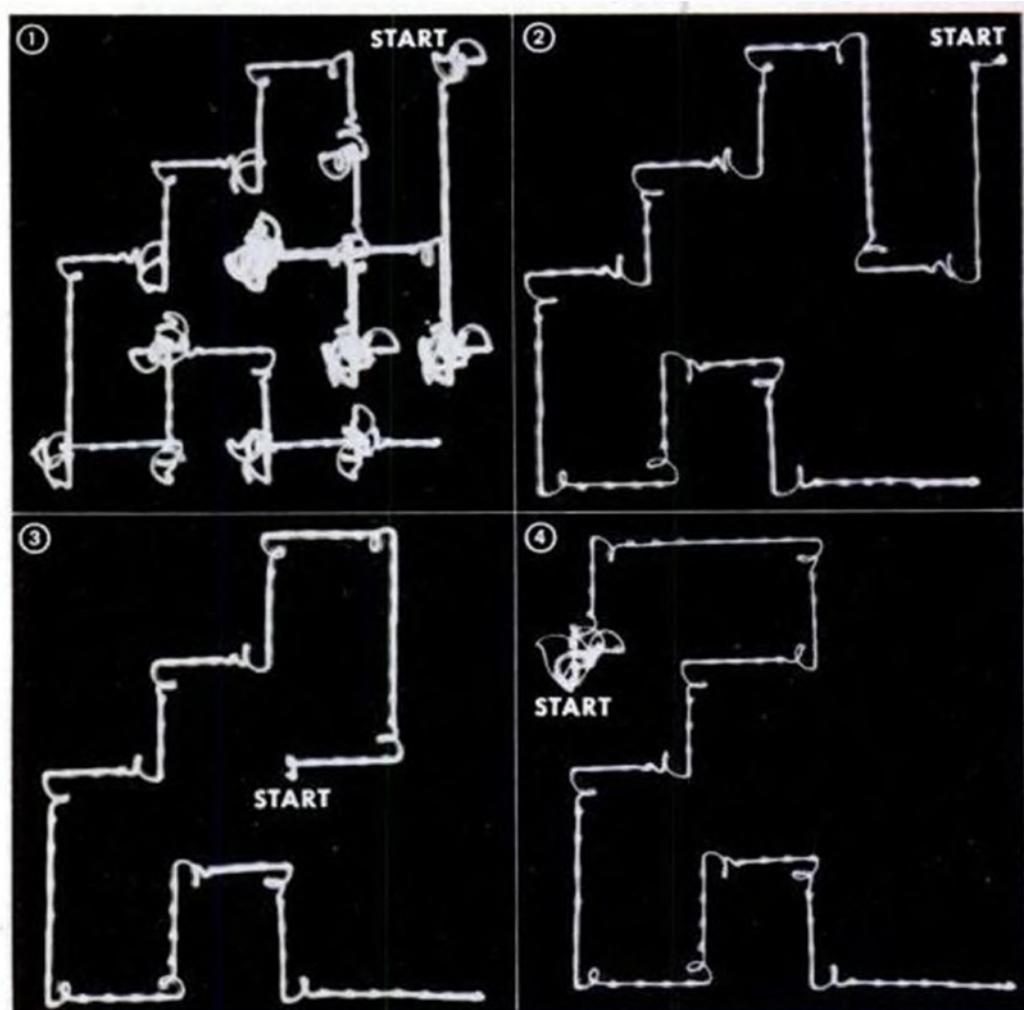
# Theseus: Shannon's Mouse-in-Maze

A life-sized mouse robot in 1952... The **Maze** solves the **Mouse**!



<http://cyberneticzoo.com/mazesolvers/1952-%E2%80%93-theseus-maze-solving-mouse-%E2%80%93-claude-shannon-american/>

# Theseus: Shannon's Mouse-in-Maze



**MEMORY TESTS** show how Theseus learns. In first trial the mouse makes wrong turns, leaves complicated trail. Second time he starts from the same place, goes straight to the goal. In third trial he is started from different spot but is on the original trail, so has no trouble. The fourth time he is put in an unfamiliar square, blunders around until he gets on the course he has learned.

- Theseus learns by experience and trial-and-error
- Memory of its route such that, when placed in a new spot that was on the previous route, Theseus can ignore blind alleys (i.e., previous errors made) and navigate correctly to end point.
- When maze topology changes, Theseus *forgets outdated knowledge, relearns and incorporates new knowledge* to existing ones in memory
- This **Shannon's maze** opens door to new results in many fields such as graph theory (breadth-first-search) and AI applications (e.g., our Internet!).

# Theseus: Shannon's Mouse-in-Maze

The trail of **Theseus** is highlighted by trial-and-error means. It does not necessarily choose the best way if there are two different ways to reach the target, although choosing the shorter one is highly probable.

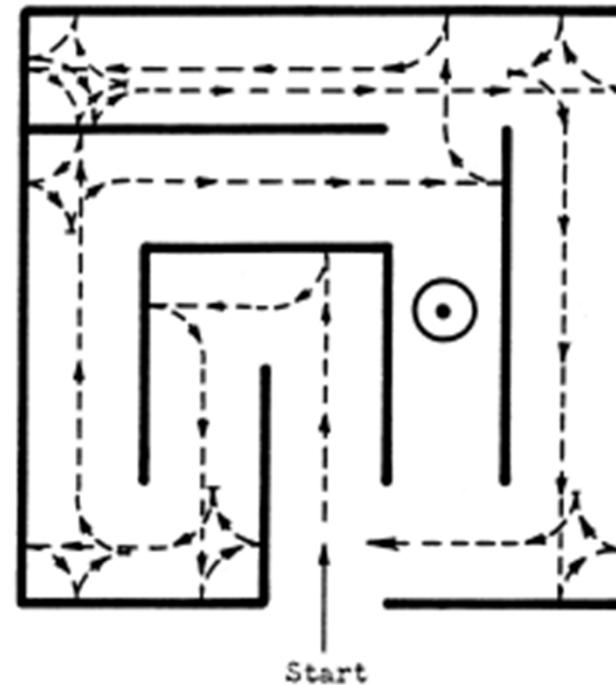
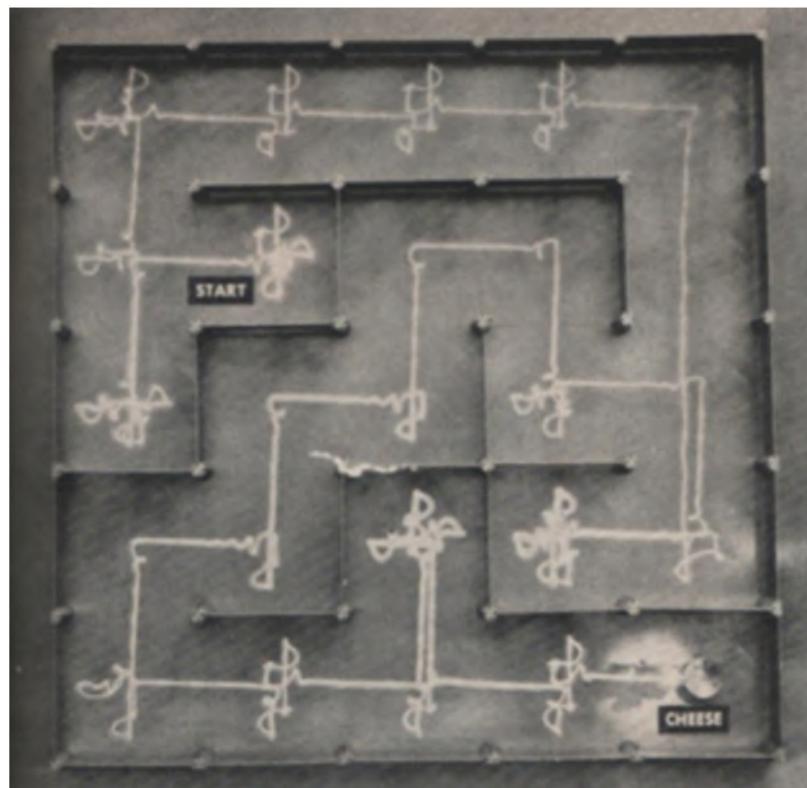


Fig. 6--Shannon's 1952 Maze

<http://cyberneticzoo.com/mazesolvers/1952-%E2%80%93-theseus-maze-solving-mouse-%E2%80%93-claude-shannon-american/>

# Our Internet is a-Maze-ing



Another that I learned was that in building self-learning systems it is equally important to forget, as it is to learn. For example, when you destroy parts of a network, the network must quickly adapt to routing traffic entirely differently. I found that by using two different time constants, one for learning and the other for forgetting provided the balanced properties desired. And, I found it helpful to view the network as an organism, as it had many of the characteristics of an organism as it responds to overloads, and sub-system failures.

Dynamic Routing, 1961

**Baran:**

I first thought that it might be possible to build a system capable of smart routing through the network after reading about [Shannon's](#) mouse through a maze mechanism . But instead of remembering only a single path, I wanted a scheme that not only remembered, but also knew when to forget, if the network was chopped up. It is interesting to note that the early simulation showed that after the hypothetical network was 50% instantly destroyed, that the surviving pieces of the network reconstituted themselves within a half a second of real world time and again worked efficiently in handling the packet flow.

Hochfelder.

How would the packets know how to do that?

**Baran:**

Through the use of a very simple routing algorithm. Imagine that you are a hypothetical postman and mail comes in from different directions, North, South, East and West. You, the postman would look at the cancellation dates on the

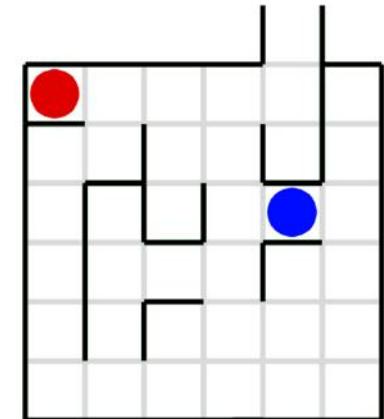
[Paul Baran: Father of Packet Switching](#)

Packet switching is the method by which the Internet works, as it features delivery of packets of data between devices over a shared network.

[https://ethw.org/Oral-History:Paul\\_Baran](https://ethw.org/Oral-History:Paul_Baran)

# Summary of Maze-Solving

- Maze design and maze-solving
  - Pac-man game, word ladder game, algebra maze, Rubik cube, Logic maze
  - Graph theory representations and its relevance
- Backward reasoning in maze-solving strategy
  - Dijkstra's Algorithm for shortest path between two vertices in a graph
  - Backward reasoning (retrograde analysis)
- The Internet has deep roots in AI
  - Shannon's Theseus mouse
  - Internet engineers implement Dijkstra's Algorithm as Open Shortest Path First (OSPF)
- Curation of new maze games and AI
  - Human aspect of a maze game is important!
  - Think like Daedalus (maze-design, give clues)
  - Solve like Theseus (maze-solving, make errors)



Theseus and the Minotaur (1990)

<https://www.logicmazes.com/theseus.html>