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# Exploiting Hidden Convexity For Flexible And Robust Resource Allocation In Cellular Networks

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***Abstract*— A systematic approach to solve seemingly nonconvex resource allocation problems in wireless cellular networks is studied in this paper. By revealing and exploiting the hidden convexity in the problem formulations, we obtain solutions that can tackle a variety of objective functions, provide robustness to resource allocations such as power, and be obtained often through distributed algorithms. The advantages of such flexibility and robustness are demonstrated through comparisons with the state-of-the-art in recent research literature.**

**First we show how to distributively solve a variety of resource allocation problems in FDMA and interference limited CDMA channels with quality of service constraints, such as meeting minimum queueing delay or energy per bit requirement. Then, for uplink transmission in a CDMA cellular network, we propose an optimal power control scheme with congestion-aware active link protection. In particular, the tradeoff between power expenditure and the protection margin of the SIR-balancing power algorithm is optimized.**

## I. Introduction

### A. Motivation

Data transmission in wireless cellular networks requires efficient and robust resource control in order to achieve quality and bandwidth efficiency, especially as wireless networks support an increasingly wide variety of applications with different Quality of Service (QoS) constraints. Resource allocation problems in wireless network often can be formulated as optimization problems with network-wide cost functions [1], [2], [3], [4], [5], [6]. Different choices of cost functions provide a variety of metrics with which to define optimality of resource allocation efficiency. These cost functions, together with constraint functions, capture the dependence on competing resource, e.g., transmit power, or other performance metrics such as average throughput or delay.

However, the objectives and constraint sets in these optimization problems are often nonconvex, which makes it hard to solve efficiently for the global optimum. In most cases, suboptimal solutions or heuristics have to be used. In our previous work [5], [6], we solve resource allocation problems using geometric programming for functions in the so-called posynomial form. In this paper, we generalize the basic idea of geometric programming to optimization of log-convex functions, and to the application of robust network protocol against unmodeled disturbances in the network. Despite the apparent nonconvexity of these optimization problems, we demonstrate flexible resource control in terms of different objective functions, and robust resource control in terms of tradeoff between the objective function and feasibility of the constraint set.

For our applications in flexible and robust resource control, the recurring theme is on revealing and exploiting the hidden convexity of problems by reformulating the problem, and then leveraging the associated tools of sensitivity analysis and Lagrange duality.

1) Conflicting goals that arise due to competition for shared resources can be modeled through general objective functions. By exploiting the inherent hidden convexity, we show how useful but seemingly nonconvex problems associated with FDMA and CDMA air interfaces can be made convex. This allows a comparison between suboptimal heuristics and the globally optimal solution.
2) Attaining optimality for each user in a wireless network can sometimes be in conflict with ensuring robustness for the overall system due to underlying dynamics in the system that cannot be adequately modeled in optimization problems. This tradeoff needs to be balanced. For the second set of applications in Section III and IV, we show how to design a robust power control and active link protection scheme for a CDMA network to minimize network costs (power expenditure). In this paper, we use the term 'robust' in the sense of robust protection scheme design with safety margin and not in the commonly used sense of robustness against channel uncertainties. Active link protection provides a protection margin for active users to maintain their QoS requirement even as new users access the channel. By taking network traffic load into account, we examine quantitatively the tradeoff between power expenditure and protection margin allocation.

### B. Related work

Previous work related to Section II of our paper include [3], [4] that focuses on optimization of different performance objectives over a Gaussian channel, but are limited by nonconvexity. The authors in [4] formulated this problem with FDMA transmission, but could not solve it satisfactorily due to the apparent nonconvexity in the problem formulation. In [3], the authors considered CDMA transmission with successive decoding and interference cancellation. In the case of an interference limited channel, i.e., without multi-user decoding, the results in [3] are somewhat restricted: some of the cost functions considered are quasi-convex, which cannot be solved as efficiently as convex optimization, especially in a distributed setting.

The second part of this paper considers a CDMA uplink transmission problem. Optimal power control is considered in [1] for downlink transmission. But, uplink transmission differs from downlink transmission in several aspects. For example, downlink transmission has a total power budget, which limits the total interference in the cell. Without a total power budget in the uplink, power control has to ensure that inter-and-intra cell interference is within tolerance. In [2], the authors proposed a joint optimal route, power and rate allocation in wireless networks, which was based on the sensitivity of the optimal cost with respect to a perturbation in the minimum average data rate. Our work is similar to [2] in the sense that sensitivity analysis is used to quantify the relationship between system design parameters and the optimal cost of the system.

Most of the previous work on power control, e.g., [1], [2], [3], [4], [7] and references in [8] considered mainly perfect power control (to attain system optimality). Due to user mobility where users enter and leave the cell randomly and other unmodeled system dynamics, perfect power control is hard to achieve in practice. Moreover, in a dynamic setting, power may grow unbounded due to infeasible power allocation when new users access the channel. Robust protection schemes, e.g., the active link protection scheme in [8] and the admission control scheme in [9], are thus necessary to ensure that users already in the channel meet their QoS thresholds when new users access the channel (to control the loss in system optimality). Our work in this paper can make robust the state-of-the-art power control algorithms proposed in the aforementioned previous work.

### C. Summary of contributions

We show that, by recognizing and exploiting the hidden convexity of cellular resource allocation problems, flexible, robust, and distributed solutions can be obtained even when prior work have resorted to suboptimal heuristics. Our contribution in Section II expands the variety of useful QoS performance metrics, originally appeared to make the optimization problems in [3], [4] hard to solve. Extending our work in [5], [6], we show that these QoS performance metrics are applicable to flexible wireless resource allocation.

In Section III, we propose a robust power control scheme in CDMA uplink transmission. The solution can easily integrate with existing power control in CDMA networks. The notion of 'price' is integrated with the design of a robust power control scheme. Intuitively, the 'price' can be interpreted as the cost needed to satisfy a Signal-to-Interference Ratio (SIR) constraint. A higher price serves as a pre-warning to the network, since it means that it is harder to maintain the 'price', and consequently, a SIR constraint violation is likely to occur. A joint optimal power and protection margin allocation using the 'price' as the right feedback signal leads to a flexible and robust network protocol design. The implication of our result is twofold: 1) An open problem posed in [8] on power control with active link protection is solved. 2) The tradeoff between the power expenditure and the robustness of CDMA power control scheme is quantified. Furthermore, we quantify some empirical observations and open remarks previously made in the pioneering work on robust power control [8], [10].

The following notation is used. Boldface uppercase letters denote matrices, boldface lowercase letters denote column vectors and italics denote scalars. $\mathbf{x} \succeq \mathbf{y}$ denotes componentwise inequality between vectors $\mathbf{x}$ and $\mathbf{y}$. $\mathbf{X} \succeq \mathbf{Y}$ denotes matrix inequality between symmetric matrices $\mathbf{X}$ and $\mathbf{Y}$.

### D. Optimization theory overview

We first give an optimization-theoretic overview that characterizes the problem formulations in Section II and also Section III. Consider the following problem

$$\begin{array}{ll} \text{minimize} & f_0(\mathbf{x}) \\ \text{subject to} & f_i(\mathbf{x}) \leq 1 \quad \forall\, i, \\ & \mathbf{x} \succeq \mathbf{0} \end{array} \tag{1}$$

where $f_0$ is convex[1] without loss of generality[2], but the $f_i(\mathbf{x})$'s are nonconvex. In general, it is difficult to obtain the optimal solution to (1). In this paper, we focus on a special case of (1) where the nonconvex $f_i$'s take a product form, i.e., $f_i(\mathbf{x}) = \prod_j g_j(\mathbf{x}_j)$ where $\mathbf{x}_j$'s are vectors with components of $\mathbf{x}$. Log-convexity[3] is the key to solving this special case. If all $g_j(\mathbf{x}_j)$'s are log-convex, then $f_i(\mathbf{x})$ is log-convex and hence jointly convex in $\mathbf{x}$. Combining this simple observation with a variable transformation, (1) can be rendered convex. Such method, which we term log-convex transformations, will be used throughout this paper.

It turns out that the problem formulations in this paper have nonconvex product form constraints as in (1). Some of these problem formulations have previously been treated as nonconvex or quasi-convex[4] problems, which cannot be solved as efficiently as convex problems. On the other hand, using a log-convex transformation, we can turn some of these seemingly nonconvex problems into convex problems, which can be solved efficiently in polynomial time (cf. [11, Sec. 11]). Under mild conditions, (1) in convex form guarantees globally optimal solutions. Hence, recognizing (1) as a convex problem (if it is a truly convex problem in disguise), as opposed to just quasi-convex or even nonconvex, has an important impact on the solution methodology used to solve (1).

In the following, we consider a variety of seemingly nonconvex optimization problems in FDMA and CDMA systems for flexible resource control, and show that they can be turned into convex problems. The advantage of convex problems is that i) they can be optimally solved, ii) Lagrangian decomposition theory can be used to obtain global optimal solutions distributively, and iii) sensitivity analysis and design can be carried out to enhance robustness.

[1] A function $f : \mathbf{R}^n \to \mathbf{R}$ is convex if it satisfies the inequality $f(\alpha x + (1-\alpha)y) \leq \alpha f(x) + (1-\alpha) f(y)$ for all $x, y \in \mathbf{R}^n$ and all $0 \leq \alpha \leq 1$.

[2] If $f_0$ is nonconvex, we can move the objective function to the constraints by introducing auxiliary scalar variable $t$ and writing minimize $t$ subject to the additional constraint $f_0(\mathbf{x})/t \leq 1$.

[3] A function $f : \mathbf{R}^n \to \mathbf{R}$ is log-convex if $\log f$ is convex.

[4] A quasi-convex problem is minimizing a quasi-convex function over a convex set where a function $f : \mathbf{R}^n \to \mathbf{R}$ is quasi-convex if its domain and all sublevel sets $S_\alpha = \{\mathbf{x} \in \mathbf{dom} f \,|\, f(\mathbf{x}) \leq \alpha\}$, for $\alpha \in \mathbf{R}$, are convex [11]. Convexity implies quasi-convexity, but not the other way round.

## II. Flexible resource control

### A. Orthogonal FDMA systems

Consider a wireless network with $L$ logical links (equivalently, transceiver pairs). Transmit power for the $l$th link is denoted by $p_l$ for $1 \le l \le L$. For FDMA transmission, we let the bandwith of the disjoint frequency bands at the $l$th link be $w_l$. The Shannon formula is given by

$$c_l(w_l, p_l) = w_l \log\left(1 + \frac{p_l}{n_l w_l}\right) \text{ bits per transmission} \quad (2)$$

for all $l$ where $n_l$ is the additive white Gaussian noise for receiver $l$. Consider the following problem of minimizing a given cost function $f_0$:

$$\begin{array}{ll} \text{minimize} & f_0(\mathbf{t}, \mathbf{w}, \mathbf{p}) - \sum_l V_l(t_l) \\ \text{subject to} & t_l \le c_l(w_l, p_l) \quad \forall\, l, \\ & \sum_l p_l \le P_T, \\ \text{variables:} & t_l, p_l, w_l \quad \forall\, l \end{array} \quad (3)$$

where $t_l$ and $V_l$ is the total offered load and a general demand function at link $l$ respectively. For simplicity, we assume $V_l(t_l) = \log t_l$ for all $l$ in this paper. In FDMA transmission, the first constraint in (3) is convex. The last constraint in (3) is the total power constraint $P_T$ for all links. Examples for the cost functions $f_0$ in (3) include

(a) Total delay cost function: $f_0 = \sum_l \dfrac{t_l}{c_l(w_l, p_l) - t_l}$,

(b) Maximum link utilization: $f_0 = \max_l \dfrac{t_l}{c_l(w_l, p_l)}$,

(c) Minimum energy cost function: $f_0 = \sum_l \dfrac{p_l}{c_l(w_l, p_l)}$,

(d) Feasibility study: $f_0 = 0$. (4)

We note that (4)a is widely used in network flow problem where the $M/M/1$ queueing model is adopted at each link [12, pp. 212], and (4)b measures the worst case congestion at any link in the network [4]. At link $l$, the number of transmissions per bit is given by $1/c_l$, thus (4)c is the total sum of the energy per bit, $\sum_l p_l/c_l$ (in Joules).

*Remark 1:* The authors in [4] treated a problem with cost function (4)b as a quasi-convex optimization problem ($t_l/c_l$ is quasi-convex in $t_l$, $p_l$ and $w_l$). Then, a sequence of convex approximations to the quasi-convex problem is used to solve this problem. However, we can transform this problem with FDMA transmission to convex form using the following result.

*Theorem 1:* Using the link capacity $c_l(w_l, p_l)$ in (2) and any convex combination of (4)a-d as the objective, problem (3) is a convex optimization problem after a log transformation $\tilde{t}_l = \log t_l$, $\tilde{p}_l = \log p_l$, and $\tilde{w}_l = \log w_l$ for all $l$.

*Proof:* We first need the following result.

*Lemma 1:* The function $\log(e^{f(\mathbf{x})} - 1)$ is convex in $\mathbf{x}$ if $f(\mathbf{x})$ is log-convex.

*Proof:* Let $g = \log(e^{f(\mathbf{x})} - 1)$ and observe that, since $f(\mathbf{x})$ is log-convex, we have $f(\mathbf{x})\nabla^2 f(\mathbf{x}) \succeq \nabla f(\mathbf{x})\nabla f(\mathbf{x})^T$. Then we have

$$\begin{aligned} \nabla^2 g &= ((e^{f(\mathbf{x})} - 1)\nabla^2 f(\mathbf{x}) - \nabla f(\mathbf{x})\nabla f(\mathbf{x})^T)\tfrac{e^{f(\mathbf{x})}}{(e^{f(\mathbf{x})}-1)^2} \\ &\succeq ((e^{f(\mathbf{x})} - 1)/f(\mathbf{x}) - 1)\nabla f(\mathbf{x})\nabla f(\mathbf{x})^T \tfrac{e^{f(\mathbf{x})}}{(e^{f(\mathbf{x})}-1)^2} \\ &\succeq \mathbf{0} \end{aligned}$$

where we used $e^x \ge x + 1$ for $x \ge 0$ in the last inequality. ■

We continue with the proof of Theorem 1. After the log transformation of variables, the constraints $t_l \le c_l(w_l, p_l)$ in (3) can be simply rewritten as $\log(\exp\{e^{\tilde{t}_l - \tilde{w}_l}\} - 1) \le \tilde{p}_l - \tilde{w}_l - \log n_l$, which is convex by Lemma 1. Let us next consider the cost functions. For cost functions (4)a-c, we introduce a scalar auxiliary variable $\eta_l$ to replace the $l$th term in $f_0$, and add extra constraints such that the $l$th term is upper bounded by $\eta_l$, e.g., for (4)a, we have $t_l/(c_l(w_l, p_l) - t_l) \le \eta_l$. After log transformation on all variables including $\eta_l$ for all $l$, each of these constraints can be rewritten such that Lemma 1 can be readily applied, e.g., for (4)a, $e^{\tilde{t}_l}/(c_l(\tilde{w}_l, \tilde{p}_l) - e^{\tilde{t}_l}) \le e^{\tilde{\eta}_l}$ is rewritten as $\log(\exp\{e^{\tilde{t}_l - \tilde{\eta}_l - \tilde{w}_l} + e^{\tilde{t}_l - \tilde{w}_l}\} - 1) \le \tilde{p}_l - \tilde{w}_l - \log n_l$, which is convex by Lemma 1. Likewise, (4)b and (4)c can be rewritten similarly. Lastly, we note that log-convexity is preserved under summation and log-convexity implies convexity, thus any convex combination of the transformed functions in (4)a-c is convex. ■

Hence, optimal power and bandwidth allocation with FDMA transmission based on cost functions in (4)a-d is a convex problem which can be solved efficiently.

### B. Interference limited CDMA systems

Most CDMA systems in practice can be modeled as interference limited channels that are non-ideal and hence non-orthogonal. Assuming a conventional matched filter receiver, the SIR for the receiver on link $l$ can be written as

$$\mathsf{SIR}_l(\mathbf{p}) = \frac{G_{ll} p_l}{\sum_{j \ne l} G_{lj} p_j + n_l} \quad (5)$$

where $G_{lj}$ are the channel gains from transmitter $j$ to receiver $l$. Assuming Gaussian noise and interference, the attainable data rates at each link are approximated by

$$c_l(\mathbf{p}) = \frac{1}{T}\log(1 + K_l \mathsf{SIR}_l(\mathbf{p})) \text{ bits per transmission} \quad (6)$$

for all $l$ where the constant $T$ represents a symbol period, which will be assumed to be one unit without loss of generality, and constant $K_l = (-\kappa_1)/(\log(\kappa_2 \text{BER}))$ where $\kappa_1, \kappa_2$ are constants depending on the modulation and BER is the required bit-error-rate in channels with additive white Gaussian noise (AWGN) [5]. Problem (3) is nonconvex due to the first constraint and the objective functions in (4).

An approach to turn the first constraint in (3) with the link capacity formula in (6) into a convex one is to use high SIR approximation [3], [5] under the assumption that the spreading gain $G_{ll}$ is much larger than $G_{kl}, k \ne l$, and not too many close-by nodes transmit simultaneously. We show below that the constraints $t_l \le c_l(\mathbf{p})$ for all $l$ of (3) can be made convex without any approximation.

*Lemma 2:* The constraint set of Problem (3) with capacity function (5)-(6) is convex in $\tilde{t}_l$ and $\tilde{p}_l$ for all $l$ where $\tilde{t}_l = \log t_l$ and $\tilde{p}_l = \log p_l$ respectively.

*Proof:* We can rewrite the constraint $t_l \le \log(1 + K_l G_{ll} p_l / (\sum_{j \ne l} G_{lj} p_j + n_l))$ as

$$\left(\sum_{j \ne l} G_{lj} p_j + n_l\right)\left(e^{t_l} - 1\right)\left(\frac{1}{K_l G_{ll} p_l}\right) \le 1, \quad (7)$$

which is a nonconvex product form constraint. However, after a log transformation of the variables, each product term in (7) becomes log-convex and (7) is thus a convex constraint. ■

*Theorem 2:* Using the link capacity $c_l(\mathbf{p})$ in (5)-(6) and any convex combination of (4)a, (4)b, (4)c and (4)d as the objective, problem (3) becomes a convex optimization problem after a log transformation: $\tilde{p}_l = \log p_l$, $\tilde{t}_l = \log t_l$ for all $l$.

*Proof:* Theorem 2 can be proved similarly to Theorem 1. For cost function (4)a, we introduce a scalar auxiliary variable $\eta_l$ and an extra constraint $t_l/(c_l(\mathbf{p}) - t_l) \leq \eta_l$ for all $l$. After log transformation on all variables and some manipulations, we have $(\exp\{(e^{\tilde{t}_l - \tilde{\eta}_l} + e^{\tilde{t}_l})\} - 1)/(K_l \mathsf{SIR}_l(\tilde{\mathbf{p}})) \leq 1$ for all $l$. Since $\log \mathsf{SIR}_l(\tilde{\mathbf{p}})$ is concave for all $l$ (see [5]), $1/\mathsf{SIR}_l(\tilde{\mathbf{p}})$ is thus log-convex. By Lemma 1, $\log(\exp\{(e^{\tilde{t}_l - \tilde{\eta}_l} + e^{\tilde{t}_l})\} - 1) - \log(K_l \mathsf{SIR}_l(\tilde{\mathbf{p}})) \leq 0$ is convex. Likewise, (4)b-c can be made convex. Lastly, log-convexity is preserved under summation and log-convexity implies convexity, so any combination of the transformed cost functions in (4)a-d is convex. ■

*Remark 2:* Restricting problem (3) with $c_l(\mathbf{p})$ in (5)-(6) to a quasi-convex optimization does not permit any convex combination of the functions in (4) because quasi-convexity is not closed under summation (cf. [11, Sec. 3.4]).

## III. Robust resource control in CDMA uplink

In this section, we consider a robust CDMA uplink transmission problem with power allocation. We propose algorithms that can co-exist with and, more importantly, make robust the current power control schemes when traffic load increases in the network. Our work is motivated by the following fact. Slightly increasing the SIR threshold offers a degree of freedom in the form of protection margin and leads to robust power control scheme as demonstrated in [8]. Similarly, intentionally compromising on the SIR threshold offers tremendous power saving and lower co-channel interference as shown by the recent work in [10]. However, the notion of precisely how much to sacrifice in order to give the network just the right amount of robustness or power saving has not been quantified in [8], [10], or any previous work. A quantitative study in this section, based on exploitation of hidden convexity, will make these important intuitions precise, and our results are further developed to solve a practical robust power control formulation (See (27) later).

### A. Review of the basic power control algorithms

The power control problem of minimizing the total power subject to SIR constraint studied by Foschini and Miljanic can be formulated as follows [8], [13], [14]:

$$\begin{array}{ll} \text{minimize} & \sum_l p_l \\ \text{subject to} & \mathsf{SIR}_l(\mathbf{p}) \geq \gamma_l \quad \forall\, l, \\ \text{variables:} & p_l \quad \forall\, l. \end{array} \tag{8}$$

The SIR constraint is the requirement that the $l$th received SIR is above a given SIR threshold $\gamma_l$ for all $l$. Problem (8) can be rewritten as a linear program (see (26) later) and thus can be solved efficiently. It is well-known that (8) can be solved using the following DPC (Distributed Power Control) algorithm [8], [13], [14]:

$$p_l(k+1) = \frac{\gamma_l}{\mathsf{SIR}_l(k)} p_l(k) \quad \forall\, l \tag{9}$$

where $k = 1, 2, \ldots,$ and $\mathsf{SIR}_l(k)$ is the received SIR at the $k$th iteration. The update in (9) is distributed as each user only needs to monitor its individual received SIR and can update by (9) independently and asynchronously [15]. Intuitively, each user $l$ increases its power when its $\mathsf{SIR}_l(k)$ is below $\gamma_l$ and decreases it otherwise. The optimal power allocation for (8) is achieved in the limit as $k \to \infty$.

The power control algorithm proposed in [8], known as Distributed Power Control With Active Link Protection (DPC/ALP), protects active users from new users that enter the channel. The two key ideas of the DPC/ALP algorithm are: 1) the gradual power-up of new users entering the channel; and 2) the introduction of a performance margin to cushion the active users already in the channel, which is accomplished by modifying the SIR constraint in (8) to

$$\mathsf{SIR}_l(\mathbf{p}) \geq \gamma_l(1+\epsilon) \quad \forall\, l \tag{10}$$

where $\epsilon > 0$ is arbitrarily chosen, and multiplying the righthand side of (9) by $(1+\epsilon)$. Clearly, the parameter $\epsilon$ serves as a *protection margin* for users that are running (9) and keeps them from falling below $\boldsymbol{\gamma}$ in a dynamic setting when new users power up. We first use sensitivity analysis to show how adjusting $\boldsymbol{\gamma}$ may affect the solution for (8) with implication on the constraint (10). This motivates us to propose an enhanced DPC/ALP algorithm to solve a practical but harder problem (27) in Section III-D.

### B. Sensitivity analysis

Recall that the SIR constraints in (8) are tight at optimality assuming that there is a feasible power allocation for all users. Hence, tightening or loosening this constraint set affects the optimal value of (8). The following result relates the sensitivity of the SIR constraint and the total power requirement in (8).

First, for the $l$th SIR constraint in (8), we define a perturbed SIR threshold $\gamma_l/u_l$ where $1/u_l$ represents a fractional perturbation of the SIR threshold $\gamma_l$, and substitute the $l$th SIR constraint in (8) by $\gamma_l/\mathsf{SIR}_l(\mathbf{p}) \leq u_l$ for all $l$. We have $0 < u_l < 1$ or $u_l > 1$ if we tighten or loosen the $l$th SIR constraint respectively. Next, we define $f^\star(\mathbf{u})$ as the optimal value of (8) with these modified perturbed constraints:

$$f^\star(\mathbf{u}) \quad = \inf\{\textstyle\sum_l p_l \mid \gamma_l/\mathsf{SIR}_l(\mathbf{p}) \leq u_l \quad \forall\, l\}. \tag{11}$$

If $f^\star(\mathbf{u})$ does not exist for some $\mathbf{u}$, we define $f^\star(\mathbf{u}) = \infty$. Though the optimization problem in (11) can be rewritten as a linear program in $\mathbf{p}$, it is expressed in a form similar to (1) with nonconvex product form constraints. This allows us to apply the log-convex transformation technique in Section I-D, which is a key step to reveal hidden convexity in (11) that will lead to our main algorithm in Section III-E (rewriting (11) as a linear program also renders the problem convex, but does not lead to the following results). Next, let $\tilde{p}_l = \log p_l$ and the parameter $\tilde{u}_l = \log u_l$, and taking the logarithm of the SIR constraints, we write $f^\star(\mathbf{u})$ in the log transformed parameter $\tilde{\mathbf{u}}$ as $\tilde{f}^\star(\tilde{\mathbf{u}})$, and $\tilde{f}^\star(\tilde{\mathbf{u}})$ is determined by solving the following convex problem

$$\begin{array}{ll} \text{minimize} & \sum_l e^{\tilde{p}_l} \\ \text{subject to} & \log(\gamma_l/\mathsf{SIR}_l(\tilde{\mathbf{p}})) \leq \tilde{u}_l \quad \forall\, l, \\ \text{variables:} & \tilde{p}_l \quad \forall\, l. \end{array} \tag{12}$$

TABLE I

VALIDATING THEOREM 3 WITH THE NUMERICAL EXAMPLE IN [10] FOR 3 USERS. THE SECOND AND THIRD COLUMN REPRESENTS THE OPERATING POINT OF $(7, 6.9, 6.9)^T$ AND $(7, 6.5, 6.5)^T$ (IN DB) IN [10] RESPECTIVELY. ROW 1 SHOWS $\boldsymbol{\nu}^\star$. ROW 2 SHOWS THE PREDICTION GIVEN BY THEOREM 3. THE NUMERICAL PERCENT DECREASE IN TOTAL POWER IS GIVEN IN ROW 3. ROWS 4 AND 5 SHOW THE PERCENT DECREASE IN INDIVIDUAL POWER AND INTERFERENCE RESPECTIVELY.

| | | |
|---|---|---|
| 1 | (128.680, 155.598, 107.532) | (8.529, 10.209, 7.066) |
| 2 | 64.650% | 74.483% |
| 3 | 64.600% | 75.200% |
| 4 | (63.600%, 64.670%, 64.300%) | (74.400%, 76.100%, 74.900%) |
| 5 | (65.074%, 64.658%, 64.774%) | (75.770%, 74.444%, 74.806%) |

*Theorem 3:* Let $\nu_l^\star$ for all $l$ be the optimal Lagrange multipliers of the unperturbed problem in (12), (i.e., $\tilde{\mathbf{u}} = \mathbf{0}$), then

$$100\frac{\tilde{f}^\star(\frac{\beta_l}{100}\mathbf{e}_l) - \tilde{f}^\star(\mathbf{0})}{\tilde{f}^\star(\mathbf{0})} = -\beta_l\nu_l^\star/\tilde{f}^\star(\mathbf{0}) + o(\beta_l) \qquad (13)$$

where $\mathbf{e}_l$ is the $l$th unit vector, and $\beta_l$ is a small positive value.

The engineering implication is as follows. Relaxing (or tightening) the $l$th SIR threshold constraint by $\beta_l$ percent in (8) reduces (or increases) the total power by approximately $\beta_l\nu_l^\star/\tilde{f}^\star(\mathbf{0})$ percent, for $\beta_l$ small. Hence, by considering all users, the total power reduction (or increment) is approximately $\sum_l \beta_l\nu_l^\star/\tilde{f}^\star(\mathbf{0})$ percent.

*Proof:* To show (13), since $\tilde{f}^\star(\tilde{\mathbf{u}})$ is differentiable, for the perturbation $\tilde{\mathbf{u}} = t\mathbf{e}_l$ where $t$ is small, we use Taylor series expansion to obtain

$$\lim_{t\to 0} \tilde{f}^\star(t\mathbf{e}_l) = \tilde{f}^\star(\mathbf{0}) + t\frac{\partial \tilde{f}^\star(\mathbf{0})}{\partial \tilde{u}_l} + o(t). \qquad (14)$$

From sensitivity analysis (cf. [11, Sec. 5.6]), we have $\partial\tilde{f}^\star(\mathbf{0})/\partial\tilde{u}_l = -\nu_l^\star$ for all $l$, which we substitute in (14). For small $\beta_l > 0$, relaxing the $l$th constraint by $\beta_l$ percent gives $t = \beta_l/100$, which is substituted in (14) to yield (13). ∎

We validate Theorem 3 using the numerical example in [10]. Table I presents the results for three mobile users. From an initial SIR thresholds $(7, 7, 7)^T$ (in dB), there are only changes in the SIR thresholds for Users 2 and 3, i.e., from 7dB to 6.90dB and to 6.50dB. The second and third columns in Table I show, respectively, the operating points corresponding to SIR thresholds $(7, 6.9, 6.9)^T$ and $(7, 6.5, 6.5)^T$ (in dB) in [10]. Row 1 shows the $\boldsymbol{\nu}^\star$, which we will show how to compute in Section III-C. Row 2 shows the prediction of Theorem 3, and row 3 shows the numerical percent decrease as reported in [10]. We see that Theorem 3 gives a good prediction on the amount of power saved with a slight compromise on the SIR threshold. Row 4 shows the percent decrease for each user, and each individual user's power is observed to have almost the same percentage reduction as the total power (cf. row 3 of Table I). The same observation is made for the interference experienced by each individual user as shown in the last row of Table I. That is, if users with large $\nu_l^\star$ compromise slightly, all users obtain both considerable power saving and lower interference simultaneously. The 'price' $\nu_l^\star$ for each user is also observed to have approximately the same percentage reduction.

The following result is the next step towards developing robust power control optimization.

*Theorem 4:* The optimal power $\mathbf{p}^\star$ in (8) satisfies

$$\nu_l^\star = p_l^\star\left(1 + \sum_{i\neq l}\frac{G_{il}\nu_i^\star}{\sum_{j\neq i}G_{ij}p_j^\star + n_i}\right) \quad \forall\, l, \qquad (15)$$

for some nonnegative $\boldsymbol{\nu}^\star$.

*Proof:* Theorem 4 is obtained by applying the Karush-Kuhn-Tucker optimality conditions to (12). ∎

### C. Perron-Frobenius Theorem based algorithms

In the following, we use Theorem 4 to reveal an interesting structural property between $\boldsymbol{\nu}^\star$ and $\mathbf{p}^\star$ to be used in our main algorithm in Section III-E. By defining $x_l^\star = \nu_l^\star/p_l^\star$ for all $l$, we can rewrite (15) in matrix form as $(\mathbf{I} - \mathbf{A})\mathbf{x}^\star = \mathbf{1}$ where $\mathbf{1}$ is a vector of appropriate dimension whose entries are all ones, $\mathbf{I}$ is the identity matrix and $\mathbf{A}$ is the matrix with entries:

$$A_{ij} = \begin{cases} 0, & \text{if } i = j \\ \frac{G_{ji}\gamma_j}{G_{jj}}, & \text{if } i \neq j \end{cases} \qquad (16)$$

where $i, j \in \{1, 2, \ldots, L\}$. Notice that we have implicitly used the fact that $\mathsf{SIR}_l(\mathbf{p}^\star) = \gamma_l$ for all $l$ in (16).

We first particularize the uplink channel gain matrices $\mathbf{G}$ to a vector $\mathbf{g}$ where $G_{ji} = g_i$ for all $j$, as in [7] (effectively assuming a code correlation coefficient of 1 for all users). Now, $\mathbf{A}$ in (16) can be simplified as

$$\mathbf{A}_g = \begin{bmatrix} 0 & g_1\gamma_2/g_2 & g_1\gamma_3/g_3 & \ldots & g_1\gamma_L/g_L \\ g_2\gamma_1/g_1 & 0 & g_2\gamma_3/g_3 & \ldots & g_2\gamma_L/g_L \\ \vdots & \ddots & \ddots & & \vdots \\ g_L\gamma_1/g_1 & \ldots & \ldots & & 0 \end{bmatrix} \quad (17)$$

which permits the use of Gaussian elimination on $(\mathbf{I} - \mathbf{A}_g)\mathbf{x}^\star = \mathbf{1}$ to yield a closed form solution for $\boldsymbol{\nu}^\star$ as follows.

*Corollary 1:* If we assume a code correlation coefficient of 1 for all users, then

$$p_l^\star = \frac{n_l\gamma_l}{g_l}\frac{1 - \sum_j \frac{\gamma_j}{1+\gamma_j}(n_l - n_j)}{(1+\gamma_l)(1 - \sum_j \frac{\gamma_j}{1+\gamma_j})} \quad \forall\, l, \qquad (18)$$

and

$$\nu_l^\star = p_l^\star\frac{1 - \sum_j \frac{\gamma_j}{1+\gamma_j}(1 - \frac{g_j}{g_l})}{(1+\gamma_l)(1 - \sum_j \frac{\gamma_j}{1+\gamma_j})} \quad \forall\, l. \qquad (19)$$

Now, coming back to the general case, to solve for $\boldsymbol{\nu}^\star$ in (15), it is interesting to note that $(\mathbf{I} - \mathbf{A})\mathbf{x}^\star = \mathbf{1}$ can be solved using the Perron-Frobenius theorem in a way similar to solving the power control problem in (8) [8], [13], [14]. The optimal power $\mathbf{p}^\star$ in (8) satisfies $(\mathbf{I} - \mathbf{F})\mathbf{p}^\star = \mathbf{v}$ where $\mathbf{F}$ is the matrix with entries [13]:

$$F_{ij} = \begin{cases} 0, & \text{if } i = j \\ \frac{G_{ij}\gamma_i}{G_{ii}}, & \text{if } i \neq j \end{cases} \qquad (20)$$

and

$$\mathbf{v} = \left(\frac{\gamma_1 n_1}{G_{11}}, \frac{\gamma_2 n_2}{G_{22}}, \ldots, \frac{\gamma_L n_L}{G_{LL}}\right)^T.$$

Now, the algorithm in (9) proposed by Foschini and Miljanic can be expressed as [13]:

$$\mathbf{p}(k+1) = \mathbf{F}\mathbf{p}(k) + \mathbf{v} \qquad (21)$$

where $k = 1, 2, \ldots,$ which converges asymptotically to $\mathbf{p}^\star$ (when that exists, which happens if and only if the simple eigenvalue of largest modulus of $\mathbf{F}$, denoted as $\rho(\mathbf{F})$, is strictly less than 1).

Also based on the Perron-Frobenius theorem, we propose the following algorithm, which will be used in Section III-E:

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{1} \tag{22}$$

where $k = 1, 2, \ldots,$ which converges asymptotically to $\mathbf{x}^\star$ (if and only if $\rho(\mathbf{A}) < 1$). Note that $\mathbf{A} = \mathbf{F}^T$. Indeed, we have $\mathbf{x}(k) = \mathbf{A}^k\mathbf{x}(0) + [\sum_{\tau=0}^{k-1}\mathbf{A}^\tau]\mathbf{1}$, which results in the limit

$$\begin{aligned}\lim_{k\to\infty}\mathbf{x}(k) &= \lim_{k\to\infty}\mathbf{A}^k\mathbf{x}(0) + \lim_{k\to\infty}\left[\sum_{\tau=0}^{k-1}\mathbf{A}^\tau\right]\mathbf{1}\\ &= \mathbf{0} + \left[\sum_{\tau=0}^{\infty}\mathbf{A}^\tau\right]\mathbf{1}\\ &= (\mathbf{I}-\mathbf{A})^{-1}\mathbf{1}.\end{aligned} \tag{23}$$

Lastly,

$$\nu_l(k+1) = x_l(k+1)p_l(k+1) \tag{24}$$

for all $l$ converges to $\boldsymbol{\nu}^\star$ in (15). However, unlike (21), which can be computed distributively as given by (9), (22) cannot be computed distributively as each user needs the cross-channel gains in (22).

It is worth noting that $\mathbf{x}^\star$ is the optimal solution to

$$\begin{array}{ll}\text{maximize} & \mathbf{v}^T\mathbf{x}\\ \text{subject to} & (\mathbf{I}-\mathbf{A})\mathbf{x} \succeq \mathbf{1},\end{array} \tag{25}$$

which is the dual linear program to

$$\begin{array}{ll}\text{minimize} & \mathbf{1}^T\mathbf{p}\\ \text{subject to} & (\mathbf{I}-\mathbf{F})\mathbf{p} \succeq \mathbf{v},\end{array} \tag{26}$$

which is simply (8) rewritten as a linear program in matrix form. Thus, an easy consequence of computing (21), (22) and (24) is that if $\mathbf{p}(k)$ and $\mathbf{x}(k)$ are feasible in (26) and (25) respectively, then $\mathbf{p}(k)$ and $\boldsymbol{\nu}(k)$ are primal and dual feasible with respect to (12) respectively, and $\mathbf{v}^T\mathbf{x}(k) \le \mathbf{1}^T\mathbf{p}(k)$.

Interestingly, based on the above results, a 'duality by mapping' in the algorithms between computing the optimal power vector $\mathbf{p}^\star$ and the Lagrange multiplier vector $\boldsymbol{\nu}^\star$ using the Perron-Frobenius theorem can be summarized as follows.

**Perron-Frobenius Theorem Duality Correspondence**

$$\begin{array}{ccc}\text{columns of } \mathbf{A} & \leftrightarrow & \text{rows of } \mathbf{F}\\ \mathbf{1} & \leftrightarrow & \mathbf{v}\\ \mathbf{x}^\star \text{ with } \nu_l^\star = x_l^\star p_l^\star & \leftrightarrow & \mathbf{p}^\star\\ \mathbf{v}^T\mathbf{x}^\star & \leftrightarrow & \mathbf{1}^T\mathbf{p}^\star\end{array}$$

### D. Robust power control formulation

In the following, we develop a system model for uplink transmission in a CDMA cell that takes robustness into account. By taking the protection margin in (10) into account, we consider minimizing the total power expenditure in the following problem

$$\begin{array}{ll}\text{minimize} & \sum_l p_l + \phi(\epsilon)\\ \text{subject to} & \mathsf{SIR}_l(\mathbf{p}) \ge \gamma_l(1+\epsilon)\quad\forall\, l,\\ & \epsilon \ge 0, p_l \ge 0\quad\forall\, l,\\ \text{variables:} & p_l\quad\forall\, l,\ \epsilon\end{array} \tag{27}$$

where $\phi(\epsilon)$ is a decreasing convex function that captures the tradeoff in adjusting the parameter $\epsilon$. A larger $\epsilon$ provides more protection to existing users, but comes at a price of higher power expenditure. At the same time, there are also operational disadvantages associated with both large $\epsilon$ (such as power growing unbounded due to an infeasible $\epsilon$) and small $\epsilon$ (such as slow user admission and convergence rate) [8]. The goal of optimizing $\epsilon$ in (27) is to remove these disadvantages in the original DPC/ALP algorithm in [8]. The first constraint is the requirement that the received SIR be above a certain SIR threshold, which in turn is related to the protection margin. Note that if we ignore $\phi(\epsilon)$ and set $\epsilon = 0$, (27) reduces to (8).

Compared to Problem (8), Problem (27) is much harder to solve, since it involves a nonconvex constraint set over $p_l$ for all $l$ and $\epsilon$. But, observe that (27) has nonconvex product form constraints. Hidden convexity, as elaborated in Section I-D, can be unveiled by transforming (27) to a convex optimization problem (Note that (27) cannot be rewritten as a linear program as can be done with (8)). First, we apply a log transformation to $p_l$ for all $l$ and $\epsilon$, and we obtain the following equivalent problem

$$\begin{array}{ll}\text{minimize} & \sum_l e^{\tilde{p}_l} + \phi(e^{\tilde{\epsilon}})\\ \text{subject to} & \log(\mathsf{SIR}_l(\tilde{\mathbf{p}})/\gamma_l) \ge \log(1+e^{\tilde{\epsilon}})\quad\forall\, l,\\ \text{variables:} & \tilde{p}_l\quad\forall\, l,\ \tilde{\epsilon}.\end{array} \tag{28}$$

*Lemma 3:* The optimization problem in (28) is convex (with strictly feasible solutions) if $\frac{\partial^2\phi(z)/\partial z^2}{\partial\phi(z)/\partial z} \ge -1/z$.

*Proof:* From the second derivative of $\phi(e^{\tilde{\epsilon}})$, it is convex if and only if the condition in Lemma 3 is true. The constraint set in (28) is convex since $\log(\mathsf{SIR}_l(\tilde{\mathbf{p}}))$ is concave in $\tilde{\mathbf{p}}$ for all $l$ and $\log(1+e^{\tilde{\epsilon}})$ is convex in $\tilde{\epsilon}$. ∎

We show a useful choice of $\phi(\epsilon)$ that satisfies Lemma 3 in Section III-G.

### E. Main algorithm

Exploiting the above property of the Lagrange multipliers $\boldsymbol{\nu}^\star$ in Section III-C, we propose the following algorithm to solve (27) based on the Lagrangian decomposition theory.

---

**Algorithm E-DPC/ALP**—*Enhanced* Distributed Power Control with Power-sensitive Active Link Protection:

- The base station initiates a $\epsilon(0)$. New users power up with sufficiently small $p_l(0)$, e.g., $p_l(0) = n_l$.

**Update by user $l$ for all $l$:**

- updates the transmitter powers $p_l(k+1)$ at the $(k+1)$th step according to the following rule:

$$p_l(k+1) = \begin{cases}\frac{(1+\epsilon(k))\gamma_l}{\mathsf{SIR}_l(k)}p_l(k), & \text{if } \mathsf{SIR}_l(k) \ge \gamma_l\\ (1+\epsilon(k))p_l(k), & \text{if } \mathsf{SIR}_l(k) < \gamma_l\end{cases} \tag{29}$$

**Update by the base station:**

- computes $x_l(k+1)$, the $l$th component of $\mathbf{x}(k+1)$, using

$$\mathbf{x}(k+1) = (1+\epsilon(k))\mathbf{A}\mathbf{x}(k) + \mathbf{1}, \tag{30}$$

- computes

$$\nu_l(k+1) = x_l(k+1)p_l(k+1)\quad\forall\, l, \tag{31}$$

- updates $\epsilon(k+1)$ by solving

$$-\left.\frac{\partial\phi(\epsilon)}{\partial\epsilon}\right|_{\epsilon=\epsilon(k+1)}(1+\epsilon(k+1)) = \mathbf{1}^T\boldsymbol{\nu}(k+1). \quad (32)$$

We have the following main theorem for the above algorithm.

*Theorem 5:* If $\{\epsilon(k)\}$ converges to a finite value $\epsilon^\star$, and $\rho(\mathbf{F}) < \dfrac{1}{1+\epsilon^\star}$, then Algorithm E-DPC/ALP converges to a global optimum $(\mathbf{p}^\star, \epsilon^\star)$ of the nonconvex problem (27).

*Remark 3:* It is important to note that the power update (29) differs from DPC/ALP in [8] in that $\epsilon$ is not a constant parameter as in [8], but varies with each iteration.

*Remark 4:* Interestingly, (32) quantifies the remark on choosing the parameter $\epsilon$ in [8]: $\epsilon$ should be chosen such that $(1+\epsilon)$ should be larger when the network is uncongested, so that (new) users power up fast, and grow smaller as congestion builds up to have more users power up more gently.

### F. Proof of Theorem 5

We introduce nonnegative Lagrange multipliers $\nu_l$ for all $l$ for the SIR constraints in (28), and write the Lagrangian

$$\begin{aligned} L(\tilde{\mathbf{p}}, \tilde{\epsilon}, \boldsymbol{\nu}) &= \textstyle\sum_l e^{\tilde{p}_l} - \sum_l \nu_l(\log(\mathsf{SIR}_l(\tilde{\mathbf{p}})/\gamma_l)) \\ &\quad + \phi(e^{\tilde{\epsilon}}) + (\textstyle\sum_l \nu_l)\log(1+e^{\tilde{\epsilon}}). \end{aligned}$$

We can minimize the above Lagrangian to obtain the Lagrange dual function

$$g(\boldsymbol{\nu}) = \inf_{\tilde{\mathbf{p}},\tilde{\epsilon}} L(\tilde{\mathbf{p}}, \tilde{\epsilon}, \boldsymbol{\nu}).$$

Next, $g(\boldsymbol{\nu})$ can be obtained by a Lagrangian decomposition:

$$g(\boldsymbol{\nu}) = g_{alp}(\boldsymbol{\nu}) + g_{dpc}(\boldsymbol{\nu})$$

where $g_{alp}(\boldsymbol{\nu})$ and $g_{dpc}(\boldsymbol{\nu})$ are, respectively, the optimal values of the objective function in the following subproblem of ALP

$$\begin{array}{ll} \text{minimize} & \phi(e^{\tilde{\epsilon}}) + (\sum_l \nu_l)\log(1+e^{\tilde{\epsilon}}) \\ \text{variables:} & \tilde{\epsilon} \end{array} \quad (33)$$

and the subproblem of DPC

$$\begin{array}{ll} \text{minimize} & \sum_l (e^{\tilde{p}_l} - \nu_l \log(\mathsf{SIR}_l(\tilde{\mathbf{p}})/\gamma_l)) \\ \text{variables:} & \tilde{p}_l \ \ \forall\, l. \end{array} \quad (34)$$

Solving (28) is equivalent to solving its Lagrange dual problem over $\boldsymbol{\nu}$:

$$\begin{array}{ll} \text{maximize} & g_{alp}(\boldsymbol{\nu}) + g_{dpc}(\boldsymbol{\nu}) \\ \text{subject to} & \nu_l \geq 0 \ \ \forall\, l. \end{array} \quad (35)$$

The solution of the subproblem ALP is obtained by solving the following fixed point equation for a given $\boldsymbol{\nu}$:

$$-\frac{\partial\phi(\epsilon)}{\partial\epsilon}(1+\epsilon) = \mathbf{1}^T\boldsymbol{\nu}. \quad (36)$$

The subproblem DPC has a solution that is intimately related to that of (8). In particular, $\boldsymbol{\nu}^\star$ and the optimal solution $\mathbf{p}^\star$ to (34) satisfy (15). To compute $\mathbf{p}^\star$ in (34) and $\boldsymbol{\nu}^\star$ in (35), the algorithms in (21), (22) and (24) are, respectively, modified to

$$\mathbf{p}(k+1) = (1+\epsilon(k))\left(\mathbf{F}\mathbf{p}(k) + \mathbf{v}\right), \quad (37)$$

$$\mathbf{x}(k+1) = (1+\epsilon(k))\mathbf{A}\mathbf{x}(k) + \mathbf{1}, \quad (38)$$

and

$$\nu_l(k+1) = x_l(k+1)p_l(k+1) \ \ \forall\, l. \quad (39)$$

Assuming that $\lim_{k\to\infty} \epsilon(k) \to \epsilon^\star$ in (36), and $\rho(\mathbf{A}) < \dfrac{1}{1+\epsilon^\star}$, then $\lim_{k\to\infty} \mathbf{p}(k) \to \mathbf{p}^\star$ and $\lim_{k\to\infty} \boldsymbol{\nu}(k) \to \boldsymbol{\nu}^\star$ in (37) and (39) respectively.

### G. Further discussions on choosing $\phi(\epsilon)$

In the following, we illustrate the importance of Theorem 3 and 5 on Algorithm E-DPC/ALP (as well as DPC/ALP [8]). Two interesting tradeoffs in E-DPC/ALP are:

1. What is the increase in extra power needed to cushion active users?

2. How aggressively can we dynamically tune E-DPC/ALP so that new users power up faster when traffic load increases?

The key to the first question is clearly Theorem 3, i.e., the extra power needed is $(\mathbf{1}^T\boldsymbol{\nu}^\star\epsilon^\star/\mathbf{1}^T\mathbf{p}^\star)$ percent or, from (36),

$$-\left.\frac{\partial\phi(\epsilon)}{\partial\epsilon}\right|_{\epsilon=\epsilon^\star}(1+\epsilon^\star)\epsilon^\star/\mathbf{1}^T\mathbf{p}^\star \quad (40)$$

percent. A network operator with a good estimate of (40) takes heed not to admit new users if the network operating point is given by the second column of Table I as this more than doubles the power expenditure with just three percent safety margin! This critical operating point is accounted by the empirical observation in [10] where tremendous total power saving is observed when the network scenario changes from the second column to the third column of Table I.

A particular choice of $\phi(\epsilon)$ that captures the tradeoff between the extra power needed and setting a large provision margin can be derived as follows. Suppose the network can tolerate at most an increase of $\delta/\mathbf{1}^T\mathbf{p}^\star$ percent in total power (to limit both intra-cell and inter-cell interference), then, from (40), this implies

$$\frac{\partial\phi(\epsilon)}{\partial\epsilon} = -\frac{\delta}{\epsilon(1+\epsilon)}, \quad (41)$$

which upon integration yields

$$\phi(\epsilon) = \delta\log(1+1/\epsilon). \quad (42)$$

It is easy to verify that $\phi(\epsilon)$ is strictly convex decreasing and satisfies Lemma 3. An upper bound on the relative change $\delta/\mathbf{1}^T\mathbf{p}^\star$ can instead be used as an input to Algorithm E-DPC/ALP, and simply replacing $\nu_l(k)$ in Algorithm E-DPC/ALP by $\nu_l(k)/\mathbf{1}^T\mathbf{p}(k)$.

By a suitable choice of relating $\epsilon$ and the aggregate 'price' $\mathbf{1}^T\boldsymbol{\nu}^\star$, the function $\phi(\epsilon)$ in (32) can be determined. The key to the second question is thus a suitable reverse engineering approach applied to the underlying protocol dynamics that may be integrated with E-DPC/ALP. It turns out that a useful family of $\phi(\epsilon)$ for $\epsilon \in (0,1]$ can be characterized by

$$\phi(\epsilon) = \delta\left(\sum_{i=1}^{q}(-1)^{q-i}\epsilon^{-i}/i + \log(1+1/\epsilon)\right), \quad (43)$$

parameterized by a nonnegative integer $q$. This $\phi(\epsilon)$ given by (43) also satisfies Lemma 3. We want, at optimality, $\epsilon^\star = (\delta/\mathbf{1}^T\boldsymbol{\nu}^\star)^{1/(q+1)}$, which means $\partial\phi(\epsilon)/\partial\epsilon = -\delta/(\epsilon^{q+1}(1+$

TABLE II

A COMPARISON BETWEEN DPC/ALP AND E-DPC/ALP ALGORITHM

| *The DPC/ALP algorithm [8]* | *Our E-DPC/ALP algorithm* |
|---|---|
| Primal based algorithm | Primal and Dual based algorithm |
| Heuristics in tuning $\epsilon$ | Tuning $\epsilon$ based on congestion |
| Does not discuss the extra power spent in exchange for robustness | Can quantify the extra power spent in exchange for robustness |
| Distributed power control without adaptive tuning of $\epsilon$ | Distributed power control with centralized computation to tune $\epsilon$ |

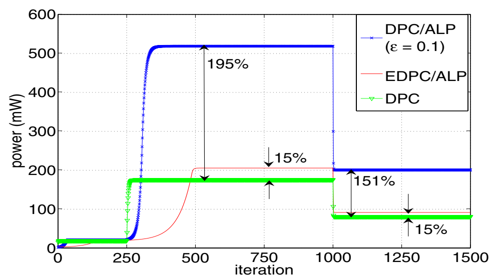


Fig. 1. Evolution of the total power with DPC/ALP, E-DPC/ALP and DPC algorithms. The percentage increases in extra power between (i) DPC/ALP and DPC, and (ii) E-DPC/ALP and DPC at two different operating points (i.e., iteration 250-1000 and 1000-1500) are shown.

$\epsilon$)), which yields (43) after integration. A larger $q$ in (43) thus models more aggressive protection margin, but also allows new users to power up faster with increasing traffic load. Hence, different reactions to the 'price' by users can thus be modeled.

Lastly, we highlight the key differences between Algorithm E-DPC/ALP and Algorithm DPC/ALP in [8] in Table II.

## IV. NUMERICAL EXAMPLES

We evaluate the performance of Algorithm E-DPC/ALP with a series of experiments.

### A. Expt. 1 (Balancing power tradeoff with protection margin)

We illustrate the dynamic power-sensitive property of Algorithm E-DPC/ALP by comparing with Algorithm DPC/ALP that uses a fixed $\epsilon = 0.1$ in a CDMA cell with three users. Using $\phi(\epsilon)$ in (42), E-DPC/ALP is configured with $\delta$ to have at most 15 percent total power increment, and the $\epsilon^\star$ computed is 0.021. From time 0-250s, only User 1 and 2 are active. From time 250s, User 3 becomes active. At time 1000, User 1 completes data transmission, and departs from the cell leaving User 2 and 3. Figure 1 shows that Algorithm DPC/ALP results in a total power expenditure of more than 150 percent at the two network operating points at time 250 and 1000 as compared to DPC, whereas Algorithm E-DPC/ALP uses an additional extra total power of 15 percent. As compared to E-DPC/ALP and DPC/ALP, DPC has the fastest convergence to the SIR thresholds. However, in the case of DPC, the received SIR's of User 1 and 2 are observed to suffer a dip of more than 40 and 60 percent respectively at time 250 when User 3 enters the channel. In contrast, this does not happen with E-DPC/ALP and DPC-ALP.

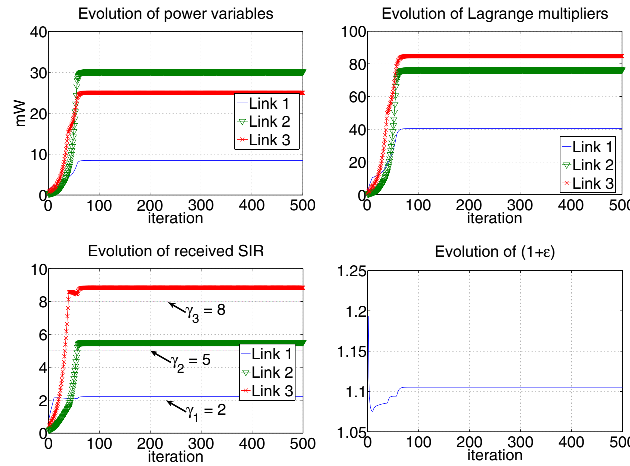


Fig. 2. A typical numerical example of power and protection margin allocation using Algorithm E-DPC/ALP in a CDMA cell with three users. The top left graph shows the power variables $\mathbf{p}$. The top right graph shows the Lagrange multipliers $\boldsymbol{\nu}$. The bottom left graph shows the achieved SIR with the thresholds, and the bottom right graph shows the parameter $\epsilon$.

### B. Expt. 2 (Power and protection margin using E-DPC/ALP)

A series of simulations are conducted to show the performance of Algorithm E-DPC/ALP in a CDMA cell with three users. We use the SIR thresholds $\boldsymbol{\gamma} = (2, 5, 8)^T$. We update $\epsilon$ using the $\phi(\epsilon)$ in (42) and select $\delta$ such that at most an extra total power of $33.3\%$ is used. We observe that E-DPC/ALP converges to the global optimum that meets the percent increase specified by $\delta$ with different input parameters by varying the channel gains and noise powers. We update $\mathbf{p}$, $\boldsymbol{\nu}$ and $\epsilon$ at each iteration. Figure 2 shows the evolution of the variables of the three users with time for a typical numerical example.

### C. Expt. 3 (Accelerate convergence speed in E-DPC/ALP)

In this experiment, we propose a simple heuristic to achieve the goal of Experiment 2 with a faster convergence speed. We use the SIR thresholds $(2, 5, 8)^T$ as in Experiment 2. Note that $\phi(\epsilon)$ in (42) is a special case of (43) for $q = 0$. The heuristic is simply to initialize a moderately large $q$ in (43) that decrements by 1 at every update of $\epsilon$ using $\phi(\epsilon)$ in (43) till $q = 0$ is reached, whereupon $\epsilon$ is updated using $\phi(\epsilon)$ in (42) thereafter. Hence, the goal of Experiment 2 in achieving the SIR thresholds with a $33.3\%$ increase in extra power is achieved after convergence. Figure 3 shows the convergence speed of the heuristic with different initial $q$'s, and the baseline case of $q = 0$ (without the heuristic) is also included for comparison. As shown in Figure 3, a larger $q$ allows all the received SIR's to rise and stay above the SIR thresholds in fewer iterations than in the baseline case.

### D. Expt. 4 (Different timescale update in E-DPC/ALP)

In our previous experiments, we update $(\mathbf{p}, \boldsymbol{\nu}, \epsilon)$ at the same timescale. In this experiment, we repeat Experiment 2, but the base station updates $(\boldsymbol{\nu}, \epsilon)$ at different slower timescales as compared to the timescale of power update by each user. In

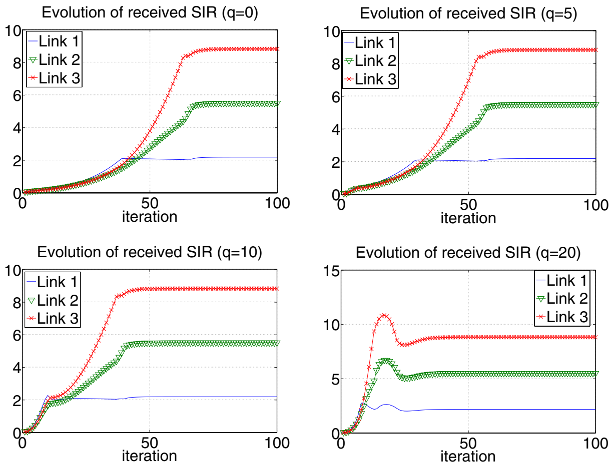


Fig. 3. Accelerating convergence speed in the Algorithm E-DPC/ALP in a CDMA cell with three users using $\phi(\epsilon)$ in (43) for different initial choice of $q$. The top left graph shows the baseline case for $q = 0$, i.e., the heuristic is not used. The titles of the other graphs indicate the initial $q$ used in the heuristic. As shown, in the case of $q = 20$, only 10 iterations are required to satisfy all the SIR thresholds, whereas roughly 70 iterations are required in the baseline case. The heuristic with a larger initial $q$ converges faster.

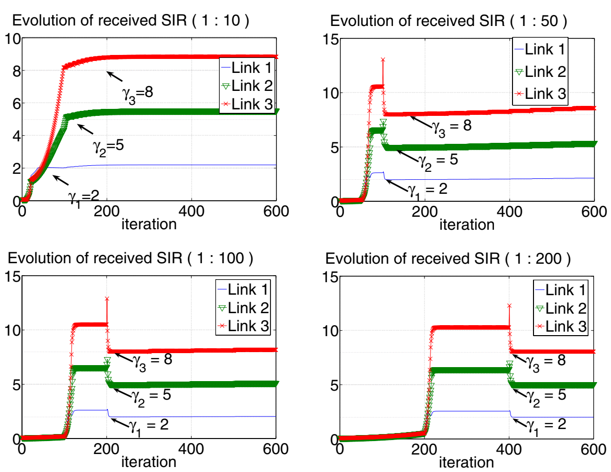


Fig. 4. A typical numerical example of using Algorithm E-DPC/ALP at different timescales of $(\boldsymbol{\nu}, \epsilon)$ update. An iteration on the x-axis of each graph refers to a power update iteration. The top left graph shows an update of $(\boldsymbol{\nu}, \epsilon)$ for every 10 iterations $(1 : 10)$. The top right graph shows an update of $(\boldsymbol{\nu}, \epsilon)$ for every 50 iterations $(1 : 50)$. The bottom left graph shows an update of $(\boldsymbol{\nu}, \epsilon)$ every 100 iterations $(1 : 100)$, and the bottom right graph shows an update of $(\boldsymbol{\nu}, \epsilon)$ every 200 iterations $(1 : 200)$.

practice, the base station needs a relatively long time interval to measure the channel gains in order to update $(\boldsymbol{\nu}, \epsilon)$. Figure 4 shows the convergence of the received SIR's for different timescale updates of $(\boldsymbol{\nu}, \epsilon)$. Interestingly, a slower timescale update of $(\boldsymbol{\nu}, \epsilon)$ may not mean a slower convergence to the SIR thresholds $\boldsymbol{\gamma}$, but only to the enhanced SIR thresholds $(1 + \epsilon^{\star})\boldsymbol{\gamma}$. As shown in Figure 4, all the received SIR's rise and stay above the SIR thresholds $\boldsymbol{\gamma}$ after the second update of $(\boldsymbol{\nu}, \epsilon)$ before converging to the enhanced SIR thresholds for the case of (1:50), (1:100) and (1:200).

## V. Conclusions

We have considered a wide variety of seemingly nonconvex resource allocation problems in FDMA and interference limited CDMA channels with QoS constraints, and showed that, despite the nonconvexity and conflicting goals to be optimized, flexible resource control can be achieved. Furthermore, we also demonstrate that, by systematically unveiling hidden convexity in a seemingly nonconvex CDMA uplink transmission problem, the tradeoff between achieving global optimality and attaining robustness in a system can be balanced and quantified. In particular, Algorithm E-DPC/ALP can easily integrate with and strengthen the robustness of the existing power control scheme used in a CDMA network.

Future work includes extending the results to CDMA cells with transmit beamforming [16] and to the case of intentional power reduction (mirroring the use of $\epsilon$ in E-DPC/ALP but with $\epsilon < 0$) by exploiting the fact that data applications often have time-varying BER requirements. Algorithm E-DPC/ALP can possibly be made distributed in a time division duplex CDMA scheme by exploiting the Perron-Frobenius theorem duality correspondence in Section III-C and the uplink-downlink duality in [16].


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