1.

First change $\Gamma$ to CNF and standardize apart variables

1. $Man(Marcus)$ Given
2. $Roman(Marcus)$ Given
3. $\neg Man(x) \vee Person(x)$ Given
4. $Ruler(Caesar)$ Given
5. $\neg Roman(x_1) \vee Loyal(x_1, Caesar) \vee Hate(x_1, Caesar)$ Given
6. $Loyal(x_2, f(x_2))$ Given
7. $\neg Person(x_3) \vee \neg Ruler(y) \vee \neg Tryassasin(x_3, y) \vee \neg Loyal(x_3, y)$ Given
8. $Tryassasin(Marcus, Caesar)$ Given

Next negate what is to be proven, convert to CNF and standardize apart variables

$\neg \exists x\ Hate(x, Caesar) \equiv \forall x \neg Hate(x, Caesar)$

9. $\neg Hate(x_4, Caesar)$

Perform resolution

10. $\neg Roman(x_5) \vee Loyal(x_5, Caesar)$ 5, 9 $\{x_1/x_4\} \rightarrow x_5$
11. $Loyal(Marcus, Caesar)$ 2, 10, $\{x_5/Marcus\}$
12. $\neg Person(Marcus) \vee \neg Ruler(Caesar) \vee \neg Tryassasin(Marcus, Caesar)$ 7, 11, $\{x_3/Marcus, y/Caesar\}$

continued on next page

13. Person(Marcus) 1, 3, {x/Marcus}

14. ¬Ruler(Caesar) ∨ ¬Tryassasin(Marcus, Caesar) 12, 13

15. ¬Tryassasin(Marcus, Caesar) 4, 14

16. □ 8, 15

line 13 could have been

13a. ¬Man(Marcus) ∨ ¬Ruler(Caesar)
∨ ¬Tryassasin(Marcus, Caesar)
3, 12 {x/Marcus}

14. ¬Ruler(Caesar) ∨ ¬Tryassasin(Marcus, Caesar) 1, 13a

2. Learned clause:

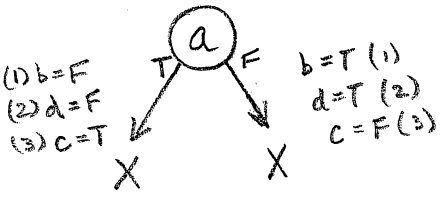
$(\neg X_4 \vee X_8 \vee X_9)$

Backtrack to:

node $X_9$

3.

| | a=T | a=F |
|---|---|---|
| a ∨ b | ✓ | (1) ✓ |
| ¬a ∨ ¬b | (1) ✓ | ✓ |
| c ∨ d | | |
| ¬c ∨ ¬d | | |
| a ∨ d | ✓ | (2) ✓ |
| ¬a ∨ ¬d | (2) ✓ | ✓ |
| b ∨ c | (3) ✓ | ✓ |
| ¬b ∨ ¬c | ✓ | (3) ✓ |
| ¬a ∨ ¬c | X | ✓ |
| ¬b ∨ ¬d | | X |




(could have chosen b, c, or d as first decision node; the tree would look the same but the unit propagation would be different)

The numbers give the order of unit values
The check marks say clause is true
The X shows the clause that is false
(Single scan through the clauses so some clauses are not marked with a ✓ even though they are true with the valuation on the branch)

4. For the 3×3 board labelled:

$$\begin{matrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{matrix}$$

3 rows

$$\begin{cases} (a_{11} \vee a_{12} \vee a_{13}) \wedge \neg(a_{11} \wedge a_{12}) \wedge \neg(a_{11} \wedge a_{13}) \wedge \neg(a_{12} \wedge a_{13}) \wedge \\ (a_{21} \vee a_{22} \vee a_{23}) \wedge \neg(a_{21} \wedge a_{22}) \wedge \neg(a_{21} \wedge a_{23}) \wedge \neg(a_{22} \wedge a_{23}) \wedge \\ (a_{31} \vee a_{32} \vee a_{33}) \wedge \neg(a_{31} \wedge a_{32}) \wedge \neg(a_{31} \wedge a_{33}) \wedge \neg(a_{32} \wedge a_{33}) \wedge \end{cases}$$

3 columns

$$\begin{cases} (a_{11} \vee a_{21} \vee a_{31}) \wedge \neg(a_{11} \wedge a_{21}) \wedge \neg(a_{11} \wedge a_{31}) \wedge \neg(a_{21} \wedge a_{31}) \wedge \\ (a_{12} \vee a_{22} \vee a_{32}) \wedge \neg(a_{12} \wedge a_{22}) \wedge \neg(a_{12} \wedge a_{32}) \wedge \neg(a_{22} \wedge a_{32}) \wedge \\ (a_{13} \vee a_{23} \vee a_{33}) \wedge \neg(a_{13} \wedge a_{23}) \wedge \neg(a_{13} \wedge a_{33}) \wedge \neg(a_{23} \wedge a_{33}) \wedge \end{cases}$$

6 diagonals ↗ ↘

$$\begin{cases} \neg(a_{21} \wedge a_{12}) \wedge \neg(a_{31} \wedge a_{22}) \wedge \neg(a_{31} \wedge a_{13}) \wedge \neg(a_{22} \wedge a_{13}) \wedge \neg(a_{32} \wedge a_{23}) \wedge \\ \neg(a_{21} \wedge a_{32}) \wedge \neg(a_{11} \wedge a_{22}) \wedge \neg(a_{11} \wedge a_{22}) \wedge \neg(a_{22} \wedge a_{33}) \wedge \neg(a_{12} \wedge a_{23}) \wedge \end{cases}$$

Conversion to CNF is straightforward — just apply DeMorgan's Law to each negated conjunct, for example:

$\neg(a_{11} \wedge a_{12})$ becomes $(\neg a_{11} \vee \neg a_{12})$

Give this CNF to the MiniSat solver using the Python wrapper

satispy

and the answer is "false"