91

Cs 2209a - Assignment2

Student Name: MingCong, Zhou

Student Number: 250945414

Student Account: mzhou272

Department of Computer Science

CS 2209A — Applied Logic for Computer Science

Assignment 2

Assigned: 29 October 2018
Due: 5 November 2018 (on OWL before midnight)

1. Convert the following First Order sentences to Conjunctive Normal Form. Show all intermediate conversion steps. Include steps that are required to incorporate the clauses into an empty set ready to do resolution.

   (a) $\exists x \forall y L(x, y)$

   x depends on 0 var, f() = c

   $\equiv \forall y \exists x L(x, y)$
   $\equiv \forall y L(f(y), y)$
   $\equiv L(f(y), y)$

   (b) $\forall x \exists y L(y, x)$

   $\equiv \forall x L(f(x), x)$
   $\equiv L(f(x), x)$

   (c) $\forall z\{Q(z) \Rightarrow \{\neg \forall x \exists y[P(y) \Rightarrow P(g(z,x))]\}\}$

   $\equiv \forall z\{\ \neg Q(z) \vee \{\exists x \neg \exists y[\neg P(y) \vee P(g(z,x))]\}\ \}$
   $\equiv \forall z\{\ \neg Q(z) \vee \{\exists x \forall y \neg[\neg P(y) \vee P(g(z,x))]\}\ \}$
   $\equiv \forall z\{\ \neg Q(z) \vee \{\exists x \forall y [P(y) \wedge \neg P(g(z,x))]\}\ \}$
   $\equiv \forall z\{\ \neg Q(z) \vee \{\forall y \exists x[P(y) \wedge \neg P(g(z,x))]\}\ \}$ −2
   $\equiv \forall z\{\ \neg Q(z) \vee \{\forall y[P(y) \wedge \neg P(g(z, f(y,z)))]\}\ \}$ −1
   // ∵ $\forall y$ is in the scope of the $\forall z$ ∴ drop it
   $\equiv \forall z \forall y\{\ \neg Q(z) \vee [P(y) \wedge \neg P(g(z, f(y)))]\ \}$
   $\equiv \neg Q(z) \vee [P(y) \wedge \neg P(g(z, f(y)))]$
   $\equiv (\neg Q(z) \vee P(y)) \wedge (\neg Q(z) \vee \neg P(g(z, f(y)))$ −2 make clauses, separate variables

   15/20

2. With the clauses given by Γ show $\Gamma \vdash \exists x \neg B(x)$ using resolution.

$$\Gamma = \Big\{\neg B(x) \vee C(x),\quad \neg C(a) \vee D(b),\quad \neg C(c) \vee E(d),\quad \neg D(w) \vee \neg E(y)\Big\}$$

∵ we need to prove $\exists x \neg B(x)$ using resolution
∴ we can get the ¬ of it as a clause
$\neg \exists x \neg B(x) \equiv \forall x B(x) \equiv B(x)$

1. $\neg B(x) \vee C(x)$
2. $\neg C(a) \vee D(b)$
3. $\neg C(c) \vee E(d)$
4. $\neg D(w) \vee \neg E(y)$
5. B(x) −1
6. $\neg B(a) \vee D(b)$ 1,2 {x/a}
7. $\neg B(a) \vee \neg E(y')$ 4,6 {w/b} −1
8. $\neg B(a) \vee \neg C(a)$ 3,7 {y/d} −1
9. $\neg B(a)$ 1,8 {x/a}
10. $\square$ 9,5 {x/a}

17/20

3. Show using resolution that the following statement is valid.

$$\forall x P(x) \rightarrow \exists y P(y)$$

$F = \forall x P(x) \rightarrow \exists y P(y)$

~~//Equivalence~~
~~$F = \forall x P(x) \rightarrow \exists y P(y)$~~
~~$= \neg\forall x P(x) \vee \exists y P(y)$~~
~~$= \exists x \neg P(x) \vee \exists y P(y)$~~
~~//Existential instantiation~~
~~$= \neg P(a) \vee P(b)$~~

Negate F & get rid of "→" & quantifiers:
$\neg(\ \forall x P(x) \rightarrow \exists y P(y)\ )$
$\neg(\ \forall x P(x) \rightarrow \exists y P(y)\ )$
$\neg(\ \neg\forall x P(x) \vee \exists y P(y)\ )$
$\neg(\ \exists x \neg P(x) \vee \exists y P(y)\ )$
$(\ \neg\exists x \neg P(x) \wedge \neg\exists y P(y)\ )$
$(\ \forall x P(x) \wedge \forall y \neg P(y)\ )$
$(\ P(x) \wedge \neg P(y)\ )$ ✓

Clauses:
1. $P(x)$
2. $\neg P(y)$ ∃ x/y
3. ~~$\neg P(a) \vee P(b)$~~
4. ~~$P(b)$ 1,3 {x/a}~~
5. ~~□ 2,4 {y/b} therefore valid~~

18/20

4. Show a bottom up derivation of $\Gamma \vdash R(a1, a1)$ using the bottom up proof algorithm given the definite clauses (written as head $\leftarrow$ body) in set Γ.

$$\Gamma = \left\{ \begin{array}{ll} P(a2, a1) & \\ P(a3, a2) & \\ P(a4, a3) & \\ P(a1, a4) & \\ Q(x, y) \leftarrow P(x, y) & *1 \\ Q(x, y) \leftarrow Q(x, z) \wedge P(z, y) & *2 \\ R(x, y) \leftarrow Q(y, x) & *3 \end{array} \right\}$$

C = {
P(a1,a4), Q(a1,a4) *1 {x/a1,y/a4}
P(a4,a3), Q(a1,a3) *2 {x/a1,z/a4,y/a3}
p(a3,a2), Q(a1,a2), *2 {x/a1,z/a3,y/a2}
P(a2,a1), Q(a1,a1), *2 {x/a1,z/a2,y/a1}
R(a1,a1) *3 {y/a1,x/a1}
}

20/20

therefore Γ is provable by R(a1, a1)

5. Show a top down derivation of $\Gamma \vdash R(a1, a1$ using the top down proof algorithm given the definite clauses (written as head $\leftarrow$ body) in set Γ from the previous question.

yes ← R(a1,a1)
yes ← Q(a1,a1)
yes ← Q(a1,a2) ∧ P(a2,a1)
yes ← Q(a1,a3) ∧ P(a3,a2) ∧ P(a2,a1)
yes ← Q(a1,a4) ∧ P(a4,a3) ∧ P(a3,a2) ∧ P(a2,a1)
yes ← P(a1,a4) ∧ P(a4,a3) ∧ P(a3,a2) ∧ P(a2,a1)
yes ←

20/20

therefore Γ is provable by R.