# Cs 2209a - Assignment 1

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**Department of Computer Science**

**CS 2209A — Applied Logic for Computer Science**

**Assignment 1**

Assigned: 1 October 2018

Due: 15 October 2018 (on OWL before midnight)

1. For the following proposition, give the unique readability tree and the truth table derived from that tree. What type of statement (tautology, contradiction, satisfiable) is it? Why?

$$\Big(\big((p \to q) \to p\big) \to p\Big)$$

The unique readability tree :

```
            →
           / \
          →   p
         / \
        →   p
       / \
      p   q
```

The truth table:

| p | q | p → q | (p → q) → p | ((p → q) → p) →p |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | F | T |

∵ the statement is always true (the last column from the above table) ∴ It is tautology.

2. Show the following logical equivalence using the logical identities and tautologies such as the law of the excluded middle.

$$\Big(\big((p \wedge q) \vee (p \wedge s)\big) \vee \big((r \wedge q) \vee (r \wedge s)\big)\Big) \equiv \big((p \vee r) \wedge (q \vee s)\big)$$

= (p ∧ (q ∨ s)) ∨ (r ∧ (q ∨ s))

Set (q ∨ s) to b

= (p ∧ b) ∨ (r ∧ b)

= (b ∧ (p ∨ r))

B is equal to (q ∨ s)

= (q ∨ s) ∧ (p ∨ r)

which is equivalent to the right

∴ ((p ∧ q) ∨ (p∧s)) ∨ ((r ∧ q) ∨ (r ∧ s)) ≡ (q ∨ s) ∧ (p ∨ r)

3. Convert the following proposition to Conjunctive Normal Form. Show the steps required to get your answer.

$$\Big((p \leftrightarrow q) \rightarrow (p \wedge r)\Big)$$

((p → q) ∧ (q → p)) → (p ∧ r)

(((¬p) ∨ q) ∧ ((¬q) ∨ p)) → (p ∧ r)

((¬p ∨ q) ∧ (¬q ∨ p)) → (p ∧ r)

Set ((¬p ∨ q) ∧ (¬q ∨ p)) to b, (p ∧ r) to c

b → c

(¬b ∨ c)

( ¬[(¬p ∨ q) ∧ (¬q ∨ p)] ∨ (p ∧ r))

( (p ∧ ¬q) ∨ (q ∧ ¬p) ∨ (p ∧ r)) is DNF

need to Convert to CNF

¬¬( (p ∧ ¬q) ∨ (q ∧ ¬p) ∨ (p ∧ r))

¬( ¬(p ∧ ¬q) ∧ ¬ (q ∧ ¬p) ∧ ¬(p ∧ r))

¬( (¬p ∨ q) ∧ (¬q ∨ p) ∧ (¬p ∨ ¬r))

4. Show the following using natural deduction.

$$\Big\{(p \rightarrow q)\Big\} \vdash \Big((r \vee p) \rightarrow (r \vee q)\Big)$$

| | | |
|---|---|---|
| 1. | (p → q) | Premise |
| 2. | ¬((p ∨ r) → (q ∨ r)) | assumption |
| 3. | ¬(¬(p ∨ r) ∨ (q ∨ r)) | equivalence 2 |
| 4. | ((p ∨ r) ∧ ¬(q ∨ r)) | De Morgan's law 3 |
| 5. | ((p ∨ r) ∧ (¬q ∧ ¬r)) | De Morgan's law 4 |
| 6. | ((r ∧ ¬r ∧ ¬q) ∨ (p ∧ ¬r ∧ ¬q)) | equivalence 5 |
| 7. | ⊥ ∨ (p ∧ ¬r ∧ ¬q) | equivalence 6 |
| 8. | ⊥ ∨ (⊥ ∧ ¬r) | ¬ - intro 1 |
| 9. | ⊥ | identity |
| 10. | (p ∨ r) → (q ∨ r) | 2-9 |

5. With the clauses given by $\Gamma$ show $\Gamma \vdash \neg r$ using resolution.

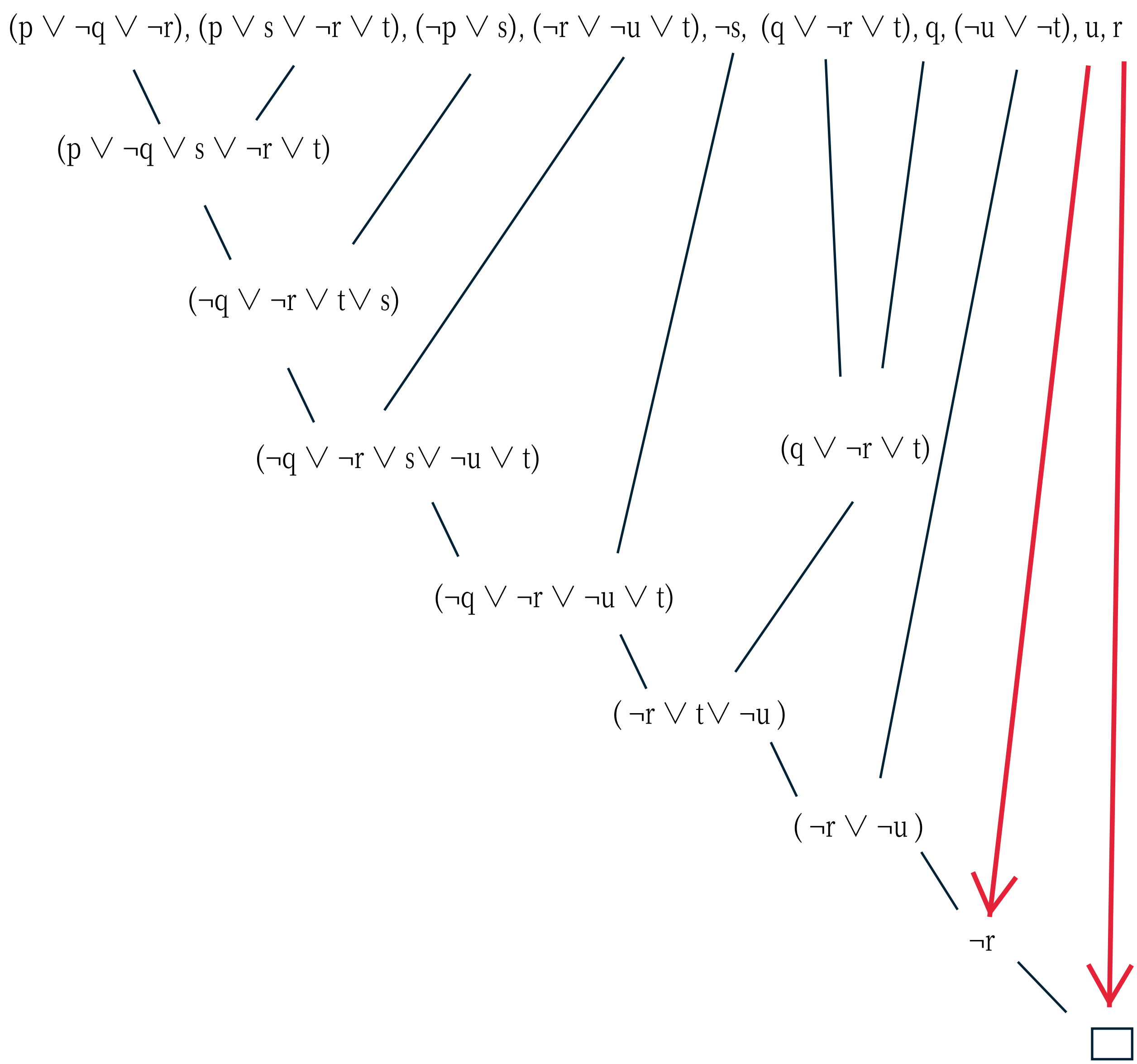
$$\Gamma = \Big\{(p \vee \neg q \vee \neg r), (\neg p \vee s), (q \vee \neg r \vee t), (\neg r \vee \neg u \vee t), (p \vee s \vee \neg r \vee t), \neg s, q, u, (\neg u \vee \neg t)\Big\}$$

On the next page.

5. With the clauses given by $\Gamma$ show $\Gamma \vdash \neg r$ using resolution.

$$\Gamma = \Big\{(p \vee \neg q \vee \neg r), (\neg p \vee s), (q \vee \neg r \vee t), (\neg r \vee \neg u \vee t), (p \vee s \vee \neg r \vee t), \neg s, q, u, (\neg u \vee \neg t)\Big\}$$

number of variable: p: 2 ¬p:1 q: 2 ¬q :1 r: 1 ¬r: 4 s: 2 ¬s: 1 t: 3 ¬t:1 ¬u:2 u:1



6. Show a bottom up derivation of $\Gamma \vdash h$ given the definite clauses (written as head $\leftarrow$ body) in set $\Gamma$.

$$\Gamma = \left\{ \begin{array}{l} h \leftarrow f \wedge g \\ g \leftarrow a \wedge c \\ d \\ e \leftarrow c \\ a \leftarrow d \\ f \leftarrow d \wedge e \wedge b \\ b \leftarrow e \\ c \leftarrow a \end{array} \right\}$$

C = { }
C = { d }
C = { d, a }
C = { d, a, c}
C = { d, a, c, e}
C = { d, a, c, e, b }
C = { d, a, c, e, b, f, }
C = { d, a, c, e, b, f, g }
C = { d, a, c, e, b, f, g, h }

$\because$ h is belong to the set C
$\therefore \Gamma$ entails h

7. Show a top down derivation of $\Gamma \vdash h$ given the definite clauses (written as head $\leftarrow$ body) in set $\Gamma$.

$$\Gamma = \left\{ \begin{array}{l} h \leftarrow f \wedge g \\ g \leftarrow a \wedge c \\ d \\ e \leftarrow c \\ a \leftarrow d \\ f \leftarrow d \wedge e \wedge b \\ b \leftarrow e \\ c \leftarrow a \end{array} \right\}$$

yes $\leftarrow$ h
yes $\leftarrow$ f $\wedge$ g
yes $\leftarrow$ d $\wedge$ e $\wedge$ b $\wedge$ g
yes $\leftarrow$ d $\wedge$ c $\wedge$b $\wedge$ g
yes $\leftarrow$ d $\wedge$ c $\wedge$ e $\wedge$ g
yes $\leftarrow$ d $\wedge$ a $\wedge$ e $\wedge$ g
yes $\leftarrow$ d $\wedge$d $\wedge$ e $\wedge$ g
yes $\leftarrow$ d $\wedge$ e $\wedge$ g
yes $\leftarrow$ d $\wedge$ c $\wedge$ g
yes $\leftarrow$ d$\wedge$ a $\wedge$ g
yes $\leftarrow$ d $\wedge$ d $\wedge$ g
yes $\leftarrow$ d $\wedge$ g
yes $\leftarrow$ d $\wedge$ a $\wedge$ c
yes $\leftarrow$ d $\wedge$ d $\wedge$ a
yes $\leftarrow$ d
yes $\leftarrow$

therefore $\Gamma$ entails h.