1. a) $\exists x \forall y\, L(x,y)$

$\forall y\, L(c,y)$ choose skolem constant for $\exists x$

$L(c,y)$ drop $\forall$

b) $\forall x \exists y\, L(y,x)$

$\forall x\, L(f(x),x)$ choose skolem function of one argument ($\forall x$) for $\exists x$

$L(f(x),x)$ drop $\forall$

c) $\forall z \{ Q(z) \rightarrow \{ \neg \forall x \exists y [P(y) \rightarrow P(g(z,x))]\}\}$

$\forall z \{ \neg Q(z) \vee \{ \neg \forall x \exists y [\neg P(y) \vee P(g(z,x))]\}\}$

convert $\rightarrow$ to $\vee$

$\forall z \{ \neg Q(z) \vee \{ \exists x \forall y [P(y) \wedge \neg P(g(z,x))]\}\}$

negations as literal negations + De Morgan's Law

$\forall z \{ \neg Q(z) \vee \{ \forall y [P(y) \wedge \neg P(g(z,f(y)))]\}\}$

choose skolem function of one argument ($\forall z$) for $\exists x$

$\neg Q(z) \vee [P(y) \wedge \neg P(g(z,f(y)))]$ drop $\forall$

$(\neg Q(z) \vee P(y)) \wedge (\neg Q(z) \vee \neg P(g(z,f(y))))$ Distribution

$(\neg Q(z) \vee P(y))$ $(\neg Q(z_1) \vee \neg P(g(z_1, f(y_1))))$ clauses with variables standardized apart

2.

1.	$\neg B(x) \vee C(x)$	premise
2.	$\neg C(a) \vee D(b)$	premise
3.	$\neg C(c) \vee E(d)$	premise
4.	$\neg D(w) \vee \neg E(y)$	premise
5.	$B(x')$	negation of sentence to be proved in clause form
6.	$C(x_1)$	$1, 5, \{x/x'\}$
7.	$D(b)$	$2, 6, \{x_1/a\}$
8.	$E(d)$	$3, 6, \{x_1/c\}$
9.	$\neg E(y_1)$	$4, 7, \{w/b\}$
10.	$\square$	$8, 9, \{y_1/d\}$

3. validity means

$$\models \forall x\, P(x) \to \exists y\, P(y)$$

using the converse of the Deduction Theorem

$\vdash \forall x\, P(x) \to \exists y P(y)$ implies this entailment relation

$\neg(\forall x\, P(x) \to \exists y\, P(y))$ in CNF

$\neg(\neg \forall x\, P(x) \vee \exists y\, P(y))$

$\forall x P(x) \wedge \neg \exists y\, P(y)$

$\forall x P(x) \wedge \forall x\, \neg P(y)$

$P(x) \wedge \neg P(y)$

1. $P(x)$
2. $\neg P(y)$

} clauses from CNF of sentence to be proved (Note: no premises)

3. $\square$ $1, 2, \{x/y\}$

4. $C = \{\}$

$C = \{P(a_2,a_1), P(a_3,a_2), P(a_4,a_3), P(a_1,a_4)\}$

add all of the facts

(*) $C = \{P(a_2,a_1), P(a_3,a_2), P(a_4,a_3), P(a_1,a_4),$
$Q(a_2,a_1), Q(a_3,a_2), Q(a_4,a_3), Q(a_1,a_4)\}$

add all the Qs from the first rule

(**) $C = \{$ all of the atoms in (*) plus

these are derived bottom up using the 2nd rule and the atoms in C above $\longrightarrow$ $Q(a_2,a_4), Q(a_3,a_1), Q(a_4,a_2), Q(a_1,a_3)\}$

(***) $C = \{$ all of the atoms in (**) plus

ditto $\longrightarrow$ $Q(a_2,a_3), Q(a_3,a_4), Q(a_4,a_1), Q(a_1,a_2)\}$

(****) $C = \{$ all of the atoms in (***) plus

$Q(a_2,a_2), Q(a_3,a_3), Q(a_4,a_4), Q(a_1,a_1)\}$

[no more Qs can be added]

$C = \{$ all of the atoms in (****) plus

$R(a_1,a_1), R(a_2,a_2), R(a_3,a_3), R(a_4,a_4),$
$R(a_1,a_2), R(a_1,a_3), R(a_1,a_4), \ldots\}$

$\Gamma \vdash R(a_1,a_1)$ because $R(a_1,a_1) \in C$

5.

$$\text{yes} \leftarrow R(a_1, a_1)$$

$$\text{yes} \leftarrow Q(a_1, a_1)$$

Rename variables in rule 3 and apply mgu $\{x_1/a_1, y_1/a_1\}$

This is not the answer. It simply shows certain paths don't work.

$$\leftarrow Q(a_1, z_1) \wedge P(z_1, a_1)$$

Rename variables in rule 2 and apply mgu $\{x_2/a_1, y_2/a_1\}$

gets stuck

$$\leftarrow P(a_1, z_1) \wedge P(z_1, a_1)$$

$\{y_3/z_1\}$ (rule 1)

$$\leftarrow P(a_4, a_1)$$

$\{z_1/a_4\}$ (fact 4)

$$\text{yes} \leftarrow Q(a_1, a_1)$$

$$\text{yes} \leftarrow P(a_1, a_1) \qquad \{x_1/a, y_1/a\} \quad \text{(rule 1)}$$

This gets stuck as well

This is the answer

Note I missed a line

Note: each of these would get stuck if either of the two uses of rules 1 or 2 are used

$$\text{yes} \leftarrow Q(a_1, a_1) \qquad \{x_1/a_1, y_1/a_1\}$$

$$\text{yes} \leftarrow Q(a_1, z_1) \wedge P(z_1, a_1) \qquad \{x_2/a_1, y_2/a_1\}$$

$$\text{yes} \leftarrow Q(a_1, z_2) \wedge P(z_2, z_1) \wedge P(z_1, a_1) \qquad \{x_3/a_1, y_3/a_1\}$$

$$\text{yes} \leftarrow Q(a_1, z_3) \wedge P(z_3, z_2) \wedge P(z_2, z_1) \wedge P(z_1, a_1) \qquad \{x_4/a_1, y_4/a_1\}\ 2$$

$$\text{yes} \leftarrow P(a_1, z_3) \quad " \qquad \{x_5/a_1, y_5/z_3\}$$

Note: if the variables were not renamed these would not work

$$\text{yes} \leftarrow P(a_1, a_4) \wedge P(a_4, z_2) \wedge P(z_2, z_1) \wedge P(z_1, a_1)\ 5 \qquad \{x_5/a_1, y_5/a_4\}$$

$$\text{yes} \leftarrow P(a_4, z_2) \wedge P(z_2, z_1) \wedge P(z_1, a_1) \qquad \{\}$$

$$\text{yes} \leftarrow P(a_3, z_1) \wedge P(z_1, a_1) \qquad \{z_2/a_3\}$$

$$\text{yes} \leftarrow P(a_2, a_1) \qquad \{z_1/a_2\}$$

$$\text{yes} \leftarrow$$

Note: it is often the case that for resolutions to succeed the resolvents must build to the point that it can collapse