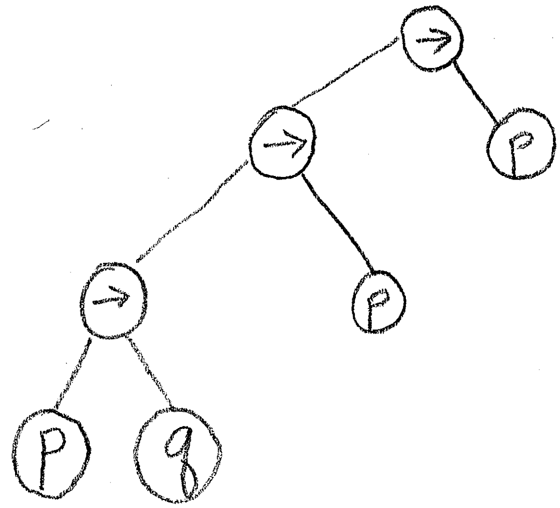
1.



p	q	① $(p \to q)$	② $(① \to p)$	✱ $(② \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

tautology
because each row in ✱ is T

2.

$$((p \wedge q) \vee (p \wedge s)) \vee ((r \wedge q) \vee (r \wedge s))$$

$$\equiv \quad (p \wedge (q \vee s)) \vee (r \wedge (q \vee s)) \qquad \text{Distribution}$$

$$\equiv \quad (p \vee r) \wedge (q \vee s) \qquad \text{Distribution}$$

3.

$$(p \leftrightarrow q) \to (p \wedge r)$$

$$[(p \to q) \wedge (q \to p)] \to (p \wedge r) \qquad \leftrightarrow$$

$$[(\neg p \vee q) \wedge (\neg q \vee p)] \to (p \wedge r) \qquad \to$$

$$\neg[(\neg p \vee q) \wedge (\neg q \vee p)] \vee (p \wedge r) \qquad \to$$

$$[\neg(\neg p \vee q) \vee \neg(\neg q \vee p)] \vee (p \wedge r) \qquad \text{DeM}$$

$$[(p \wedge \neg q) \vee (q \wedge \neg p)] \vee (p \wedge r) \qquad \text{DeM}$$

$$(p \vee (q \wedge \neg p)) \wedge (\neg q \vee (q \wedge \neg p)) \vee (p \wedge r) \qquad \text{Dist} \downarrow$$

$$((p \vee q) \wedge \underbrace{(p \vee \neg p)}_{T}) \wedge (\underbrace{(\neg q \vee q)}_{T} \wedge (\neg q \vee \neg p)) \vee (p \wedge r)$$

$$((p \vee q) \wedge (\neg q \vee \neg p)) \vee (p \wedge r)$$

$$((p \vee q) \vee (p \wedge r)) \wedge ((\neg q \vee \neg p) \vee (p \wedge r))$$

$$(p \vee q \vee p) \wedge (p \vee q \vee r) \wedge (\neg q \vee \neg p \vee p) \wedge (\neg q \vee \neg p \vee r)$$

$$\underbrace{(p \vee q) \wedge (p \vee q \vee r)} \wedge T \wedge (\neg q \vee \neg p \vee r)$$

$$(p \vee q) \wedge (\neg q \vee \neg p \vee r) \qquad \text{Simplify (Identity)}$$

4.

1. $(p \rightarrow q)$ Premise
2. $(\neg q \rightarrow \neg p)$ Contrapositive, 1
3. $(r \vee p)$ Assumption
4. $\neg(r \vee q)$ Assumption
5. $(\neg r \wedge \neg q)$ De Morgan, 4
6. $\neg r$ $\wedge$-Elim, 5
7. $\neg q$ Commutativity, $\wedge$-Elim, 5
8. $\neg p$ $\rightarrow$ Elim, 7,2
9. $(r \vee p) \wedge \neg p$ $\wedge$-Intro, 3,8
10. $(r \wedge \neg p) \vee (p \wedge \neg p)$ De Morgan, 9
11. $(r \wedge \neg p)$ Identity, Contradiction, 10
12. r $\wedge$-Elimination, 11
13. $\perp$ $\perp$-Introduction, 6, 12
14. $\neg\neg(r \vee q)$ $\neg$-Introduction, 4–13
15. $(r \vee q)$ Double negation, 14
16. $(r \vee p) \rightarrow (r \vee q)$ $\rightarrow$-Introduction, 3-15

5.

$(p \vee \neg q \vee r)$ $(\neg p \vee s)$ $(q \vee \neg r \vee t)$ $(\neg r \vee \neg u \vee t)$ $(p \vee s \vee \neg r \vee t)$ $\neg s$

$(\neg u \vee \neg t)$ q u

$(\neg r \vee \neg u)$

Step 1: want to prove $\neg r$
add r to set of clauses :

r

$\neg r$

Step 2: use resolution to achieve $\square$

$\square$

This is the simplest proof that I have found (my first attempts found slightly more resolutions needed)

Of course there are different orders of resolvents that will produce the same result.

There are many other ways to get $\square$. (My first attempt used 7 resolution steps!)

6. $C = \{\ \}$

$\{d\}$

$\{d, a\}$

$\{d, a, c\}$

$\{d, a, c, e\}$

$\{d, a, c, e, b\}$

$\{d, a, c, e, b, f\}$

$\{d, a, c, e, b, f, g\}$

$\{d, a, c, e, b, f, g, h\}$

$h \in C \quad \therefore \Gamma \vdash h$

7.

$yes \leftarrow h$

$yes \leftarrow f \wedge g$

$yes \leftarrow d \wedge e \wedge b \wedge g$

$yes \leftarrow e \wedge b \wedge g$

$yes \leftarrow c \wedge b \wedge g$

$yes \leftarrow a \wedge b \wedge g$

$yes \leftarrow d \wedge b \wedge g$

$yes \leftarrow b \wedge g$

$yes \leftarrow e \wedge g$

$yes \leftarrow c \wedge g$

$yes \leftarrow a \wedge g$

$yes \leftarrow d \wedge g$

$yes \leftarrow g$

$yes \leftarrow a \wedge c$

$yes \leftarrow d \wedge c$

$yes \leftarrow c$

$yes \leftarrow a$

$yes \leftarrow d$

$yes \leftarrow$ $\quad\therefore\ \Gamma \vdash h$