

Cs 2209a - Assignment2

Student Name: MingCong, Zhou

Student Number: 250945414

Student Account: mzhou272

Department of Computer Science
CS 2209A — Applied Logic for Computer Science

Assignment 2

Assigned: 29 October 2018

Due: 5 November 2018 (on OWL before midnight)

1. Convert the following First Order sentences to Conjunctive Normal Form. Show all intermediate conversion steps. Include steps that are required to incorporate the clauses into an empty set ready to do resolution.

(a) $\exists x \forall y L(x, y)$

$$\begin{aligned} &\equiv \forall y \exists x L(x, y) \\ &\equiv \forall y L(f(y), y) \\ &\equiv L(f(y), y) \end{aligned}$$

(b) $\forall x \exists y L(y, x)$

$$\begin{aligned} &\equiv \forall x L(f(x), x) \\ &\equiv L(f(x), x) \end{aligned}$$

(c) $\forall z \{ Q(z) \Rightarrow \{ \neg \forall x \exists y [P(y) \Rightarrow P(g(z, x))] \} \}$

$$\begin{aligned} &\equiv \forall z \{ \neg Q(z) \vee \{ \exists x \neg \exists y [\neg P(y) \vee P(g(z, x))] \} \} \\ &\equiv \forall z \{ \neg Q(z) \vee \{ \exists x \forall y \neg [\neg P(y) \vee P(g(z, x))] \} \} \\ &\equiv \forall z \{ \neg Q(z) \vee \{ \exists x \forall y [P(y) \wedge \neg P(g(z, x))] \} \} \\ &\equiv \forall z \{ \neg Q(z) \vee \{ \forall y \exists x [P(y) \wedge \neg P(g(z, x))] \} \} \\ &\equiv \forall z \{ \neg Q(z) \vee \{ \forall y [P(y) \wedge \neg P(g(z, f(y)))] \} \} \\ &\quad // \because \forall y \text{ is in the scope of the } \forall z \therefore \text{ drop it} \\ &\equiv \forall z \forall y \{ \neg Q(z) \vee [P(y) \wedge \neg P(g(z, f(y)))] \} \\ &\equiv \neg Q(z) \vee [P(y) \wedge \neg P(g(z, f(y)))] \\ &\equiv (\neg Q(z) \vee P(y)) \wedge (\neg Q(z) \vee \neg P(g(z, f(y)))) \end{aligned}$$

2. With the clauses given by Γ show $\Gamma \vdash \exists x \neg B(x)$ using resolution.

$$\Gamma = \{ \neg B(x) \vee C(x), \quad \neg C(a) \vee D(b), \quad \neg C(c) \vee E(d), \quad \neg D(w) \vee \neg E(y) \}$$

\therefore we need to prove $\exists x \neg B(x)$ using resolution

\therefore we can get the \neg of it as a clause

$$\neg \exists x \neg B(x) \equiv \forall x B(x) \equiv B(x)$$

1. $\neg B(x) \vee C(x)$

2. $\neg C(a) \vee D(b)$

3. $\neg C(c) \vee E(d)$

4. $\neg D(w) \vee \neg E(y)$

5. $B(x)$

6. $\neg B(a) \vee D(b)$ 1,2 $\{x/a\}$

7. $\neg B(a') \vee \neg E(y')$ 4,6 $\{w/b\}$

8. $\neg B(a) \vee \neg C(a)$ 3,7 $\{y/d\}$

9. $\neg B(a)$ 1,8 $\{x/a\}$

10. \square 9,5 $\{x/a\}$

3. Show using resolution that the following statement is valid.

$$\forall x P(x) \rightarrow \exists y P(y)$$

$$F = \forall x P(x) \rightarrow \exists y P(y)$$

//Equivalence

$$\begin{aligned} F &= \forall x P(x) \rightarrow \exists y P(y) \\ &= \neg \forall x P(x) \vee \exists y P(y) \\ &= \exists x \neg P(x) \vee \exists y P(y) \\ //Existential instantiation \\ &= \neg P(a) \vee P(b) \end{aligned}$$

Negate F & get rid of " \rightarrow " & quantifiers:

$$\begin{aligned} &\neg(\forall x P(x) \rightarrow \exists y P(y)) \\ &\neg(\neg \forall x P(x) \vee \exists y P(y)) \\ &\neg(\neg \neg \forall x P(x) \vee \exists y P(y)) \\ &\neg(\exists x \neg P(x) \vee \exists y P(y)) \\ &(\neg \exists x \neg P(x) \wedge \neg \exists y P(y)) \\ &(\forall x P(x) \wedge \forall y \neg P(y)) \\ &(P(x) \wedge \neg P(y)) \end{aligned}$$

Clauses:

1. $P(x)$
2. $\neg P(y)$
3. $\neg P(a) \vee P(b)$
4. $P(b)$ 1,3 {x/a}
5. \square 2,4 {y/b} therefore valid

4. Show a bottom up derivation of $\Gamma \vdash R(a1, a1)$ using the bottom up proof algorithm given the definite clauses (written as head \leftarrow body) in set Γ .

$$\Gamma = \left\{ \begin{array}{l} P(a2, a1) \\ P(a3, a2) \\ P(a4, a3) \\ P(a1, a4) \\ Q(x, y) \leftarrow P(x, y) \quad *1 \\ Q(x, y) \leftarrow Q(x, z) \wedge P(z, y) \\ R(x, y) \leftarrow Q(y, x) \quad *3 \end{array} \right\} *2$$

$$\begin{aligned} C = \{ & \\ &P(a1, a4), Q(a1, a4) \quad *1 \{x/a1, y/a4\} \\ &P(a4, a3), Q(a1, a3) \quad *2 \{x/a1, z/a4, y/a3\} \\ &p(a3, a2), Q(a1, a2), \quad *2 \{x/a1, z/a3, y/a2\} \\ &P(a2, a1), Q(a1, a1), \quad *2 \{x/a1, z/a2, y/a1\} \\ &R(a1, a1) \quad *3 \{y/a1, x/a1\} \\ &\} \end{aligned}$$

therefore Γ is provable by $R(a1, a1)$

5. Show a top down derivation of $\Gamma \vdash R(a1, a1)$ using the top down proof algorithm given the definite clauses (written as head \leftarrow body) in set Γ from the previous question.

yes \leftarrow R(a1,a1)
yes \leftarrow Q(a1,a1)
yes \leftarrow Q(a1,a2) \wedge P(a2,a1)
yes \leftarrow Q(a1,a3) \wedge P(a3,a2) \wedge P(a2,a1)
yes \leftarrow Q(a1,a4) \wedge P(a4,a3) \wedge P(a3,a2) \wedge P(a2,a1)
yes \leftarrow P(a1,a4) \wedge P(a4,a3) \wedge P(a3,a2) \wedge P(a2,a1)
yes \leftarrow

therefore Γ is provable by R.