UWO CS2214 April 1st, 2019

Assignment #3
Due: April 9, 2019, by 23:55
Submission: on the OWL web site of the course

Format of the submission. You must submit a **single** file which must be in **PDF** format. All other formats (text or Miscrosoft word format) will be **ignored** and considered as **null**. You are strongly encouraged to type your solutions using a text editor. To this end, we suggest the following options:

1. Miscrosoft word and convert your document to PDF
2. the typesetting system LaTeX; see `https://www.latex-project.org/` and `https://en.wikipedia.org/wiki/LaTeX#Example` to learn about LaTeX; see `https://www.tug.org/begin.html` to get started
3. using a software tool for typing mathematical symbols, for instance `http://math.typeit.org/`
4. using a Handwriting recognition system such as those equipping tablet PCs

Hand-writing and scanning your answers is allowed but not encouraged:

1. if you go this route please use a scanning printer and **do not take a picture of your answers with your phone**,
2. if the quality of the obtained PDF is too poor, your submission will be **ignored** and considered as **null**.

Problem 1 (Relations) *[25 marks]*

1. Show that the relation

$$R = \{(x, y) |\ (x - y) \text{ is an even integer}\}$$

   is an equivalence relation on the set $\mathbb{R}$ of real numbers.
2. Show that the relation

$$\boxed{R = \{((x_1, y_1), (x_2, y_2))\ (x_1 < x_2) \ \text{ or } \ ((x_1 = x_2) \ \text{ and } \ (y_1 \leq y_2))\}}$$

   is a total ordering relation on the set $\mathbb{R} \times \mathbb{R}$.

Problem 2 (Basic probability calculations) *[25 marks]* In a roulette, a wheel with 38 numbers is spun. Of these, 18 are red, and 18 are black. The other two numbers, which are neither black nor red, are 0 and 00. The probability that when the wheel is spun it lands on any particular number is 1/38.

1. What is the probability that the wheel lands on a red number?
2. What is the probability that the wheel lands on a black number twice in a row?
3. What is the probability that the wheel lands on 0 or 00?
4. What is the probability that in five spins the wheel never lands on either 0 or 00?

Provide detailed justifications of your answers.

Problem 3 (Bayes theorem) *[25 marks]* Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time (this is a "false positive" test result).

1. What is the probability that a randomly selected bicylist who tests positive for steroids actually uses steroids?
2. What is the probability that a randomly selected bicyclist who tests negative for steroids did not use steroids?

Problem 4 (Graphs) *[25 marks]* For each of the following two graphs, determine whether or not it has an Euler circuit. Justify your answers. If the graph has an Euler circuit, use the algorithm described in class to find it, including drawings of intermediate subgraphs.