Cs 2214b- Assignment1

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Assignment #1

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Submission: on the OWL web site of the course

Problem 1 (Undertsamding implication) *[20 marks] Let p, q be two Boolean variables. By definition, the implication $p \longrightarrow q$ is true if and only if p is false or q is true. Based on that, we have established the following practical tautologies:*

1. $(p \longrightarrow q) \iff (\neg q \longrightarrow \neg p)$
2. $(p \leftrightarrow q) \iff ((p \longrightarrow q) \wedge (q \longrightarrow p))$

Would these two tautologies still be true if we were changing the truth value of the implication $p \longrightarrow q$ to that of

1. $p \wedge q$?
2. $p \vee q$?

1.1 p ∧ q and ¬q ∧¬p has a different truth table. False.

p ∧ q ⇔ ¬q ∧¬p					
p	q	**¬q**	**¬p**	p ∧ q	**¬q ∧¬p**
T	T	F	F	T	F
T	F	T	F	F	F
F	T	F	T	F	F
F	F	T	T	F	T

1.2 p ∨ q and ¬q ∨¬p has a different truth table. False.

p ∨ q ⇔ ¬q ∨¬p					
p	q	**¬q**	**¬p**	**p ∨ q**	**¬q ∨ ¬p**
T	T	F	F	T	F
T	F	T	F	T	T
F	T	F	T	T	T
F	F	T	T	F	T

2.1 **P ↔ q and (p ∧ q) ∧ (q ∧ p) have a different truth table. False.**

p ↔ q ⇔ (p ∧ q) ∧ (q ∧ p)					
p	q	**q ∧ p**	**p ∧ q**	**P ↔ q**	**(p ∧ q) ∧ (q ∧ p)**
T	F	F	F	F	F
T	T	T	T	T	T
F	F	F	F	T	F
F	T	F	F	F	F

2.2 **P ↔ q and (p ∨ q) ∧ (q ∨ p) have a different truth table. False.**

p ↔ q ⇔ (p ∨ q) ∧ (q ∨ p)					
p	q	**p ∨ q**	**q ∨ p**	**P ↔ q**	**(p ∨ q) ∧ (q ∨ p)**
T	F	T	T	F	T
T	T	T	T	T	T
F	F	F	F	T	F
F	T	T	T	F	T

Problem 2 (Proving theorems!) *[20 marks]* For each of the following statements, translate it into predicate logic and prove it, if the statement is true, or disprove it, otherwise:

1. for any two even integers, there exists a third integer (even or odd) the double of which is equal to the sum of the first two integers.
2. for any two odd integers, there exists a third integer (even or odd) the triple of which is equal to the sum of the first two integers.

1.

X: any even integer
Y: any even integer
Z: any integer

$\forall X \forall Y \exists Z (2Z = X + Y)$
X is even, if there exists an integer K:
$X = 2K$
Follow the same instruction, $Y = 2L$.
$X + Y = 2(K + L) \in$ any integer $\in Z$
Therefore, the statement is true.

2.

X: any odd integer
Y: any odd integer
Z: any integer
$\forall X \forall Y \exists Z (3Z = X + Y)$
X is odd, if there exists an integer K:
$X = 2 K + 1$
In the same instruction $Y = 2 L + 1$
$X + Y = 2(K + L) + 2$
$2(K + L) + 2$ is always even, therefore $\notin Z$
The statement is false.

Problem 3 (Finding a treasure!) *[20 marks]* In the back of an old cupboard you discover a note signed by a pirate famous for his bizarre sense of humour and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements and challenged the reader to use them to figure out the location of the treasure.

1. If there is an old shipwreck near the beach, then the treasure is buried under a coconut palm tree.
2. There is a coconut palm tree growing either at the far end of the island or near the cave.
3. Either there is a shipwreck near the beach, or the treasure is hidden in a cave.
4. If there is a coconut palm tree at the far end of the island, then there is no shipwreck on the beach
5. There is no coconut palm tree near the cave.

All five sentences are true statement, therefore at least one argument from each of the sentence has to be true.
Let Near (X, Y) denote X near Y.
Red denotes F, green denotes T

1. Near(shipwreck, beach) → Near(treasure, tree)
2. Near(tree, end of island) $\vee$ Near(tree, cave)
3. Near(shipwreck, beach) $\vee$ Near(treasure, cave)
4. Near(tree, end of island) → ¬Near(shipwreck, beach)
5. Near(tree, cave) → F, therefore in sentence 2 Near(tree, end of island) → T
6. Near(tree, end of island) → T, therefore ¬Near(shipwreck, beach) → T
7. ¬Near(shipwreck, beach) → T, therefore Near(shipwreck, beach) → F
8. Near(shipwreck, beach) → F, therefore Near(treasure, cave) → T
9. Finally the treasure is hidden in the cave.

Problem 4 (Deciding consistency) *[20 marks]* A set of propositions is *consistent* if there is an assignment of truth values to each of the propositional variables, that makes all propositions true. Is the following set of propositions consistent?

1. The system is in multiuser state if and only if it is operating normally.
2. If the system is operating normally, the kernel is functioning.
3. The kernel is not functioning or the system is in interrupt mode.
4. If the system is not in multiuser state, then it is in interrupt mode.
5. The system is in interrupt mode.

P: the system is in multiuser state.
Q: the system is operating normally.
R: the kernel is functioning.
$\neg P$: the system is not in multiuser state.
$\neg Q$: the system is in interrupt mode.
$\neg R$: the kernel is not functioning.

Translating the sentence into logic sentence:

1. $P \leftrightarrow Q$
2. $Q \rightarrow R$
3. $\neg R \lor \neg Q$
4. $\neg P \rightarrow \neg Q$
5. $\neg Q$

Simplify the above sentence we get:

1. T $\neg P \lor Q$ F
2. T $\neg Q \lor P$
3. T $\neg Q \lor R$
4. $\neg R \lor \neg Q$ T
5. F $P \lor \neg Q$ T
6. $\neg Q$ T

the set of propositions is consistent if there is an assignment of truth values to each of the propositional variables that makes all propositions true.
By this definition assign $\neg Q = T$ assign truth value to the above formula using red pen.

After assigning truth value to each variable.
Every formula contains at least one T.

Therefore the set is consistent.

Problem 5 (Deciding satisfiability) *[20 marks]* Let p, q, r be three Boolean variables. For each of the following propositional formulas determine whether it is satisfiable or not.

1. $p \wedge (q \vee \neg p) \wedge (\neg q \vee \neg r)$
2. $p \wedge (q \vee \neg p) \wedge (\neg q \vee \neg p)$

1. Assign T to p, T to q and F to r, the formula is satisfiable.
2. Not satisfiable. if P=T, the middle and the last part is not suit by either q is T or F. if P = F, the front part make the entire formula false.