Assignment #5

UWO CS2214 | Due: April 9, 2019, by 23:55 | April 1st, 2019

Submission: on the OWL web site of the course

Problem 1 (Relations) *[25 marks]*

1. Show that the relation

$$R = \{(x, y) | \ (x - y) \text{ is an even integer}\}$$

is an equivalence relation on the set $\mathbb{R}$ of real numbers.

Given a set R={(x, y) | x – y is an even number}
To prove equivalence, we need to prove the relation are reflexive, symmetric and transitive.

1. To prove reflexive, we need to prove:
   a) $x \in A \rightarrow (x, x) \in R$
      x is belonging to a not empty set therefore $x \in A$
      for all x, x – x is always 0, and 0 is an even number
      therefore $(x, x) \in R$

   The relation is reflexive
2. To prove symmetric, we need to prove:
   a) $(x, y) \in R \rightarrow (y, x) \in R$
      We know that $(x, y) \in R$ therefore:
         $x - y = a$ (where a is an even number)
      $y - x = -a$
      $-a$ is also an even number, $(y, x) \in R$

   The relation is symmetric
3. To prove transitive, we need to prove:
   a) $(x, y) \in R \land (y, z) \in R \rightarrow (x, z) \in R$
      By $(x, y) \in R$, we can have:
         $x - y = k$ (where k is an even number)
         because k is an even number k=2a (where a is an integer)
         $x - y = 2a$
      Similarly, by having $(y, z) \in R$, we can have:
         $y - z = 2b$ (where b is an integer)
      $x - z = (y + 2a) - z = (2b + z + 2a) - z = 2(a + b)$
         $2(a + b)$ is always an even number
      Thus, $(x, z) \in R$.

   The relation is transitive

Because the relation of set R are reflexive, symmetric and transitive, it is an equivalence relation.

2. Show that the relation

$$R = \{((x_1, y_1), (x_2, y_2))\ (x_1 < y_1)\ \text{ or }\ ((x_1 = y_1)\ \text{ and }\ (x_2 < y_1))\}$$

is a total ordering relation on the set $\mathbb{R} \times \mathbb{R}$.

R = {((x1, y1), (x2, y2)) (x1 < y1) or ((x1 = y1) and (x2 < y1)) } is a total ordering relation on the set R × R.

To prove a set is total ordering set, we need to prove that:

every two elements of S are comparable

This is the more readable version of the above question:

R = { (x1, y1), (x2, y2) | (x1 < y1) ∨ (x1 = y1 ∧ x2 < y1) }

In this set, either (x1 < y1) or x1 = y1 and x2 < y1

therefore, this is a comparable set.

To prove it is a total order, we need to prove it is a binary relation set which also every set element is antisymmetric, transitive and connex relation

(see this https://en.m.wikipedia.org/wiki/Total_order).

4. To prove reflexive, we need to prove:
   a) x ∈ A → (x, x) ∈ R
      if (x1, y1) == (x2, y2), then it satisfies x1 == x2 and y1 ≤ y2
      Therefore, it is reflexive
5. To prove antisymmetric, we need to prove:
   a) not (x, y) ∈ R → (y, x) ∈ R
      if x1 < x2 then x1 is smaller than x2, x2 is not smaller than x1

   Therefore it is antisymmetric
6. To prove connex, we need to prove
   a) ∀x∀y (x ∈ X ∧ y ∈ X) ⇒ (xRy ∨ yRx).
      x1 ∈ X and y1 ∈ X and
      we can find that x1y1 is in the set R and so yRx
      Therefore, the set is connex

Overall the set R is in total order.

Problem 2 (Basic probability calculations) *[25 marks]* In a roulette, a wheel with 38 numbers is spun. Of these, 18 are red, and 18 are black. The other two numbers, which are neither black nor red, are 0 and 00. The probability that when the wheel is spun it lands on any particular number is 1/38.

1. What is the probability that the wheel lands on a red number?
2. What is the probability that the wheel lands on a black number twice in a row?
3. What is the probability that the wheel lands on 0 or 00?
4. What is the probability that in five spins the wheel never lands on either 0 or 00?

In this sample space S, if we spun only one time,
we can only land on either red, black, 00 or 0.
|S| = 38, |red| = 18, |black| = 18, |00| = 1, |0| = 1
Therefore 1 = P(red) + P(black) + P(00) + P(0) and 38 = 18 + 18 + 1 + 1

1) P(red) = |red| / |S| = 18 / 38 ~= 0.47
2) P(black) = |black| / |S| = 18 / 38 ~= 0.47
   a) because it is twice, the answer is (0.47)*0.47
3) P(00 and 0) = |00 and 0| / |S| = 2/ 38 ~= 0.05
4) P(not 00 and 0) = |not 00 and 0| / |S| = 36/38 ~= 0.94
   the above is the probability for one spin. the five spin is $(36/38)^5$ ~= 0.73

Problem 3 (Bayes theorem) *[25 marks]* Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time (this is a "false positive" test result).

1. What is the probability that a randomly selected bicylist who tests positive for steroids actually uses steroids?
2. What is the probability that a randomly selected bicyclist who tests negative for steroids did not use steroids?

S denotes that bicyclist use steroid. $P(S) = 0.08$
T denote test is positive. $P(\neg S) = 0.92$
By reading the content I gather that: $P(S \cap T) = 0.96$
$P(S \cap \neg T) = 0.04$
$P(\neg S \cap T) = 0.09$
$P(\neg S \cap \neg T) = 0.91$

1) Already know that the test is positive, how many of them use steroid?

$P(S|T) = P(T|S) * P(S) / P(T|S) * P(S) + P(T|\neg S) * P(\neg S)$

$P(T|S) = P(T \cap S) / P(S) = 0.96/0.08 = 12$

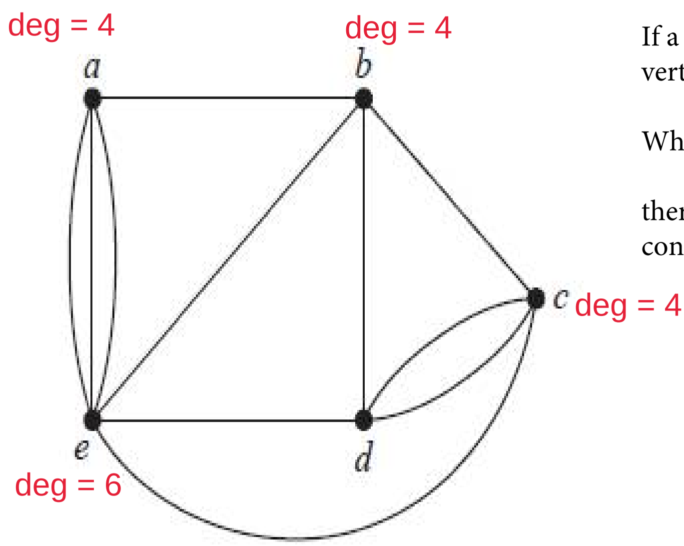
$P(T|\neg S) = P(T \cap \neg S) / P(\neg S) = 0.09/0.92 = 98\%$

Therefore, $P(S|T)$ is equal to

$= 12 * 0.08 / 12 * 0.08 + 0.098 * 0.92 = 91.4\%$

2) Already know that the test is negative, how many of them did not use steroid?$P(\neg S|\neg T) = P(\neg T |\neg S) * P(\neg S) / P(\neg T |\neg S) * P(\neg S) + P(\neg T |S) * P(S)$ $P(\neg T |\neg S) * P(\neg S) = P(\neg T \cap \neg S) / P(\neg S) * P(\neg S) = P(\neg T \cap \neg S) = 0.91$
$P(\neg T |S) * P(S) = P(\neg T \cap S) / P(S) * P(S) = P(\neg T \cap S) = 0.04$
$P(\neg S|\neg T) = 0.91 / (0.91 + 0.04) = 0.958$

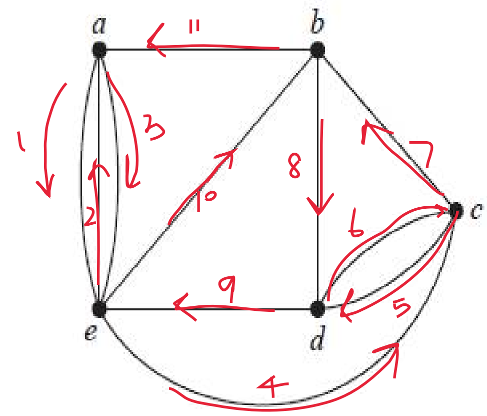
Problem 4 (Graphs) *[25 marks]* For each of the following two graphs, determine whether or not it has an Euler circuit. Justify your answers. If the graph has an Euler circuit, use the algorithm described in class to find it, including drawings of intermediate subgraphs.

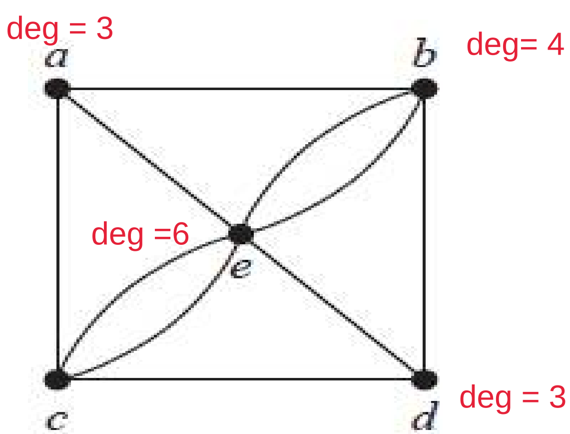


If a graph has an Euler circuit then the degree of every vertex must be even.

Which means:

If the degree of one vertex are not even, then there will be no Euler circuit exist. This graph satisfy the condition. Then we can use the algorithm to test it.





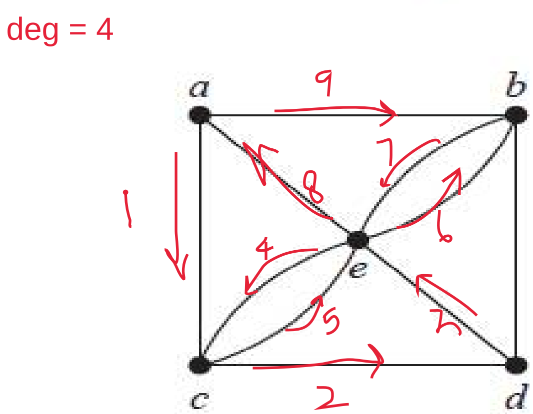
Theorem: A connected multigraph with

at least two vertices has an Euler circuit

if and only if each of its vertices has an even degree

and

it has an Euler path if and only if it has exactly two vertices of odd degree.



This graph does not have Euler circuit, because not all deg is even.

but there is a Euler Path exists