Cs 2214b- Assignment2

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Assignment #2
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Submission: on the OWL web site of the course

Problem 1 (Proving properties about the integers) *[15 marks]* Prove or disprove the following properties:

1. For every integer n we have $n \leq n^2$.
2. For every integer n, the integer $n^2 + n + 1$ is odd.

Problem 1

X: any integer

Prove: $\forall x\ (x \leq x^2)$

$X^2 - X \geq 0$

$X (X-1) \geq 0$

Case 1:

$X < 0$

$X(X-1)$

$X < 0$ and $(X-1) < 0$

A negative number times a negative number is positive

Therefore, $X (X-1) \geq 0$

Case 2:

$X = 0$

$X(X-1)$ is always 0

Zero times everything is zero.

Therefore, $X (X-1) \geq 0$

Case 3:

$X > 0$ and $(X-1) \geq 0$

A positive number times a positive number is always positive

Therefore, $X (X-1) \geq 0$

Problem 2

Prove: $\forall n$ (n^2+n+1 is odd)

x: any integer

let $f(n) = n^2+n+1$

Case 1: if x is odd, and there exist an integer n, x=2n+1

$f(2n+1) = (2n+1)^2+(2n+1) + 1$

$f(2n+1) = 4n^2+6n+3$

$4n^2$ is always even and 6n+3 is odd

$4n^2+6n+3$ returns a odd number

Therefore, when x is odd n^2+n+1 is odd

Case 2: if x is even, and existing an integer n, x=2n

$f(2n) = (2n)^2+2n+1$

$f(2n) = 4n^2 + 2n+1$

$4n^2$ is always even and 2n+1 is odd

$4n^2 + 2n + 1$ is odd

Therefore, when x is even n^2+n+1 is odd

Therefore n^2+n+1 is odd for all integer n.

Problem 2 (Proving properties about real numbers) *[15 marks]* Prove or disprove the following properties:

1. For every real number x, if $x \leq 0$ or $1 \leq x$ holds, then $x \leq x^2$ holds as well.
2. For all real number x we have $\lfloor 2x \rfloor = 2\lfloor x \rfloor$

Part1:

X: 0≤ real number ≤ 1

<u>Prove</u> ∀n (x ≤ x^2)

∀n (x(x-1) ≤ 0)

<u>Case 1</u>: x = 0

Zero times everything is zero Therefore x(x-1) ≤ 0 is true

<u>Case 2</u>: x = 1

1(1-1) = 0

Therefore x(x-1) ≤ 0 is true

<u>Case 3</u>: 0< x < 1

x(x-1)

x > 0 and

x < 1, therefore (x - 1) < 0

x(x-1) < 0

x(x-1) ≤ 0 is true

Finally, ∀n (x ≤ x^2) is true

Part2:

x: any real number

m: integer

<u>Prove</u> (⌊2x⌋ = 2⌊x⌋)

<u>Case 1</u>: when x is a integer

⌊2x⌋ = m

⌊x⌋ = m/2 and 2⌊x⌋ = m

Therefore ⌊2x⌋ = 2⌊x⌋ is true

<u>Case 2</u>: when x is not a integer, then x = m + ε, and 0 < ε < 1

⌊x⌋ = m and 2⌊x⌋ = 2m

⌊2x⌋ = ⌊ 2(m + ε) ⌋

⌊ 2(m + ε) ⌋ = 2m +1 when ε ≥ 0.5

2m +1 ≠ 2m

Therefore ⌊2x⌋ = 2⌊x⌋ is false

Finally ⌊2x⌋ = 2⌊x⌋ is not true for all real number

Problem 3 (Properties of preimage sets) *[20 marks]* Let f be a function from a set A to a set B. Let S and T be two subsets of B. Prove the following properties

1. $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
2. $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

Part1:

Assume x is a preimage of S ∪ T

X ∈ f^{-1}(S ∪ T) then equivalently f(x) ∈ S ∪ T

either f(x) ∈ S or f(x) ∈ T

equivalently x ∈ f^{-1}(S) or x ∈ f^{-1}(T)

x ∈ f^{-1}(S) or x ∈ f^{-1}(T) is the same as x ∈ f^{-1}(S ∪ T)

reversely, assume x ∈ f^{-1}(S) ∪ f^{-1}(T)

then x ∈ f^{-1}(S) or x ∈ f^{-1}(T), which is the same as X ∈ f^{-1}(S ∪ T)

Therefore, the property is true.

Part2

Assume x is a preimage of S ∩ T, X ∈ f^{-1}(S ∩ T).

Then f(x) ∈ S ∩ T. So, f(x) ∈ S and f(x) ∈ T.

Therefore x ∈ f^{-1}(S) and x ∈ f^{-1}(T), and it is the same as

X ∈ f^{-1}(S) ∩ f^{-1}(T).

reversely, assume x ∈ f^{-1}(S) ∩ f^{-1}(T)

then x ∈ f^{-1}(S) and x ∈ f^{-1}(T), which is the same as X ∈ f^{-1}(S ∩

T) Therefore, the property is true.

Problem 4 (Properties of functions) *[30 marks]* Which of the functions below is injective? surjective? When the function is bijective, determine its inverse

1. $f_1 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ n & \longmapsto & 2019n+1 \end{array}$

2. $f_2 : \begin{array}{ccc} \mathbb{Z} & \to & \mathbb{Z} \\ n & \longmapsto & \lfloor n/2 \rfloor + \lceil n/2 \rceil \end{array}$

3. $f_3 : \begin{array}{ccc} [1,2) & \to & [0,1) \\ x & \longmapsto & x - \lfloor x \rfloor \end{array}$

4. $f_4 : \begin{array}{ccc} [1,2) & \to & [0,1) \\ x & \longmapsto & (f_3(x))^2 \end{array}$

f1:

z → z

n → 2019n + 1

switch n to x

x → 2019x + 1

Prove Injection

f(x) = f(y)

2019x+1= 2019y+1

x = y

therefore, the function is injective.

Prove surjection

f(x) = y

2019x+1 =y

x = (y-1)/2019

therefore, the function is surjective

and it is bijective

f3:

[1, 2) → [0, 1)

x → x – ⌊x⌋

Prove Injective:

f(x) = f(y)

x – ⌊x⌋ = y – ⌊y⌋

because 1 ≤ ⌊x⌋ ≤ 2, therefore ⌊x⌋ = 1

x - 1 = y -1

x = y

therefore, the function is injective.

Prove surjective:

f(x) = y

x – ⌊x⌋ = y

x = y +1

therefore surjective.

and it is bijective

f2:

z → z

n → ⌊n/2⌋ + ⌈n/2⌉

switch n to x, just look clearer x → ⌊x/2⌋ + ⌈x/2⌉

Prove Injective

f(x) = f(y)

⌊x/2⌋ + ⌈x/2⌉ = ⌊y/2⌋ + ⌈y/2⌉

when x and y are even:

x/2+x/2 = y/2 + y/2

and x= y

when x and y are odd

x = 2k+1

y = 2j +1

⌊(2k+1)/2⌋ + ⌈(2k+1)/2⌉ = ⌊(2j +1)/2⌋ + ⌈(2j +1)/2⌉ ⌊k +1/2⌋ + ⌈k+1/2⌉ = ⌊j +1/2⌋ + ⌈j +1/2⌉

k+(k+1) = j + (j+1)

k = j

therefore, the function is injective.

Prove Surjective

f(x) = y

⌊x/2⌋ + ⌈x/2⌉ = y

when x is even x = 2k

⌊2k /2⌋ + ⌈2k /2⌉ = y

k + k = y

x = y

when x is odd x = 2k + 1

⌊(2k+1) /2⌋ + ⌈(2k+1) /2⌉ = y

⌊k+1/2⌋ + ⌈k+1/2⌉ = y

k+(k+1) = y

x = y -1

therefore, is surjective

and it is bijective

f4:

$[1, 2) \to [0, 1)$

$x \to (f_3(x))^2$

$f(x) = (f_3(x))^2$

$f(x) = (x - 1)^2$

Prove injective:

$f(x) = f(y)$

$(x - 1)^2 = (y - 1)^2$

$(x – 1)^2 - (y - 1)^2 = 0$

$(x – y)(x+y-2) = 0$

$x – y = 0$ (only way that works)

$x = y$

therefore, it is injective

Prove surjective:

$f(x) = y$

$(x - 1)^2 = y$

$x = (y -1)^2$

$y – 1 = \sqrt{x}$ and $y = \sqrt{x} + 1$

therefore, it is surjective.

and the function is bijective.

Problem 5 (Properties of functions) *[20 marks]* Let f be a surjective function from a set A to a set B and a g be a function from B to a set C. Prove or disprove the following properties:

1. if g is surjective then so is gof.
2. if f and g are both injective, then so is gof.

1.

f and g are both surjective therefore, there are more than one entry from A that can reach Band from B that can reach A. For g f there exist at least one element in domain that imply 2 or more elements in co-domain from B to A and from C to B. By this definition the property is false.

2.

2. Suppose g f(x1) = g f(x2) for x1,x2 ∈ A

g(f(x1)) = g(f(x2))

∵ g is injective and g(f(x1)) = g(f(x2))

∴ f(x1) = f(x2) is true.

∵ f is injective

∴ x1 = x2 is true.

If f and g are both injective. g f is true.