CS3331 – Assignment 2
due Oct. 17, 2019 (latest to submit: Oct. 20)

1. (30pt) Consider the alphanumeric alphabet $\Sigma = \{a, b, \ldots, z, A, B, \ldots, Z, 0, 1, \ldots, 9\}$ and let L be the language of all regular expressions over Σ:

$$L = \{w \in \left(\Sigma \cup \{\emptyset, (,), \cup, \cdot, {}^*\}\right)^* \mid w \text{ is a syntactically legal regular expression over } \Sigma\} .$$

   (a) Give an unambiguous context-free grammar that generates L. The grammar should use the following precedence levels, from highest to lowest:

   (1) * (Kleene star) – highest precedence

   (2) $\cdot$ (concatenation)

   (3) $\cup$ (union) – lowest precedence

   (b) Show the parse tree that your grammar produces for the string $a(a \cup b)^*$.

2. (30pt) For each of the following languages L, prove whether L is regular, context-free but not regular, or not context-free:

   (a) $L = \{xy \mid x, y \in \{a, b\}^* \text{ and } |x| = |y|\}$.

   (b) $\{a^m b^n \mid m, n \geq 0 \text{ and } m \geq 2n\}$.

   (c) $\{w^R w w^R \mid w \in \{a, b, c\}^*\}$.

3. (10pt) Consider the language $L = \{ww^R | w \in \{a, b\}^*\}$. Below are two proofs, one showing L is context free, the other showing the opposite. Which proof is correct and why?

   L is context-free. Here is a context free grammar that generates L:

   $$\begin{aligned} S &\longrightarrow aA \\ A &\longrightarrow Sa \\ S &\longrightarrow bB \\ B &\longrightarrow Sb \\ S &\longrightarrow \varepsilon \end{aligned}$$

   L is not context-free. Consider the string $w_k = a^k bb a^k \in L$. Because $|w| \geq k$, using pumping theorem, w_k can be written as $w_k = uvxyz$. Put $v = a^p, y = a^q$, where at least one of p and q is not 0. Then, according to the pumping theorem, $uv^2 xy^2 z \in L$. But $uv^2 xy^2 z = a^{k+p+q} bb a^k$ cannot be written as ww^R, for any string w, therefore $uv^2 xy^2 z \notin L$, a contradition. This implies that L is not context-free.

4. (30pt) Show that the following problem is decidable: Given a context-free grammar G, does G generate any odd-length, nonempty strings?

Note Submit your solution as a pdf file on `owl.uwo.ca`.