HEAPS

What is a heap?

Binary tree with additional properties

What is a binary tree? subset of trees where every node can have at most 2 children

Properties of a heap:

SHAPE PROPERTY AND ORDER OF NODE VALUE PROPERTY

So what makes a binary tree a HEAP?

It has to satisfy 2 properties:

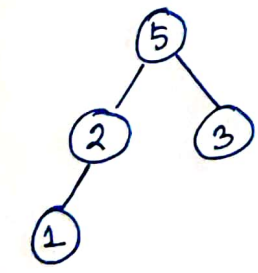
1. shape of the tree

In order to be a heap, a Binary Tree has to be complete.

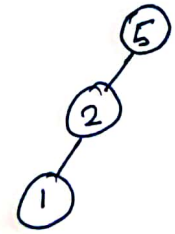
What's a COMPLETE Binary Tree?

A binary tree where EVERY Level is full except MAYBE the last level and at all levels nodes are filled-in from the left to right.

EXAMPLES:

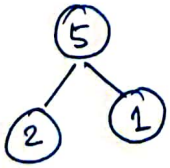


← VALID HEAP

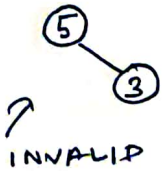


← INVALID HEAP as it is NOT COMPLETE BINARY TREE

← VALID



← VALID HEAP



INVALID

2. Order of node values

For a heap, the node values should follow an order at each level

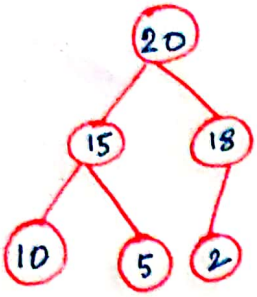
MAX HEAPS: Parent node val > child node value

max node value at the top

MIN HEAP: Parent node val < child node values

min node value at top.

REPRESENT HEAP AS AN ARRAY



index of node (15) → 2*(0) + 1 → 1

index of node (2) → 2 * (2) + 1 → 5

index of node (5) → 2 * (1) + 2 → 4

index of parent of (5) → floor((4-1)//2))

↓

floor(1) → 1

0	1	2	3	4	5
20	15	18	10	5	2

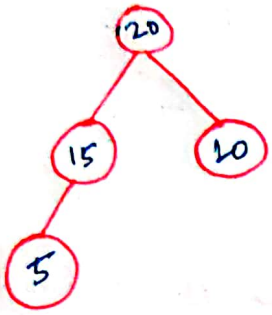
- the root node of heap will always be at 0th index
- index of left child for a node at index i

  ↳ 2i + 1
- index of right child for a node at index i:

  ↳ 2i + 2
- Formula to determine parent node for a node at index i : floor[(i - 1)//2]

MAJOR CONCEPTS FOR PROBLEMS AND HEAP IMPLEMENTATION

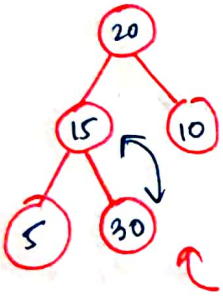
ADDING A NEW NODE : (restore - ~~down~~ up)



0	1	2	3	4
20	15	10	5	

insert (30)

1. insert new node at the end



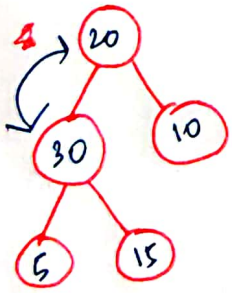
0	1	2	3	4	5
20	15	10	5	30	

parent of (30) → floor((4-1)//2))
→ floor(1) → 1

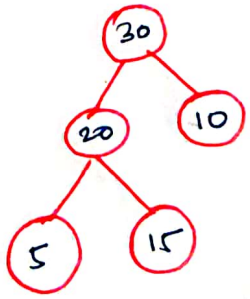
↑ Now heap property does not hold as parent of (30) < (30)

↓

↳ so we move (30) up till it is at correct place. by mapping with parent node



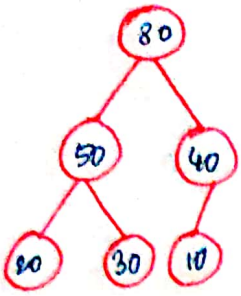
→



Now heap is VALID

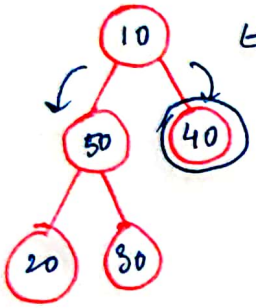
REMOVING MAX/MIN NODE FROM HEAP

restore-down



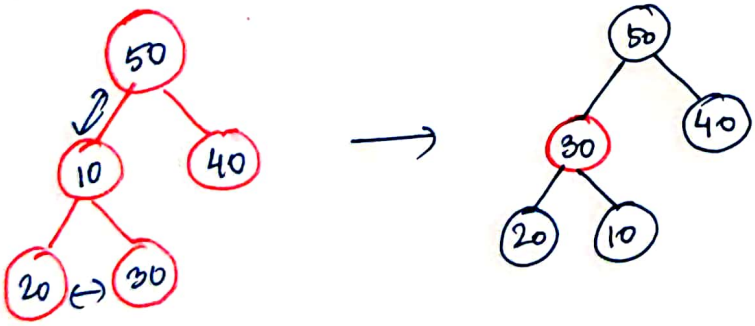
→ remove (80)

1. Swap the max-node with min node. Remove max node



← Heap property not valid. To make the heap property valid, restore down (10) till it is at right position

2. To do this compare the max node with left and right node. If max node is < left or right then swap. Keep comparing and swapping till parent > left and right node



Now the heap property is restored