Priority Queues

Priority Queue Operations

- Enqueue items with a priority
- Dequeue item with the highest priority
- Highest - return item with the highest priority

Priority Queue Operations

- Uses for a priority queue
  - Printers
  - Office hours
  - Emergency room

Priority Queue Implementations

- In class exercise - Evaluate and choose the best ADT implementation
  - Data Structure
    - Array based list
    - Linked list based list
    - Tree
  - Operations
    - Enqueue - enqueue with a priority
    - Dequeue - dequeue highest priority
    - Highest - return highest priority

Priority Queue Implementations

	Array	Linked List	BST
Insert at front or end (unsorted)			n/a
Search for item & deQ			n/a
Search for highest			n/a

OR

Insert sorted
deQ highest item
Return highest item

Priority Queue Implementations

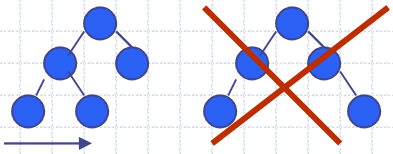
	Array	Linked List	BST
Insert at front or end (unsorted)			n/a
Search for item to deQ			n/a
Search for highest			n/a

OR

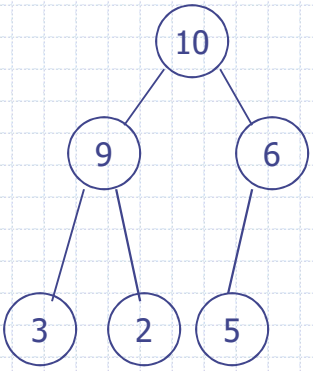
Insert sorted
deQ highest item
Return highest item

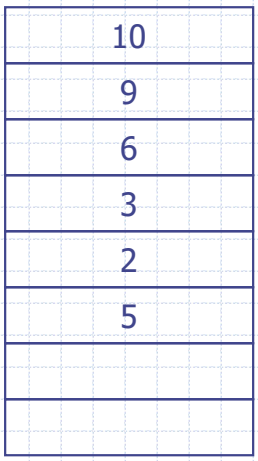
Binary Heaps/Heaps

- Similar to a BST
- Heap is sorted in a much weaker sense however the sorting is sufficient for a priority queue
- Tree is always a complete tree
  - Bottom level may not be filled but fills from left to right



Max Heap





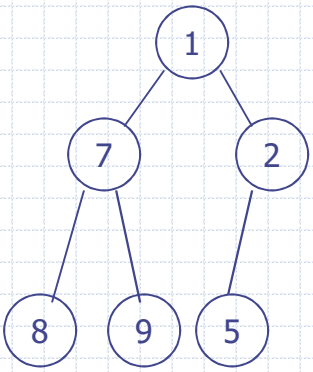
Heaps

- Full binary trees are easily stored in an array
  - Must know max size but no wasted space
    - Parent = (i-1)/2
    - Left Child = 2i + 1
    - Right Child = 2i + 2

Heap Ordering Properties

- Min Heap
  - For each node X, X.item >= (X.parent).item
    - Smallest number is at the root
- Max Heap
  - For each node X, X.item <= (X.parent).item
    - Largest number is at the root

Min Heap



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Heap ADT

HEAP class

int MAXNODES	*set to current array size*
int numNodes	*current #of items in heap*
*itemtype *Arr*	*dynamically allocated space for MAXNODES items (can realloc if needed)*

Heap - Enqueue

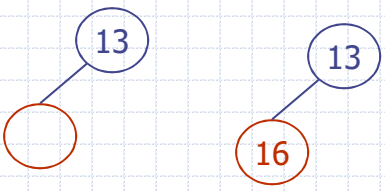
- Create a hole (empty node) in the next available complete tree location
  - Remember to fill bottom layer from left to right
- If the item can be inserted into the hole without violation of the heap property, insert item
- Otherwise, copy the holes parent item into the hole then trickle up

Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

Enqueue 13

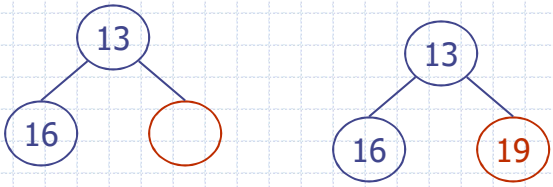
Enqueue 16



Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

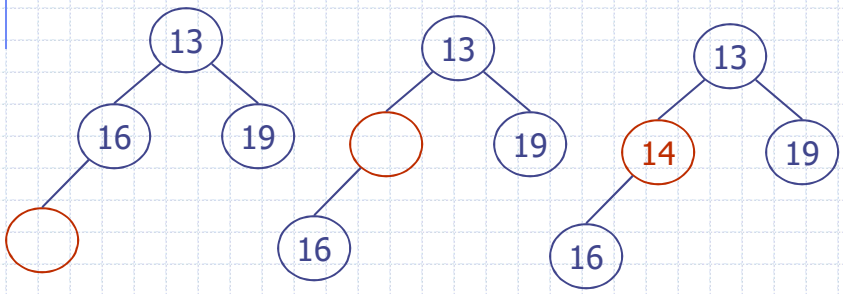
Enqueue 19



Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

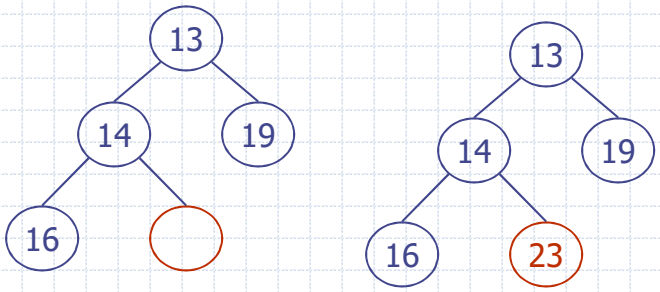
Enqueue 14



Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

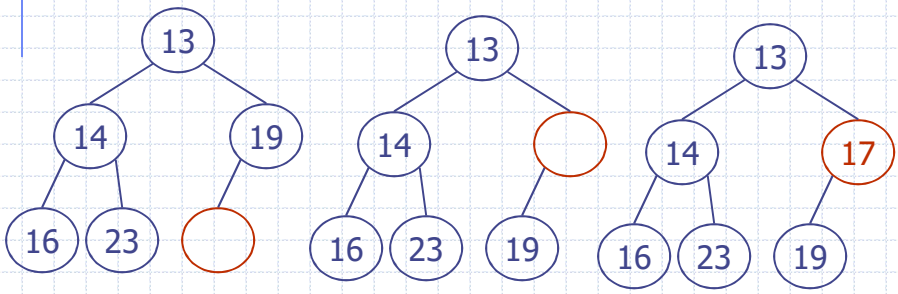
Enqueue 23



Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

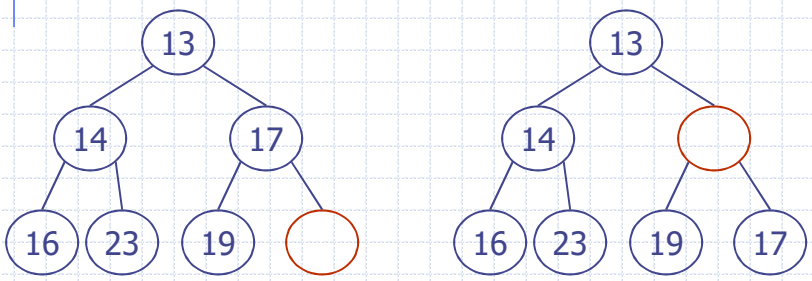
Enqueue 17



Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

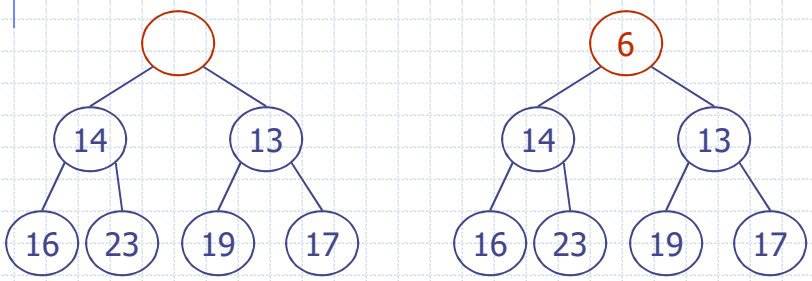
Enqueue 6



Min Heap Enqueue

13, 16, 19, 14, 23, 17, 6

Enqueue 6



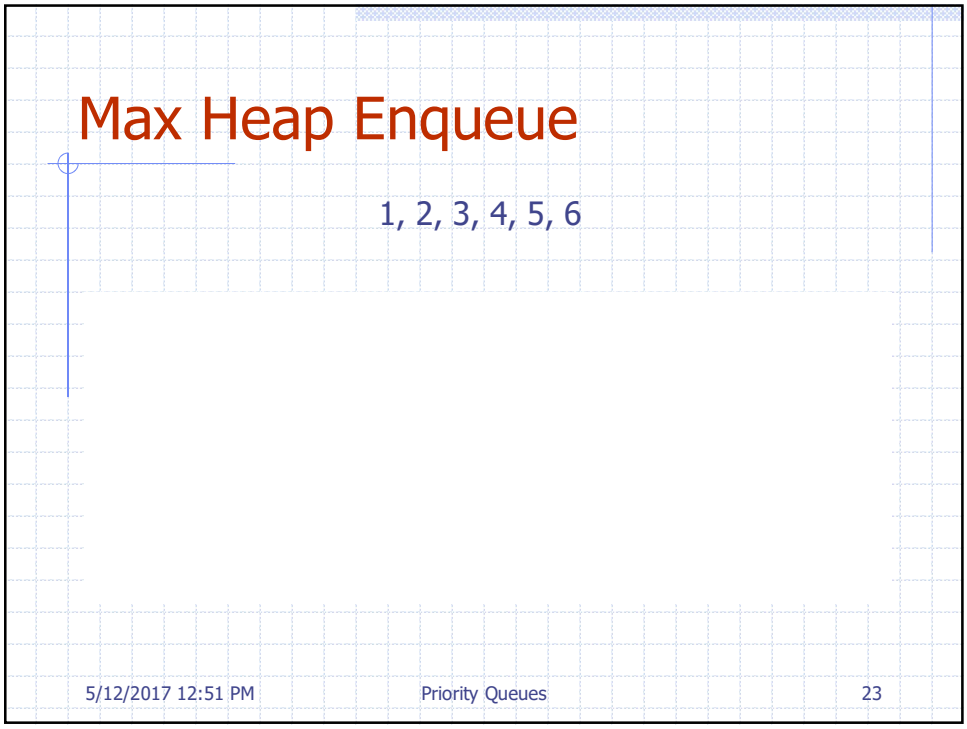
Max Heap Enqueue

- In class exercise - Enqueue the following numbers into a max heap (remember that the **largest** number is at the root)

1, 2, 3, 4, 5, 6

Max Heap Enqueue

1, 2, 3, 4, 5, 6


Max Heap Enqueue
1, 2, 3, 4, 5, 6
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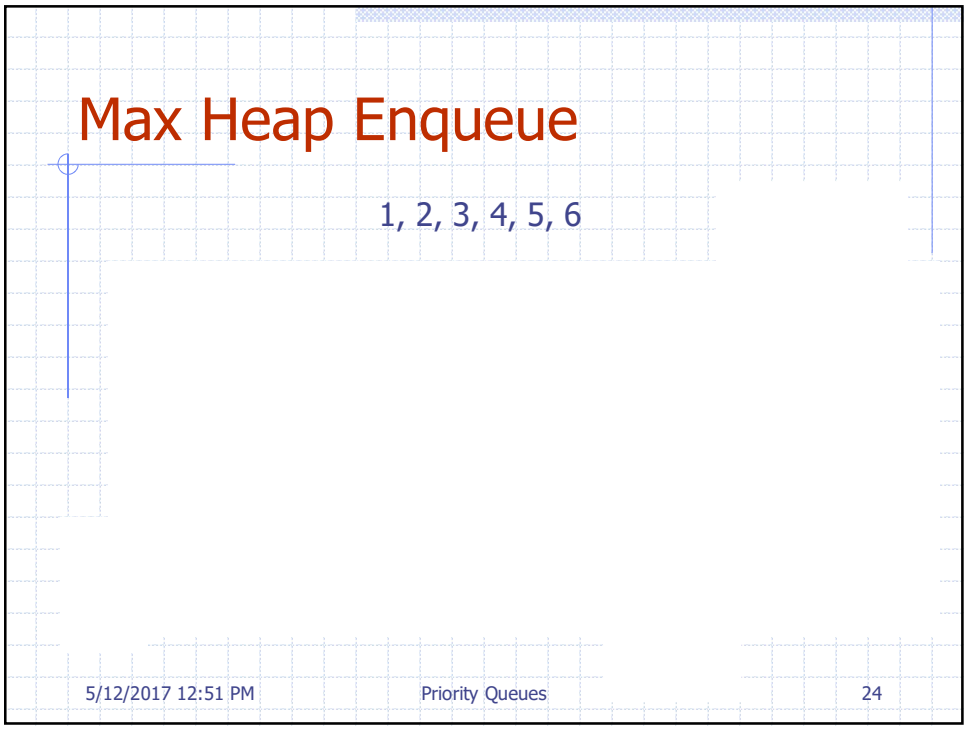
Priority Queues
23


Max Heap Enqueue
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Priority Queues
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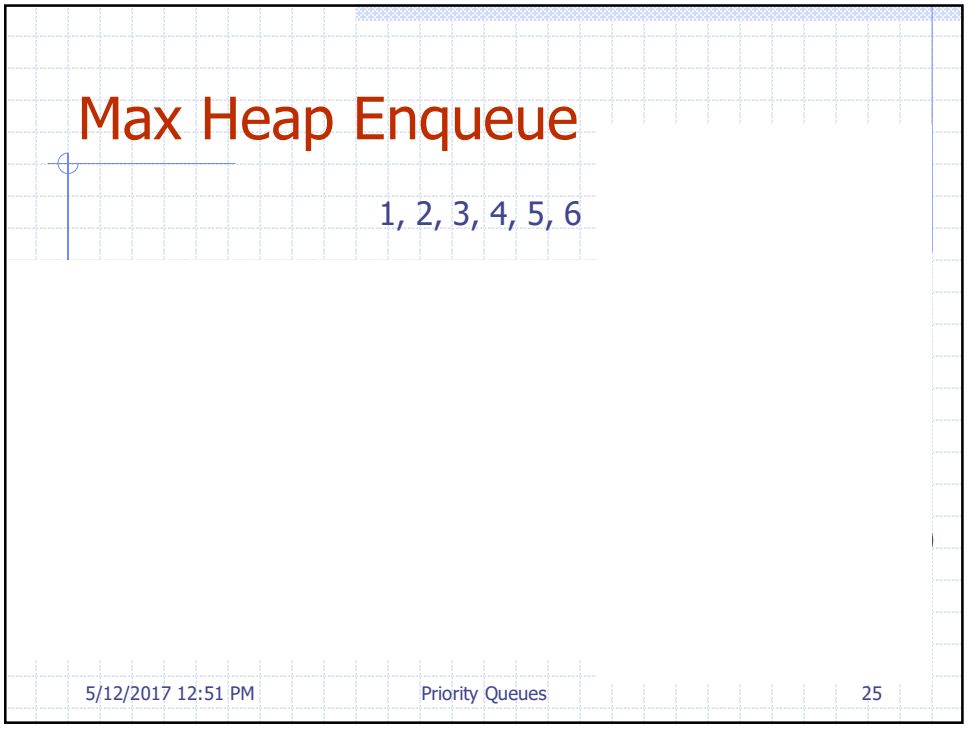
Max Heap Enqueue
1, 2, 3, 4, 5, 6
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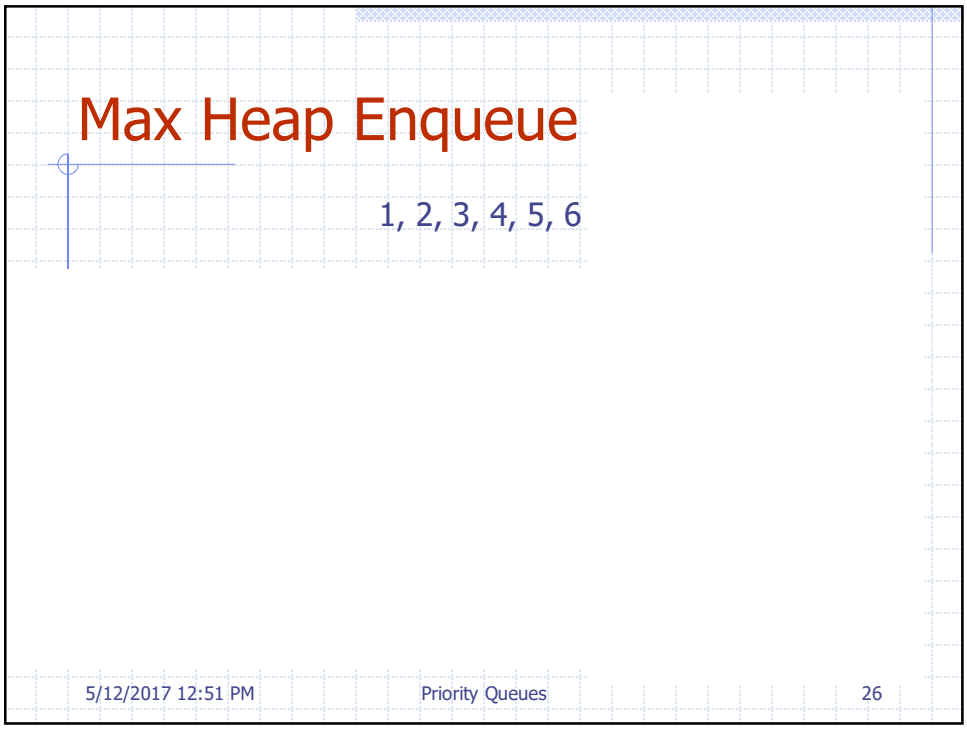
Priority Queues
25


Max Heap Enqueue
1, 2, 3, 4, 5, 6
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Priority Queues
26

Min Heap Enqueue

```
void enqueueHeap ( itemtype item )
    if heap not full
        if heap empty
            arr[numNodes] = item
            ++numNodes
        else
            x = numNodes
            while (x > 0 && arr[(x-1)/2] > item)
                arr[x] = arr[(x-1)/2]
                x = (x-1)/2
            arr[x] = item
            ++numNodes
```

Running time =

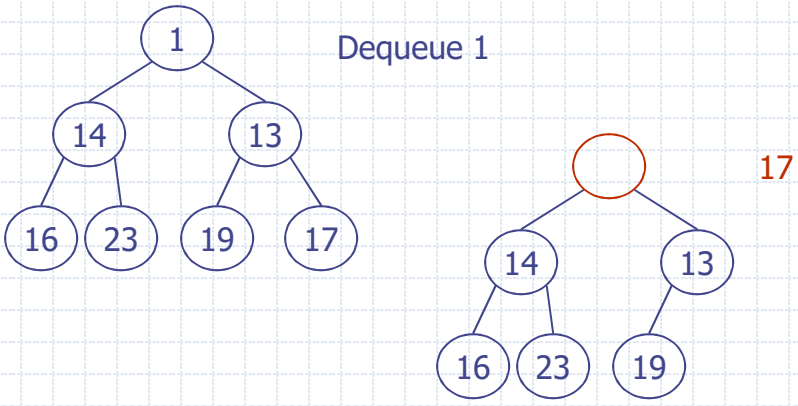
Heap

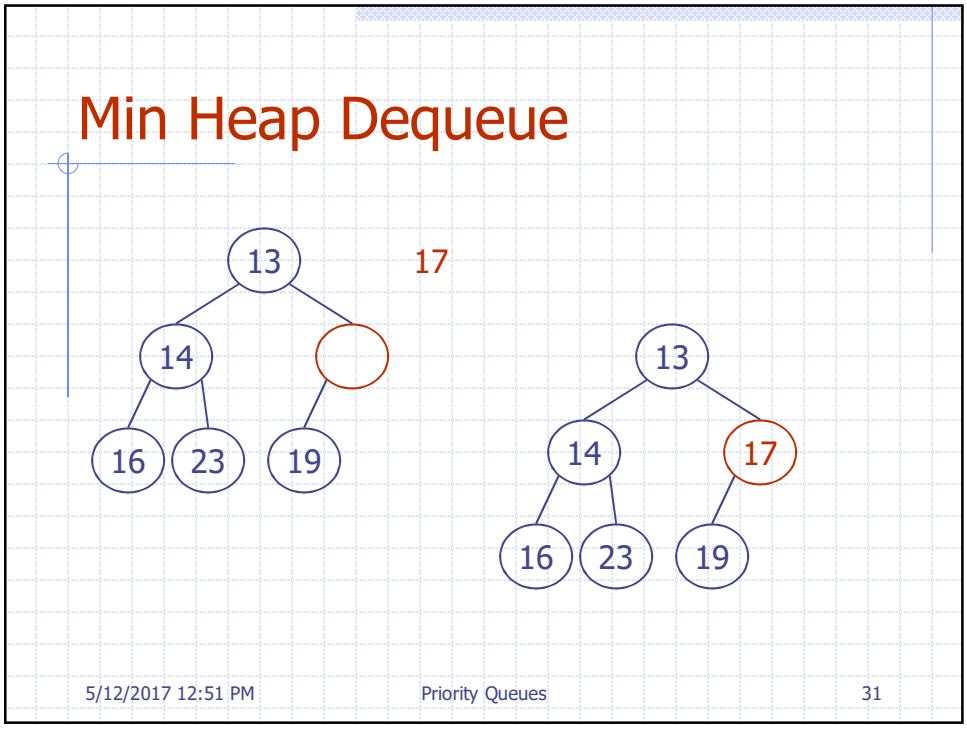
- Finding the node with the highest priority is easy - it is just at the root
  - Running time = O(1)

Heap Dequeue

- Dequeue highest priority item
  - Makes a hole at the root
  - Want to remain a complete tree so place last item in the heap into the hole
    - If item can be placed in hole without violation of the heap property, then done
    - Otherwise, trickle down. Put the highest priority of the holes children into the hole

Min Heap Dequeue




Min Heap Dequeue
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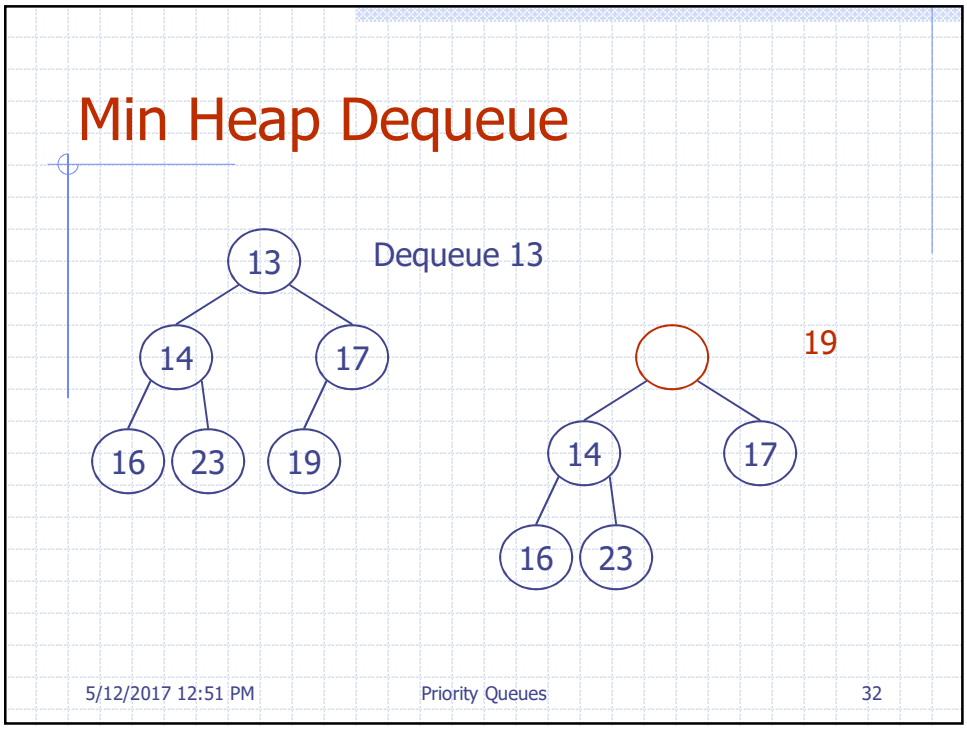
Priority Queues
31


Min Heap Dequeue
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Priority Queues
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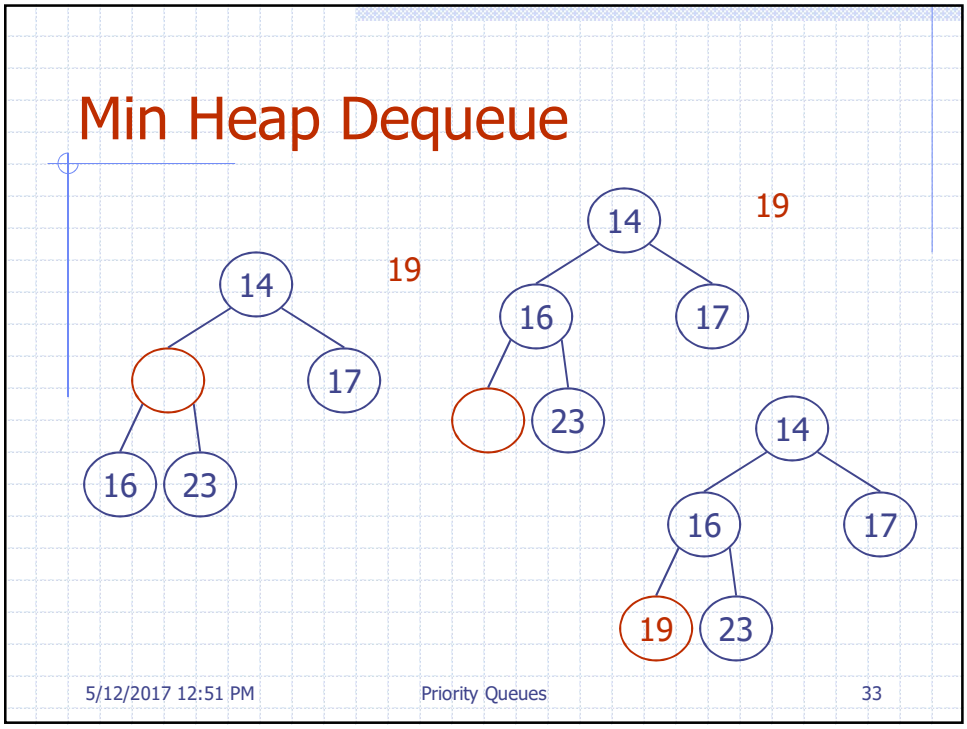
Min Heap Dequeue
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Priority Queues
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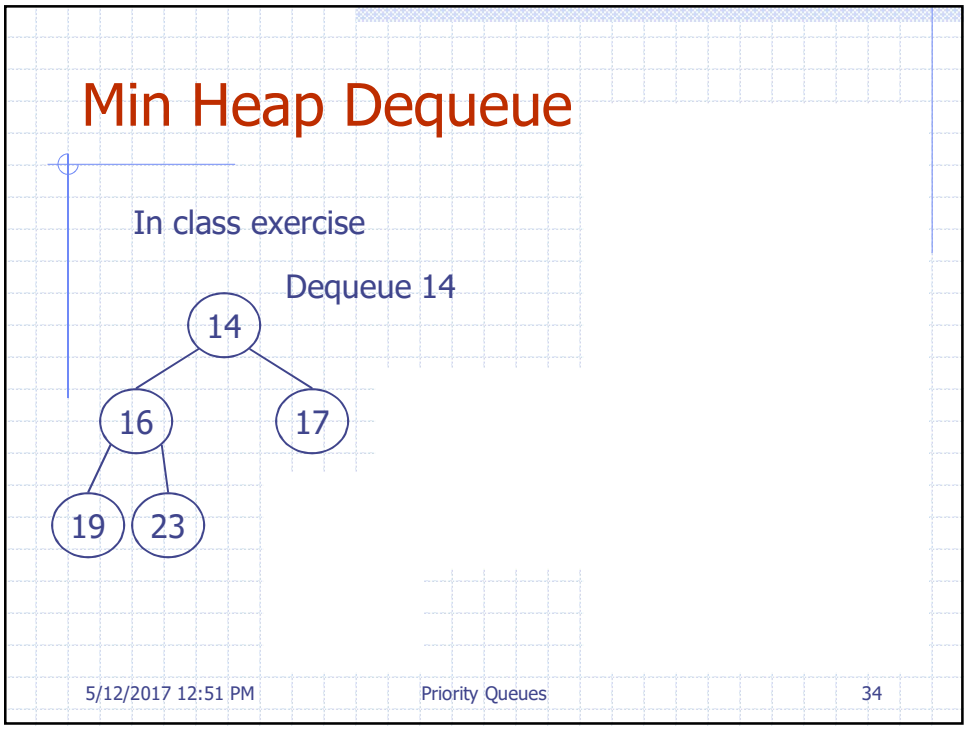
Min Heap Dequeue
In class exercise
Dequeue 14
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5/12/2017 12:51 PM
Priority Queues
34

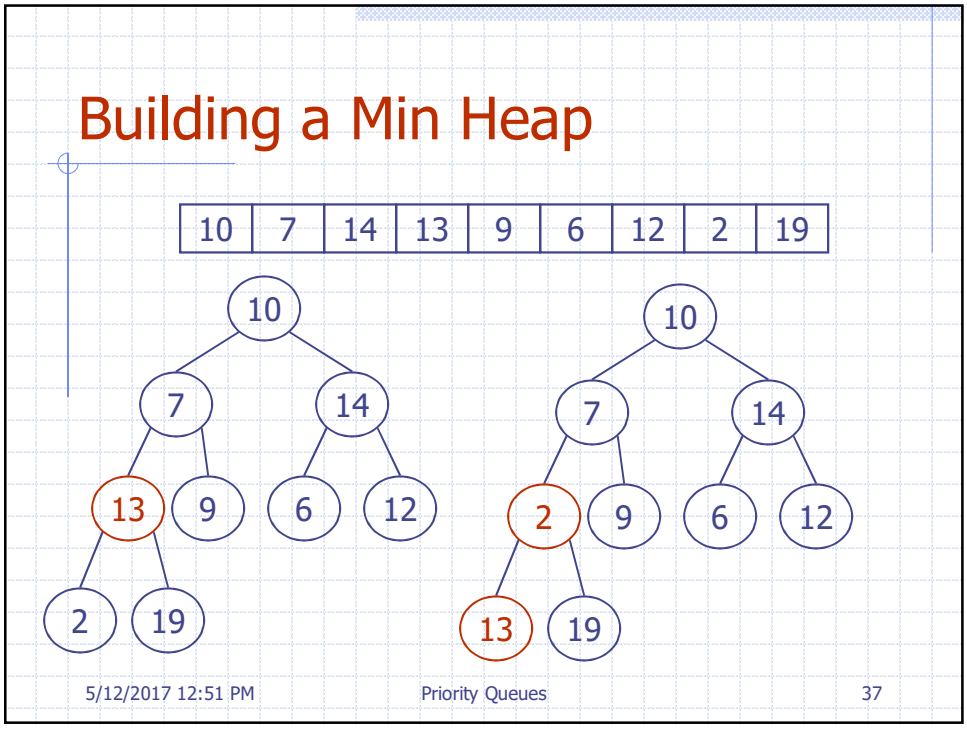
Heap Notes

- Common error when writing code for dequeue
  - Check number of children before finding child with highest priority value
- Searching for an item requires looking at all nodes since there is no particular ordering
  - Search is not too common

Building a Heap

- Naïve method
  - Perform N enqueue operations - O(n lg n)
- Better solution
  - Given an array of numbers:

for (x = (N/2)-1; x >= 0; --x)
Trickle down


Building a Min Heap
10 7 14 13 9 6 12 2 19
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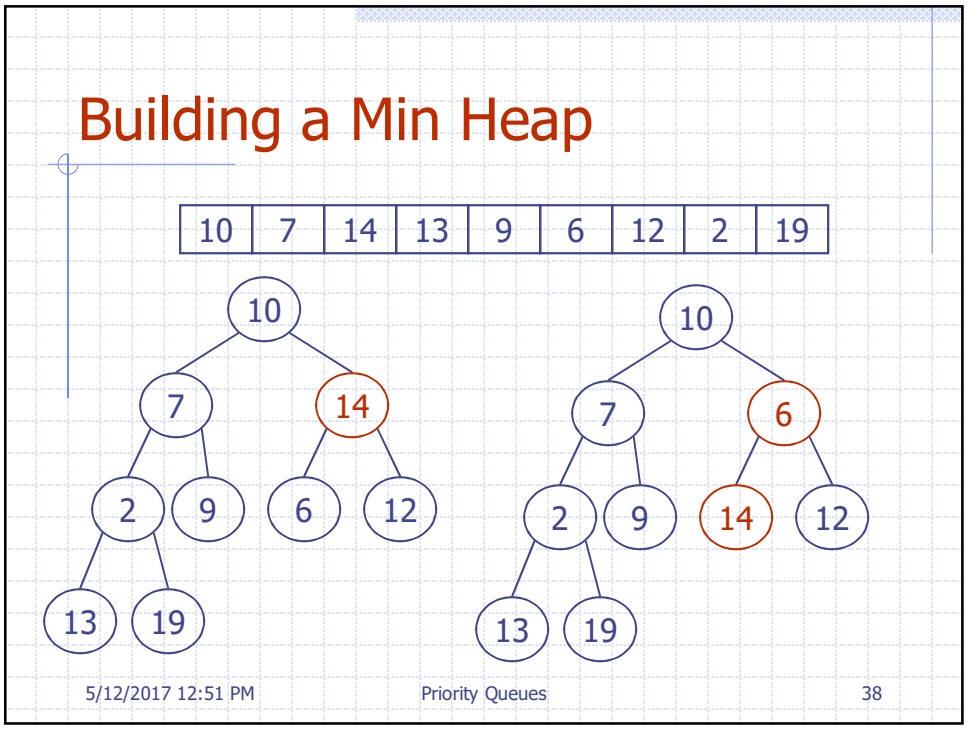
Priority Queues
37


Building a Min Heap
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Priority Queues
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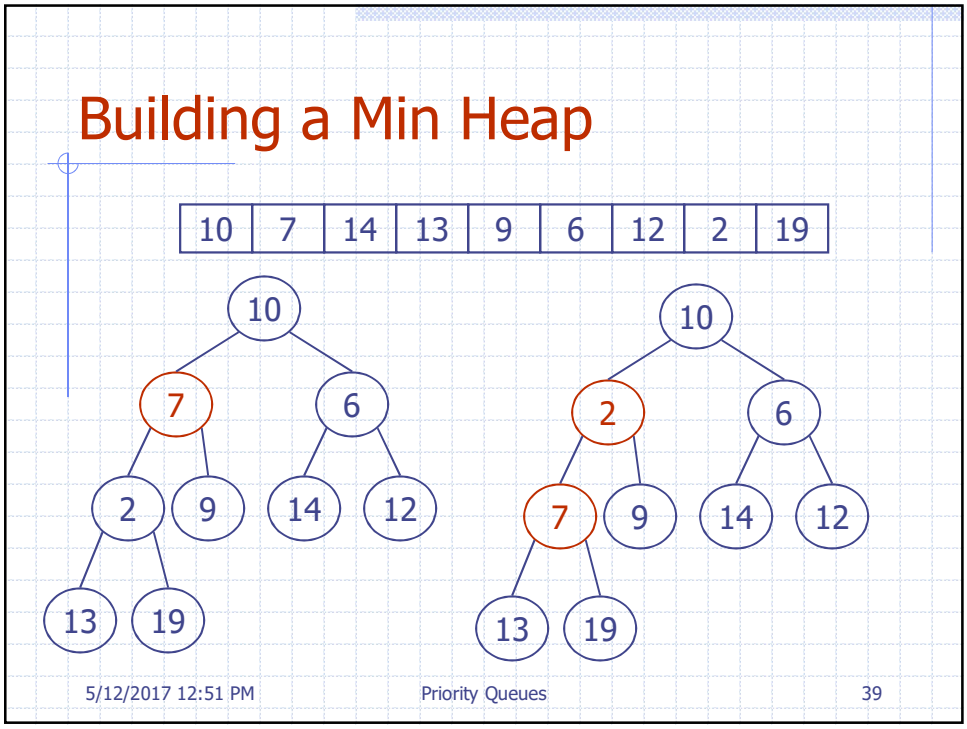
Building a Min Heap
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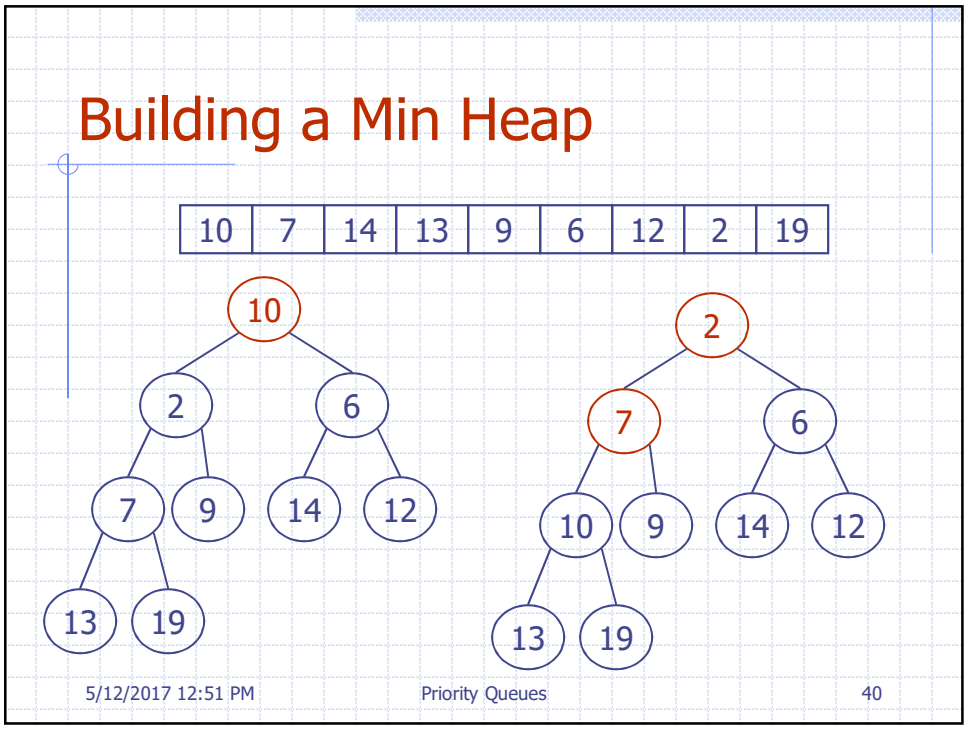
Priority Queues
39


Building a Min Heap
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Priority Queues
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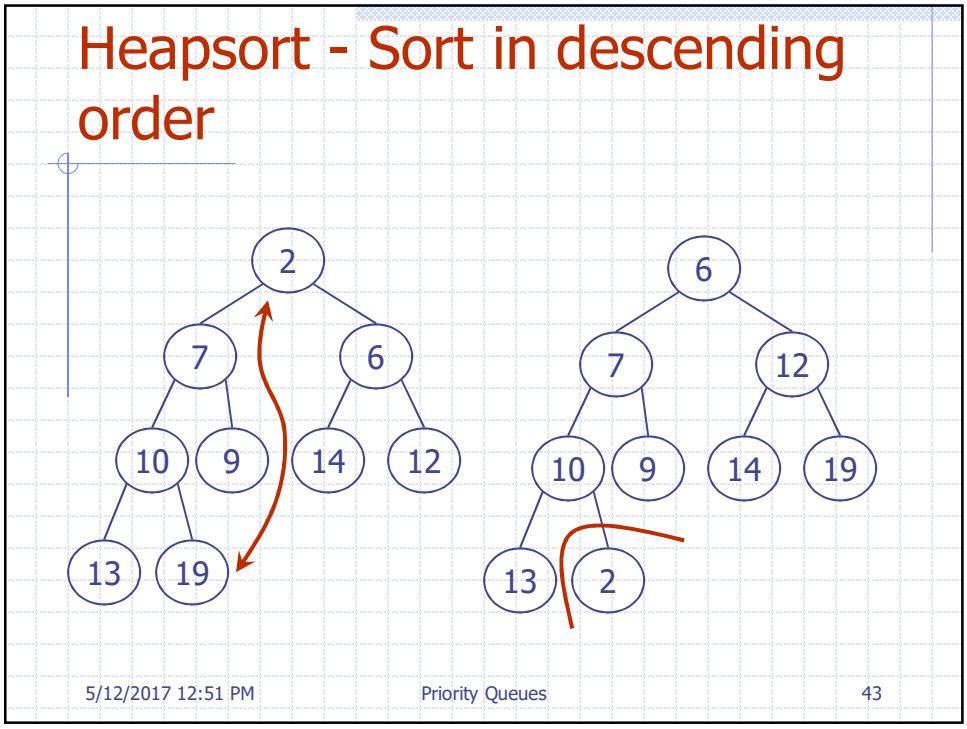
Building a Heap

- Build heap algorithm actually runs in O(n) time
  - Beyond the scope of the class to prove

Heapsort

- Build heap for the opposite of what you want
  - Max heap for ascending order
  - Min heap for descending order
- Take root and place in last array position, then think of array as 1 smaller
- Trickle down from root to rebuild heap
- Continue until all items are moved


Heapsort - Sort in descending order
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Priority Queues
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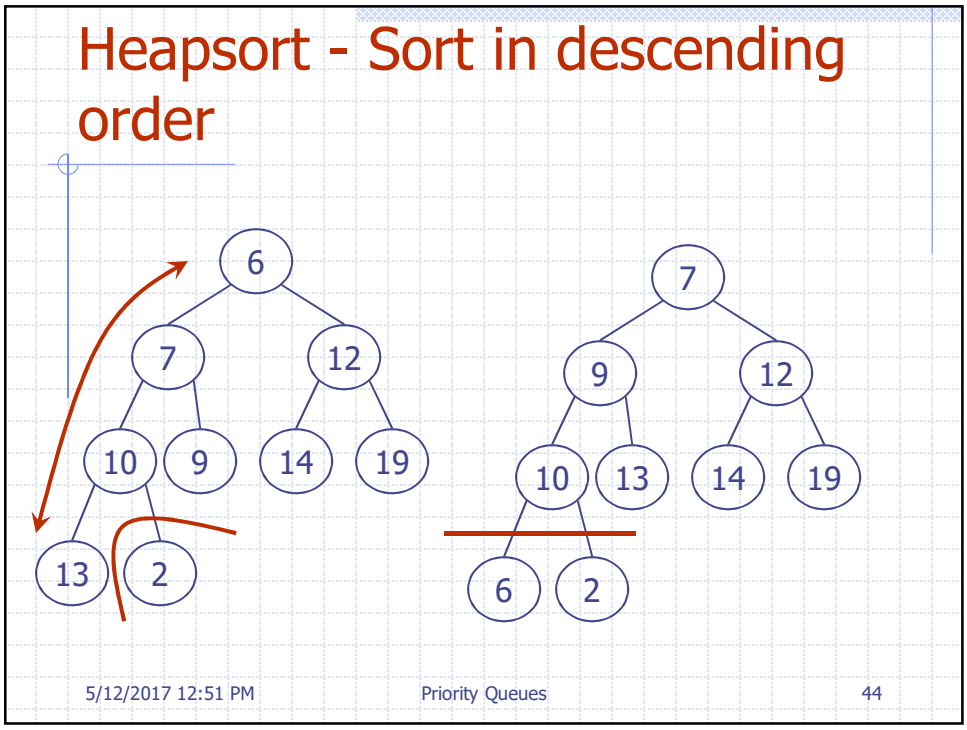
Heapsort - Sort in descending order
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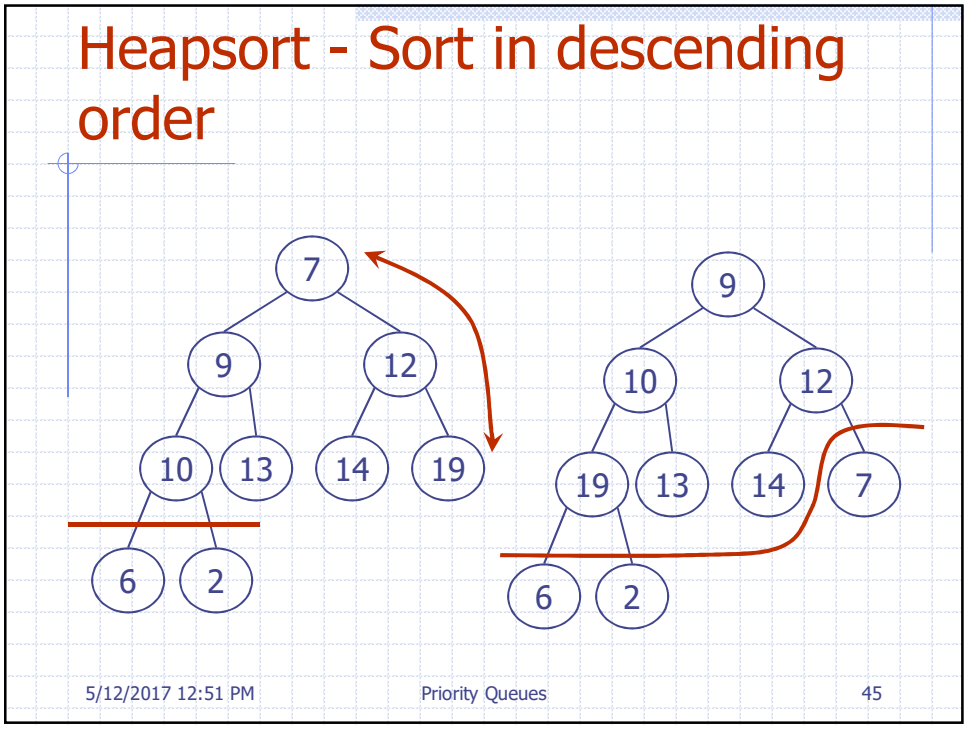
5/12/2017 12:51 PM
Priority Queues
44


Heapsort - Sort in descending order
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Priority Queues
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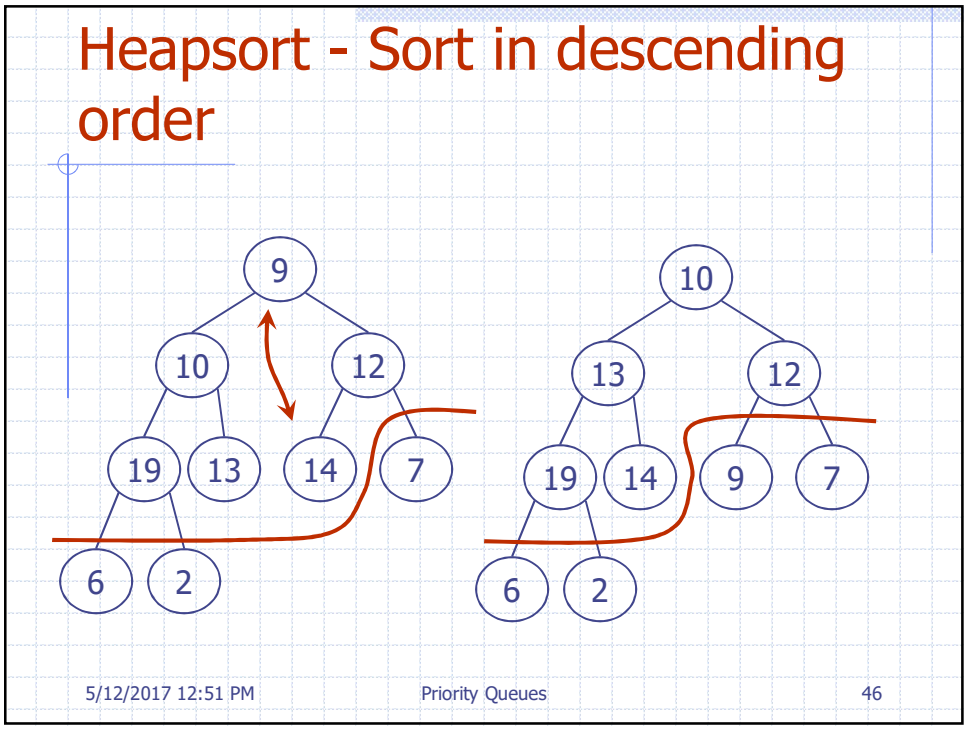
Heapsort - Sort in descending order
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Priority Queues
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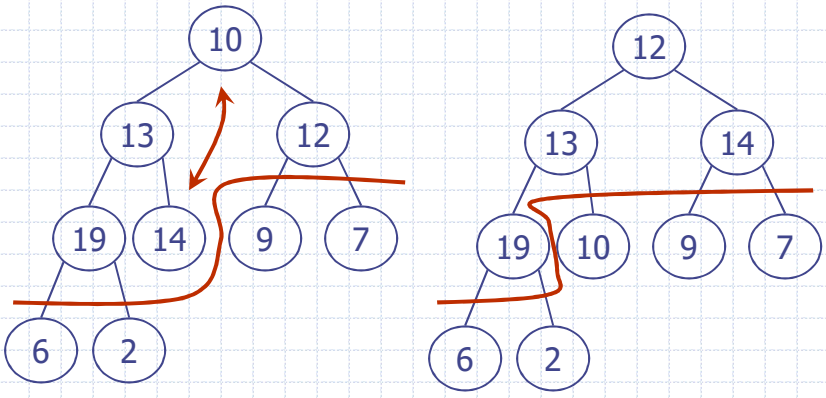
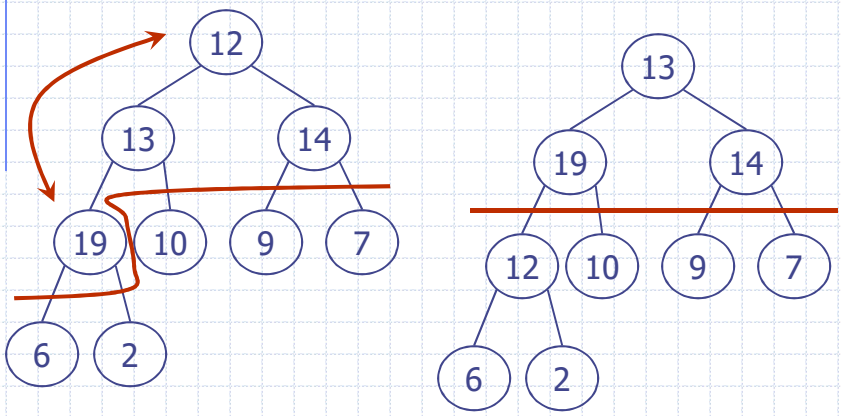
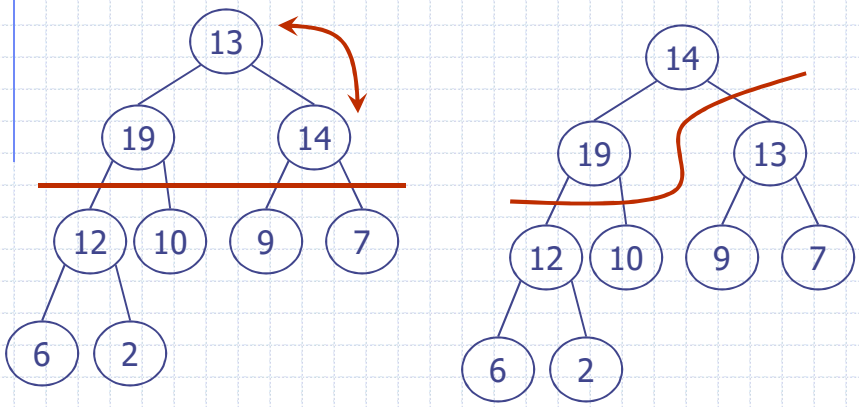
Heapsort - Sort in descending order



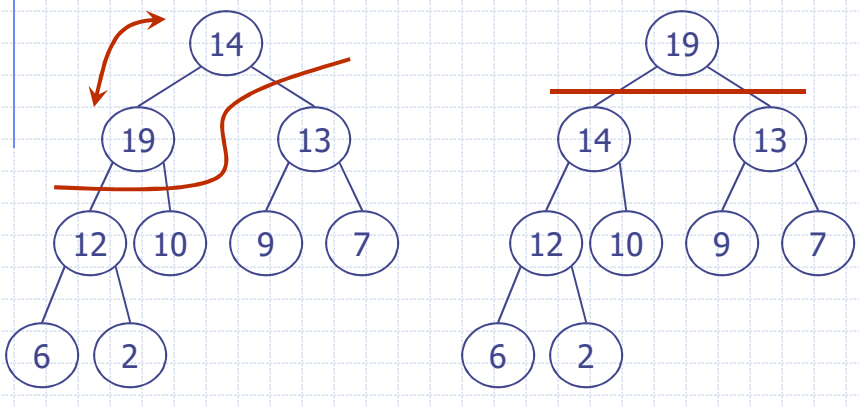
Heapsort - Sort in descending order

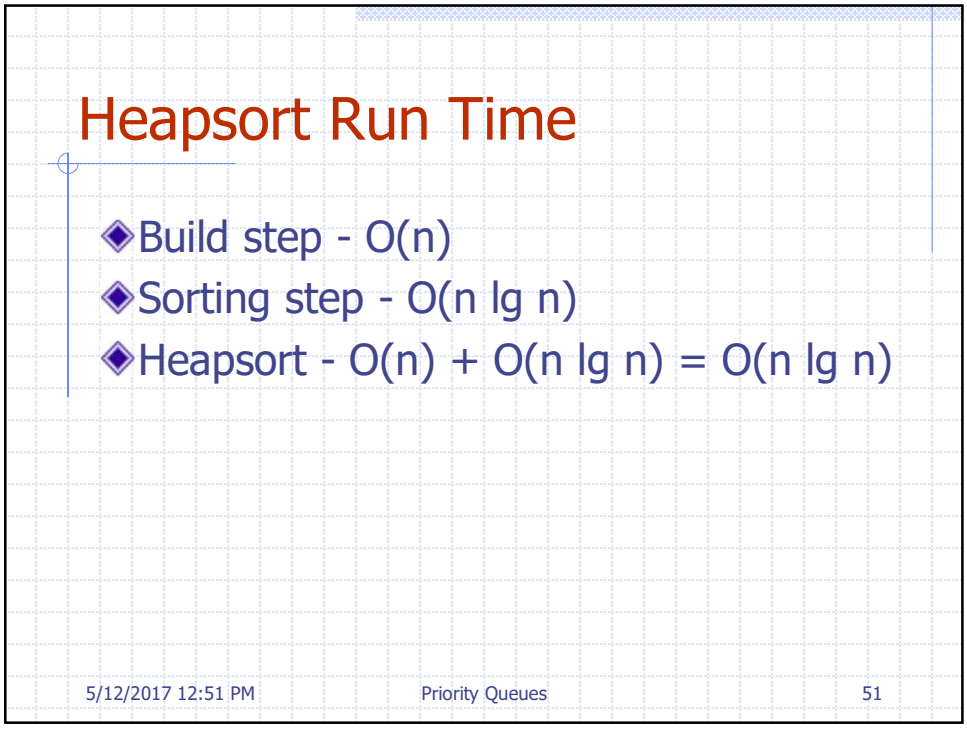


Heapsort - Sort in descending order



Heapsort - Sort in descending order




Heapsort Run Time
Build step - O(n)
Sorting step - O(n lg n)
Heapsort - O(n) + O(n lg n) = O(n lg n)
5/12/2017 12:51 PM
Priority Queues
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