```
%OLS

N = 20;  % Number of observations
R = 200;  % Number of realizations
X1 = randn(N,1);
X2 = randn(N,1);
phi = [X1 X2 ones(N,1)];
Y_data = zeros(N, R);

n_iter = 10000;
lr = 0.01;

% Generate R realizations of the data
for r = 1:R
    e = randn(N, 1);
    Y_gen = 3 * X1 + 5 * X2 + e;
    Y_data(:, r) = Y_gen;
end

theta_true = [3; 5; 0];  % True parameter values

[MSE, RB, MAD, avg_theta] = OLS(Y_data, phi, N, R, n_iter, lr, theta_true);

disp('Average Estimated Parameters using OLS:');
```

```
Average Estimated Parameters using OLS:
```

```
disp(avg_theta);
```

```
    2.9965
    4.9961
    0.0162
```

```


disp('Mean Square Error (MSE) for OLS:');
```

```
Mean Square Error (MSE) for OLS:
```

```
disp(MSE);
```

```
    0.0274
    0.0378
    0.0559
```

```


disp('Robust Bias (RB): for OLS');
```

```
Robust Bias (RB): for OLS
```

```
disp(RB);
```

```
    0.0083
    0.0081
    0.0171
```

```
disp('Median Absolute Deviation (MAD) for OLS:');
```

```
Median Absolute Deviation (MAD) for OLS:
```

```
disp(MAD);
```

```
    0.1170
    0.1279
    0.1481
```

```
function [MSE, RB, MAD, avg_theta] = OLS(Y_data, phi, N, R, n_iter, lr,
theta_true)
    % Initialize storage for parameter vectors and gradients
    theta_all = zeros(3, R);
    grad_all = zeros(3, R);

    % Perform parameter estimation for each realization
    for r = 1:R
        Y = Y_data(:, r);
        theta = zeros(3, 1);
        grad = zeros(3, 1);

        % Perform Gradient Descent (OLS estimation)
        for iter = 1:n_iter
            % Compute predicted values based on current parameters (theta)
            y_pred = phi * theta;

            % Compute gradient of the loss function (MSE) with respect to
theta
            grad = -1 * phi' * (Y - y_pred) / N;

            % Update parameters (theta) using gradient descent
            theta = theta - lr * grad;
        end

        % Store the parameter vector (theta) and gradient for the current
realization
        theta_all(:, r) = theta;
        grad_all(:, r) = grad;
    end

    % Compute average parameter vector and gradient across realizations
    avg_theta = mean(theta_all, 2);
    avg_grad = mean(grad_all, 2);

    % Initialize storage for metrics
    MSE = zeros(3, 1);
```

```
    RB = zeros(3, 1);
    MAD = zeros(3, 1);

    % Compute metrics for each parameter across all realizations
    for i = 1:3
        % Extract estimated parameter values (theta_i) across all
realizations
        theta_est = theta_all(i, :);

        % Compute bias (mean) and variance of the estimated parameter values
        bias_theta = mean(theta_est) - theta_true(i);
        var_theta = var(theta_est);

        % Compute Mean Square Error (MSE) for the parameter
        MSE(i) = bias_theta^2 + var_theta;

        % Compute Robust Bias (RB) for the parameter
        RB(i) = median(theta_est) - theta_true(i);

        % Compute Median Absolute Deviation (MAD) for the parameter
        MAD(i) = median(abs(theta_est - theta_true(i)));
    end
end
```

```
%LMS
beta_true = [3; 5; 0]; %True parameters

[MSE, RB, MAD, avg_beta] =LMS(Y_data, phi, N, R, n_iter, lr, beta_true);

disp('Average Estimated Parameters using LMS:');
```

```
Average Estimated Parameters using LMS:
```

```
disp(avg_beta);
```

```
    2.9550
    4.9320
    0.0484
```

```


disp('Mean Square Error (MSE) for LMS:');
```

```
Mean Square Error (MSE) for LMS:
```

```
disp(MSE);
```

```
    0.0622
    0.0941
    0.1131
```

```


disp('Robust Bias (RB) for LMS:');
```

```
Robust Bias (RB) for LMS:
```

```
disp(RB);
```

```
   -0.0326
   -0.0446
    0.0379
```

```
disp('Median Absolute Deviation (MAD) for LMS:');
```

```
Median Absolute Deviation (MAD) for LMS:
```

```
disp(MAD);
```

```
    0.1926
    0.1741
    0.2219
```

```
function [MSE, RB, MAD, avg_beta] = LMS(Y_data, phi, N, R, n_iter,
lr,theta_true)
    beta_all = zeros(3, R);
    grad_all = zeros(3, R);

    for r = 1:R
        Y = Y_data(:, r);
        beta = zeros(3, 1);
        gradient = zeros(3, 1);

        for iter = 1:n_iter
            y_pred = phi * beta;
            residuals = Y - y_pred;

            % Square the residuals
            squared_residuals = residuals.^2;
            huber_delta = median(squared_residuals);
            weights = min(1, huber_delta ./ squared_residuals);

            gradient = -1 * phi' * (weights .* residuals) / N;
            beta = beta - lr * gradient;
        end

        beta_all(:, r) = beta;
        grad_all(:, r) = gradient;
    end

    avg_beta = mean(beta_all, 2);
    avg_grad = mean(grad_all, 2);
```

```
    MSE = zeros(3, 1);
    RB = zeros(3, 1);
    MAD = zeros(3, 1);

    for i = 1:3
        beta_est = beta_all(i, :);

        % Compute bias (mean) and variance of the estimated parameter values
        bias_beta = mean(beta_est) - theta_true(i);
        var_beta = var(beta_est);

        % Compute Mean Square Error (MSE) for the parameter
        MSE(i) = bias_beta^2 + var_beta;

        % Compute Robust Bias (RB) for the parameter
        RB(i) = median(beta_est) - theta_true(i);

        % Compute Median Absolute Deviation (MAD) for the parameter
        MAD(i) = median(abs(beta_est - theta_true(i)));
    end
end
```

```
%LTS

gamma_true = [3; 5; 0]; %True parameters
q = floor(N/2 + 1);
[MSE, RB, MAD, avg_gamma] =LTS(Y_data, phi, N, R, n_iter, lr,q,gamma_true);

disp('Average Estimated Parameters using LTS:');
```

```
Average Estimated Parameters using LTS:
```

```
disp(avg_gamma);
```

```
    2.7850
    4.8234
   -0.5633
```

```
disp('Mean Square Error (MSE) for LTS:');
```

```
Mean Square Error (MSE) for LTS:
```

```
disp(MSE);
```

```
    0.1543
    0.1486
    0.3972
```

```
disp('Robust Bias (RB) for LTS:');
```

```
Robust Bias (RB) for LTS:
```

```
disp(RB);
```

```
   -0.1764
   -0.2060
   -0.5249
```

```
disp('Median Absolute Deviation (MAD) for LTS:');
```

```
Median Absolute Deviation (MAD) for LTS:
```

```
disp(MAD);
```

```
    0.2626
    0.2838
    0.5249
```

```
function [MSE, RB, MAD, avg_gamma] = LTS(Y_data, phi, N, R, n_iter, lr, q,
gamma_true)
    gamma_all = zeros(3, R);
    grad_all = zeros(3, R);

    for r = 1:R
        Y = Y_data(:, r);
        gamma = zeros(3, 1);
        grad = zeros(3, 1);

        for iter = 1:n_iter
            % Compute residuals
            res = Y - phi * gamma;

            abs_res = abs(res);
            sorted_res = sort(abs_res);

            % Select the q smallest residuals
            threshold = sorted_res(q);
            inliers_mask = (res <= threshold);

            if sum(inliers_mask) > 0
                grad = -1 * phi' * (inliers_mask .* res) / N;
            else
                grad = zeros(size(gamma)) / N;
            end

            % Update parameters (gamma) using gradient descent
            gamma = gamma - lr * grad;
        end

        gamma_all(:, r) = gamma;
        grad_all(:, r) = grad;
    end
```

```
    avg_gamma = mean(gamma_all, 2);
    avg_grad = mean(grad_all, 2);

    MSE = zeros(3, 1);
    RB = zeros(3, 1);
    MAD = zeros(3, 1);

    for i = 1:3
        gamma_est = gamma_all(i, :);

        % Compute bias (mean) and variance of the estimated parameter values
        bias_gamma = mean(gamma_est) - gamma_true(i);
        var_gamma = var(gamma_est);

        % Compute Mean Square Error (MSE) for the parameter
        MSE(i) = bias_gamma^2 + var_gamma;

        % Compute Robust Bias (RB) for the parameter
        RB(i) = median(gamma_est) - gamma_true(i);

        % Compute Median Absolute Deviation (MAD) for the parameter
        MAD(i) = median(abs(gamma_est - gamma_true(i)));
    end
end
```

```
%Applying Custom OLS,LMS and LTS on Real Dataset

data=readtable("medical_insurance.csv");

age = data.age;
bmi = data.bmi;
charges = data.charges;
sex = data.sex;
smoker = data.smoker;
region = data.region;

% Normalize numeric columns (age, bmi, charges)
age_normalized = normalize(age);
bmi_normalized = normalize(bmi);
charges_normalized = normalize(charges);

data.sex = categorical(data.sex);
data.sex = double(data.sex == 'male'); % Convert to numeric (1 for male, 0
for female)

data.smoker = categorical(data.smoker);
data.smoker = double(data.smoker == 'yes'); % Convert to numeric (1 for
smoker, 0 for non-smoker)
```

```
% Perform categorical encoding for 'region'
region_encoded = grp2idx(region);

% Combine all encoded features into a single matrix
features = [age_normalized, bmi_normalized,data.smoker,data.sex,
region_encoded,data.children,ones(2772,1)];
target=[charges_normalized];
```

```
%Splitting into traing and testing data
trainRatio = 0.8;

% Create a random partition for training and testing
c = cvpartition(size(features, 1), 'HoldOut', 1 - trainRatio);

% Get indices for training and testing data
trainIdx = training(c); % Logical indices for training data
testIdx = test(c); % Logical indices for testing data

% Split features into training and testing sets
X_train = features(trainIdx, :)
```

```
X_train = 2218×7
   -1.4281   -0.4570    1.0000         0    1.0000         0    1.0000
   -1.4991    0.5006         0    1.0000    2.0000    1.0000    1.0000
   -0.7890    0.3750         0    1.0000    2.0000    3.0000    1.0000
   -0.4339   -1.3046         0    1.0000    3.0000         0    1.0000
   -0.5049   -0.2971         0    1.0000    3.0000         0    1.0000
   -0.5759   -0.8094         0         0    2.0000         0    1.0000
    0.4893    0.4468         0         0    2.0000    1.0000    1.0000
   -0.1498   -0.4831         0         0    3.0000    3.0000    1.0000
   -0.1498   -0.1422         0    1.0000    4.0000    2.0000    1.0000
    1.4835   -0.7931         0         0    3.0000         0    1.0000
     ⋮
```

```
X_test = features(testIdx, :);

% Split target into training and testing sets
Y_train = target(trainIdx);
Y_test = target(testIdx);
n=size(Y_train);
```

```
n_iter = 10000;
lr = 0.01;

theta=zeros(7,1);

[avg_theta] = modified_OLS(Y_train,X_train,n(1),1, n_iter, lr)
```

```
avg_theta = 7×1
    0.2996
```

```
    0.1676
    1.9764
   -0.0222
    0.0308
    0.0405
   -0.5114
```

```
%Calculating the MSE for the model
y_test_pred=X_test*avg_theta;
error=(y_test_pred-Y_test);
mse_per_example = (error.^2);

average_mse = mean(mse_per_example);
disp(['Average Mean Squared Error (MSE) on Test Set using OLS: ',
num2str(average_mse)]);
```

```
Average Mean Squared Error (MSE) on Test Set using OLS: 0.22689
```

```
function [avg_theta] = modified_OLS(Y_data, phi, N, R, n_iter, lr)
    % Initialize storage for parameter vectors and gradients
    theta_all = zeros(7, R);
    grad_all = zeros(7, R);

    % Perform parameter estimation for each realization
    for r = 1:R
        Y = Y_data(:, r);
        theta = zeros(7, 1);
        grad = zeros(7, 1);

        % Perform Gradient Descent (OLS estimation)
        for iter = 1:n_iter
            % Compute predicted values based on current parameters (theta)
            y_pred = phi * theta;

            % Compute gradient of the loss function (MSE) with respect to
theta
            grad = -1 * phi' * (Y - y_pred) / N;

            % Update parameters (theta) using gradient descent
            theta = theta - lr * grad;
        end

        % Store the parameter vector (theta) and gradient for the current
realization
        theta_all(:, r) = theta;
        grad_all(:, r) = grad;
    end

    % Compute average parameter vector and gradient across realizations
    avg_theta = mean(theta_all, 2);
    avg_grad = mean(grad_all, 2);
```

```
end
```

```
n_iter = 10000;
lr = 0.01;

beta=zeros(7,1);

[avg_beta] = modified_LMS(Y_train,X_train,n(1),1, n_iter, lr)
```

```
avg_beta = 7×1
    0.3025
   -0.0011
    0.1233
   -0.0385
    0.0142
    0.0341
   -0.5762
```

```
%Calculating the MSE for the model
y_test_pred=X_test*avg_beta;
error=(y_test_pred-Y_test);
mse_per_example = (error.^2);

average_mse = mean(mse_per_example);
disp(['Average Mean Squared Error (MSE) on Test Set using LMS: ',
num2str(average_mse)]);
```

```
Average Mean Squared Error (MSE) on Test Set using LMS: 0.97284
```

```
function [avg_beta] = modified_LMS(Y_data, phi, N, R, n_iter, lr)
    beta_all = zeros(7, R);
    grad_all = zeros(7, R);

    for r = 1:R
        Y = Y_data(:, r);
        beta = zeros(7, 1);
        gradient = zeros(7, 1);

        for iter = 1:n_iter
            y_pred = phi * beta;
            residuals = Y - y_pred;

            % Square the residuals
            squared_residuals = residuals.^2;
            huber_delta = median(squared_residuals);
            weights = min(1, huber_delta ./ squared_residuals);

            gradient = -1 * phi' * (weights .* residuals) / N;
            beta = beta - lr * gradient;
        end
```

```
        beta_all(:, r) = beta;
        grad_all(:, r) = gradient;
    end

    avg_beta = mean(beta_all, 2);
    avg_grad = mean(grad_all, 2);
end
```

```
n_iter = 10000;
lr = 0.01;
q = floor(n(1)/2 + 1);
gamma=zeros(7,1);

[avg_gamma] = modified_LTS(Y_train,X_train,n(1),1, n_iter, lr,q)
```

```
avg_gamma = 7×1
    0.3118
   -0.0009
    0.0015
   -0.0406
    0.0137
    0.0424
   -0.6031
```

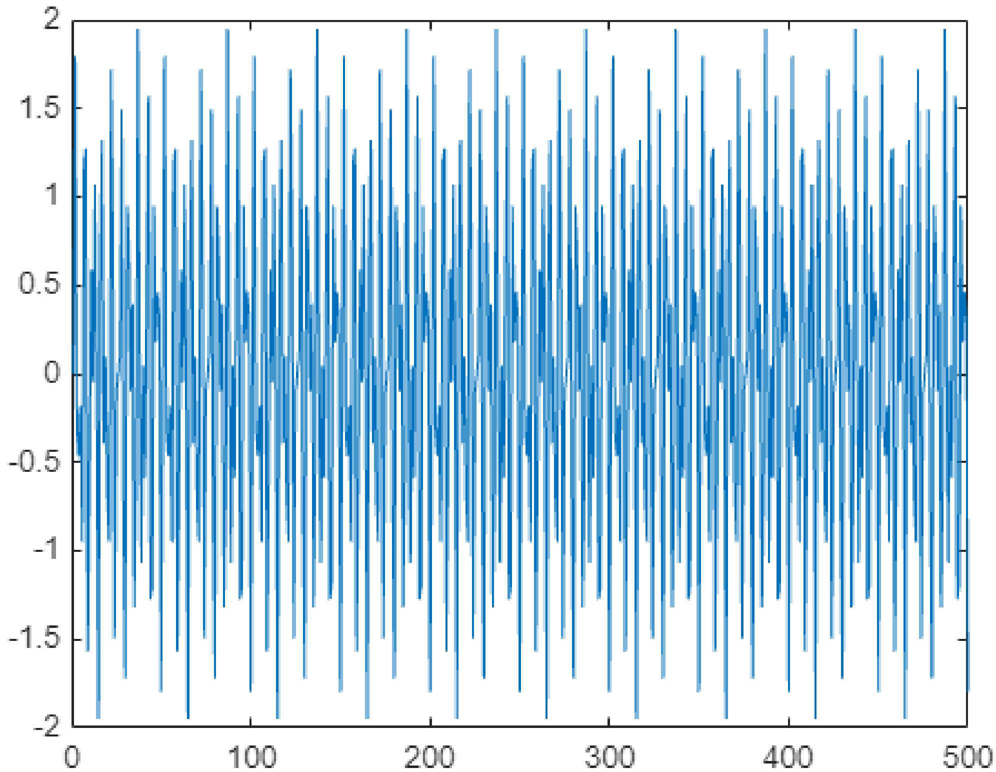
```
%Calculating the MSE for the model
y_test_pred=X_test*avg_gamma;
error=(y_test_pred-Y_test);
mse_per_example = (error.^2);

average_mse = mean(mse_per_example);
disp(['Average Mean Squared Error (MSE) on Test Set using LTS: ',
num2str(average_mse)]);
```

```
Average Mean Squared Error (MSE) on Test Set using LTS: 1.0817
```

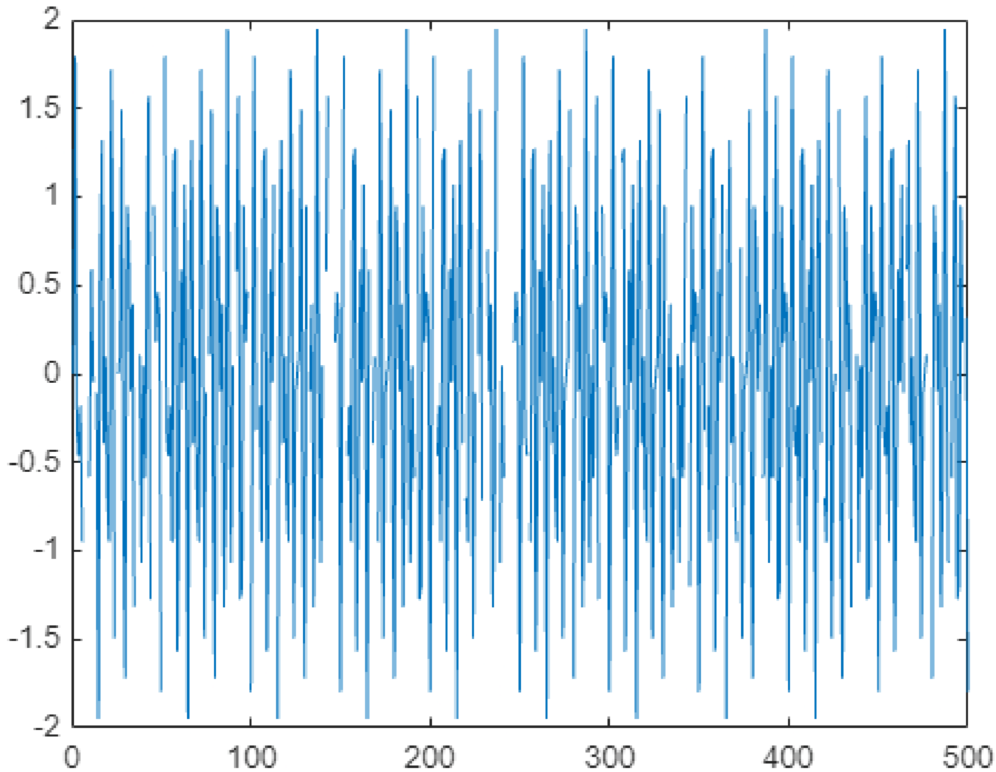
```
function [avg_gamma] = modified_LTS(Y_data, phi, N, R, n_iter, lr,q)
    gamma_all = zeros(7, R);
    grad_all = zeros(7, R);

    for r = 1:R
        Y = Y_data(:, r);
        gamma = zeros(7, 1);
        grad = zeros(7, 1);

        for iter = 1:n_iter
            % Compute residuals
            res = Y - phi * gamma;

            abs_res = abs(res);
            sorted_res = sort(abs_res);
```

```
            % Select the q smallest residuals
            threshold = sorted_res(q);
            inliers_mask = (res <= threshold);

            if sum(inliers_mask) > 0
                grad = -1 * phi' * (inliers_mask .* res) / N;
            else
                grad = zeros(size(gamma)) / N;
            end

            % Update parameters (gamma) using gradient descent
            gamma = gamma - lr * grad;
        end

        gamma_all(:, r) = gamma;
        grad_all(:, r) = grad;
    end

    avg_gamma = mean(gamma_all, 2);
    avg_grad = mean(grad_all, 2);
end
```

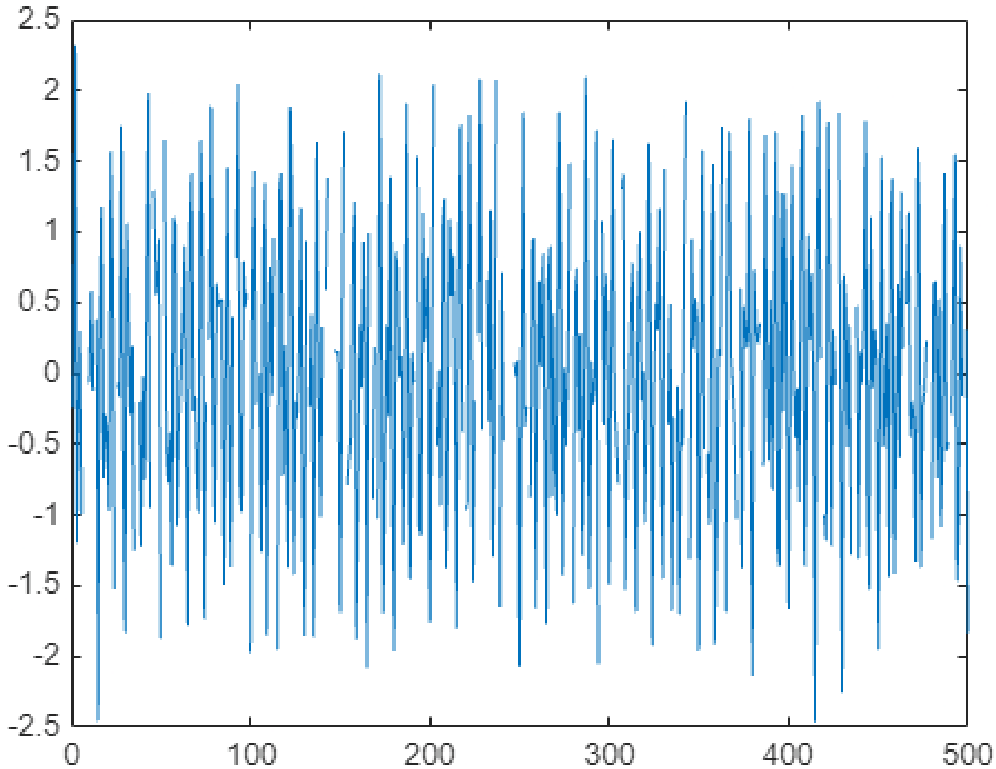
```
%LSP PART (a)
Fs = 50;
N = 500;
f1 = 10;
f2 = 17;
missing_percentage = 0.1;
SNR_dB = 10;
t = (0:N-1) / Fs;


signal = sin(2*pi*f1*t) + sin(2*pi*f2*t);
plot(signal);
```



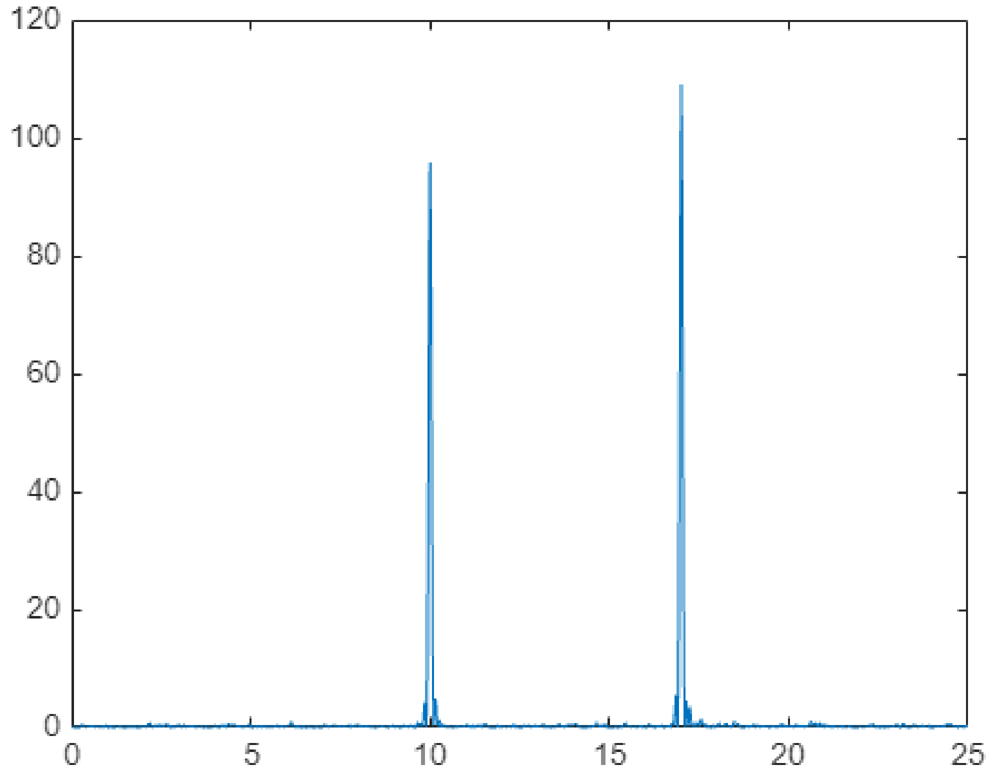

```
missing_data_indices = randperm(N, round(missing_percentage*N));
signal(missing_data_indices) = NaN;
plot(signal);
```




```
noise_power = nanvar(signal) / (10^(SNR_dB/10));

% Add noise to the signal
noise = sqrt(noise_power) * randn(size(signal));
noisy_signal = signal + noise;
plot(noisy_signal);
```



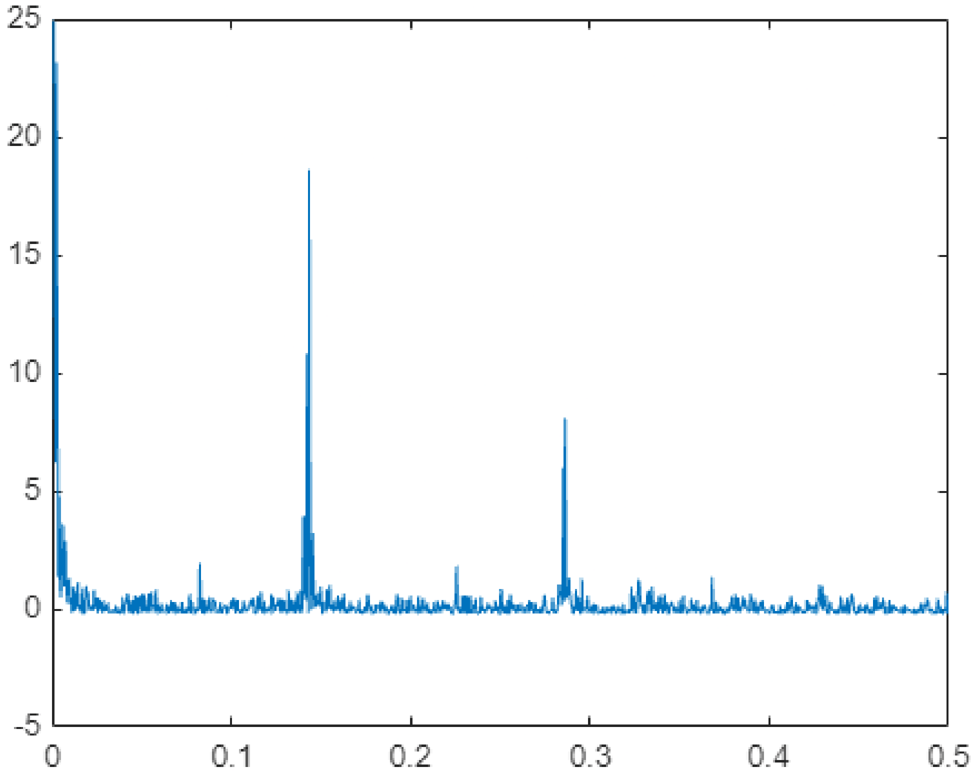

```
frequencies=linspace(1/Fs,Fs/2,1000);
[periodogram,ai,bi]=lomb_scargle_periodogram(noisy_signal,t,frequencies,0.01,
1000);
plot(frequencies,periodogram);
```




```
%LSP part (b)
%Importing the tesla stock data in csv format
table_data = readtable('tesla_csv.csv');


%Encoding date to make it easier to inerpret
date=datenum(datetime(table_data.Date,"InputFormat","dd-MM-yyyy"));
date=date-date(1)+1;
charges=normalize(table_data.Close);


%Splitting the data into test and train
trainRatio = 0.8;

% Create a random partition for training and testing
c = cvpartition(size(date, 1), 'HoldOut', 1 - trainRatio);

% Get indices for training and testing data
trainIdx = training(c);
testIdx = test(c);

% Split features into training and testing sets
Xtrain = date(trainIdx, :);
Xtest = date(testIdx, :);
```

```
% Split target into training and testing sets
Ytrain = charges(trainIdx);
Ytest = charges(testIdx);
```

```
%Appying lomb scargle periodogram on train data
%(i)

frequencies=linspace(0.001,0.5,1000);

[periodogram,ai,bi]=lomb_scargle_periodogram(Ytrain,Xtrain,frequencies,0.01,1
000);
plot(frequencies,periodogram);
```




```
%Parameters from the model
freq_1=frequencies(286);
freq_2=frequencies(572);
a_1=ai(286);
b_1=bi(286);
a_2=ai(572);
b_2=bi(572);

y_pred=a_1* cos(2*pi*freq_1 *Xtest) + b_1 * sin(2*pi*freq_1 * Xtest)+a_2*
cos(2*pi*freq_2 *Xtest) + b_2 * sin(2*pi*freq_2 *Xtest);
```

```
Ytest;

%Calculating the evaluation metrics

% Calculate NMSE (Normalized Mean Square Error)
mse = mean((y_pred - Ytest).^2);
variance = var(Ytest);
nmse = mse / variance;
disp(['The Normalised Mean Square Error using Lomb Scargle Periodogram: ',
num2str(nmse)]);
```

```
The Normalised Mean Square Error using Lomb Scargle Periodogram: 1.0009
```

```
%Calculaitng the mean absolute percentagre error
mape = mean(abs((y_pred - Ytest) ./ Ytest) * 100);
disp(['The Normalised Mean Absolute Percentage using Lomb Scargle
Periodogram: ', num2str(mape)]);
```

```
The Normalised Mean Absolute Percentage using Lomb Scargle Periodogram: 100.1085
```

```
%Testing ARIMA MODEL
%(ii)
mdl = arima(2, 1, 1); % Specify the appropriate p, d, q values
EstMdl = estimate(mdl, Ytrain);
```

```
Warning: Nonlinear inequality constraints are active; standard errors may be inaccurate.
```

```
    ARIMA(2,1,1) Model (Gaussian Distribution):
```

	Value	StandardError	TStatistic	PValue
Constant	4.9232e-05	6.237e-05	0.78935	0.42991
AR{1}	1.0251	0.022035	46.522	0
AR{2}	-0.02514	0.017922	-1.4028	0.16069
MA{1}	-0.96985	0.014511	-66.836	0
Variance	0.0043211	5.4345e-05	79.512	0

```
Ypred = forecast(EstMdl, numel(Ytest), 'Y0', Ytest);
% Calculate NMSE (Normalized Mean Square Error)
mse = mean((Ypred - Ytest).^2);
variance = var(Ytest);
nmse = mse / variance;
disp(['The Normalised Mean Square Error using ARIMA(2,1,1) model: ',
num2str(nmse)]);
```

```
The Normalised Mean Square Error using ARIMA(2,1,1) model: 524.7689
```

```
%Calculaitng the mean absolute percentagre error
mape = mean(abs((Ypred - Ytest) ./ Ytest) * 100);
disp(['The Mean Absolute Percentage using ARIMA(2,1,1) model: ',
num2str(mape)]);
```

```
The Mean Absolute Percentage using ARIMA(2,1,1) model: 6378.1207
```

```
function [periodogram,ai,bi] = lomb_scargle_periodogram(y, t, frequencies,
learning_rate, max_iterations)
    periodogram = zeros(size(frequencies));
    ai = zeros(size(frequencies));
    bi = zeros(size(frequencies));
    nan_indices = isnan(y);
    y(nan_indices) = 0;

    for i =1: length(frequencies)
        for iter = 1:max_iterations
            f= frequencies(i);
            residual = y - (ai(i) * cos(2*pi*f * t) + bi(i) * sin(2*pi*f *
t));
            gradient_a = -2 * sum(residual .* cos(2*pi*f * t));
            gradient_b = -2 * sum(residual .* sin(2*pi*f * t));
            ai(i) = ai(i) - learning_rate * gradient_a/1000;
            bi(i) = bi(i) - learning_rate * gradient_b/1000;
        end
        periodogram(i) = periodogram(i) +sum(residual.^2);



    end
    for iter = 1:max_iterations
            omega = 0;
            residual = y - (ai(i) * cos(omega * t) + bi(i) * sin(omega * t));
            gradient_a = -2 * sum(residual .* cos(omega * t));
            gradient_b = -2 * sum(residual .* sin(omega * t));
            ai(i) = ai(i) - learning_rate * gradient_a/1000;
            bi(i) = bi(i) - learning_rate * gradient_b/1000;
    end
    zero_chi_square=sum(residual.^2);
    for i =1:length(frequencies)
        periodogram(i)=(zero_chi_square-periodogram(i))/2;
    end
    y(nan_indices) = NaN;
end
```