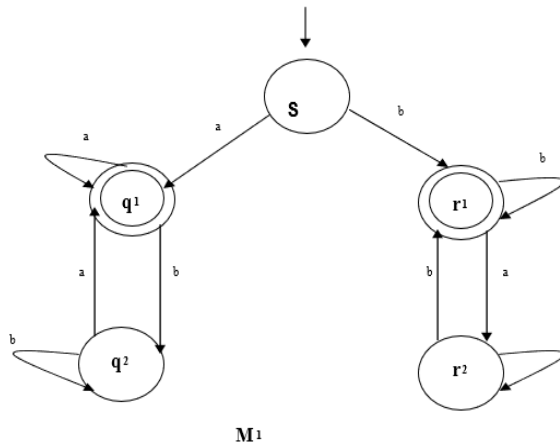


The figure below shows the state diagram of a finite automaton  $M_1$ . Give the formal description of this automaton. (5 Marks)

Provide a conclusion on the language accepted by this machine



| Input Alphabet |    |    |
|----------------|----|----|
| States         | a  | b  |
| S              | q1 | r1 |
| q1             | q1 | q2 |
| q2             | q1 | q2 |
| r1             | r2 | r1 |
| r2             | r2 | r1 |

$Q = \{S, q1, q2, r1, r2\}$

$\Sigma = \{a, b\}$

$\delta =$  Transition Symbol

$q_0 = \{S\}$

$F = \{q1, r1\}$

| Possible path  | Pattern |
|--|---------|
| $S \rightarrow b \rightarrow r1$   | $b$     |
| $S \rightarrow b \rightarrow r1 \rightarrow b \rightarrow r1$  | $bb$    |
| $S \rightarrow b \rightarrow r1 \rightarrow a \rightarrow r2 \rightarrow b \rightarrow r1$   | $bab$   |
| $S \rightarrow b \rightarrow r1 \rightarrow a \rightarrow r2 \rightarrow a \rightarrow r2 \rightarrow b \rightarrow r1$                              | $baab$  |
| $S \rightarrow b \rightarrow r1 \rightarrow b \rightarrow r1 \rightarrow a \rightarrow r2 \rightarrow b \rightarrow r1$                              | $bbab$  |
| $S \rightarrow b \rightarrow r1 \rightarrow b \rightarrow r1 \rightarrow a \rightarrow r2 \rightarrow a \rightarrow r2 \rightarrow b \rightarrow r1$ | $bbaab$ |
| $S \rightarrow a \rightarrow q1$   | $a$     |
| $S \rightarrow a \rightarrow q1 \rightarrow b \rightarrow q2 \rightarrow a \rightarrow q1$   | $aba$   |
| $S \rightarrow a \rightarrow q1 \rightarrow a \rightarrow q1$  | $aa$    |
| $S \rightarrow a \rightarrow q1 \rightarrow a \rightarrow q1 \rightarrow b \rightarrow q2 \rightarrow a \rightarrow q1$                              | $aaba$  |
| $S \rightarrow a \rightarrow q1 \rightarrow a \rightarrow q1 \rightarrow b \rightarrow q2 \rightarrow b \rightarrow q2 \rightarrow a \rightarrow q1$ | $aabba$ |

### Conclusion

The machine accepts a language that either starts with an "a" and ends with an "a" or a language that starts with a "b" and ends with a "b".

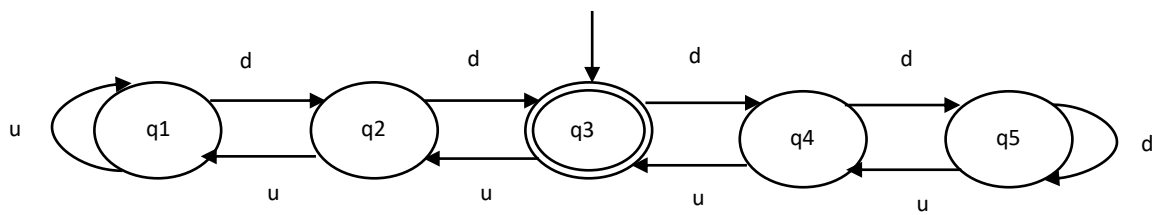
$L(M_1) = \sum W \in W$ ,  $W$  starts and ends with the same symbol.

**Question Seven (from Logic & Truth Tables Lesson):**

The formal description of a DFA is  $(\{q_1, q_2, q_3, q_4, q_5\}, (u, d), \delta, q_3, \{q_3\})$ , where  $\delta$  is given by the following transition table. Give the state diagram of this machine. (8 Marks).

|                | u              | d              |
|----------------|----------------|----------------|
| q <sub>1</sub> | q <sub>1</sub> | q <sub>2</sub> |
| q <sub>2</sub> | q <sub>1</sub> | q <sub>3</sub> |
| q <sub>3</sub> | q <sub>2</sub> | q <sub>4</sub> |
| q <sub>4</sub> | q <sub>3</sub> | q <sub>5</sub> |
| q <sub>5</sub> | q <sub>4</sub> | q <sub>5</sub> |

**State diagram of this machine**



| Possible path  | Pattern |
|--|---------|
| q <sub>3</sub> --u--> q <sub>0</sub> --u--> q <sub>1</sub> --u--> q <sub>1</sub> --d--> q <sub>2</sub> --d--> q <sub>3</sub> | uuudd   |
| q <sub>3</sub> --u--> q <sub>0</sub> --u--> q <sub>1</sub> --d--> q <sub>2</sub> --d--> q <sub>3</sub>                       | uudd    |
| q <sub>3</sub> --d--> q <sub>4</sub> --d--> q <sub>5</sub> --d--> q <sub>5</sub> --u--> q <sub>4</sub> --u--> q <sub>3</sub> | ddduu   |
| q <sub>3</sub> --d--> q <sub>4</sub> --d--> q <sub>5</sub> --u--> q <sub>4</sub> --u--> q <sub>3</sub>                       | dduu    |

**Conclusion**

The machine accepts a language that starts with double 'u' and ends with double 'd' or starts with double 'd' and ends with double 'u'.