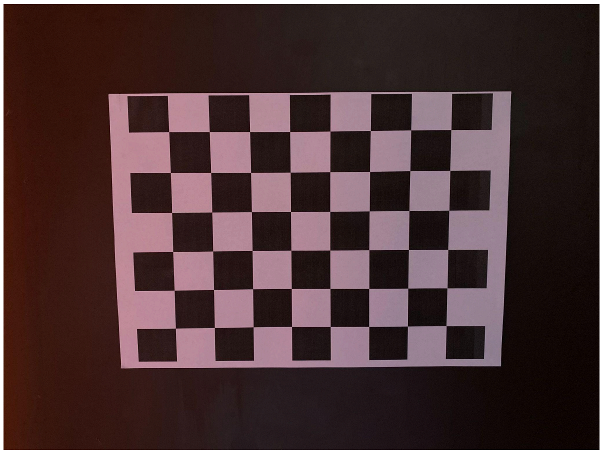
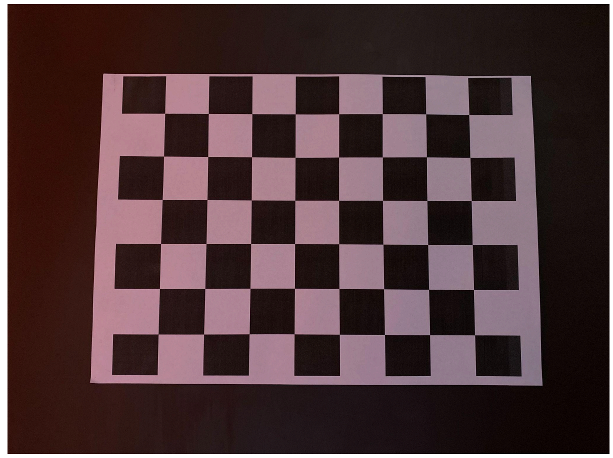
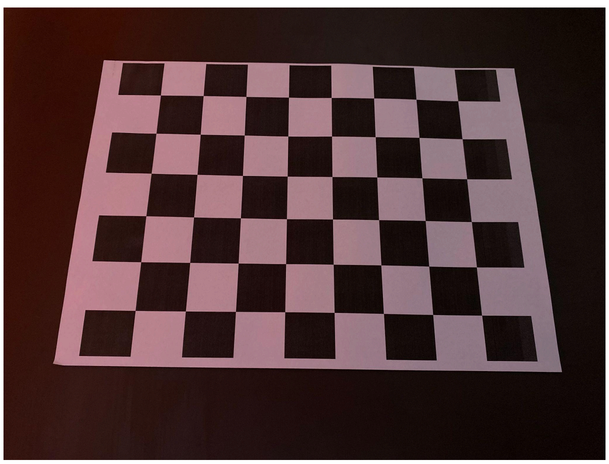
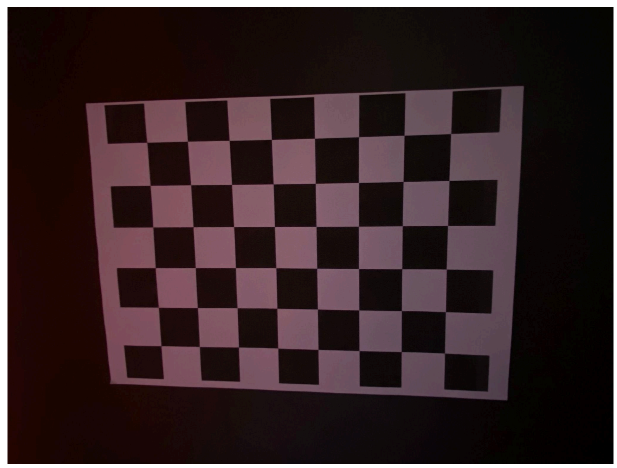
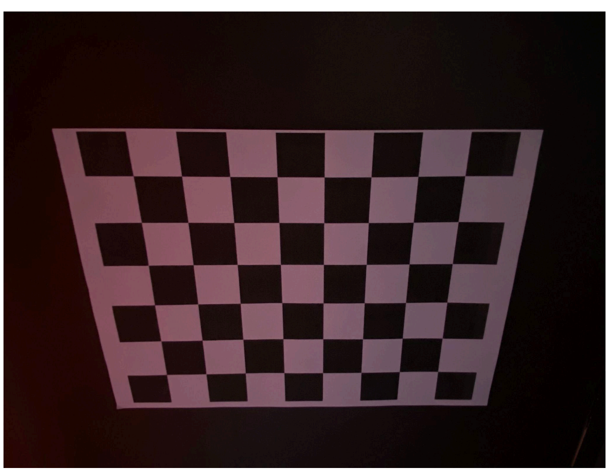
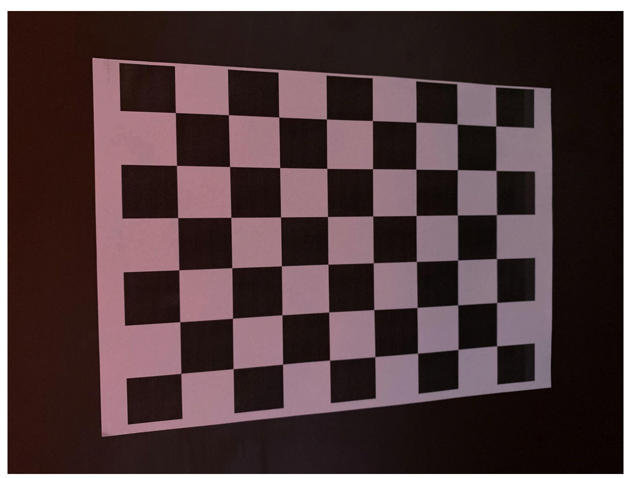
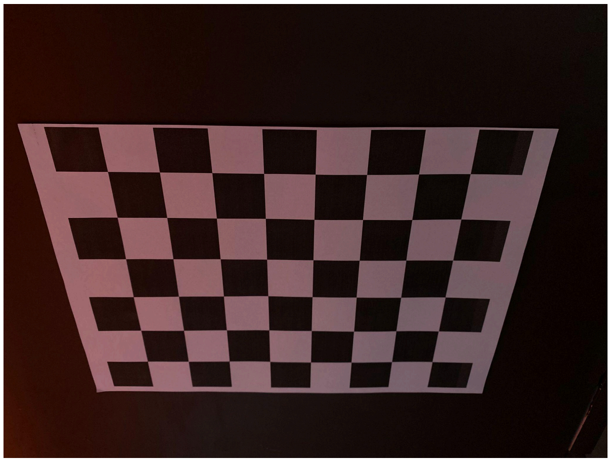
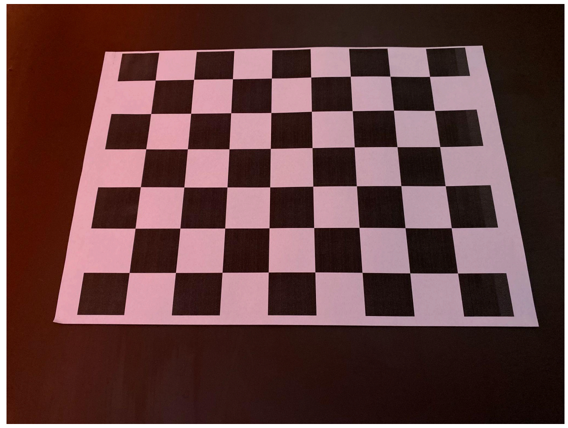
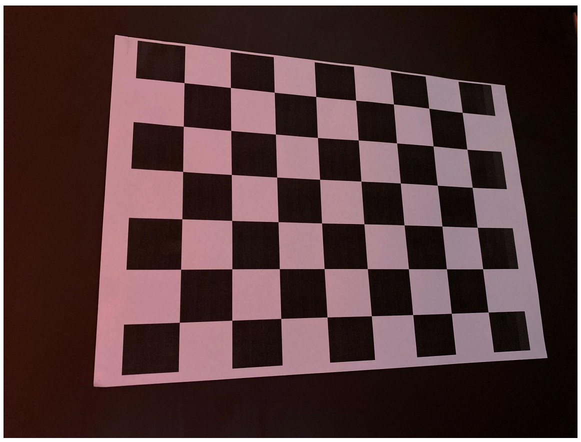
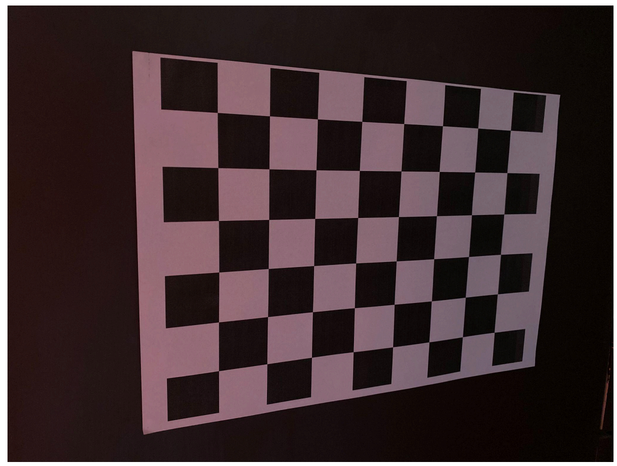
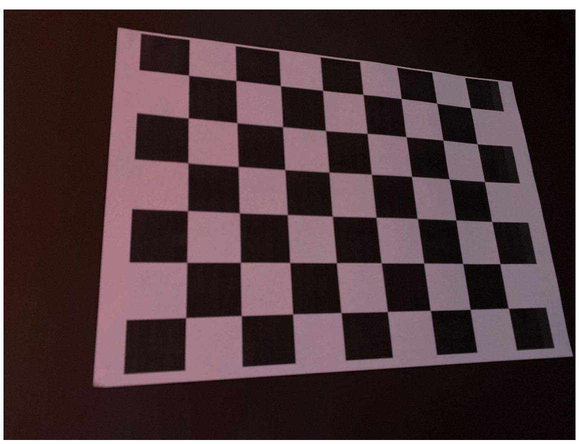
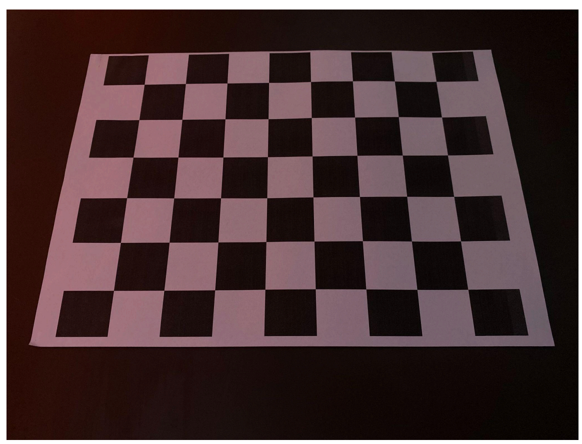
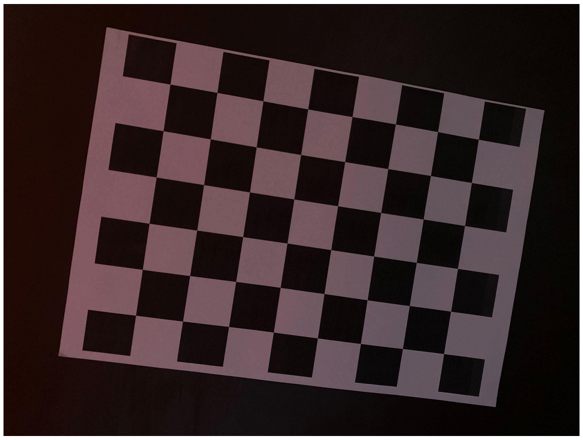
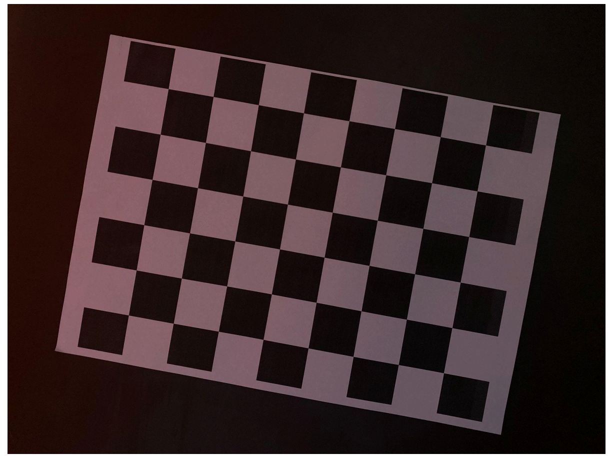
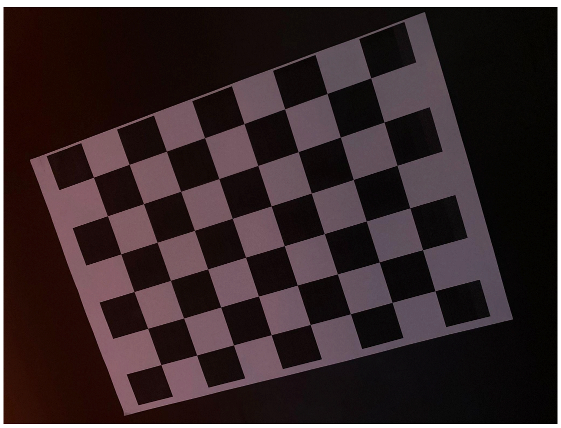
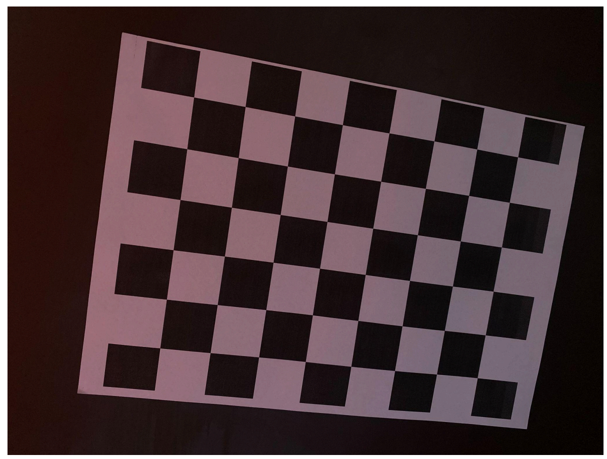
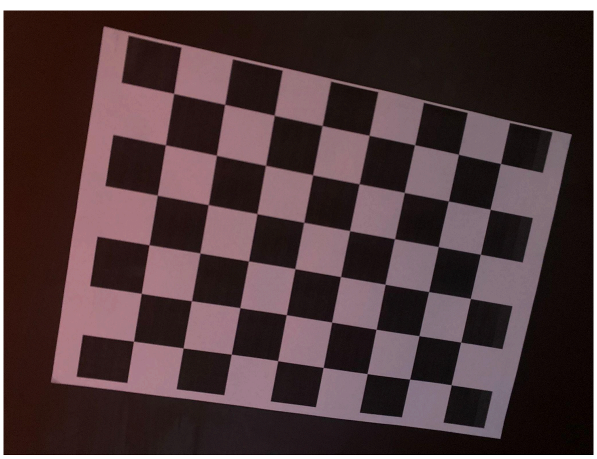
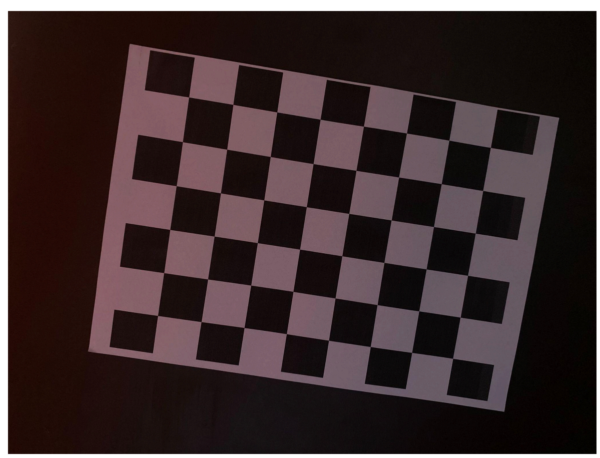
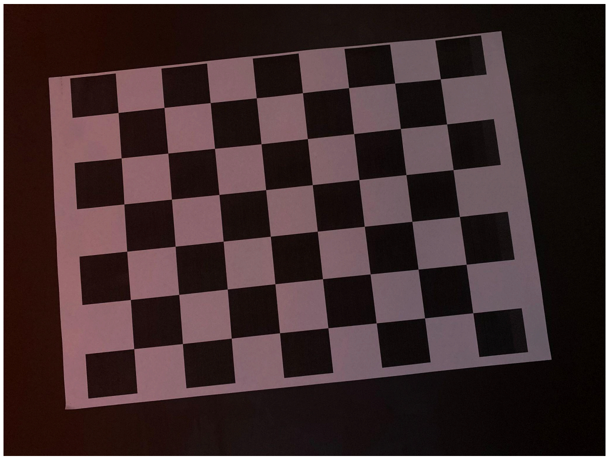
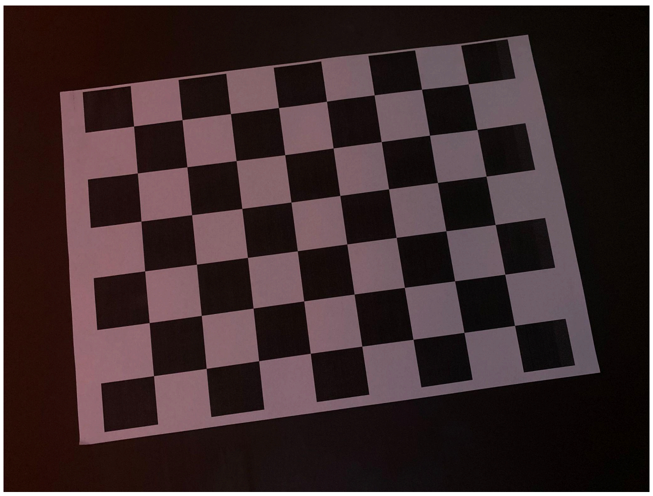
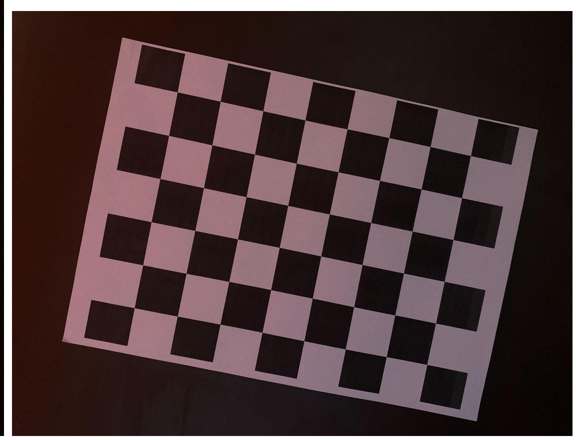
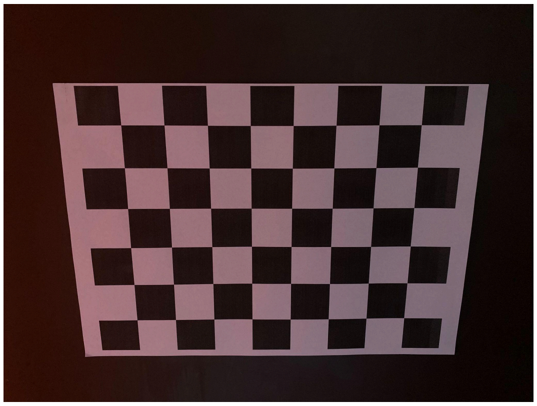
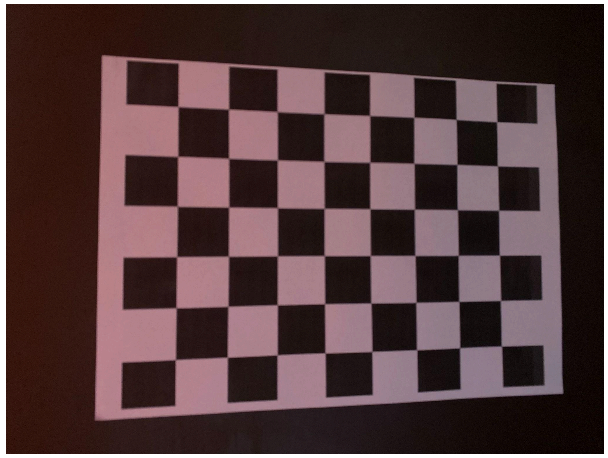
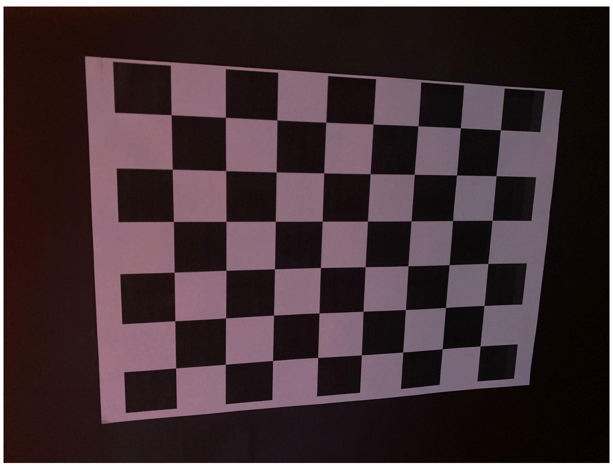
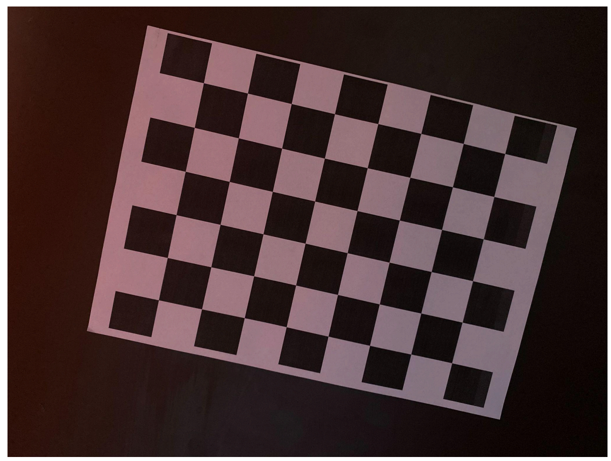
4.

1. Focal length (estimated):

fx = 2991.2243176031193

fy = 3000.6012208756274

Skew parameter: 0.0

Principal point (cx, cy): (1976.1676828886625, 1514.762272815991)

2.

2. Extrinsic parameters for image 1:
3. Rotation matrix:
4. [[0.10325465 -0.99455217 0.01429895]
5. [0.99016203 0.10414246 0.09345319]
6. [-0.0944332   0.0045088   0.99552099]]
7. Translation vector:
8. [[3.08022838]
9. [-3.25579771]
10. [9.77307077]]
11.
12. Extrinsic parameters for image 2:
13. Rotation matrix:
14. [[6.57418467e-03 9.99978311e-01 3.97385171e-04]

```
 [-8.65901474e-01  5.89148805e-03 -5.00179896e-01]
 [-5.00171389e-01  2.94417860e-03  8.65921425e-01]]
Translation vector:
[[-3.71795494]
 [ 3.19074481]
 [11.2505986 ]]

Extrinsic parameters for image 3:
Rotation matrix:
[[ 0.0095127  -0.99995293  0.00190776]
 [ 0.92757372  0.00811147 -0.37355214]
 [ 0.37351909  0.00532308  0.92760722]]
Translation vector:
[[ 3.5576885 ]
 [-1.33540534]
 [ 9.65038502]]

Extrinsic parameters for image 4:
Rotation matrix:
[[ 0.00173855 -0.98845408 -0.15151076]
 [ 0.99923176 -0.0042148   0.03896323]
 [-0.03915195 -0.1514621   0.98768738]]
Translation vector:
[[ 4.488038  ]
 [-2.40911331]
 [10.12883973]]

Extrinsic parameters for image 5:
Rotation matrix:
[[ 0.00171259 -0.99998591 -0.00502425]
 [ 0.96984948  0.00288534 -0.24368764]
 [ 0.2436987  -0.00445543  0.96984076]]
Translation vector:
[[ 3.71029589]
 [-1.82066331]
 [ 9.3015547 ]]

Extrinsic parameters for image 6:
Rotation matrix:
[[-0.00922304 -0.99925332  0.03751986]
 [ 0.93734313  0.00442931  0.34837945]
 [-0.34828551  0.0383821   0.93660238]]
Translation vector:
[[ 3.7981136 ]
 [-3.45955513]
```

```
 [10.57286293]]

Extrinsic parameters for image 7:
Rotation matrix:
[[ 0.00400832  0.99997207  0.00630846]
 [-0.99675471  0.00348806  0.0804232 ]
 [ 0.08039895 -0.00661035  0.99674084]]
Translation vector:
[[-3.17636545]
 [ 2.31936512]
 [10.05454967]]

Extrinsic parameters for image 8:
Rotation matrix:
[[-0.02602416 -0.97738253 -0.20987171]
 [ 0.98582569  0.00971613 -0.16749121]
 [ 0.16574213 -0.21125575  0.96327595]]
Translation vector:
[[ 4.43150994]
 [-2.293934  ]
 [10.02349448]]

Extrinsic parameters for image 9:
Rotation matrix:
[[ 0.17689143  0.98398933  0.02178103]
 [-0.98410909  0.17717371 -0.01177989]
 [-0.01545032 -0.01935115  0.99969336]]
Translation vector:
[[-3.76163844]
 [ 1.87563203]
 [ 9.92204104]]

Extrinsic parameters for image 10:
Rotation matrix:
[[ 0.04749269 -0.96672775 -0.2513601 ]
 [ 0.96049006 -0.02488867  0.27719921]
 [-0.27423219 -0.25459382  0.92735036]]
Translation vector:
[[ 4.77224464]
 [-2.96391415]
 [10.89916388]]

Extrinsic parameters for image 11:
Rotation matrix:
[[-0.00169481 -0.99977324  0.02122727]
```

```
 [ 0.93256946  0.00608261  0.36093933]
 [-0.3609866   0.02040763  0.93234768]]
Translation vector:
[[ 3.57856968]
 [-3.40790124]
 [10.24661   ]]

Extrinsic parameters for image 12:
Rotation matrix:
[[ 0.3122449  -0.94052781  0.13383038]
 [ 0.94671321  0.29634979 -0.12613838]
 [ 0.07897605  0.16608506  0.98294381]]
Translation vector:
[[ 2.11803715]
 [-3.01010294]
 [ 9.164977  ]]

Extrinsic parameters for image 13:
Rotation matrix:
[[ 0.15097352  0.97866185 -0.13938427]
 [-0.98598571  0.13895408 -0.09232516]
 [-0.07098709  0.15136956  0.98592499]]
Translation vector:
[[-2.80208447]
 [ 2.34747543]
 [ 9.08565918]]

Extrinsic parameters for image 14:
Rotation matrix:
[[ 0.00825375 -0.9999531   0.00506609]
 [ 0.99513673  0.00771648 -0.09820056]
 [ 0.09815686  0.00585198  0.99515375]]
Translation vector:
[[ 3.86720663]
 [-2.41482486]
 [11.12040459]]

Extrinsic parameters for image 15:
Rotation matrix:
[[-0.01995807 -0.9373556  -0.3478019 ]
 [ 0.98907554  0.03230667 -0.14382578]
 [ 0.14605222 -0.34687284  0.92647071]]
Translation vector:
[[ 4.95415113]
 [-2.45356484]
```

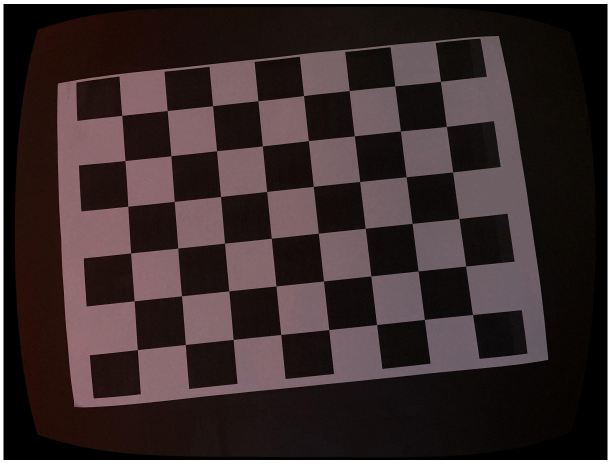
```
 [10.86448753]]

Extrinsic parameters for image 16:
Rotation matrix:
[[ 0.15210427  0.98461648 -0.08599238]
 [-0.98808439  0.14941344 -0.03694411]
 [-0.02352736  0.09058708  0.99561059]]
Translation vector:
[[-3.21953415]
 [ 2.04750574]
 [ 9.38972325]]

Extrinsic parameters for image 17:
Rotation matrix:
[[ 0.21967487  0.97548377 -0.01320454]
 [-0.97286039  0.21803528 -0.07748083]
 [-0.07270224  0.02986676  0.9969064 ]]
Translation vector:
[[-3.27169192]
 [ 1.5599954 ]
 [10.43192675]]

Extrinsic parameters for image 18:
Rotation matrix:
[[ 0.0311539  -0.96697885 -0.25294532]
 [ 0.97607796 -0.02504788  0.21597318]
 [-0.21517724 -0.25362276  0.94306641]]
Translation vector:
[[ 4.60752222]
 [-2.80233721]
 [11.11392833]]

Extrinsic parameters for image 19:
Rotation matrix:
[[ 0.18489988  0.9764022  -0.11158303]
 [-0.97901255  0.17310168 -0.10756505]
 [-0.08571154  0.12912995  0.98791649]]
Translation vector:
[[-3.23689278]
 [ 2.19903735]
 [ 9.1094352 ]]

Extrinsic parameters for image 20:
Rotation matrix:
[[ 0.01521126 -0.97282668 -0.23103434]
```

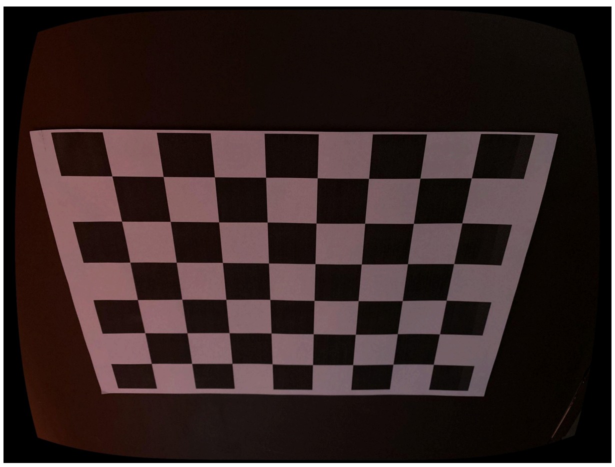
```
 [ 0.99937629  0.02215674 -0.02749759]
 [ 0.03186936 -0.23047197  0.97255695]]
Translation vector:
[[ 4.25380535]
 [-2.62831679]
 [10.49349556]]

Extrinsic parameters for image 21:
Rotation matrix:
[[ 0.20022001  0.9792901  -0.03004742]
 [-0.97959492  0.19954662 -0.02397793]
 [-0.01748549  0.03423517  0.99926083]]
Translation vector:
[[-3.50176679]
 [ 1.92009434]
 [ 9.78653749]]

Extrinsic parameters for image 22:
Rotation matrix:
[[ 0.0113447   0.99987378 -0.01112332]
 [-0.9539176   0.01415774  0.2997345 ]
 [ 0.29985415  0.00721033  0.95395781]]
Translation vector:
[[-3.27201221]
 [ 1.53696504]
 [ 9.13277965]]

Extrinsic parameters for image 23:
Rotation matrix:
[[ 0.0208192  -0.99233489  0.12181144]
 [ 0.98899551  0.00259126 -0.14792286]
 [ 0.14647337  0.1235506   0.9814687 ]]
Translation vector:
[[ 3.21712087]
 [-2.20308626]
 [10.01579876]]

Extrinsic parameters for image 24:
Rotation matrix:
[[ 1.33265239e-01 -9.91080349e-01  3.42620007e-04]
 [ 9.80019248e-01  1.31829401e-01  1.48940530e-01]
 [-1.47657200e-01 -1.95128211e-02  9.88846096e-01]]
Translation vector:
[[ 3.1176337 ]
 [-3.42829478]
```

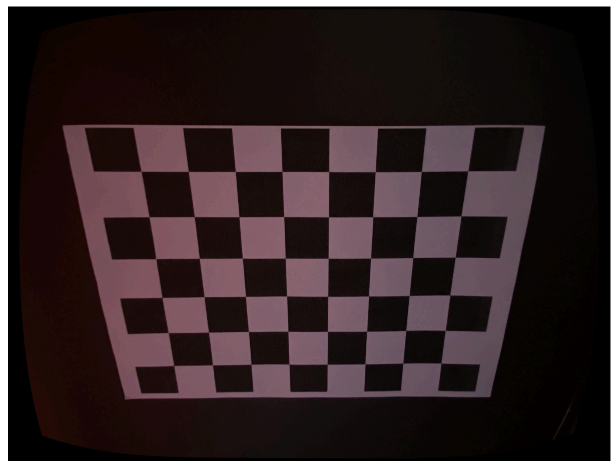
```
 [10.15474549]]

Extrinsic parameters for image 25:
Rotation matrix:
[[ 0.15498992  0.98763037 -0.02375669]
 [-0.98617063  0.15324252 -0.06312072]
 [-0.0586994   0.03321122  0.99772311]]
Translation vector:
[[-3.18594042]
 [ 1.90287386]
 [10.27183273]]

```

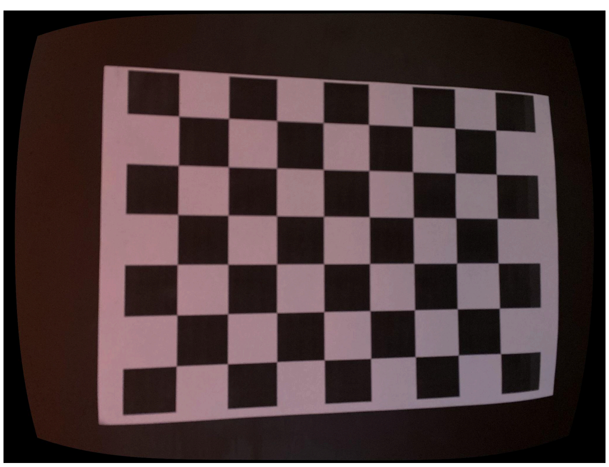
3. Radial distortion coefficients: [[3.01337914e-01 -1.73023034e+00 4.72778162e-05 -3.64819056e-03 2.93455213e+00]]

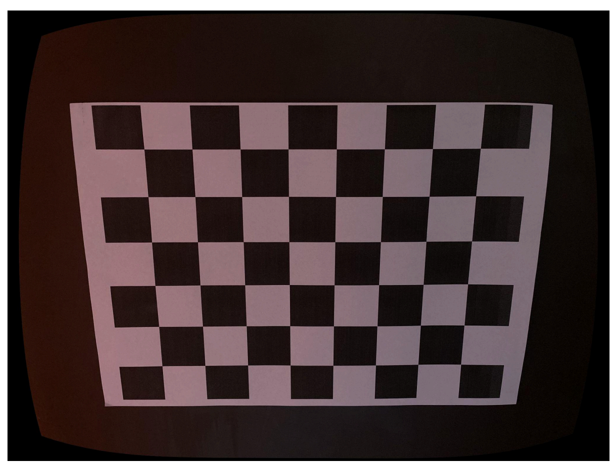
As we can see in the images below, the straight lines at the edges have curved outward. The [redacted]





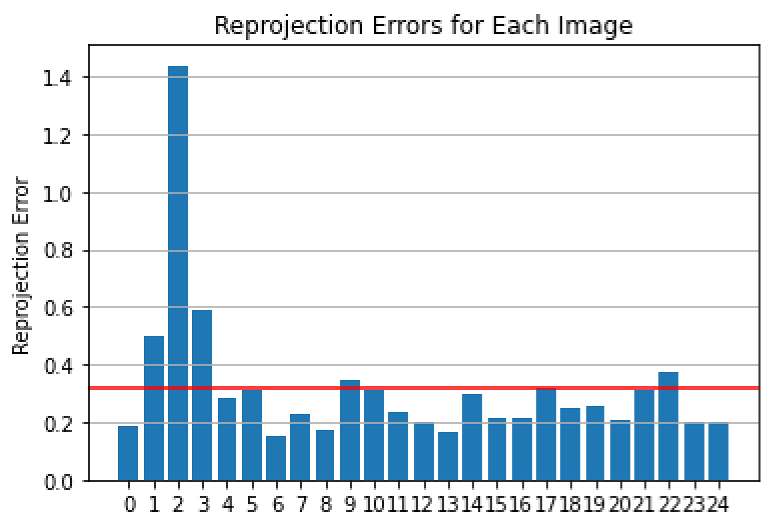






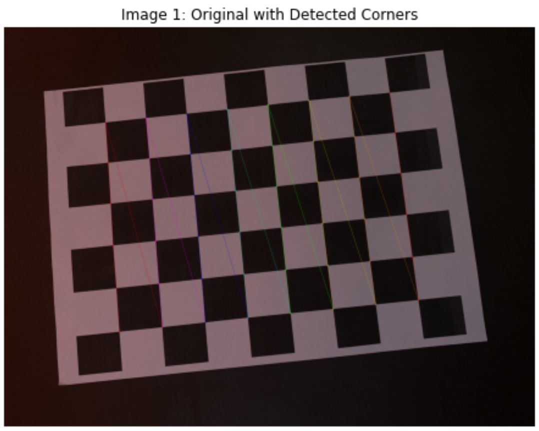
4. Reprojection error for image 1: 0.18815845251083374
Reprojection error for image 2: 0.49887120723724365
Reprojection error for image 3: 1.4376713434855144
Reprojection error for image 4: 0.5919148127237955
Reprojection error for image 5: 0.2864372332890828
Reprojection error for image 6: 0.30933888753255206
Reprojection error for image 7: 0.14918930331865946

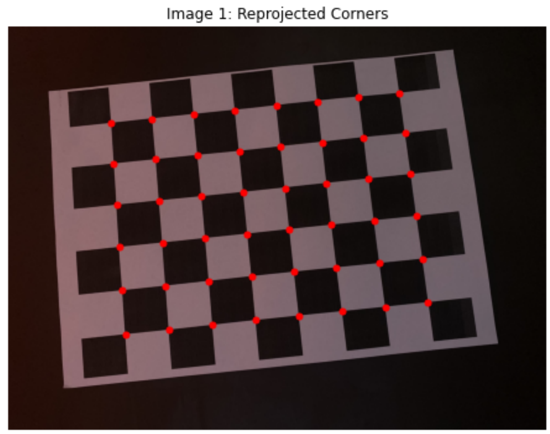
```
Reprojection error for image 8: 0.22535043954849243
Reprojection error for image 9: 0.1706577936808268
Reprojection error for image 10: 0.3470464547475179
Reprojection error for image 11: 0.30841777722040814
Reprojection error for image 12: 0.235300083955129
Reprojection error for image 13: 0.2022481362024943
Reprojection error for image 14: 0.16284451882044473
Reprojection error for image 15: 0.2968143622080485
Reprojection error for image 16: 0.21077015002568564
Reprojection error for image 17: 0.21432522932688394
Reprojection error for image 18: 0.3188309669494629
Reprojection error for image 19: 0.2488251527150472
Reprojection error for image 20: 0.25513505935668945
Reprojection error for image 21: 0.2064374883969625
Reprojection error for image 22: 0.30831992626190186
Reprojection error for image 23: 0.371770183245341
Reprojection error for image 24: 0.2008621096611023
Reprojection error for image 25: 0.19757423798243204
```

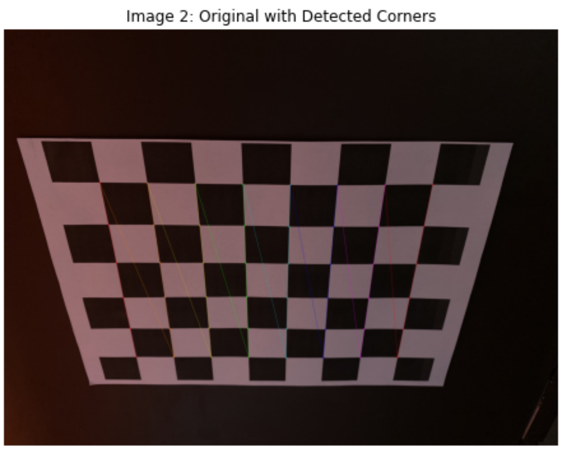


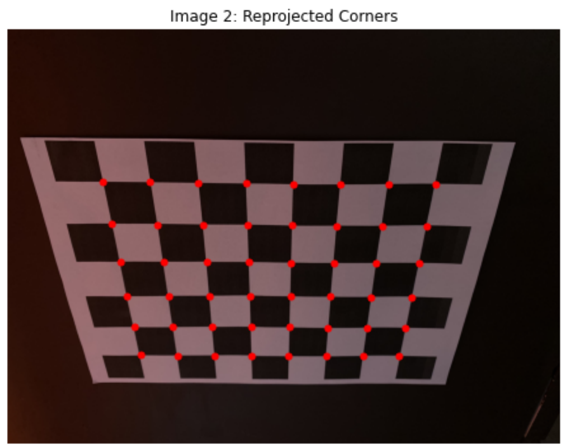

```
Mean Reprojection Error: 0.32
Standard Deviation of Reprojection Error: 0.25
```

5.









Rest in code.

First we projected the points onto the image plane and then calculated the euclidean distance between the actual observed and reprojected image points that gives us the reprojection error.

6.

checkerboard plane normal for image 1: [0.01429895 0.09345319 0.99552099]

Checkerboard plane normal for image 2: [3.97385171e-04 -5.00179896e-01 8.65921425e-01]

Checkerboard plane normal for image 3: [0.00190776 -0.37355214 0.92760722]

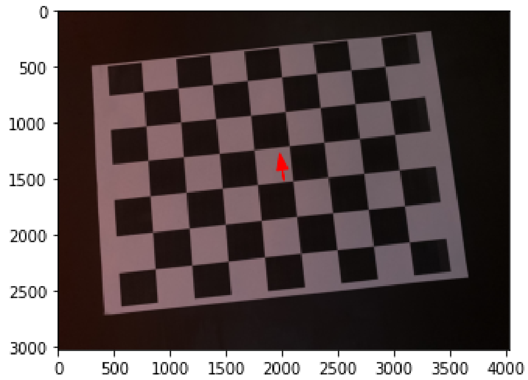
Checkerboard plane normal for image 4: [-0.15151076 0.03896323 0.98768738]

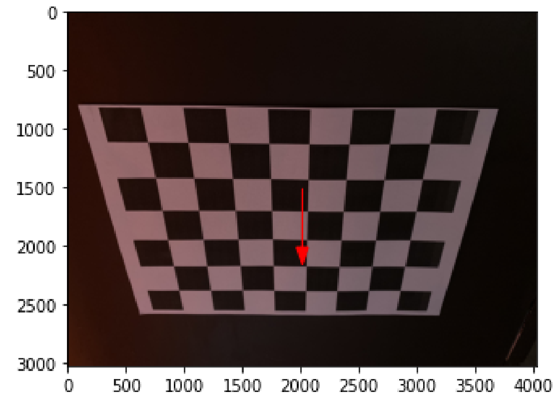
Checkerboard plane normal for image 5: [-0.00502425 -0.24368764 0.96984076]

Checkerboard plane normal for image 6: [0.03751986 0.34837945 0.93660238]

Checkerboard plane normal for image 7: [0.00630846 0.0804232 0.99674084]

Checkerboard plane normal for image 8: [-0.20987171 -0.16749121 0.96327595]
Checkerboard plane normal for image 9: [0.02178103 -0.01177989 0.99969336]
Checkerboard plane normal for image 10: [-0.2513601 0.27719921 0.92735036]
Checkerboard plane normal for image 11: [0.02122727 0.36093933 0.93234768]
Checkerboard plane normal for image 12: [0.13383038 -0.12613838 0.98294381]
Checkerboard plane normal for image 13: [-0.13938427 -0.09232516 0.98592499]
Checkerboard plane normal for image 14: [0.00506609 -0.09820056 0.99515375]
Checkerboard plane normal for image 15: [-0.3478019 -0.14382578 0.92647071]
Checkerboard plane normal for image 16: [-0.08599238 -0.03694411 0.99561059]
Checkerboard plane normal for image 17: [-0.01320454 -0.07748083 0.9969064]
Checkerboard plane normal for image 18: [-0.25294532 0.21597318 0.94306641]
Checkerboard plane normal for image 19: [-0.11158303 -0.10756505 0.98791649]
Checkerboard plane normal for image 20: [-0.23103434 -0.02749759 0.97255695]
Checkerboard plane normal for image 21: [-0.03004742 -0.02397793 0.99926083]
Checkerboard plane normal for image 22: [-0.01112332 0.2997345 0.95395781]
Checkerboard plane normal for image 23: [0.12181144 -0.14792286 0.9814687]
Checkerboard plane normal for image 24: [3.42620007e-04 1.48940530e-01 9.88846096e-01]
Checkerboard plane normal for image 25: [-0.02375669 -0.06312072 0.99772311]





5.
   1. Normals and offsets respectively:
      4.9938116
      4.72078
      5.2043056
      4.793502
      5.2208033
      5.702525
      6.323572
      6.445439
      5.6742587
      5.743694
      5.062352
      4.9540153
      5.254984
      6.1036453
      5.436721
      8.213132
      7.4355464
      8.020993
      9.499026
      10.309979
      8.140582
      8.724273
      7.271752
      6.5950584
      6.9878364
   2.

2.

According to the thesis (5.2)

$\theta_c = [\theta_{c,1}, \dots \theta_{c,n}]^T$, $\alpha_c = [\alpha_{c,1}, \dots \alpha_{c,n}]^T$

Camera normals and offsets

& obtained from given txt files

$\theta_l = [\theta_{l,1}, \dots \theta_{l,n}]^T$, $\alpha_l = [\alpha_{l,1}, \dots \alpha_{l,n}]^T$

Lidar normals and offsets

computed in a part above

$$t_1 = (\theta_c^T \theta_c)^{-1} \theta_c^T (\alpha_c - \alpha_l)$$

$$R_1 = V U^T$$

where $\theta_l \theta_c^T = U S V^T$ (the svd)

These are used to estimate the transformation matrix as

$${}^{C}T_{L} = \begin{bmatrix} R_1 & t_1 \\ 0 & 1 \end{bmatrix}$$

3.

```
The estimated transformation matrix is:
 [[-0.17934806 -0.98377389 -0.00481772  0.15016984]
 [ 0.01810312  0.00159606 -0.99983485 -0.41198299]
 [ 0.98361911 -0.17940566  0.01752312 -0.60030899]
 [ 0.          0.          0.          1.        ]]
```

4. Most points are, not all points

5.

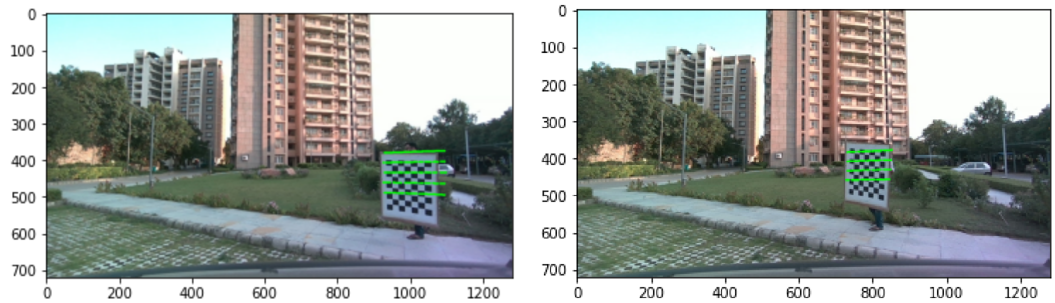


Image 1 with Camera and LiDAR Normals

Image 2 with Camera and LiDAR Normals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Mean error:  0.0006586086344746269
Standard deviation of error:  0.0027892694145765676
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CV Assignment 2

1. $(2,5,1)^T$

$\frac{\pi}{2}$ Y axis, $-\frac{\pi}{2}$ X axis, $(-1,3,2)^T$ translation

$\cancel{R_Y =}$

For Y axis: $\begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

For X axis: $\cancel{\begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}}$

$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Translation: $\begin{bmatrix} 1 & 0 & 0 & T_X \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

1. $T \cdot R_X \cdot R_Y = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2. $$\begin{bmatrix} x_n \\ y_n \\ z_n \\ 1 \end{bmatrix} = \cancel{\begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix}} \begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 1 \\ -3 \\ 1 \end{bmatrix}$$

$(0, 1, -3)$ new coordinate

origin:

$$\begin{bmatrix} 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 3 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

3. Combined rotation $= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{pmatrix}$$

~~Axis is $(0, -1, 0)$~~

$$\theta = \cos^{-1}\left(\frac{Tr(R) - 1}{2}\right) = \cos^{-1}\left(\frac{-1}{2}\right)$$

$$\theta = \frac{2\pi}{3}$$ (Rodrigues formula)

$$n = \frac{1}{2 \sin\theta} \begin{bmatrix} R_{32} - R_{23} \\ R_{13} - R_{31} \\ R_{21} - R_{12} \end{bmatrix}$$

$$= \frac{1}{2\sin\left(\cancel{\frac{2\pi}{3}}\right)} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\frac{\sqrt{3}}{2}$$

$$= \frac{1}{\sqrt{3}} \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$$

axis is $\frac{1}{\sqrt{3}} \times [-1, 1, -1]^T$

4. using Rodrigue's formula:

$$R = I + (\sin\theta) N + (1-\cos\theta) N^2$$

$$N = \begin{pmatrix} 0 & -n_3 & n_2 \\ n_3 & 0 & -n_1 \\ -n_2 & n_1 & 0 \end{pmatrix}$$

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{pmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \frac{\cancel{\sqrt{3}}}{2} \times \frac{1}{\cancel{\sqrt{3}}} \begin{bmatrix} 0 & 1 & 1 \\ -1 & 0 & 1 \\ -1 & -1 & 0 \end{bmatrix} + \left(\frac{\cancel{3}}{2}\right) \times \frac{1}{\cancel{3}} \begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$ = same as the combined rotation matrix

2. R, θ, $u \rightarrow$ given a unit vector

Rotated vector x

$= x + (\sin\theta)(\hat{n}\times x) + (1-\cos\theta)\, \hat{n}\times(\hat{n}\times x)$

$= x + (\sin\theta)(\hat{u}\times x) + (1-\cos\theta)\, ((\hat{n}^T x)\hat{n} - (\underbrace{\hat{n}^T\hat{n}}_{I})x)$ (using triple product)

$= x + (\sin\theta)(\hat{u}\times x) + (1-\cos\theta)\,((\hat{n}^T x)\hat{n} - x)$

$= \cancel{x} + (\sin\theta)(\hat{u}\times x) + (1-\cos\theta)(\hat{u}^T x)\hat{u} - (1-\cos\theta)\cancel{x}$

$= (\sin\theta)(\hat{u}\times x) + x\cos\theta + (1-\cos\theta)(\hat{u}^T x)\hat{u}$

$= x\cos\theta + \sin\theta(\hat{u}\times x) + (1-\cos\theta)(\hat{u}^T x)\hat{u}$

Hence proved

3. $x = K\,[\overbrace{R|t}^{\text{extrinsic}}]\,X$ $\rightarrow$ 3D point

$K \rightarrow$ Intrinsic parameter matrix

C_1 C_2

K_1 K_2

x_1 x_2

3D-R applied to 1

T.P. $x_1 = Hx_2$

$H \rightarrow$ 3x3 invertible and find

$K_1[R_1|t_1] = HK_2[R_2|t_2]$

$[R_2|t_2]$ can be written as

$R[R_1|t_1]$

comparing both sides,

$K_1 = HK_2R$

$H = \underbrace{K_1}_{3\times3}\underbrace{R^T}_{3\times3}\underbrace{K_2^{-1}}_{3\times3}$

Here H is 3x3

Invertible? prove non 0 determinant

$$|H| = |K_1| \, |R^T| \, |K_2|^{-1}$$

$$|R^T| \rightarrow |R| \qquad |K_2|^{-1} \rightarrow \frac{1}{|K_2|}$$

$|R| = 1$ (Rotation matrix)

$|K_1|, |K_2|$ are invertible (intrinsic matrices)

Hence $|H| \neq 0$ and H is invertible.