1.

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1.

Null space will satisfy $Ex = 0$

$E = [t_x]R$

$[t_x]Rx = 0 \Rightarrow Rx \parallel t$

$\Rightarrow Rx = \lambda t$

$\Rightarrow x = R^{-1}\lambda t \qquad (\lambda \in \mathbb{R})$

$= \lambda(R^{-1}t) = \lambda R^T t$

$(R^{-1} = R^T)$

$\hookrightarrow$ first epipole

Left null-space will satisfy

$E^T y = 0$

$(t_x)^T = -(t_x)$

skew sym.

$([t_x]R)^T y = 0 \Rightarrow R^T [t_x] y = 0$

As R is full rank, so is R^T and hence

$(t_x) y = 0$

$\Rightarrow y \parallel t$

$\Rightarrow y = \mu t \qquad (\mu \in \mathbb{R})$

2. $R = I$

$t = [t_x, 0, 0]^T$

$$E = [t_x] R = \begin{bmatrix} 0 & -t_z & +t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix} I \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \cancel{\begin{bmatrix} 0 & 0 & 0 \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}}$$

$$= \begin{bmatrix} 0 & -t_z & t_y \\ t_z & 0 & -t_x \\ -t_y & t_x & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \quad \text{as } t_z, t_y = 0$$

Let the 2 points be x_1 & x_2, T.P. $y_1 = y_2$

$$x_2^T E x_1 = 0$$

$$\begin{bmatrix} x_2 & y_2 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -t_x \\ 0 & t_x & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & t_x & -y_2 t_x \end{bmatrix} \cancel{\begin{bmatrix} 0 \\ t_x \\ -y_2 t_x \end{bmatrix}} \begin{bmatrix} x_1 \\ y_1 \\ 1 \end{bmatrix} = 0$$

$$y_1 t_x - y_2 t_x = 0$$

$$\Rightarrow y_1 = y_2$$

3 . T.F. R_{rect}

use

E

By SVD

$$E = [t_x] R = U \Sigma V^T$$

to get epipole e we do SVD and
t is taken from E.
e_l is column of V corresponding to null SV
e_r " U "

After estimating the epipoles, build R_{rect}
by,

$$R_{rect} = \begin{bmatrix} r_1^T \\ r_2^T \\ r_3^T \end{bmatrix}$$

using given

$$r_1 = e_1 = \frac{T}{\|T\|}$$ → translation vector

$$r_2 = \frac{1}{\sqrt{T_x^2 + T_y^2}} [-T_y \;\; T_x \;\; 0]$$

$$r_3 = r_1 \times r_2$$