MTH 371: Assignment I - Solutions

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Question 1 (10 points)

Suppose a coin is tossed 20 times. We model the process as a Bernoulli process, so all the assumptions of the Bernoulli process are satisfied.

(a)

For $p = 0.8$, we simulated the process 1000 times and generated the corresponding histogram.

(b)

For $p = 0.5$, we simulated the process 1000 times and generated the corresponding histogram. The histograms are shown in Figure 1.

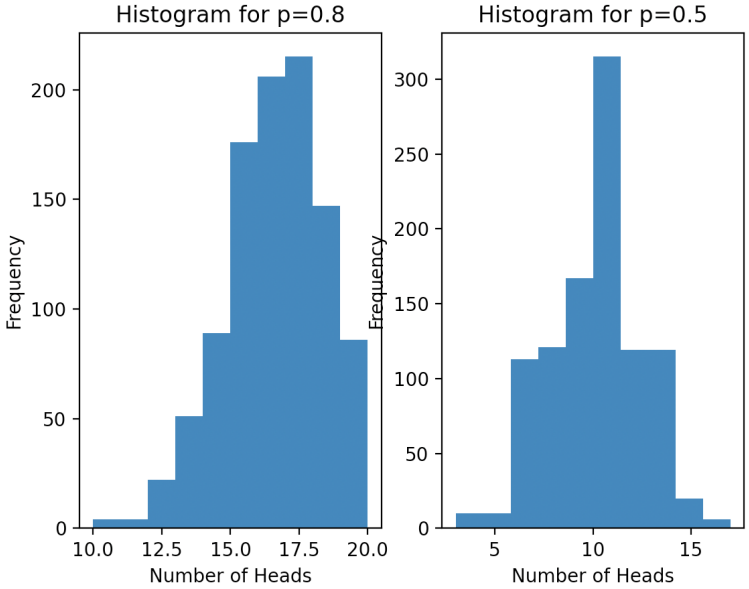

Figure 1: Histogram for $p = 0.8$ and $p = 0.5$

Inference:

$p = 0.8$ shows that the distribution is towards higher number of heads and when $p = 0.5$ the distribution is relatively symmetric and centered around the mean due to equal probability of both outcomes

Question 2 (15 points)

Assume the patients arrive at a hospital's emergency room at a rate of 5 patients per hour. We are interested in studying the number of visitors in the time interval $(0, t]$ (where t is considered to be continuous). Let us suppose we can model it as a Poisson process.

(a)

We simulated the density of the number of arrivals until time 10. The graph is shown in Figure 2. We also calculated the mean value of the number of arrivals in the process, which is λt.

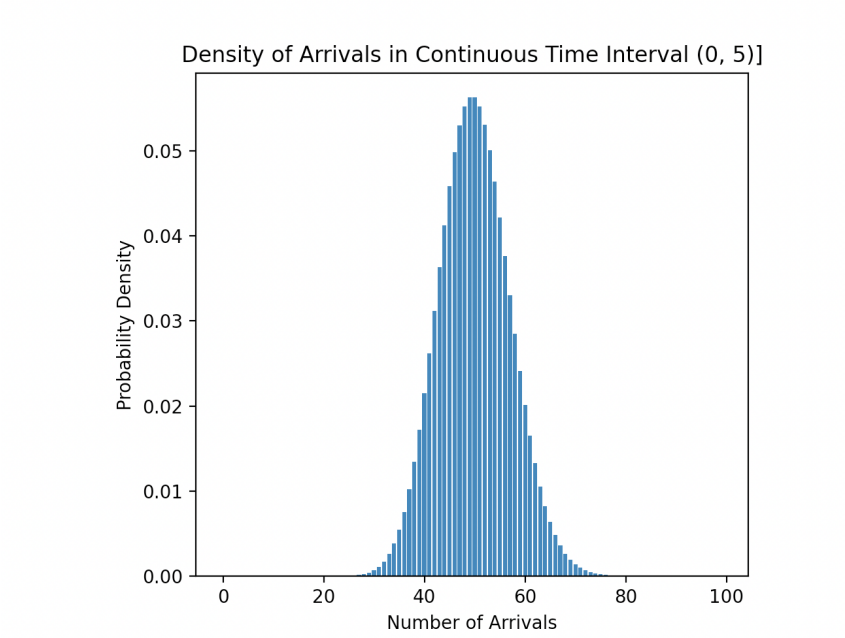

Figure 2: Density of Number of Arrivals for $\lambda = 5$

Verification:

The theoretical mean is 50 which is also observed by the graph

(b)

We simulated the density of the number of arrivals until time t for $\lambda = 15$ and compared the results with part (a) (when $\lambda = 5$). The comparison is shown in Figure 3.

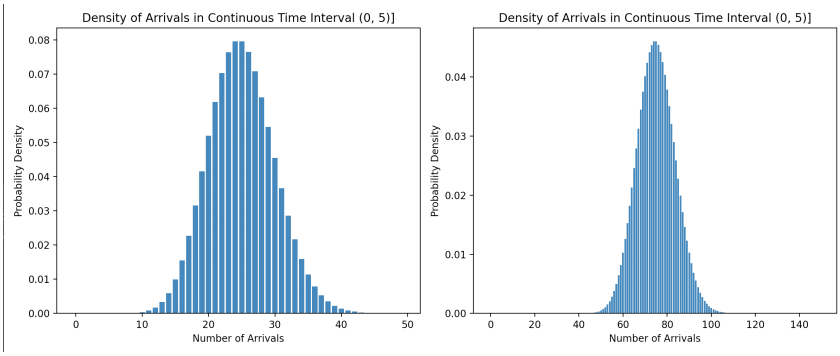

Figure 3: Density of Number of Arrivals for $\lambda = 5$ and $\lambda = 15$

Inference:

We see a similar distribution and the density of arrivals shifts to the right, indicating a higher average number of arrivals within the same time interval. The distribution for $\lambda = 15$ is more skewed to the right and has a higher peak compared to the distribution for $\lambda = 5$.

(c)

We simulated the first inter-arrival time of the process. The graph is shown in Figure 4.

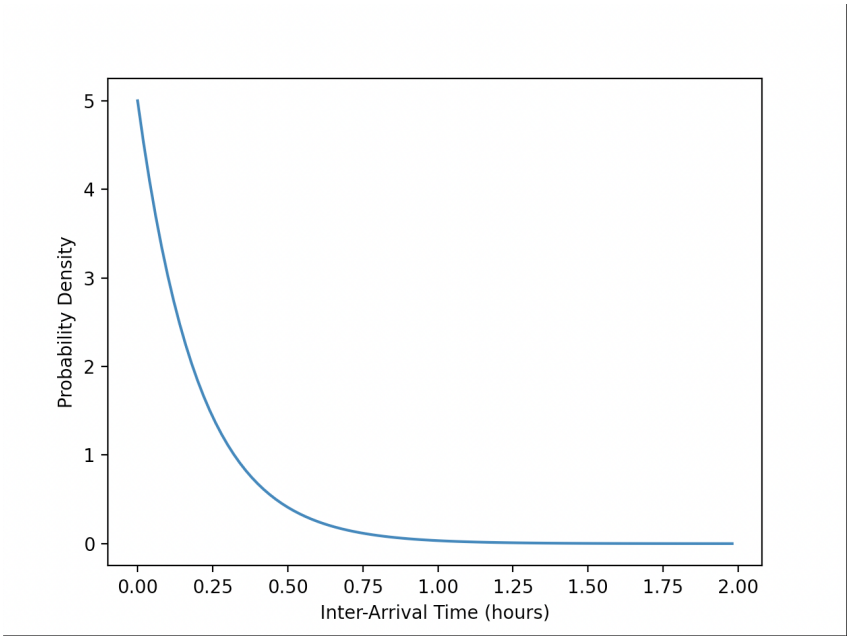

Figure 4: First Inter-arrival Time

Inference:

It follows an exponential distribution as expected