ASSIGNMENT [X] ON [COURSE NAME]		
Student's Code		Deadline
[Your Code]		**[Date, Time]**
November 27, 2023		2019-2020
		Lecturer: [Lecturer Name]

1 Solution of Exercise 1

Solve the following systems of equations
1.

$$\begin{cases} x+2y+3z=10 & (1) \\ 2x+y-z=3 & (2) \\ -x+3y+2z=5 & (3) \end{cases}$$

If we fixe (1) and try to annul x in (2) and (3). We multiply the equation (1) by -2, and we will have:

$$\begin{cases} -2(x+2y+3z=10) & (4) \\ 2x+y-z=3 & (5) \end{cases}$$

$\Longrightarrow$

$$\begin{cases} -2x-4y-6z=-20) & (4) \\ 2x+y-z=3 & (5) \end{cases}$$

Hence, after sum of equation (4) and (5), the result will be:
We will have:

$$\begin{cases} x+2y+3z & =10 \\ -3y-7z & =-17 \\ 5y+5z & =15 \end{cases}$$

Secondly, we note the new second equation (4) and the last (5). If we multiply (4) by 5 and (5) by 3, we will have.

$$\begin{cases} -15y-35z & =-85 \\ 15y+15z & =45 \end{cases}$$

Then, It will give us z = 2, and if we replace the value of z in (4), hence y = 1. To have the value of x, we can replace in (1). The result is x = 2. The

solution is given by: S $= \{(2,1,2)\}$
2.

$$\begin{cases} 3x-4y+5z &= 1 \\ 7x-2y-4z &= 3 \\ -x-6y+14z &= 8 \end{cases}$$

Firstly, we rename the first equation (1), the second (2) and the third (3). If we fixe (1) and try to annule x in (2) and (3). We will have:

$$\begin{cases} 3x-4y+5z &= 1 \\ -22y+47z &= -2 \\ -22y+42z &= 25 \end{cases}$$

$$\begin{cases} 3x+\ \ y+2z = 6 & (1) \\ x+3y+2z = -6 & (2) \\ x+\ \ y+\ \ z = 1 & (3) \end{cases}$$

2 Solution of exercice

Let

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 3 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

1) Verify that the matrix A is invertible and compute its inverse A^{-1}.

   We have to determine the determinant of A

$$det(A) = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 3 & -1 \\ 2 & 2 & 0 \end{vmatrix}$$

$$= 2\begin{vmatrix} 3 & -3 \\ 2 & 0 \end{vmatrix}$$

$$= 2 \times (2) = 4 \neq 0$$

   Therefore A is **inversible**.

Now we want to find the inverse of the matrix A.
We denote $A^c = (c_{ij})_{1\leq i,j\leq 2}$ the matrix of cofactors of A. We determine the coefficients of A^c:
$A^{-1} = \frac{1}{det(A)} \times [cofactorsA]^t$

$$[cofactorsA] = \begin{bmatrix} 2 & 2 & -4 \\ 0 & 0 & 4 \\ 0 & -2 & 6 \end{bmatrix}$$

$$[cofactorsA]^t = \begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & -2 \\ -4 & 4 & 6 \end{bmatrix}$$

So A^{-1} is equal to:

$$A^{-1} = \frac{1}{4} \times \begin{bmatrix} 2 & 0 & 0 \\ 2 & 0 & -2 \\ -4 & 4 & 6 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & -2 \\ -1 & 1 & \frac{3}{2} \end{bmatrix}$$

2) Compute its characteristic polynomial $P_A(x)$.

We have:

$$P_A(X) = det \begin{pmatrix} 2-X & 0 & 0 \\ 1 & 3-X & -1 \\ 2 & 2 & -X \end{pmatrix}$$

$$= -X \begin{bmatrix} 2-X & 0 \\ 1 & 3-X \end{bmatrix} + 1 \begin{bmatrix} 2-X & 0 \\ 2 & 2 \end{bmatrix}$$

$$= -X(2-X)(3-X) + 2(2-X) = (2-X)(-X(3-X)+2)$$

$$= (2-X)(-3X + X^2 + 2)$$

$$\Rightarrow \quad P_A(X) = (2-X)(X-1)(3X+2)$$

The eigenvalues of A are $\lambda_1 = 2, \quad \lambda_2 = 1 \quad and \quad \lambda_3 = -\frac{2}{3}$

3) Compute the sets of all eigenvalues and eigenvectors.

The set of eigenvalues is: $Spect_A = \{2, 1, 3\}$.
To find the eigenvectors, we must each eigenvalue and try to find his eigenvectors associated.
This is given by:

If $\lambda_1 = 2 \implies E_{\lambda_1} = E_2 = Ker(A - 2.I)$

Let x_1, x_2, x_3 are $\in E_2$. Then one has:

$$(A - I) \, . \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$\Rightarrow$

$$\begin{pmatrix} 0 & 0 & 0 \\ 1 & 1 & -1 \\ 2 & 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{cases} 0 &= 0 \\ x_1 + x_2 - x_3 &= 0 \\ 2x_1 + 2x_2 - 2x_3 &= 0 \end{cases}$$